ELEC6530 HOMEWORK 2 - DERIVATION OF DIFFERENTIAL DRIVE KINEMATICS

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1. Model Derivation

1.1. **Kinematics - Body Frame.** First, we must derive the kinematics of the model in the body frame. The coordinate frame of the robot is x forward, y left, and z up. Theta represents the rotation of the body about the z-axis. The axes form a right-handed coordinate system, so positive theta indicates counter-clockwise rotation.

The velocity of the robot can be represented as a pair of vectors. $\vec{v}$ represents the linear velocity (forwards and backwards) of the robot, and $\vec{w}$ represents the angular velocity (rotation) of the robot.

Given ω_R and ω_L, the wheel speeds of the right and left wheels, we can represent the linear and angular velocity of the differential drive robot as follows.

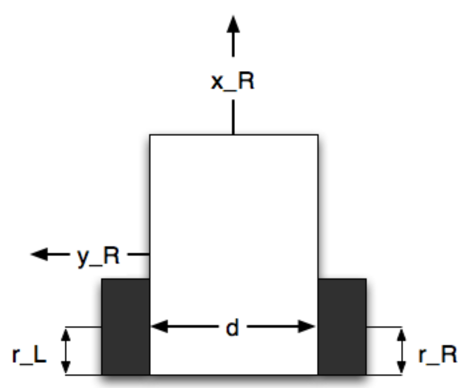

Figure 1. Physical Configuration of the Robot

$$
\begin{aligned}
v &= \frac{r_R}{2}\omega_R + \frac{r_L}{2}\omega_L \\
w &= \frac{r_L}{d}\omega_R - \frac{r_L}{d}\omega_L
\end{aligned}
\tag{1}
$$

Where r_R and r_L are the wheel radii of the left and right wheels, and d is the width of the wheelbase.

1.2. **Kinematics - Inertial Frame.** Second the kinematics of the robot in the global coordinate frame. The global coordinate frame is represented in the figure below.

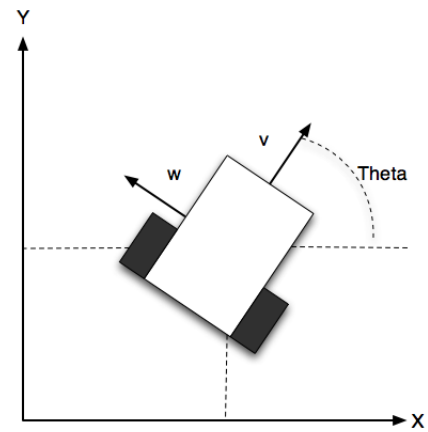

FIGURE 2. Robot in the Inertial Coordinate Frame

Since v and w are velocities, it is possible to derive the velocity and angular velocity of the robot in the inertial coordinate frame. The equations of motion are shown below.

$$\begin{aligned} \dot{x} &= v \cos\theta \\ \dot{y} &= v \sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{2}$$

To find the position of the robot, then these time-differentials must be integrated from a beginning time T_0 to the current time T. Additionally, a constant offset may be added if the robot does not start at the origin of the intertial coordinate frame.

$$\begin{aligned} x(t) &= \int_{T_0}^{T} v(t)\cos(\theta(t))dt + X_0 \\ y(t) &= \int_{T_0}^{T} v(t)\sin(\theta(t))dt + Y_0 \\ \theta(t) &= \int_{T_0}^{T} w(t)dt + \theta_0 \end{aligned} \tag{3}$$

1.3. **Discrete Time Model.** Often, it is more useful to represent the kinematics of the robot via a discrete-time model. This model assumes that the continuous values of the motor speeds are sampled at a constant rate $1/\Delta t$ and represented as v_k and w_k. This makes the equations of motion as follows.

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{k+1} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{k} + \begin{bmatrix} \Delta t v_k \cos(\theta_k + \Delta t w_k/2) \\ \Delta t v_k \sin(\theta_k + \Delta t w_k/2) \\ \Delta t w_k \end{bmatrix} \tag{4}$$