

Koornwinder polynomials from Greengard et al:

$$K_{nm}(u, v) = c_{nm}(1 - v)^m P_{n-m}^{(0, 2m+1)}(1 - 2v) P_m \left(\frac{2u + v - 1}{1 - v} \right), \quad m \leq n, \quad (1)$$

$$c_{nm} = [2(1 + 2m)(n + 1)]^{1/2}. \quad (2)$$

Jacobi polynomial recursions (DLMF):

$$P_{n-m+1}^{(0, 2m+1)}(x) = (A_n x + B_n) P_{n-m}^{(0, 2m+1)}(x) - C_n P_{n-m-1}^{(0, 2m+1)}(x), \quad (3)$$

$$A_n = \frac{(n + 1)(2n + 3)}{(n - m + 1)(n + m + 2)}, \quad (4)$$

$$B_n = -\frac{(2m + 1)^2(n + 1)}{(n - m + 1)(n + m + 2)(2n + 1)}, \quad (5)$$

$$C_n = \frac{(n - m)(n + m + 1)(2n + 3)}{(n - m + 1)(n + m + 2)(2n + 1)}, \quad (6)$$

$$\begin{aligned} \frac{dP_{n-m}^{0, 2m+1}(x)}{dx} &= 2(n - m)(n + m + 1)P_{n-m-1}^{0, 2m+1}(x) \\ &\quad - (n - m)((2n + 1)x + 2m + 1)P_{n-m}^{0, 2m+1}. \end{aligned} \quad (7)$$

Legendre polynomials:

$$P_{n+1}(x) = \frac{2n + 1}{n + 1} x P_n(x) - \frac{n}{n + 1} P_{n-1}(x), \quad (8)$$

$$\frac{dP_{m+1}}{dx} = (m + 1) \frac{P_m(x) - x P_{m+1}(x)}{1 - x^2}. \quad (9)$$

Spherical patch test integrals, on triangular patch (θ_0, ϕ_0) , (θ_1, ϕ_0) , (θ_0, ϕ_1) of radius ρ :

$$\int_{\theta} \int_{\phi} f(\rho) dS = \rho^2 f(\rho) \left\{ \frac{1}{a} [\sin(\gamma - a\theta_1) - \sin(\gamma - a\theta_0)] + (\theta_1 - \theta_0) \cos \phi_0 \right\}, \quad (10)$$

$$\gamma = \frac{\theta_1 \phi_1 - \theta_0 \phi_0}{\theta_1 - \theta_0}, \quad (11)$$

$$a = \frac{\phi_1 - \phi_0}{\theta_1 - \theta_0}. \quad (12)$$