Chapter 2 | Identifiability

- Given $f(\underline{x};\vec{\theta})$, where $\underline{x}$ is known but $\vec{\theta}$ unknown.
  - We consider whether $\vec{\theta}$ can be found at all.

Def | Parameter Identifiability

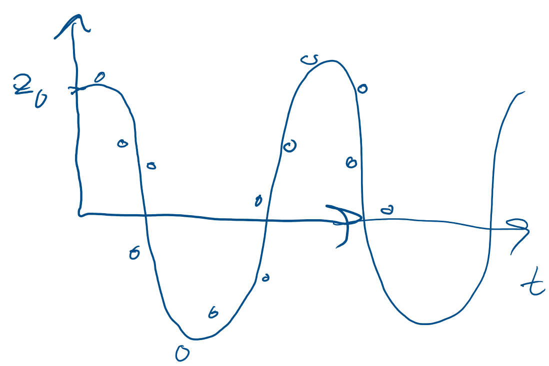
- Let $y(\underline{x}) = f(\underline{x};\vec{\theta})$ denote observations from f, where $\vec{\theta} \in \Gamma$ is the parameter vector.
  - We say that $\vec{\theta}$ are identifiable if $f(\underline{x};\vec{\theta})$ are uniquely determined by $\vec{\theta}$.

Ex) Consider a spring problem with $c=0$

$$\Rightarrow m\frac{d^2z}{dt^2} + kz = 0, \quad z(0) = z_0, \quad \left.\frac{dz}{dt}\right|_{t=0} = 0.$$

- Here, $\vec{\theta} = [m, k] \in \Gamma = (0,\infty) \times (0,\infty)$
- Note: Characteristic equation gives $r = \pm\sqrt{k/m}\, i$

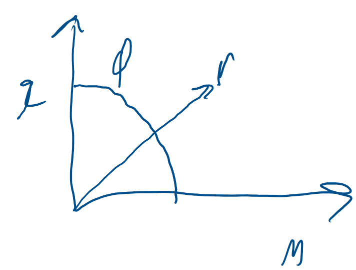
$$\Rightarrow z(t) = z_0 \cos\left(\sqrt{k/m}\; t\right)$$



- Note, if z_0 is known, the ratio $k/m = K$ is not unique.
  - There exist identifiable + non-identifiable manifolds

$$I(\theta) = \left\{ \varphi = \arctan(k/m) \;\middle|\; 0 < \varphi < \pi/2 \right\}$$

$$NI(\theta) = \left\{ r = \sqrt{k^2+m^2} \;\middle|\; r > 0 \right\}$$



Ex) SIR Model

$$\frac{dS}{dt} = \mu(N-S) - \gamma k I S$$
$$\frac{dI}{dt} = \gamma k I S - (\mu + r) R$$
$$\frac{dR}{dt} = r I - \mu R$$

γk is not identifiable

Identifiability Definitions

- there are 4 types of identifiability

i) Structural

ii) Practical

iii) Statistical

iv) Sensitivity-based