

Hash Tables

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- ▶ Skip List (SkipMap): add, find, and remove in $O(\log n)$ expected time.
- ▶ 2-3 Tree (BTree): add, find, and remove in $O(\log n)$ time.



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- ▶ Sometimes it is important that the entries be in alphabetical or numerical order.
- ▶ When you look up a name, it is nice to be able to go forward or back a few names in case you misspelled it.
- ▶ If you want Milenkovic in a hash table, you better not look for Milenkovich because it will be far far away.



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- ▶ We will look at the one for String:

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public int hashCode()
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Returns a hash code for this string. The hash code for a String object is computed as

$$s[0]*31^{(n-1)} + s[1]*31^{(n-2)} + \dots + s[n-1]$$

using int arithmetic, where $s[i]$ is the i th character of the string, n is the length of the string, and $^$ indicates exponentiation. (The hash value of the empty string is zero.)

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An int can only hold integers in the range from -2147483648 to 2147483647.



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```
int hashIndex (String name, int m) {  
    int code = name.hashCode();  
    int index = code % m;  
    if (index < 0)  
        index += m;  
    return index;  
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- ▶ But it is also -3 remainder 2 . Check $-3*7+2=-19$.



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- ▶ Open Addressing



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- ▶ Entries in the same list in the first table will be in different lists in the second table.



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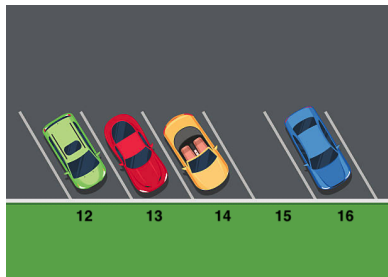
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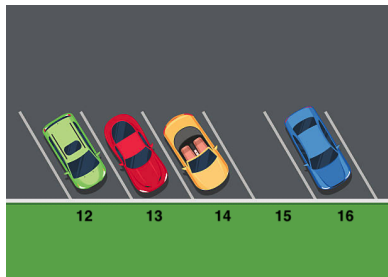
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- ▶ If you reach the end of the lot, you go back to the beginning.



Add

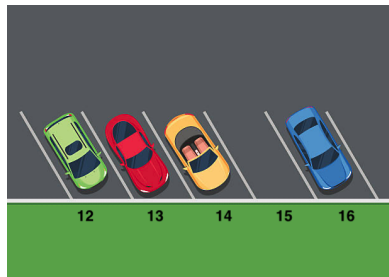


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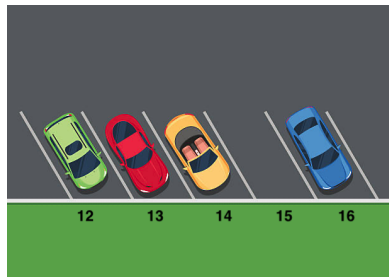
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- ▶ Suppose my assigned number is 12.

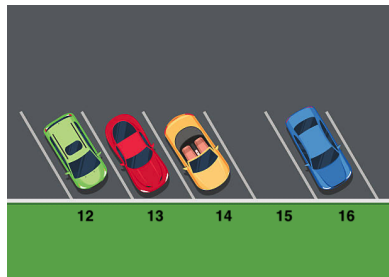
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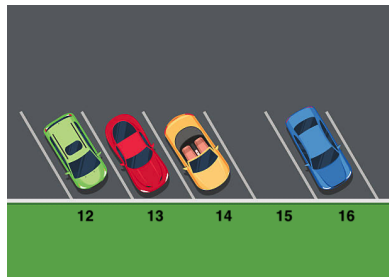
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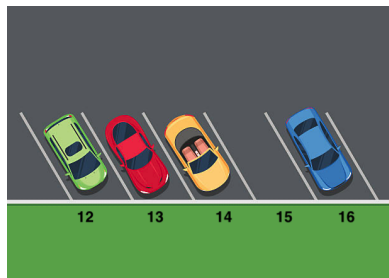
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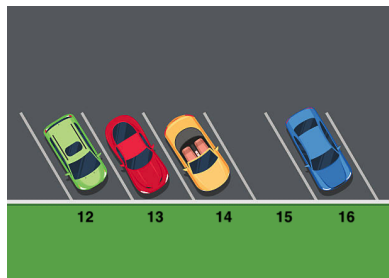
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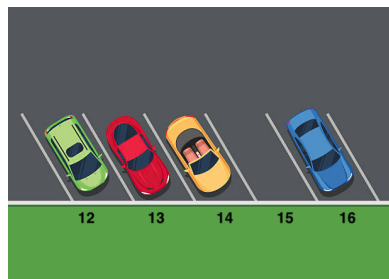


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Here is how to find Victor.

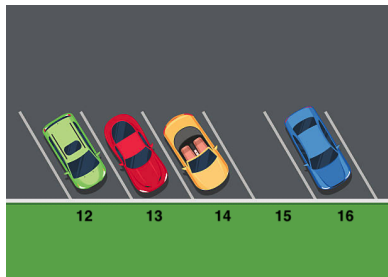
Find



Here is how to find Victor.

- ▶ You know my assigned number (hash index of Victor) is 12.

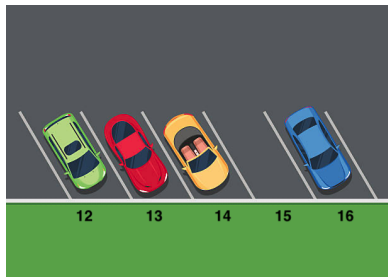
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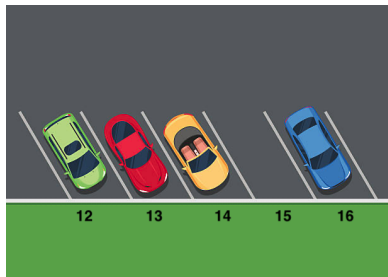
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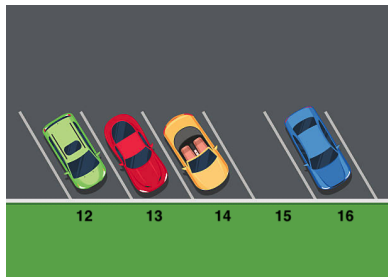
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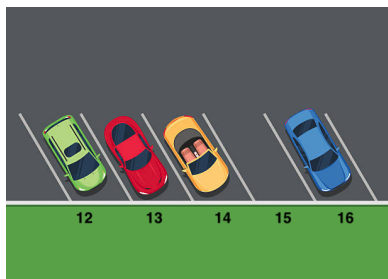
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- ▶ If I am not there (stayed home) you get to an empty space and stop.

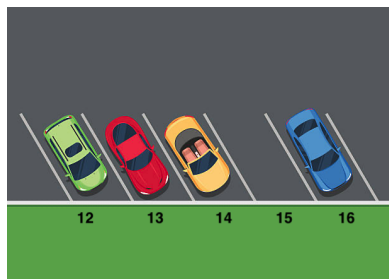
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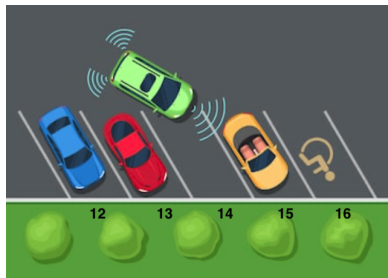


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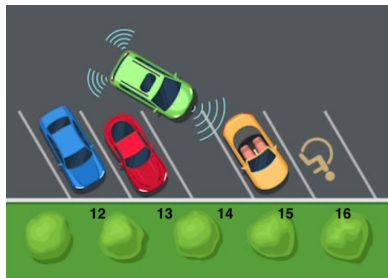
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- ▶ So if I am there, I must be before the first empty space you see.



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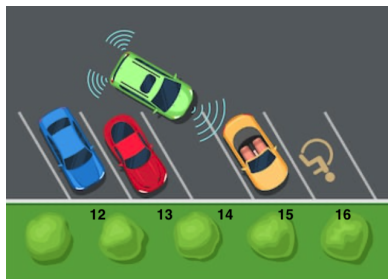


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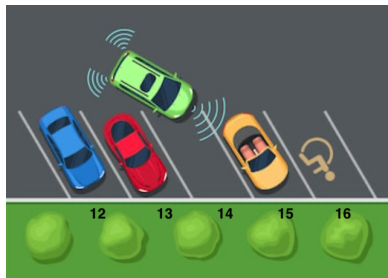
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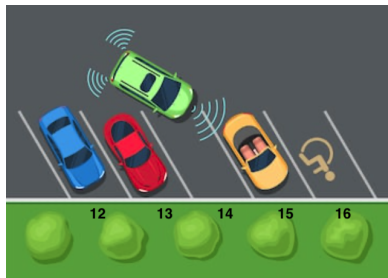
Problem with Find



There is a problem with find.

- ▶ What if someone was in a spot that I skipped, in this case 14.
- ▶ And left before you came looking for me.

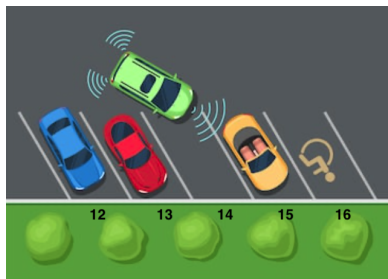
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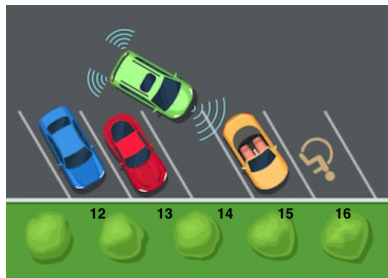
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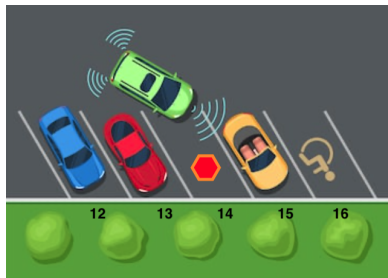
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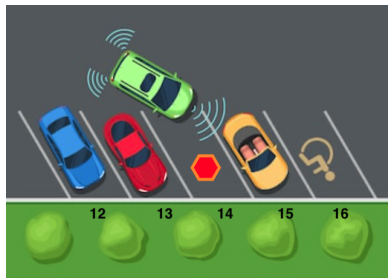
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- ▶ How do we fix this?

Marking DELETED spaces

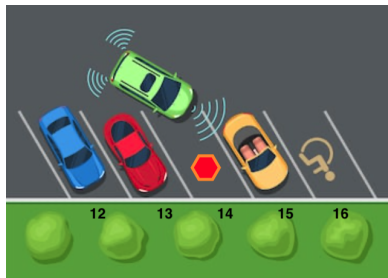


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The fix is to mark DELETED spaces.

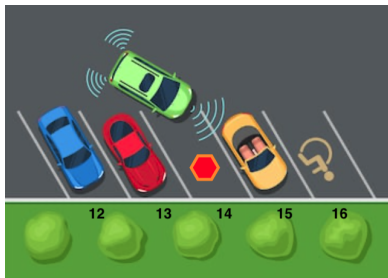
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- ▶ If you leave, put a traffic cone in the space.

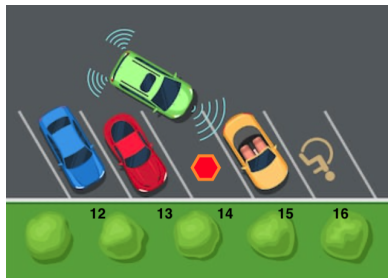
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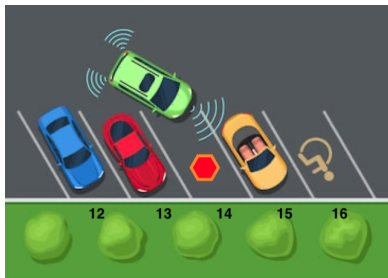
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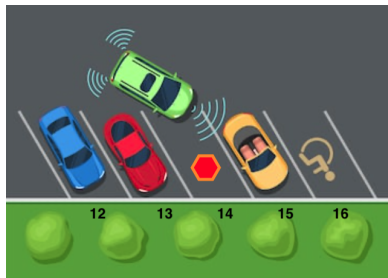
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- ▶ I park in the first space that is empty or has a traffic cone.

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- ▶ You have to keep $n \leq m/2$.



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- ▶ More suitable for real-time programming or operating systems.
- ▶ Item cycles through the array until it finds an empty spot.



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- ▶ add, find, remove are n/m expected time.
- ▶ Need to reallocate and rehash when $n > m$.

Open Addressing

- ▶ More suitable for real-time programming or operating systems.
- ▶ Item cycles through the array until it finds an empty spot.
- ▶ Removed items need to leave a “traffic cone”.



Summary

The hash code of an item is an int,

- ▶ possibly negative,
- ▶ usually unique (for up to 65536).
- ▶ Don't use ^ or pow when calculating the hash code of a String.

The hash index of an item is an int

- ▶ in the range 0 to $m-1$.
- ▶ Mod the hash code by m ,
- ▶ add m if negative.
- ▶ Not unique!

A hash table is an array of length m .

- ▶ Item with hash index i is stored at index i ,
- ▶ unless there is an item there already.

Separate Chaining

- ▶ stores a linked list at each index.
- ▶ add, find, remove are n/m expected time.
- ▶ Need to reallocate and rehash when $n > m$.

Open Addressing

- ▶ More suitable for real-time programming or operating systems.
- ▶ Item cycles through the array until it finds an empty spot.
- ▶ Removed items need to leave a “traffic cone”.
- ▶ Needs to rehash and possibly reallocate when less than $m/2$ spaces are empty.

