`laplace_G`	G_L	
`laplace_dG`	$\partial G_L/\partial n$	evaluated for normal at field point
`helmholtz_Gr`	$\mathcal{R}(G_H)$	
`helmholtz_Gi`	$\mathcal{I}(G_H)$	
`helmholtz_dGr`	$\mathcal{R}(\partial G_H/\partial n)$	evaluated for normal at field point
`helmholtz_dGi`	$\mathcal{I}(\partial G_H/\partial n)$	evaluated for normal at field point
`helmholtz_ring_r`	$\mathcal{R}(G_n)$	
`helmholtz_ring_i`	$\mathcal{I}(G_n)$	
`sphere_scattered_r`	$\mathcal{R}(p_s(k,a,r,\theta))$	scattered potential from sphere under plane wave excitation
`sphere_scattered_i`	$\mathcal{I}(p_s(k,a,r,\theta))$	

Table 1: Source terms for generation of boundary conditions; syntax for expressions is given by the help function of the solver

(Notes on some details of implementation for reference during development)

Local correction integrals are implemented as matrix multiplications of the source vector at patch nodes:

$$\phi(\mathbf{x}_i) = \sum_j A_{ij}\sigma_j, \tag{1}$$

where A_{ij} is a matrix of size $N_n \times n_p$, with N_n the number of neighbours, including the patch nodes, and n_p the number of patch nodes. The result of the matrix multiplication is given at nodes $\mathbf{x}_i$, the location of the ith entry in the neighbour list of the patch. The correction matrices are computed for the single and double layer potentials and packed together.

$$G_L(\mathbf{x}) = \frac{1}{4\pi R}, \tag{2}$$

$$G_H(\mathbf{x}) = \frac{\mathrm{e}^{\mathrm{j}kR}}{4\pi R}, \tag{3}$$

$$R = |\mathbf{x}|, \tag{4}$$

$$G_n(\mathbf{x}) = \int_0^{2\pi} \frac{\mathrm{e}^{\mathrm{j}(kR+n\theta_1)}}{4\pi R}\,\mathrm{d}\theta_1, \tag{5}$$

$$R^2 = (x - a\cos\theta_1)^2 + (y - a\sin\theta_1)^2 + z^2. \tag{6}$$

In Helmholtz problems, k is wavenumber. For ring sources, a is ring radius.

Scattering by sphere of radius a due to incident plane wave $\exp[\mathrm{j}kz]$

$$p_s(r,\theta) = \sum_{m=0}^{M} A_m P_m(\cos\theta) h_m(kr) \tag{7}$$

$$A_m = -(2m+1)\mathrm{j}^m \frac{m j_{m-1}(ka) - (m+1) j_{m+1}(ka)}{m h_{m-1}(ka) - (m+1) h_{m+1}(ka)}, \tag{8}$$

with $j_n(\cdot)$ and $h_n(\cdot)$ the spherical Bessel and Hankel functions of the first kind, respectively.

For reference:

$$\frac{\partial G_H}{\partial R} = \frac{\mathrm{e}^{\mathrm{j}kR}}{4\pi R^2}(\mathrm{j}kR - 1), \tag{9}$$

$$\frac{\partial R}{\partial x} = \frac{x}{R}. \tag{10}$$