Markov Chains

- First, we need to define random processes (Similar to KL-expansions

Def Random Process

- A random process is defined as $X = \{X(t), t \in T\}$, which is an indexed set of R.V.'s all from the same probability space $(\Omega, \mathcal{F}, P)$
  - Let $X_t(\omega)$ denote a realization of the process at time $t \in T$ with realization $\omega \in \Omega$.

Def Second-order Random process

- A Second-order Random process satisfies

$$E[X_t^2] < \infty, \quad \forall t \in T$$

- All Second-order Random processes have

$$E[X_t] = \mu_t, \quad t \in T$$

$$c(s,t) = Cov(X_s, X_t) = E[(X_s - \mu_s)(X_t - \mu_t)], \quad s, t \in T$$

Def Gaussian Process (GP)

- A GP is a continuous random process s.t. all finite dimensional collections of realizations, e.g., $\vec{X}_t = [X_{t_1}, X_{t_2}, \ldots, X_{t_N}]$, have a multivariate Normal distribution, i.e.,

$$\vec{X}_t \sim N(\vec{\mu}(t), \underset{\sim}{C}(s,t))$$

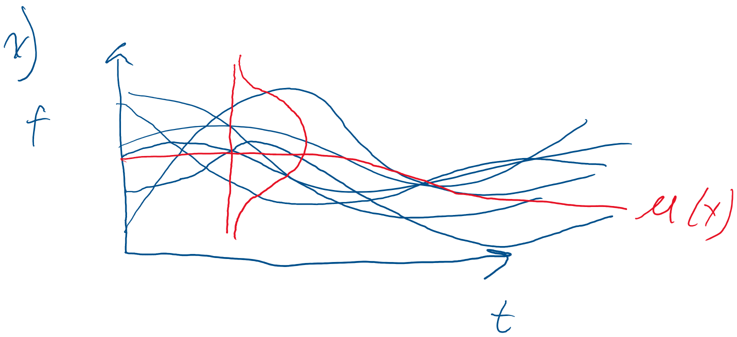
where $\vec{\mu}(t) = [E[X_{t_1}], E[X_{t_2}], \ldots, E[X_{t_N}]]$

where $\mu(t) = [\mu(t_1), \mu(t_2), \ldots, \mu(t_N)]$

+ $\underset{\sim}{C}(s,t) = Cov(X_s, X_t)$ is the covariance.

Ex) $x \sim N(0,1)$

Ex)



Markov Chains

- A random process, X_t, satisfies the Markov process if the probability of future events, X_{t+1}, only depends on current state, X_t

$$\Rightarrow P(X_{t+1} = x_{t+1} \mid X_t = x_t, X_{t-1} = x_{t-1}, \ldots, X_0 = x_0) = P(X_{t+1} = x_{t+1} \mid X_t = x_t)$$

- A Markov Chain is characterized by:
  i) the state space, S $\quad (X_t \in S, \forall t)$
  ii) the initial probability, $\vec{P}^0$
  iii) a transition kernel

- Assume we have k finite states, i.e. $S = \{x_1, x_2, \ldots, x_k\}$
- Let P_{ij} be defined as follows

$$P_{ij} = P(X_{N+1} = x_j \mid X_N = x_i) \quad (\text{i.e. } x_i \to x_j)$$

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