#### Social Statistics

Categorical Associations

November 7, 2023

Test if there is a significant difference at the 95% confidence level in the proportion of immigrants vs native born respondents who say hard work is essential.

```
X-squared = 1.518, df = 1, p-value = 0.2179
alternative hypothesis: two.sided
95 percent confidence interval:
   -0.02604693   0.12186679
sample estimates:
   prop 1     prop 2
0.5817308   0.5338208
```

- Do not reject the null hypothesis:
  - → The p-value is not less than .05
  - → The confidence interval includes the null hypothesis value of zero
  - → Ignore the X-squared test statistic for prop tests

Test if there is a significant difference at the 95% confidence level in the average age between immigrants and native born respondents.

```
1 t.test(assignment_05$age[assignment_05$born=="Native"],
2          assignment_05$age[assignment_05$born=="Immigrant"])

Welch Two Sample t-test

data: assignment_05$age[assignment_05$born == "Native"] and assignment_05$age[assignment_05$born == "Immigrant"]
t = 7.0254, df = 515.38, p-value = 6.784e-12
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
4.251933 7.553053
sample estimates:
mean of x mean of y
52.87196 46.96947
```

- Reject the null hypothesis
  - → test statistic is more extreme than 1.96
  - → p-value is less than .05
  - → confidence interval does not include null hypothesis value of zero

Consider immigrants only, and test if there is a significant difference at the 95% confidence level in average years in the US between immigrants who say hard work is essential and immigrants who do not say hard work is essential.

Welch Two Sample t-test

```
data: assignment_05$yearsusa[assignment_05$born == "Immigrant" &
   assignment_05$ophrdwrk_essential == 1] and assignment_05$yearsusa[assignment_05$born ==
   "Immigrant" & assignment_05$ophrdwrk_essential == 0]
   t = -0.14191, df = 193.37, p-value = 0.8873
   alternative hypothesis: true difference in means is not equal to 0
   95 percent confidence interval:
        -5.229543      4.527533
   sample estimates:
   mean of x mean of y
   26.17442   26.52542
```

- Do not reject the null hypothesis
  - → test statistic is not more extreme than -1.96
  - → p-value is greater than .05
  - → confidence interval includes null hypothesis value of zero

Consider immigrants only, and create a binary variable distinguishing those who have been in the US less than fifteen years from those who have been in the US fifteen years or more. Test if there is a significant difference between these groups at the 95% confidence level in the proportion saying knowing the right people is essential for getting ahead.

2-sample test for equality of proportions with continuity correction

```
data: q4_table
X-squared = 6.4255, df = 1, p-value = 0.01125
alternative hypothesis: two.sided
95 percent confidence interval:
   -0.30736217 -0.02540019
sample estimates:
   prop 1     prop 2
0.6825397 0.8489209
```

- Reject the null hypothesis
  - → p-value is less than .05
  - → confidence interval does not include null hypothesis value of zero
  - → do not consider X-squared test statistic for prop.test

From your chosen variable, create one new binary variable distinguishing responses of "Essential" or "Very Important" from all other responses. Then use this new variable to test if there is a significant difference at the 99% confidence level in the proportion responding "Essential" or "Very Important" (rather than a different response) between immigrants and native born respondents.

2-sample test for equality of proportions with continuity correction

```
data: opwlth_table
X-squared = 9.8702, df = 1, p-value = 0.00168
alternative hypothesis: two.sided
99 percent confidence interval:
   -0.105158213 -0.004348905
sample estimates:
   prop 1    prop 2
0.8198198 0.8745734
```

Variable	p-value	ci_lower	ci_upper	Reject?
opwlth	0.002	0.820	0.875	Yes
oppared	0.004	-0.117	-0.002	Yes
opeduc	0.596	-0.079	0.503	No
opclout	0.003	-0.081	0.00023	?
oprace	0.493	-0.042	0.024	No
opreligion	0.091	-0.046	0.012	No

## Significance and Association

- Testing significance of differences of proportions works when we only have two levels
- Today: statistical significance of distributions of categorical variables with two or more levels

	Not Important At All	Not Very Important	Fairly Important	Very Important	Essential
Immigrant	0.085	0.191	0.322	0.307	0.095
Native	0.116	0.289	0.320	0.214	0.060

# Significance and Association

- Are variables dependent or independent?
  - → Dependent = Association. Knowing value of one variable helps predict value of the other variable
  - → Independent = No association. Knowing value of one variable does not help predict value of the other variable
- Will also want to know if association is strong or weak
  - → Often more important than only knowing about statistical significance

### Association and Chi-Squared Test

- With means and proportions, used t- and z-distributions
- Difference based on sample size and degrees of freedom
- Assumed a normal distribution
  - → CLT says distribution of sample means or sample proportions are normally distributed

### Association and Chi-Squared Test

- Not the same when variables have more than two options
  - → Political Party = Democrat, Independent, Republican
  - → Race, Class, Religion, Region, Marital Status, Labor Force Status, etc.
- For today's tests, should use at least one non-ordered variable
- If we do not use a mean or create a binary variable, how do we measure significance of distribution?

# Chi-Squared Test

- Like previous tests, we will calculate a test statistic and convert it to a probability of getting a more extreme value
- Also like previous tests, we will see if that probability allows us to reject a null hypothesis
- ullet  $H_0$ : X and Y are independent (no association)
- $\bullet$   $H_A$ : X and Y are dependent (association)
- Test Statistic Formula:  $\mathbf{X}^2 = \sum \frac{(f_o f_e)^2}{f_e}$

# Chi-Squared Test

- Two key points:
  - → Use frequencies (counts) rather than proportions
  - → Compare observed frequencies to expected frequencies
- Distribution is not normally distributed
  - → No negative test statistics, so only consider the "right-tail"
  - → Shape changes based on DF (but always skewed right)
  - $\rightarrow$  DF = (rows 1) \* (columns 1)
- Test-statistic not like z-score or t-score, but p-value similar to what we have seen
- Still need big sample size (expected>5 in each cell)

# Trump Leads in 5 Critical States as Voters Blast Biden, Times/Siena Poll Finds

Voters in battleground states said they trusted Donald J. Trump over President Biden on the economy, foreign policy and immigration, as Mr. Biden's multiracial base shows signs of fraying.



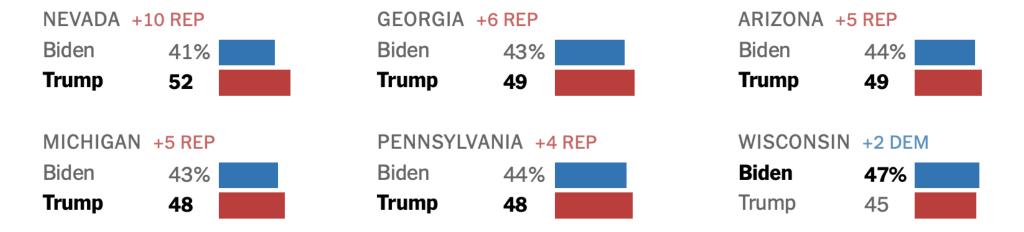
By Shane Goldmacher

The New York Times

Nov. 5, 2023

#### Trump Is Ahead in Five of Six Swing States

Margins are calculated using unrounded figures.



Based on New York Times/Siena College polls of 3,662 registered voters from Oct. 22 to Nov. 3 • By Ashley Wu and Molly Cook Escobar

								The	New	llork	Time	:s											
(Includes leaners) Thinking about the upcoming presidential election in 2024, if the election were held today, who would you vote for if the candidates were Joe Biden, the Democrat, and Donald Trump, the Republican?  FOLLOW UP: Are you				ST	ATE			GEI	NDER		А	GE			RACE/E	THNICITY		EDUC	ATION		RACE/EL	DUCATION	
definitely or probably voting for them?																							Nev
[IF NEEDED: If you had to decide today]	REGISTERED VOTERS	AZ	GA	MI	NV	PA	WI	MEN	WOMEN	18-29	30-44	45-64	65+	WHITE	BLACK	HISPANIC	отн.	B.A.+	NO B.A.	WHITE,	WHITE, NO COLLEGE	NON- WHITE, COLLEGE	NON- WHITE, NO COLLEGE
Joe Biden	44%	44%	43%	43%	41%	44%	47%	37%	50%	47%	43%	42%	46%	39%	71%	50%	39%	52%	39%	51%	32%	58%	54%
Donald Trump	48%	49%	49%	48%	52%	48%	45%	55%	42%	46%	47%	52%	46%	54%	22%	42%	51%	38%	54%	40%	62%	32%	38%
[VOL] Another candidate	2%	2%	3%	2%	<1%	2%	3%	2%	2%	2%	3%	2%	2%	2%	2%	<1%	3%	3%	2%	3%	2%	2%	1%
[VOL] Not going to vote/wouldn't vote if those were the choices	3%	2%	2%	4%	4%	3%	2%	3%	3%	2%	4%	3%	2%	3%	3%	2%	4%	3%	3%	3%	3%	4%	3%
[VOL] Don't know/Refused	3%	3%	3%	3%	2%	3%	3%	3%	3%	3%	3%	1%	3%	2%	3%	5%	2%	3%	2%	3%	2%	4%	4%
Number of respondents	3,662	603	629	616	611	600	603	1,791	1,827	646	868	1,234	793	2,487	403	463	225	1,605	2,036	1,158	1,320	400	685
Percent of total electorate	100%	17%	17%	17%	17%	17%	17%	48%	51%	17%	23%	31%	26%	68%	11%	13%	6%	35%	64%	25%	42%	9%	21%

[IF NEEDED: If you had to decide today]	REGISTERED VOTERS	18-29	30-44	45-64	65+
Joe Biden	44%	47%	43%	42%	46%
Donald Trump	48%	46%	47%	52%	46%
[VOL] Another candidate	2%	2%	3%	2%	2%
[VOL] Not going to vote/wouldn't vote if those were the choices	3%	2%	4%	3%	2%
[VOL] Don't know/Refused	3%	3%	3%	1%	3%
Number of respondents	3,662	646	868	1,234	793
Percent of total electorate	100%	17%	23%	31%	26%

Age	Biden	Trump	Other/DK
18-29	47%	46%	7%
30-44	43%	47%	10%
45-64	42%	52%	6%
65+	46%	46%	7%

We need a table of frequencies. We'll build it with separate rows for each age group. Here's the first row...

```
1 age_18_29 <- c(.47, .46, .07)*646
```

Build the rest!

```
1 age_18_29 <- c(.47, .46, .07)*646
2 age_30_44 <- c(.43, .47, .10)*868
3 age_45_64 <- c(.42, .52, .06)*1234
4 age_65_plus <- c(.46, .46, .07)*793</pre>
```

```
observed table <- rbind(age 18 29,
 2
                          age 30 44,
 3
                          age 45 64,
                          age 65 plus)
 5
    observed table
             [,1]
                    [,2] [,3]
age 18 29
         303.62 297.16 45.22
age 30 44 373.24 407.96 86.80
age 45 64 518.28 641.68 74.04
age 65 plus 364.78 364.78 55.51
    observed table <- round(observed table, 0)
 2
   observed table
           [,1] [,2] [,3]
age 18 29
           304 297
                       45
age 30 44 373 408
                     87
age 45 64
         518 642
                      74
age 65 plus
            365
                 365
                       56
```

```
1 colnames(observed_table) <- c("biden", "trump", "other")
2
3 addmargins(observed_table)</pre>
```

```
biden trump other
                             Sum
age 18 29
             304
                  297
                         45
                             646
age 30 44
         373
                  408
                         87
                             868
age 45 64
             518
                 642
                       74 1234
age 65 plus
             365
                 365
                         56 786
Sum
            1560
                 1712
                        262 3534
```

- If distribution of age is equal across candidate, expected value of each cell should be:
  - → (total in row \* total in column) / (total in table)
- What is the expected value for the "age\_18\_29, Biden" cell?

```
1 (646*1560) / 3534
```

For the test statistic, we need the expected values in every cell.
 For now, calculate them for the other two columns in the first row...

```
1  # For age_18_29, Trump
2
3  (646*1712) / 3534

[1] 312.9462

1  # For age_18_29, Other
2
3  (646*262) / 3534

[1] 47.89247
```

Make a table of all the expected frequencies

```
1 expected_age_18_29 <- c(285.1613, 312.9462, 47.8925)
2 expected_age_30_44 <- c(383.1579, 420.4912, 64.3509)
3 expected_age_45_64 <- c(544.7199, 597.7951, 91.4850)
4 expected_age_65_plus <- c(346.9610, 380.7674, 58.2717)
5
6 expected_table <- rbind(expected_age_18_29, expected_age_30_44, expected_age_45_64, expected_age_65_plus)
10
11 colnames(expected_table) <- c("biden", "trump", "other")</pre>
```

	biden	trump	other
expected_age_18_29	285.1613	312.9462	47.8925
expected_age_30_44	383.1579	420.4912	64.3509
expected_age_45_64	544.7199	597.7951	91.4850
expected_age_65_plus	346.9610	380.7674	58.2717

 The difference between each observed and expected value is the residual

1 residual_table <- observed_table - expected_table												
	biden	trump	other									
age_18_29	18.8387	-15.9462	-2.8925									
age_30_44	-10.1579	-12.4912	22.6491									
age_45_64	-26.7199	44.2049	-17.4850									
age_65_plus	18.0390	-15.7674	-2.2717									

• Formula: 
$$x^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

- For each cell, square the residual and divide it by the expected frequency
- Squaring the difference always gives a positive value, which is why we are only working with the right-tail probabilities

```
1 chi2_table <- (residual_table^2)/expected_table</pre>
```

	biden	trump	other
age_18_29	1.2445469	0.8125400	0.1746945
age_30_44	0.2692961	0.3710662	7.9716326
age_45_64	1.3106792	3.2688009	3.3418071
age_65_plus	0.9378735	0.6529207	0.0885614

• Test statistic is the sum of all the values of  $\frac{(f_0-f_e)^2}{f_e}$ 

```
1 sum(chi2_table)
```

[1] 20.44442

Degrees of Freedom = (#rows - 1)\*(#columns - 1)

```
1 (4 - 1)*(3 - 1)
```

[1] 6

- If test statistic is greater than our cutoff, we can reject the null hypothesis that the variables are independent
- To find the cutoff, use **qchisq()** with the degrees of freedom. Note that in the chi-squared test, we only use the area to the right, so for .05 we use .95 (not .975 like the two-tailed t-test):

```
1 qchisq(.95, df = 6)
[1] 12.59159
```

- With DF=6, need a chi-squared test statistic at least as big as 12.59159 to reject the null hypothesis.
- With our test statistic of 20.444, we can reject the null
- Can also convert to a p-value...

```
1 1 - pchisq(20.444, df = 6)
```

[1] 0.002307742

• There is a .002307 chance of getting a test statistic more extreme than our test statistic. That is less than .05 so we can reject the null hypothesis that the variables are independent.

Shortcut in R:

```
1 chisq.test(observed_table)
```

#### Pearson's Chi-squared test

```
data: observed table
X-squared = 20.444, df = 6, p-value = 0.002307
```

Test statistic in red, degrees of freedom in blue, p-value in green

### Chi-Squared Test - Exercise

- Try one more: "Regardless of who you support for president, tell me whether you strongly agree, somewhat agree, somewhat disagree or strongly disagree with the statement that Joe Biden is just too old to be an effective president."
- Are state and belief in Biden being too old dependent at the 99% confidence level?

Joe Biden is just too old to be an effective president.				ST	ATE	
[IF NEEDED: Tell me whether you strongly agree, somewhat agree, somewhat disagree or strongly disagree]	REGISTERED VOTERS	AZ	GA	МІ	PA	WI
Strongly agree	47%	47%	45%	46%	45%	45%
Somewhat agree	24%	23%	24%	25%	24%	27%
Somewhat disagree	14%	13%	13%	13%	16%	15%
Strongly disagree	13%	15%	14%	14%	13%	12%
[VOL] Don't know/Refused	2%	3%	4%	3%	1%	<1%
<b>NET</b> Agree	71%	70%	69%	70%	69%	72%
<b>NET</b> Disagree	27%	28%	28%	27%	30%	27%
Number of respondents	3,662	603	629	616	600	603
Percent of total electorate	100%	17%	17%	17%	17%	17%

#### Chi-Squared Test - Exercise

```
1 az <-c(.47, .23, .13, .15, .03)*603
 2 ga < c(.45, .24, .13, .14, .04)*629
 3 \text{ mi} \leftarrow c(.46, .25, .13, .14, .03)*616
 4 pa <-c(.45, .24, .16, .13, .01)*600
 5 wi < c(.45, .27, .15, .12, .01)*603
 6
   states <- rbind(az, ga, mi, pa, wi)
 8
   colnames(states) <- c("Strongly Agree",</pre>
10
                           "Somewhat Agree",
                           "Somewhat Disagree",
11
                           "Strongly Disagree",
12
13
                           "DK/Refused")
14
   states <- round(states, 0)</pre>
```

### Chi-Squared Test - Exercise

```
1 chisq.test(states)

Pearson's Chi-squared test

data: states
X-squared = 26.301, df = 16, p-value = 0.04994
```

Remove "Don't Know/Refused" responses?

```
1 states[,1:4] # include all rows, columns 1:4
   Strongly Agree Somewhat Agree Somewhat Disagree Strongly Disagree
               283
                              139
                                                   78
                                                                      90
az
               283
                              151
                                                   82
                                                                      88
qa
              283
                              154
                                                                      86
mi
                                                   80
              270
                              144
                                                   96
                                                                      78
pa
wi
               271
                              163
                                                   90
                                                                      72
  1 chisq.test(states[,1:4])
```

```
Pearson's Chi-squared test

data: states[, 1:4]

X-squared = 8.0897, df = 12, p-value = 0.7781
```

#### More Real World Relevance

Philip N. Cohen

HUSI	BANL
1	2

	WIFE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	Agriculture	19.6	3.4	.5	.0	.4	1.0	.6	1.1	.8	.0	5.3	.4	.0	1.2	.9	.0	.8	.0	.4	.7	.9	1.0	1.5	.4	.7	1.7	.7	.5
2	Environment NatRes	1.9	11.4	2.3	2.0	.9	.6	.4	.9	.2	.0	.0	2.4	.9	1.3	1.2	.0	.9	1.7	.0	1.2	.7	.5	.6	1.3	.6	2.3	.5	.6
3	Architecture	.2	.0	17.9	1.2	.4	1.2	.3	1.6	.0	.8	.0	.5	1.6	.8	1.5	.0	.5	2.7	.0	.3	.1	.1	.6	.8	1.0	.0	.7	.6
4	Area/Ethnic/CivStud	.7	1.7	2.2	6.9	1.3	1.5	.9	.6	.0	2.6	.0	1.0	.2	1.3	1.4	.6	.1	1.4	.0	1.0	.3	.0	.6	1.2	1.8	.6	.8	2.1
5	Communications	.7	.9	.4	1.2	2.6	.6	.8	.7	.8	.5	1.3	1.2	.9	.6	.9	1.1	.9	1.2	1.2	.7	.9	1.0	1.2	1.2	.8	.8	1.2	1.2
6	Computer/InfoSci	.4	.2	.4	.0	.8	4.7	.1	2.1	4.6	.1	.0	.3	.2	.5	.7	.4	.1	.5	.1	1.0	.4	.3	.7	.3	.4	.7	.4	.2
7	Education Admin	1.7	.9	.7	.7	.9	.8	3.3	.7	.9	.7	1.4	.8	1.3	.7	.8	1.0	1.2	.7	2.1	.6	.9	1.4	.4	.7	.9	1.0	1.0	1.3
8	Engineering	.2	.6	.9	.2	.4	1.8	.3	3.5	1.1	.5	.0	.3	.1	.6	.5	.4	.5	.2	.3	1.2	.2	.4	.9	.4	.4	.6	.4	.2
9	Engineering Techn	1.3	.0	.0	.0	.1	2.2	1.3	2.5	7.0	1.5	.0	.0	.0	1.1	.5	1.1	.3	.7	.7	.0	.2	.0	.0	.4	.8	1.4	.5	.0
10	Linguistics/Foreign	.2	1.6	1.2	1.2	.9	.9	.8	.9	.9	5.0	.0	1.9	.1	1.1	2.0	1.6	1.4	1.5	.7	.8	.7	.6	1.3	1.4	1.3	.4	.7	1.4
11	Family/Consumer Sci	.5	1.6	1.7	2.2	1.5	1.0	.5	1.0	.7	.5	2.5	1.3	2.4	.6	.8	1.5	2.5	.5	.3	.7	1.3	1.3	1.6	1.1	.7	.2	1.0	.6
12	English Language/Lit	.9	1.2	.8	1.4	1.3	1.1	.7	.7	.5	1.3	.4	2.9	1.6	.9	1.7	1.5	.6	.9	.9	.9	1.3	.6	.8	1.3	1.3	.4	.8	1.8
13	Liberal Arts/Human	.3	.4	3.6	3.1	1.3	1.0	.8	.8	.6	1.2	.0	1.2	9.4	.7	1.6	1.3	.5	.2	1.0	1.5	.8	1.5	.3	1.1	.8	.8	.7	1.2
14	Biology and LifeSci	.9	1.2	1.2	.9	.6	.9	.5	1.0	1.0	.9	.8	.8	.6	3.0	.8	.8	1.0	.7	.8	1.8	1.1	.8	.8	1.0	.6	1.2	.7	.8
15	Math/Stats	.0	.3	.8	.0	.5	1.6	1.3	1.3	2.1	1.7	.0	1.3	.8	.6	5.7	1.7	.5	1.2	1.8	2.0	.5	.3	.8	.7	.9	1.1	.7	.5
16	Interdisc/multidisc	1.5	.5	.3	1.8	1.0	1.1	.6	.9	2.5	1.3	.0	.9	.8	1.6	.7	2.9	1.2	1.6	1.7	1.0	1.8	.5	.0	1.1	.9	.3	.8	1.2
17	PhysFit, Park/Rec	1.3	.7	.5	.4	.6	.5	1.3	.7	2.0	.8	1.6	.4	.6	1.1	.2	1.5	5.7	.5	.4	.5	.9	.8	2.5	.9	.5	.9	1.3	1.2
18	Philosophy/ReligStud	.9	1.5	1.2	1.4	.9	.7	.4	.9	1.0	3.7	.0	1.8	.7	.7	2.8	4.2	.4	8.3	1.6	.7	1.8	1.1	.0	1.0	1.2	.2	.6	1.9
19	Theology/ReligVoc	.0	.0	2.6	1.7	1.1	.3	2.4	.5	.4	1.5	.0	.4	.0	1.3	2.7	2.7	.0	4.0	31.0	.0	1.0	1.4	2.7	.3	1.7	1.5	.5	.4
20	PhysicalSci	.4	.8	.4	2.0	.4	.9	.6	.9	.5	1.9	.5	.8	.5	1.2	2.0	1.0	.6	.9	.2	7.2	.6	.6	.0	.9	.5	.8	.6	1.0
21	Psychology	.9	1.3	.8	1.0	1.0	.8	.8	.8	.4	.8	2.4	.8	.7	.8	1.0	1.4	1.0	2.0	1.2	.7	1.9	1.4	1.5	1.2	1.3	.8	1.0	1.3
22	CrimJustice/Fire	.6	.5	.5	1.5	1.0	.9	1.2	.5	.9	1.9	2.7	.2	2.3	.6	1.0	1.0	1.1	1.1	.3	.7	.5	6.3	1.1	.8	.9	1.8	1.2	.2
23	PubAff/Policy/SocWk	1.5	.6	1.1	3.9	1.0	1.1	.9	.8	1.1	1.9	2.0	1.4	1.3	.8	.3	.2	.4	.7	1.1	1.3	1.4	1.0	6.9	.9	.3	.6	1.2	.4
24	SocialSci	.4	.9	1.2	1.6	.9	.9	.5	.8	.4	.7	.3	1.2	1.1	1.0	1.1	1.4	.5	1.9	.3	1.0	1.4	.7	1.7	1.8	1.1	.4	.9	1.3
25	Fine Arts	.5	1.0	1.6	1.1	1.4	1.0	1.0	.7	.6	1.2	.5	1.8	1.3	.4	.9	.9	.8	1.0	1.1	.7	.7	.8	.6	1.0	4.4	.5	.8	.8
26	Medical/HealthSci	.8	1.1	.5	.7	.8	.7	1.1	1.0	1.0	.8	1.7	.6	.9	1.4	.8	1.0	1.6	.9	.7	.8	1.2	1.2	1.0	.7	.5	3.4	1.0	.9
27	Business	1.0	.7	.7	.5	.9	1.0	.6	1.0	1.3	1.1	.8	.7	.6	.7	.8	.5	.8	.3	.7	.6	.8	1.1	.7	.9	.6	.7	1.6	.6
28	History	.4	1.1	1.5	.5	.8	.7	1.0	.7	.2	.6	.7	2.0	.4	1.0	1.0	.8	.8	2.9	1.0	.8	1.2	.9	.7	1.6	1.0	.6	.8	2.9