Social Statistics

OLS With Multiple Variables

November 29, 2021

Warm Up In Groups

Regress age at first birth (`agekdbrn`) on years of education (`educ`)

Predict age at first birth for respondents with 16 years of education

Regress age at first birth (`agekdbrn`) on highest degree (`degree`)

Use "College Degree" as the reference group. Predict age at first birth for respondents with a graduate or professional degree

Regress having a first child at age 30 or later (`agekdbrn_30plus`) on religion (`religion`)

Use "Protestant" as the reference group. Predict probability of having a first child at age 30 or later for Jewish respondents

Warm Up 1

Regress age at first birth (`agekdbrn`) on years of education (`educ`)

```
warmup_1 <- lm(agekdbrn ~ educ, data = gss_week11)

summary(warmup_1)
```

Warm Up 1

```
##
## Call:
## lm(formula = agekdbrn ~ educ, data = gss_week11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.5223  -3.5605  -0.9757   2.8548  27.0243
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 13.46735    0.80342   16.76   <2e-16 ***
## educ         0.79237    0.05768   13.74   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.157 on 1055 degrees of freedom
##   (868 observations deleted due to missingness)
## Multiple R-squared:  0.1518,    Adjusted R-squared:  0.1509
## F-statistic: 188.7 on 1 and 1055 DF,  p-value: < 2.2e-16
```

Warm Up 1

Predict age at first birth for respondents with 16 years of education

```
13.46735 + .79237*16
```

```
## [1] 26.14527
```

Warm Up 2

Regress age at first birth (`agekdbrn`) on highest degree (`degree`). Use "College Degree" as the reference group.

```
gss_week11 <- mutate(gss_week11, degree = factor(degree,
      levels = c("Less Than HS", "HS Diploma",
      "Some College", "College Degree",
      "Grad/Prof Degree")))

gss_week11$degree <- relevel(factor(gss_week11$degree),
  ref = "College Degree")

warmup2 <- lm(agekdbrn ~ degree, data = gss_week11)

summary(warmup2)
```

Warm Up 2

```
##
## Call:
## lm(formula = agekdbrn ~ degree, data = gss_week11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.7426  -3.3598  -0.9965   2.6402  27.0035
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)            27.3598     0.4000  68.400  < 2e-16 ***
## degreeLess Than HS     -6.5101     0.5758 -11.307  < 2e-16 ***
## degreeHS Diploma       -4.3633     0.5027  -8.680  < 2e-16 ***
## degreeSome College     -3.3286     0.4917  -6.770 2.14e-11 ***
## degreeGrad/Prof Degree  0.3829     0.5941   0.645    0.519
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.122 on 1052 degrees of freedom
##   (868 observations deleted due to missingness)
## Multiple R-squared:  0.1655,    Adjusted R-squared:  0.1623
```

Warm Up 2

Predict age at first birth for respondents with a graduate or professional degree

```r
27.3598 + .3829
```

```
## [1] 27.7427
```

Warm Up 3

Regress having a first child at age 30 or later (`agekdbrn_30plus`) on religion (`religion`). Use "Protestant" as the reference group.

```
gss_week11$religion <- relevel(factor(gss_week11$religion),
                               ref="Protestant")

warmup3 <- lm(agekdbrn_30plus ~ religion, data = gss_week11)

summary(warmup3)
```

Warm Up 3

```
##
## Call:
## lm(formula = agekdbrn_30plus ~ religion, data = gss_week11)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.4815 -0.1754 -0.1499 -0.1499  0.8501
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.149915   0.015756   9.515  < 2e-16 ***
## religionCatholic 0.068709   0.028952   2.373  0.01782 *
## religionEastern  0.304631   0.082899   3.675  0.00025 ***
## religionJewish   0.331567   0.075137   4.413 1.13e-05 ***
## religionNone     0.009377   0.039216   0.239  0.81106
## religionOther    0.025524   0.052961   0.482  0.62995
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3817 on 1047 degrees of freedom
##   (872 observations deleted due to missingness)
```

Warm Up 3

Predict probability of having a first child at age 30 or later for Jewish respondents:

```
.149915 + .331567
```

```
## [1] 0.481482
```

Introducing Multiple Regression

So far, our models have had one X (even if it has more than one category)

We want to adjust for possible confounding or spuriousness like we did with descriptive tables

- How do we *control for* other variables?
- Can we *explain away* the association between X and Y by controlling for other variables?

Introducing Multiple Regression

Find another variable, *hold it constant*, and see if the association between X and Y changes

- Can be categorical (Highest Degree, Year, Religion) or continuous (Years Since Marriage, Months Since Sister's First Birth)

Introducing Multiple Regression

We already saw that each additional year of education is associated with a delay of .79 years in the age at first birth.

Perhaps religion explains some of the variation in both education and age at first birth. So let's *hold religion constant*.

In R, include more variables by linking them to the independent variable with a plus sign

```
agekd_educ_religion <- lm(agekdbrn ~ educ + religion,
  data = gss_week11)

summary(agekd_educ_religion)
```

Introducing Multiple Regression

```
##
## Call:
## lm(formula = agekdbrn ~ educ + religion, data = gss_week11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.3572  -3.4731  -0.7609   2.5773  25.9809
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       13.32194    0.81313  16.384  < 2e-16 ***
## educ               0.76259    0.05792  13.167  < 2e-16 ***
## religionCatholic   1.54599    0.38639   4.001 6.75e-05 ***
## religionEastern    3.44548    1.10633   3.114  0.00189 **
## religionJewish     2.85010    1.01669   2.803  0.00515 **
## religionNone      -0.14292    0.52325  -0.273  0.78480
## religionOther      1.24962    0.70710   1.767  0.07748 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.093 on 1046 degrees of freedom
```

Introducing Multiple Regression

Holding religion constant (or net of religion), each additional year of education is associated with a delay of .76 years in the age at first birth, on average

To find the predicted values, think of the full equation:

$$\hat{y}_{agekdbrn} = \alpha + \beta_1(educ) + \beta_2(Catholic) + \beta_3(Eastern) + \beta_4(Jewish) + \beta_5(None) + \beta_6(Other)$$

Every prediction will have a value for education. Every prediction will also have a value for each binary religious category (even though they are mutually exclusive).

Predictions From Multiple Regression

For 16 years of education and Protestant (the reference category)

```
13.32 + .76*16 + 1.55*0 + 3.45*0 + 2.85*0 - .14*0 + 1.25*0
```

```
## [1] 25.48
```

Try finding the predicted age at first birth for Catholic respondents with 16 years of education

```
13.32 + .76*16 + 1.55*1 + 3.45*0 + 2.85*0 - .14*0 + 1.25*0
```

```
## [1] 27.03
```

Is the difference of 1.55 in the predictions between Protestants and Catholics *with the same years of education*

Predictions From Multiple Regression

```
##
## Call:
## lm(formula = agekdbrn ~ educ + religion, data = gss_week11)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.3572  -3.4731  -0.7609   2.5773  25.9809
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       13.32194    0.81313  16.384  < 2e-16 ***
## educ               0.76259    0.05792  13.167  < 2e-16 ***
## religionCatholic   1.54599    0.38639   4.001 6.75e-05 ***
## religionEastern    3.44548    1.10633   3.114  0.00189 **
## religionJewish     2.85010    1.01669   2.803  0.00515 **
## religionNone      -0.14292    0.52325  -0.273  0.78480
## religionOther      1.24962    0.70710   1.767  0.07748 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.093 on 1046 degrees of freedom
```

Predictions From Multiple Regression

What is the prediction for a respondent in an Eastern religion with 13 years of education?

```
13.32 + .76*13 + 1.55*0 + 3.45*1 +2.85*0 - .14*0 + 1.25*0
```

```
## [1] 26.65
```

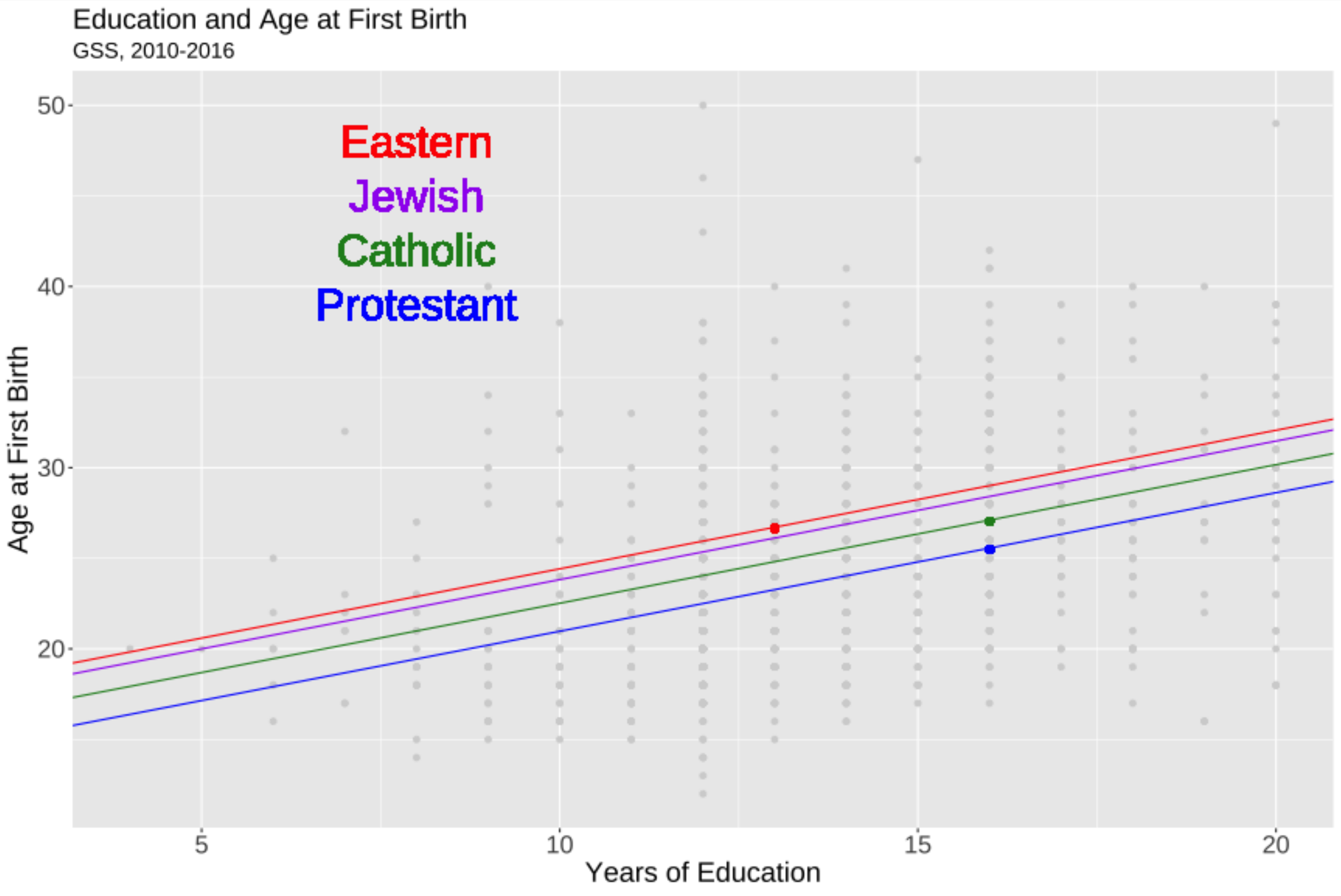
Plotting Multiple Regression

How do we make sense of this in a plot?

The beta for all groups is the coefficient for `educ`. So in this model the slopes are the same for each group.

But the intercepts are different: use the intercept coefficient for the reference group, use the intercept and the respective coefficient for each other group

Plotting Multiple Regression



More And More Variables

Models can continue adding control variables

Let's try regressing age at first birth on education, religion, and race

```
agekd_educ_religion_race_model <-
lm(agekdbrn ~ educ + religion + racehisp,
data = gss_week11)

summary(agekd_educ_religion_race_model)
```

More And More Variables

```
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)       12.8134     0.8728  14.682  < 2e-16 ***
educ               0.7420     0.0589  12.599  < 2e-16 ***
religionCatholic   1.5079     0.4110   3.669 0.000256 ***
religionEastern    3.4348     1.2394   2.771 0.005681 ** 
religionJewish     2.6802     1.0187   2.631 0.008641 ** 
religionNone      -0.1464     0.5249  -0.279 0.780304    
religionOther      1.2509     0.7072   1.769 0.077226 .  
racehispHispanic   0.2763     0.7641   0.362 0.717710    
racehispOther      0.7380     1.0481   0.704 0.481469    
racehispWhite      1.0171     0.4536   2.242 0.025175 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

More And More Variables

Holding religion and race constant, each additional year of education is associated with a delay of .74 years in age at first birth, on average

Controlling for education and race, Catholic women are 1.5 years older than Protestant women at their first birth, on average. This difference is significant.

Net of education and religion, there is no significant difference in the age at first birth between Black women and women in the other race category, on average.

Holding constant, controlling for, and net of can all be used interchangeably in these examples.

More And More Variables

Predictions still require the full equation

What is the predicted age at first birth for a Black Protestant with 17 years of education?

Black is the reference group for `racehisp` and Protestant is the reference group for `religion` so:

```
12.8134 + .7420*17
```

```
## [1] 25.4274
```

More And More Variables

What is the predicted age at first birth for a Hispanic with no religious affiliation with 14 years of education?

```
12.8134 + .7420*14 - .1464 + .2763
```

```
## [1] 23.3313
```

Comparing Models

How do we know if our model gets better when we add more control variables?

In other words: how well does our X predict our Y?

Without an X, only comparison is the difference between the observed Y and the mean of Y

With an X, the measure of fit is the residual (the difference between the observed Y and the predicted Y)

r^2 is a function of both of these in the form of a ratio

- The proportional reduction in error from using the model

R-Squared

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.41495    0.20125  -7.031 2.84e-12 ***
educ         0.22722    0.01442  15.762  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.78 on 1923 degrees of freedom
Multiple R-squared:  0.1144,    Adjusted R-squared:  0.1139
F-statistic: 248.4 on 1 and 1923 DF,  p-value: < 2.2e-16
```

R-Squared

Let's calculate r^2 for the model regressing number of memberships on years of education

$$r^2 = \frac{\sum(y - \bar{y})^2 - \sum(y - \hat{y})^2}{\sum(y - \bar{y})^2}$$

```
memnum_educ_model <-
lm(memnum ~ educ, data = gss_week11)

summary(memnum_educ_model)
```

R-Squared

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.41495    0.20125  -7.031 2.84e-12 ***
educ         0.22722    0.01442  15.762  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.78 on 1923 degrees of freedom
Multiple R-squared:  0.1144,	Adjusted R-squared:  0.1139
F-statistic: 248.4 on 1 and 1923 DF,  p-value: < 2.2e-16
```

R-Squared

```
gss_week11$pred_memnum <- memnum_educ_model$fitted.values

gss_week11$res_memnum <-
(gss_week11$memnum - gss_week11$pred_memnum)^2

gss_week11$dev_memnum <-
(gss_week11$memnum - mean(gss_week11$memnum))^2

rsquared <- ((sum(gss_week11$dev_memnum)) -
                  (sum(gss_week11$res_memnum))) /
                  sum(gss_week11$dev_memnum)

rsquared
```

```
## [1] 0.1144075
```

Properties of R-Squared

Like correlation, always between 0 and 1

Unlike correlation, always positive (since it is squared and a proportion)

Closer to 1 means observations fall more tightly around the line (in a linear association)

Will usually increase when you add variables to the model. But that does not necessarily mean the model is getting better.

Remember, parsimony is still our goal

Comparing Models

If we regress number of memberships on education and age, it looks like the model is better since r-squared increases.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.958854   0.243859  -8.033 1.65e-15 ***
educ         0.234599   0.014486  16.195  < 2e-16 ***
age          0.009605   0.002451   3.919 9.22e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.773 on 1922 degrees of freedom
Multiple R-squared:  0.1214,	Adjusted R-squared:  0.1205 
F-statistic: 132.8 on 2 and 1922 DF,  p-value: < 2.2e-16
```

Comparing Models

But be careful: R-squared will almost always go up as you add variables, even if the variables are not significant.

```
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)      -1.9264512  0.2546124  -7.566 5.93e-14 ***
educ              0.2334475  0.0146100  15.979  < 2e-16 ***
age               0.0095195  0.0024569   3.875  0.00011 ***
placeNortheast    0.0009124  0.1284526   0.007  0.99433    
placeSoutheast   -0.0480122  0.1049289  -0.458  0.64731    
placeWest         0.0250015  0.1204691   0.208  0.83561    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.774 on 1919 degrees of freedom
Multiple R-squared:  0.1217,	Adjusted R-squared:  0.1194 
F-statistic: 53.16 on 5 and 1919 DF,  p-value: < 2.2e-16
```

Adjusted R Squared

Adjusted r-squared adjusts for the number of parameters in your model (but not for how good they are)

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.41495    0.20125  -7.031 2.84e-12 ***
educ         0.22722    0.01442  15.762  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.78 on 1923 degrees of freedom
Multiple R-squared:  0.1144,	Adjusted R-squared:  0.1139 
F-statistic: 248.4 on 1 and 1923 DF,  p-value: < 2.2e-16
```

Adjusted R-Squared

```
# adjusted_rsquared =
# 1 - (((1 - rsquared)*(n-1)) / (n-k-1))

# n = sample size; k = number of variables

adjusted_rsquared <-
1 - (((1 - rsquared)*(1924-1)) / (1924-1-1))

adjusted_rsquared
```

```
## [1] 0.1139467
```

Adjusted R-Squared

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.41495    0.20125  -7.031 2.84e-12 ***
educ         0.22722    0.01442  15.762  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.78 on 1923 degrees of freedom
Multiple R-squared:  0.1144,	Adjusted R-squared:  0.1139
F-statistic: 248.4 on 1 and 1923 DF,  p-value: < 2.2e-16
```