OLS With Multiple Variables

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Setting Up

We'll use the `gss_week_11.csv` file on Canvas. Load it as a data frame called `gss_week11` and load the usual packages.

Warm Up / Review

Warm Up 1: Regress age at first birth (`agekdbrn`) on years of education (`educ`)

REPLACE THIS LINE WITH YOUR CODE

```
warmup1 <- lm(agekdbrn ~ educ, data = gss_week11)
summary(warmup1)
```

```
##
## Call:
## lm(formula = agekdbrn ~ educ, data = gss_week11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.5223  -3.5605  -0.9757   2.8548  27.0243
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 13.46735    0.80342   16.76   <2e-16 ***
## educ         0.79237    0.05768   13.74   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.157 on 1055 degrees of freedom
##   (868 observations deleted due to missingness)
## Multiple R-squared:  0.1518, Adjusted R-squared:  0.1509
## F-statistic: 188.7 on 1 and 1055 DF,  p-value: < 2.2e-16
```

Predict age at first birth for respondents with 16 years of education:

REPLACE THIS LINE WITH YOUR CODE

```
13.46735 + .79237*16
```

```
## [1] 26.14527
```

Warm Up 2: Regress age at first birth (`agekdbrn`) on highest degree (`degree`). Use "College Degree" as the reference group.

REPLACE THIS LINE WITH YOUR CODE

```
gss_week11$degree <- factor(gss_week11$degree,
                     levels = c("Less Than HS", "HS Diploma",
                              "Some College", "College Degree",
                              "Grad/Prof Degree"))
gss_week11$degree <- relevel(factor(gss_week11$degree), ref = "College Degree")
warmup2 <- lm(agekdbrn ~ degree, data = gss_week11)
summary(warmup2)
```

```
##
## Call:
## lm(formula = agekdbrn ~ degree, data = gss_week11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.7426  -3.3598  -0.9965   2.6402  27.0035
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               27.3598     0.4000  68.400  < 2e-16 ***
## degreeLess Than HS        -6.5101     0.5758 -11.307  < 2e-16 ***
## degreeHS Diploma          -4.3633     0.5027  -8.680  < 2e-16 ***
## degreeSome College        -3.3286     0.4917  -6.770 2.14e-11 ***
## degreeGrad/Prof Degree     0.3829     0.5941   0.645    0.519
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.122 on 1052 degrees of freedom
##   (868 observations deleted due to missingness)
## Multiple R-squared:  0.1655, Adjusted R-squared:  0.1623
## F-statistic: 52.14 on 4 and 1052 DF,  p-value: < 2.2e-16
```

Predict age at first birth for respondents with a graduate or professional degree:

REPLACE THIS LINE WITH YOUR CODE

```
27.3598 + .3829
```

```
## [1] 27.7427
```

Warm Up 3: Regress having a first child at age 30 or later (`agekdbrn_30plus`) on religion (`religion`). Use "Protestant" as the reference group.

REPLACE THIS LINE WITH YOUR CODE

```
gss_week11$religion <- relevel(factor(gss_week11$religion), ref = "Protestant")
summary(lm(agekdbrn_30plus ~ religion, data = gss_week11))
```

```
##
## Call:
## lm(formula = agekdbrn_30plus ~ religion, data = gss_week11)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.4815 -0.1754 -0.1499 -0.1499  0.8501
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.149915   0.015756   9.515  < 2e-16 ***
## religionCatholic 0.068709   0.028952   2.373  0.01782 *
## religionEastern  0.304631   0.082899   3.675  0.00025 ***
## religionJewish   0.331567   0.075137   4.413 1.13e-05 ***
## religionNone     0.009377   0.039216   0.239  0.81106
## religionOther    0.025524   0.052961   0.482  0.62995
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3817 on 1047 degrees of freedom
##   (872 observations deleted due to missingness)
## Multiple R-squared:  0.03204,    Adjusted R-squared:  0.02742
## F-statistic: 6.931 on 5 and 1047 DF,  p-value: 2.229e-06
```

Predict probability of having a first child at age 30 or later for Jewish respondents:

REPLACE THIS LINE WITH YOUR CODE

```
.149915 + .331567
```

```
## [1] 0.481482
```

Introducing Multiple Regression

Why do we control for other variables? The idea is to find another variable, *hold it constant*, and see if the association between X and Y changes.

In R, include more variables by linking them to the independent variable with a plus sign. In this example, we want to regress age at first birth on education, holding religion constant:

```r
agekd_educ_religion <- lm(agekdbrn ~ educ + religion,
                          data = gss_week11)

summary(agekd_educ_religion)
```

```
##
## Call:
## lm(formula = agekdbrn ~ educ + religion, data = gss_week11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.3572  -3.4731  -0.7609   2.5773  25.9809
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       13.32194    0.81313  16.384  < 2e-16 ***
## educ               0.76259    0.05792  13.167  < 2e-16 ***
## religionCatholic   1.54599    0.38639   4.001 6.75e-05 ***
## religionEastern    3.44548    1.10633   3.114  0.00189 **
## religionJewish     2.85010    1.01669   2.803  0.00515 **
## religionNone      -0.14292    0.52325  -0.273  0.78480
## religionOther      1.24962    0.70710   1.767  0.07748 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.093 on 1046 degrees of freedom
##   (872 observations deleted due to missingness)
## Multiple R-squared:  0.1764, Adjusted R-squared:  0.1716
## F-statistic: 37.33 on 6 and 1046 DF,  p-value: < 2.2e-16
```

To find the predicted values, need to use the full equation. For example, to predict the age at first birth for a Protestant respondent with 16 years of education:

```r
13.32 + .76*16 + 1.55*0 + 3.45*0 + 2.85*0 - .14*0 + 1.25*0
```

```
## [1] 25.48
```

Predict age at first birth for a Catholic respondent with 16 years of education:

REPLACE THIS LINE WITH YOUR CODE

```r
13.32 + .76*16 + 1.55*1 + 3.45*0 + 2.85*0 - .14*0 + 1.25*0
```

```
## [1] 27.03
```

Predict age at first birth for respondent from an Eastern religion with 13 years of education:

REPLACE THIS LINE WITH YOUR CODE

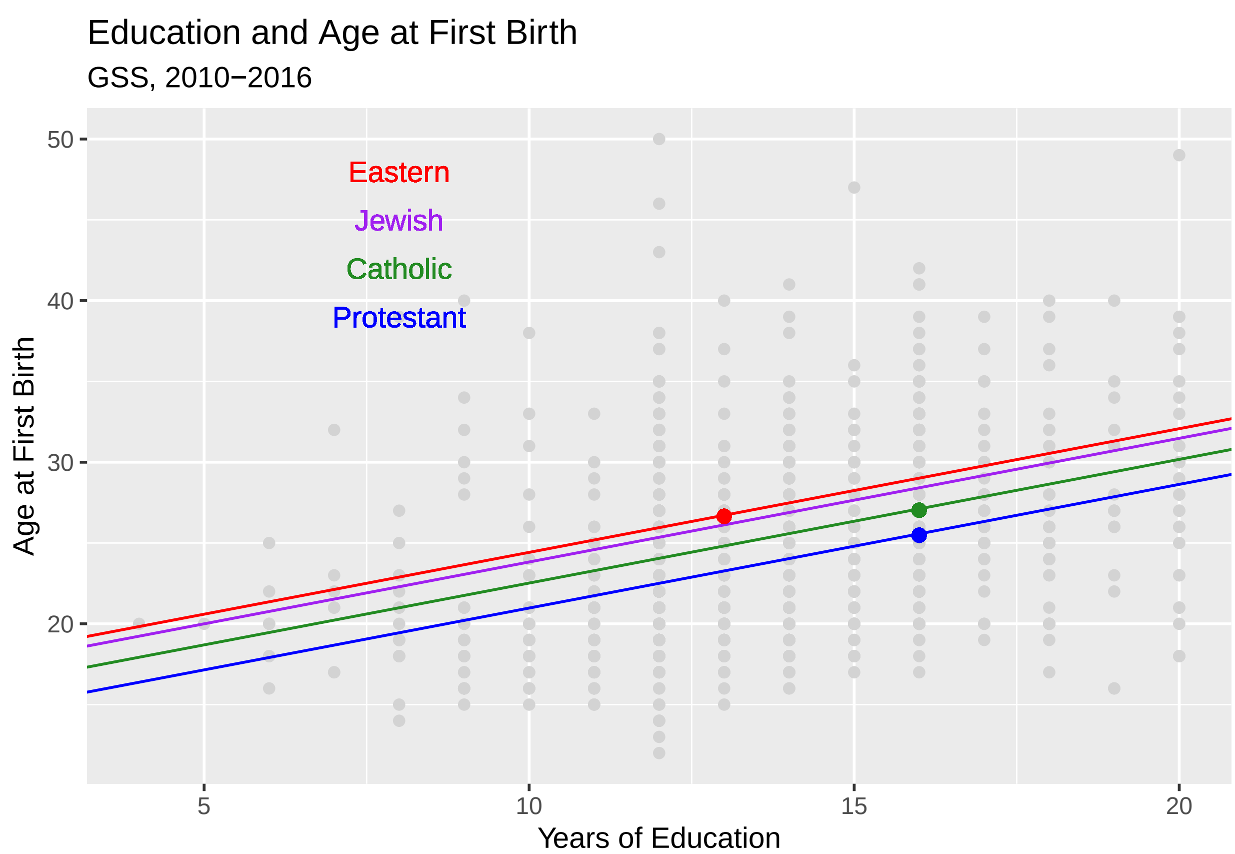
```r
13.32 + .76*13 + 1.55*0 + 3.45*1 + 2.85*0 - .14*0 + 1.25*0
```

```
## [1] 26.65
```

How do we make sense of this in a plot? The slopes (betas) are the same for every value of the control variable. But the intercepts are different.

Lots of code here; looking at the plot is more important than mastering the code!

```r
agekd_educ_religion_plot <- ggplot(gss_week11, aes(x = educ, y = agekdbrn))
agekd_educ_religion_plot + geom_point(color = "Light Gray") +
     geom_abline(slope = .76529, intercept = 13.32194, color = "Blue") +
     geom_abline(slope = .76529, intercept = 13.32194 + 1.54599, color = "Forest Green") +
     geom_abline(slope = .76529, intercept = 13.32194 + 3.44548, color = "Red") +
     geom_abline(slope = .76529, intercept = 13.32194 + 2.85010, color = "Purple") +
     geom_text(x = 8, y = 48, label = "Eastern", color = "Red") +
     geom_text(x = 8, y = 45, label = "Jewish", color = "Purple") +
     geom_text(x = 8, y = 42, label = "Catholic", color = "Forest Green") +
     geom_text(x = 8, y = 39, label = "Protestant", color = "Blue") +
     geom_point(x = 16, y = 25.48, color = "Blue", size = 2) +
     geom_point(x = 16, y = 27.03, color = "Forest Green", size = 2) +
     geom_point(x = 13, y = 26.65, color = "Red", size = 2) +
     labs(x = "Years of Education", y = "Age at First Birth",
          title = "Education and Age at First Birth",
          subtitle = "GSS, 2010-2016")
```



Multiple Control Variables

Models can have as many control variables as you want. Just continue adding them with plus signs. Let's try regressing age at first birth on education, religion, and race:

REPLACE THIS LINE WITH YOUR CODE

```
agekd_educ_religion_race_model <-
lm(agekdbrn ~ educ + religion + racehisp, data = gss_week11)

summary(agekd_educ_religion_race_model)
```

```
##
## Call:
## lm(formula = agekdbrn ~ educ + religion + racehisp, data = gss_week11)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -13.437  -3.485  -0.735   2.540  26.774
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        12.8134     0.8728  14.682  < 2e-16 ***
## educ                0.7420     0.0589  12.599  < 2e-16 ***
## religionCatholic    1.5079     0.4110   3.669 0.000256 ***
## religionEastern     3.4348     1.2394   2.771 0.005681 **
## religionJewish      2.6802     1.0187   2.631 0.008641 **
## religionNone       -0.1464     0.5249  -0.279 0.780304
## religionOther       1.2509     0.7072   1.769 0.077226 .
## racehispHispanic    0.2763     0.7641   0.362 0.717710
## racehispOther       0.7380     1.0481   0.704 0.481469
## racehispWhite       1.0171     0.4536   2.242 0.025175 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.086 on 1043 degrees of freedom
##   (872 observations deleted due to missingness)
## Multiple R-squared:  0.181,  Adjusted R-squared:  0.174
## F-statistic: 25.62 on 9 and 1043 DF,  p-value: < 2.2e-16
```

What is the predicted age at first birth for a Black Protestant with 17 years of education?

REPLACE THIS LINE WITH YOUR CODE

```
12.8134 + .7420*17
```

```
## [1] 25.4274
```

What is the predicted age at first birth for a Hispanic with no religious affiliation with 14 years of education?

REPLACE THIS LINE WITH YOUR CODE

```
12.8134 + .7420*14 - .1464 + .2763
```

```
## [1] 23.3313
```

Comparing Models, Introducing R-Squared

R-squared is the proportional reduction in the error from using the model. We'll calculate r-squared for the model regressing number of memberships on years of education.

```
memnum_educ_model <-
lm(memnum ~ educ, data = gss_week11)

summary(memnum_educ_model)
```

```
##
## Call:
## lm(formula = memnum ~ educ, data = gss_week11)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.1295 -1.3117 -0.3117  0.7794 11.6883
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.41495    0.20125  -7.031 2.84e-12 ***
## educ         0.22722    0.01442  15.762  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.78 on 1923 degrees of freedom
## Multiple R-squared:  0.1144, Adjusted R-squared:  0.1139
## F-statistic: 248.4 on 1 and 1923 DF,  p-value: < 2.2e-16
```

The r-squared value is 0.1144. Where does that number come from?

```
# Save the predicted values of Y:
gss_week11$pred_memnum <- memnum_educ_model$fitted.values

# Find the residuals (the observed values of Y minus the predicted values of Y) and square them:
gss_week11$res_memnum <- (gss_week11$memnum - gss_week11$pred_memnum)^2

#Find the deviations of each observed value of Y from the mean of Y:
gss_week11$dev_memnum <- (gss_week11$memnum - mean(gss_week11$memnum))^2

#Sum the squared deviations and subtract the sum of the squared residuals.

#Divide this difference by the sum of the squared deviations:
rsquared <- ((sum(gss_week11$dev_memnum)) -
                  (sum(gss_week11$res_memnum))) /
```

```
                 sum(gss_week11$dev_memnum)

rsquared
```

```
## [1] 0.1144075
```

To Find Adjusted R-squared:

```
# adjusted_rsquared =
# 1 - (((1 - rsquared)*(n-1)) / (n-k-1))

# n = sample size; k = number of variables

adjusted_rsquared <-
1 - (((1 - rsquared)*(1924-1)) / (1924-1-1))

adjusted_rsquared
```

```
## [1] 0.1139467
```

Comparing R-Squared Values

When we regress number of memberships on education and age it looks like the model is better since r-squared increases:

```
memnum_educ_age_model <- lm(memnum ~ educ + age, data = gss_week11)
summary(memnum_educ_age_model)
```

```
##
## Call:
## lm(formula = memnum ~ educ + age, data = gss_week11)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.2381 -1.2830 -0.4119  0.8691 11.8747
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.958854   0.243859  -8.033 1.65e-15 ***
## educ         0.234599   0.014486  16.195  < 2e-16 ***
## age          0.009605   0.002451   3.919 9.22e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.773 on 1922 degrees of freedom
## Multiple R-squared:  0.1214, Adjusted R-squared:  0.1205
## F-statistic: 132.8 on 2 and 1922 DF,  p-value: < 2.2e-16
```

But be careful: R-squared will almost always go up as you add variables, even if the variables are not significant. That does not necessarily mean the model is getting better.

```r
memnum_educ_age_place_model <-
    lm(memnum ~ educ + age + place, data = gss_week11)

summary(memnum_educ_age_place_model)
```

```
##
## Call:
## lm(formula = memnum ~ educ + age + place, data = gss_week11)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.2430 -1.2747 -0.4076  0.8771 11.9065
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -1.9264512  0.2546124  -7.566 5.93e-14 ***
## educ            0.2334475  0.0146100  15.979  < 2e-16 ***
## age             0.0095195  0.0024569   3.875  0.00011 ***
## placeNortheast  0.0009124  0.1284526   0.007  0.99433
## placeSoutheast -0.0480122  0.1049289  -0.458  0.64731
## placeWest       0.0250015  0.1204691   0.208  0.83561
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.774 on 1919 degrees of freedom
## Multiple R-squared:  0.1217, Adjusted R-squared:  0.1194
## F-statistic: 53.16 on 5 and 1919 DF,  p-value: < 2.2e-16
```