Bayesian Inference (Chapter 12)

- The fundamental difference between frequentist/classical stats + Bayes. Stats:
  - Freq/classical => We want to find a fixed, unknown $\vec{\Theta}$ that can be estimated with measurements obtained at some frequency.
    - The frequency + noise of observations dictate estimate + uncertainty.
  - Bayes: Assume that each Θ_i is a R.V. with some (unknown) distribution.
    - Now, $\vec{\Theta}$ + measurements are random.

Bayes Theorem

- Let A + B denote two events.
  - Let $P(A|B)$ denote the probability of A being "true" conditioned on B having occured.
  - So A is "conditioned" on B.
- Bayes theorem gives

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

mini proof

$$P(A|B) = P(A\cap B)/P(B) \quad + \quad P(B|A) = P(A\cap B)/P(A)$$

$$\Rightarrow P(A\cap B) = P(A|B)\,P(B) = \underbrace{P(B|A)\,P(A)}$$

$$\Rightarrow P(A|B) = \frac{P(B|A)\cdot P(A)}{P(B)}. \quad \square$$

$$\Rightarrow P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} . \quad \square$$

Bayesian Inverse Problems

- Let $\vec{\theta}$ be our unknown, random parameters + $\vec{y}$ be realizations from a random observation process $\vec{Y}$.
- Our posterior beliefs in $\vec{\theta}$ conditioned on $\vec{y}$ are written

$$\pi(\vec{\theta}|\vec{y}) = \frac{\overbrace{P(\vec{y}|\vec{\theta})}^{\text{Likelihood}} \cdot \overbrace{\pi_0(\vec{\theta})}^{\text{Prior on } \vec{\theta}}}{\underbrace{\pi_y(\vec{y})}_{\text{Evidence}}}$$

- The evidence, $\pi_y(\vec{y})$, ensures $\pi(\vec{\theta}|\vec{y})$ is a PDF + normalizes the posterior