

Koornwinder polynomials from Greengard et al:

$$K_{nm}(u, v) = c_{nm}(1 - v)^m P_{n-m}^{(0, 2m+1)}(1 - 2v) P_m \left(\frac{2u + v - 1}{1 - v} \right), \quad m \leq n, \quad (1)$$

$$c_{nm} = [2(1 + 2m)(n + 1)]^{1/2}. \quad (2)$$

Recursions (DLMF):

$$P_{n+1}^{(0, 2m+1)}(x) = (A_n x + B_n) P_n^{(0, 2m+1)}(x) - C_n P_{n-1}^{(0, 2m+1)}(x), \quad (3)$$

$$A_n = \frac{(n + m + 1)(2n + 2m + 3)}{(n + 1)(n + 2m + 2)}, \quad (4)$$

$$B_n = -\frac{(2m + 1)^2(n + m + 1)}{(n + 1)(n + 2m + 2)(2n + 2m + 1)}, \quad (5)$$

$$C_n = \frac{n(n + 2m + 1)(2n + 2m + 3)}{(n + 1)(n + 2m + 2)(2n + 2m + 1)}, \quad (6)$$

$$P_{n+1}(x) = \frac{2n + 1}{n + 1} P_n(x) - \frac{n}{n + 1} P_{n-1}(x). \quad (7)$$