DERIVATION OF THE AUTOMOW EXTENDED KALMAN FILTER

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1. Model Derivation

1.1. **Kinematics - Body Frame.** First, the kinematics of the model in the body frame. The body frame is x-forwards, y-left, and z-up. Theta represents the rotation of the body about the z-axis. Positive theta indicates counter-clockwise rotation.

$$v = \frac{r_R}{2}\omega_R + \frac{r_L}{2}\omega_L \tag{1}$$

$$w = \frac{r_L}{d}\omega_R - \frac{r_L}{d}\omega_L \tag{2}$$

1.2. **Kinematics - Earth Frame.** Second the kinematics of the robot in the global coordinate frame. The global coordinate frame is x-East, y-North, z-Up in UTM (Universal Trans-Mercator) coordinates.

$$\dot{E} = v\cos\theta \tag{3}$$

$$\dot{N} = v\sin\theta \tag{4}$$

$$\dot{\theta} = \omega \tag{5}$$

1.3. **States.** Based on the reference paper, we choose the same augmented extended kalman filter states. The states are as follows.

$$x = \begin{bmatrix} E & N & \theta & r_L & r_R & d \end{bmatrix}^T \tag{6}$$

We have also experimented with two different augmented state vectors

$$x = \begin{bmatrix} E & N & \theta & r_L & r_R & d & E_{bias} & N_{bias} \end{bmatrix}^T \tag{7}$$

$$x = \begin{bmatrix} E & N & \theta & r_L & r_R & d & E_{bias} & N_{bias} & \theta_{bias} \end{bmatrix}^T \tag{8}$$

1.4. **Update Equation - Nonlinear.** This makes the discrete update equation as follows:

$$\begin{bmatrix} E \\ N \\ \theta \\ r_L \\ r_R \\ d \end{bmatrix}_{k+1} = \begin{bmatrix} E \\ N \\ \theta \\ r_L \\ r_R \\ d \end{bmatrix}_{k} + \begin{bmatrix} Tv_k \cos\theta_k + Tw_k/2 \\ Tv_k \sin\theta_k + Tw_k/2 \\ Tw_k \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

When expanded, this becomes:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}_{k+1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}_{k} + \begin{bmatrix} Tv_k \cos\theta_k + Tw_k/2 \\ Tv_k \sin\theta_k + Tw_k/2 \\ Tw_k \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

2. Filter Equation Derivation

2.1. **F Matrix.** The Jacobian matrix at each iteration may be derived as

$$\begin{bmatrix} 1 & 0 & A_{13} & A_{14} & A_{15} & 0 \\ 0 & 1 & A_{23} & A_{24} & A_{25} & 0 \\ 0 & 0 & 1 & A_{34} & A_{35} & A_{36} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$A_{13} = \frac{\delta f_1}{\delta \theta} = \frac{\delta f_1}{\delta x_3} = -\frac{1}{2}T(r_L\omega_L + r_R\omega_R)\sin\theta \qquad = -\frac{1}{2}T(x_4u_1 + x_5u_2)\sin x_3 \tag{12}$$

$$A_{14} = \frac{\delta f_1}{\delta r_L} = \frac{\delta f_1}{\delta x_4} = \frac{1}{2}Tw_L\cos\theta \qquad = \frac{1}{2}Tu_1\cos x_3 \tag{13}$$

$$A_{15} = \frac{\delta f_1}{\delta r_R} = \frac{\delta f_1}{\delta x_5} = \frac{1}{2}Tw_R\cos\theta \qquad = \frac{1}{2}Tu_2\cos x_3 \tag{14}$$

$$A_{23} = \frac{\delta f_2}{\delta \theta} = \frac{\delta f_2}{\delta x_3} = \frac{1}{2}T(r_L\omega_L + r_R\omega_R)\sin\theta \qquad = \frac{1}{2}T(x_4u_1 + x_5u_2)\sin x_3 \tag{15}$$

$$A_{24} = \frac{\delta f_2}{\delta r_L} = \frac{\delta f_2}{\delta x_4} = \frac{1}{2}Tw_L\sin\theta \qquad = \frac{1}{2}Tu_1\sin x_3 \tag{16}$$

$$A_{25} = \frac{\delta f_2}{\delta r_R} = \frac{\delta f_2}{\delta x_5} = \frac{1}{2}Tw_R\sin\theta \qquad = \frac{1}{2}Tu_2\sin x_3 \tag{17}$$

$$A_{34} = \frac{\delta f_3}{\delta r_L} = \frac{\delta f_3}{\delta x_4} = -T\frac{\omega_L}{d} \qquad = -T\frac{u_1}{x_6} \tag{18}$$

$$A_{35} = \frac{\delta f_3}{\delta r_R} = \frac{\delta f_3}{\delta x_5} = T\frac{\omega_R}{d} \qquad = T\frac{u_2}{x_6} \tag{19}$$

$$A_{36} = \frac{\delta f_3}{\delta d} = \frac{\delta f_3}{\delta x_6} = T\frac{r_L\omega_L - r_R\omega_R}{d^2} \qquad = T\frac{x_4u_1 - x_5u_2}{x_6^2} \tag{20}$$

2.2. **G Matrix.** The G matrix is the jacobian of the system with respect to the noises. The derivation depends on the following noises.

$$\begin{bmatrix} W_L \\ W_R \\ W_\theta \\ W_{RL} \\ W_{RR} \\ W_{WB} \end{bmatrix} = \tag{21}$$