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Derivative-Based Global Sensitivity Measures (DGSMs)

- Assume $Y = F(\vec{\theta})$ is differentiable, we can construct

$$\mu_i = \int_\Gamma \left[\frac{\partial f}{\partial \theta_i}\bigg|_{\vec{\theta}} \right] \rho(\vec{\theta}) d\vec{\theta} \,, \quad \mu_i^* = \int_\Gamma \left| \frac{\partial f}{\partial \theta_i}\bigg|_{\vec{\theta}} \right| \rho(\vec{\theta}) d\vec{\theta}$$

$$\nu_i = \int_\Gamma \left[\frac{\partial f}{\partial \theta_i}\bigg|_{\vec{\theta}} \right]^2 \rho(\vec{\theta}) d\vec{\theta}$$

where $\rho(\vec{\theta})$ is the probability measure

- Note that using μ_i^* avoids cancellation erros
- We can show that the DGSMs satisfy

$$S_{T_i} \leq C_i \frac{\nu_i}{Var(Y)}$$

Where C_i are Poincaré-Constants which depend on the PDF of θ_i

- In practice

$$\mu_i \approx \frac{1}{M} \sum_{j=1}^{M} \frac{\partial f}{\partial \theta_i}\bigg|_{\vec{\theta}^j} \,, \quad \nu_i \approx \frac{1}{M} \sum_{j=1}^{M} \left(\frac{\partial f}{\partial \theta_i}\bigg|_{\vec{\theta}^j} \right)^2$$

Morris' Screening

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- Given $\vec{\theta} \in \Gamma$, the "Elementary Effects"

$$d_i^j = \frac{F(\vec{\theta}^j + \delta \vec{e}_i) - F(\vec{\theta}^j)}{\delta}$$

where $\vec{e}_i$ is a unit vector in the i-th direction, + δ is the step size.

- we care about the following measures, with R random samples

$$\mu_i = \frac{1}{R} \sum_{j=1}^{R} d_i^j(\vec{\theta}^j), \quad \mu_i^* = \frac{1}{R} \sum_{j=1}^{R} \left| d_i^j(\vec{\theta}^j) \right|, \quad \sigma = \left(\frac{1}{R-1} \sum_{j=1}^{R} \left(d_i^j(\vec{\theta}^j) - \mu_i \right)^2 \right)^{1/2}$$