Koornwinder polynomials from Greengard et al:

$$K_{nm}(u,v) = c_{nm}(1-v)^m P_{n-m}^{(0,2m+1)}(1-2v) P_m\left(\frac{2u+v-1}{1-v}\right), \quad m \le n, \tag{1}$$

$$c_{nm} = \left[2(1+2m)(n+1)\right]^{1/2}. \tag{2}$$

Recursions (DLMF):

$$P_{n+1}^{(0,2m+1)}(x) = (A_n x + B_n) P_n^{(0,2m+1)}(x) - C_n P_{n-1}^{(0,2m+1)}(x), \tag{3}$$

$$A_n = \frac{(n+m+1)(2n+2m+3)}{(n+1)(n+2m+2)}, \tag{4}$$

$$B_n = -\frac{(2m+1)^2(n+m+1)}{(n+1)(n+2m+2)(2n+2m+1)}, \tag{5}$$

$$C_n = \frac{n(n+2m+1)(2n+2m+3)}{(n+1)(n+2m+2)(2n+2m+1)}, \tag{6}$$

$$(2n+\alpha+\beta+1)P_n^{(\alpha,\beta)}(x) = (n+\alpha+\beta+1)P_n^{(\alpha+1,\beta)}(x) - (n+\beta)P_{n-1}^{(\alpha+1,\beta)}(x); \tag{7}$$

$$(n+\alpha/2+\beta/2+1)(1-x)P_n^{(\alpha+1,\beta)}(x) = -(n+1)P_{n+1}^{(\alpha,\beta)}(x) + (n+\alpha+1)P_n^{(\alpha,\beta)}(x). \tag{8}$$

$$P_{n+1}(x) = \frac{2n+1}{n+1} x P_n(x) - \frac{n}{n+1} P_{n-1}(x). \tag{9}$$

Differentiation of Koornwinder polynomials. From arXiV:1801.09099v1 (Olver, Townsend, Vasil), Corollaries 1 and 4,

$$\begin{aligned}
K_{nm}(u,v) &= (-1)^{n-m} c_{nm} P^{0,0,0}_{n,m}(v,u), && (10)\\
\frac{\partial}{\partial y} P^{0,0,0}_{n,m}(x,y) &= (m+1) P^{0,1,1}_{n-1,m-1}(x,y), && (11)\\
(2m+3)(2n+4) y P^{0,1,1}_{n,m}(x,y) &= (m+1)(n+m+3) P^{0,0,1}_{n,m}(x,y)\\
&\quad - (m+1)(n-m) P^{0,0,1}_{n,m+1}(x,y)\\
&\quad - (m+1)(n-m+1) P^{0,0,1}_{n+1,m}(x,y)\\
&\quad + (m+1)(n+m+4) P^{0,0,1}_{n+1,m+1}(x,y), && (12)\\
(2m+2)(2n+3) z P^{0,0,1}_{n,m}(x,y) &= (m+1)(n+m+2) P^{0,0,0}_{n,m}(x,y)\\
&\quad + (m+1)(n-m) P^{0,0,0}_{n,m+1}(x,y)\\
&\quad - (m+1)(n-m+1) P^{0,0,0}_{n+1,m}(x,y)\\
&\quad - (m+1)(n+m+3) P^{0,0,0}_{n+1,m+1}(x,y). && (13)
\end{aligned}$$

$$\begin{aligned}
\frac{(2m+3)(2n+4)}{m+1} yz P^{0,1,1}_{n,m}(x,y) &= \frac{1}{2}\frac{(n+m+2)(n+m+3)}{2n+3} P^{0,0,0}_{n,m}(x,y)\\
&\quad - \frac{1}{2}\frac{(n-m-1)(n-m)}{2n+3} P^{0,0,0}_{n,m+2}(x,y)\\
&\quad + \frac{1}{2}\frac{(n-m+1)(n-m+2)}{2n+5} P^{0,0,0}_{n+2,m}(x,y)\\
&\quad - \frac{1}{2}\frac{(n+m+4)(n+m+5)}{2n+5} P^{0,0,0}_{n+2,m+2}(x,y)\\
&\quad + \frac{2n+4}{(2n+3)(2n+5)}\Big[(n-m)(n+m+4) P^{0,0,0}_{n+1,m+2}(x,y)\\
&\quad - (n-m+1)(n+m+3) P^{0,0,0}_{n+1,m}(x,y)\Big], \qquad (14)
\end{aligned}$$

$$z = 1 - x - y.$$