Def | Parameter Redundancy

- A parameter set $\vec{\Theta}$ is <u>redundant</u> if $\vec{\Theta}_{sub} \subset \vec{\Theta}$ s.t.

$$f\left(y; \{\vec{\Theta}_{sub,i}; \vec{\Theta}_{\sim sub}\}\right) = f\left(y; \{\vec{\Theta}_{sub,j}; \vec{\Theta}_{\sim sub}\}\right)$$

where $i \neq j$.

OR $$L\left(\{\vec{\Theta}_{sub,i}; \vec{\Theta}_{\sim sub}\}\right) = L\left(\{\vec{\Theta}_{sub,j}; \vec{\Theta}_{\sim sub}\}\right)$$

- This usually implies that a manifold, $h(\vec{\Theta}_{sub})$, exists.
  - we can try to identify $I(\Theta)$ + $NI(\Theta)$

- We can state that $\vec{\Theta}$ is not practically (statistically) identifiable if

$$L(\vec{\Theta}_{MLE}) - L(\vec{\Theta}^*) < \eta, \quad \eta > 0,$$

for $\vec{\Theta}_{MLE} \neq \vec{\Theta}^*$.

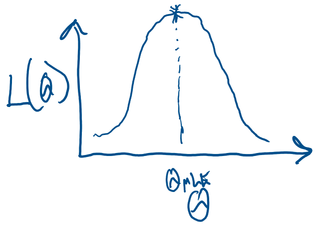
Def | Profile Likelihood

- Given $\Theta_i \subset \vec{\Theta} \in \Gamma$, the profile-likelihood (PL) is calculated as

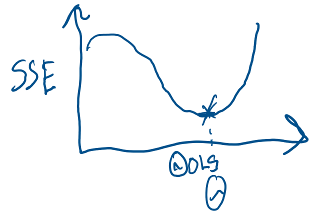
$$L_{PL}(\Theta_i^n) = \max_{\tilde{\Theta}} L(\tilde{\Theta} \mid \Theta_i^n)$$

where $\tilde{\Theta} = \vec{\Theta} \setminus \Theta_i$, $\Theta_i^n \in [\Theta_i^{min}, \Theta_i^{max}]$, $\tilde{\Theta} \in \Gamma \setminus \Gamma_{\Theta_i}$

- This approach "profiles" a single θ_i at a time to approximate the 1D likelihood landscape.



or



- Given $\vec{\theta} \in \mathbb{R}^n$, how many PL do we need? n PL
  - Optimization is for $\hat{\theta} \in \mathbb{R}^{n-1} \Rightarrow (n-1)+1$ function eval per gradient

Ex) $y_i = \theta_1 \theta_2 t + \theta_3 + \varepsilon_i, \quad \varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$

- If we can construct $L_{PL}(\theta_i)$ for each θ_i, we can define confidence intervals given

$$L(\tilde{\theta}_{MLE}) - L(\vec{\theta}) < \chi^2(\alpha, df)$$

- If we use $df = 1$

$$\Rightarrow \left| L_{PL_i}(\tilde{\theta} \mid \theta_i) - L(\vec{\theta}_{MLE}) \right| < \chi^2(\alpha, 1)$$

- If $df = 1 \Rightarrow$ Pointwise confidence intervals
- If $df = \#\theta \Rightarrow$ Simultaneous confidence intervals

<u>Def</u> Statistical Identifiability

- Consider the random sample $\vec{Y} = [Y_1, \ldots, Y_{N_y}]$ + denote the parameter-dependent joint PDF $P_Y(\vec{y} \mid \vec{\theta})$
  - The parameters $\vec{\theta}$ are statistically identifiable if

- The parameters, $\vec{\theta}$, are statistically identifiable if

$$P_y(\vec{y} \mid \vec{\theta}) = P_y(\vec{y} \mid \vec{\theta}^*) \Rightarrow \vec{\theta} = \vec{\theta}^*$$

- Recall that

$$L(\vec{\theta} \mid \vec{y}) = P_y(\vec{y} \mid \vec{\theta}) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\frac{1}{2\sigma^2}\left(\sum_i \left(y_i - f(x_i; \vec{\theta})\right)^2\right)\right)$$

for $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$.

- $L(\vec{\theta} \mid \vec{y}) \Rightarrow$ assumes $\vec{y}$ known, $\vec{\theta}$ may change
- $P_y(\vec{y} \mid \vec{\theta}) \Rightarrow$ assumes $\vec{\theta}$ known, $\vec{y}$ unknown

- Here, we can assess how model structure, choice of x_i, values of σ^2, or choice of $C(f): f \to y$ affect identifiability.