Friday, March 21, 2025 12:09 PM

- Revisit Basic Example

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)}$$

- Suppose there are events $A_1, A_2, \ldots, A_N$ that could occur

• Note that $P(B)$ is

(updated 3/21) $P(B) = P(B|A_1)\cdot P(A_1) + P(B|A_2)P(A_2) + \ldots + P(B|A_N)P(A_N)$

$$\Rightarrow \quad P(B) = \sum_{i=1} P(B|A_i)P(A_i)$$

$$P(B) = \int_{\Omega} P(B|A_i)P(A_i)\,dA_i$$

- So let $P(B) \equiv \pi_y(\vec{y})$

$$\pi(\vec{\theta}|\vec{y}) = \frac{P(\vec{y}|\vec{\theta})\cdot\pi_0(\vec{\theta})}{\pi_y(\vec{y})}$$

E.g. if each θ_i is independent

$$\Rightarrow \pi_0(\vec{\theta}) = \prod_{i=1}^{P} \pi_0(\theta_i)$$

using definition above

$$\Rightarrow \quad \boxed{\pi(\vec{\theta}|\vec{y}) = \frac{P(\vec{y}|\vec{\theta})\cdot\pi_0(\vec{\theta})}{\int_{\Omega} P(\vec{y}|\vec{\theta})\cdot\pi_0(\vec{\theta})\,d\vec{\theta}}}$$

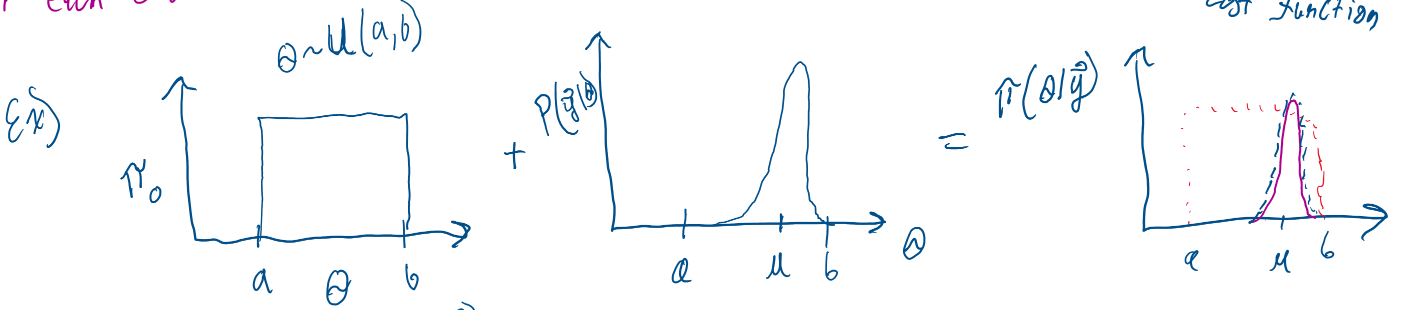
is deterministic & fixed

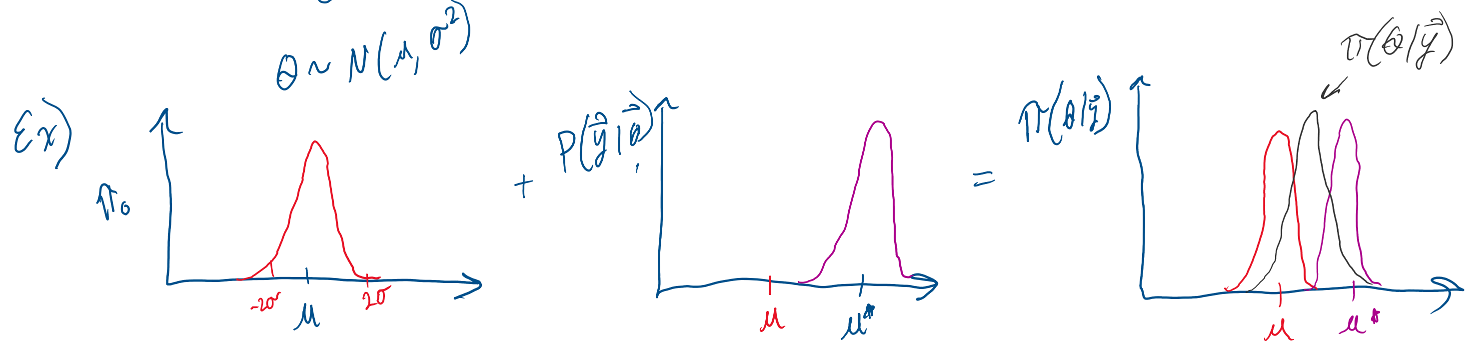
* These terms are evaluated for each $\vec{\theta}$ value

$P(\vec{y}|\vec{\theta}) \approx$ likelihood similar to a cost function

$\theta \sim \mathcal{U}(a,b)$

for each θ value   to a cost function





The likelihood

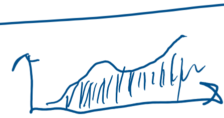
- The likelihood tells us how likely our observations are conditioned on our model, $f(\underline{x};\vec{\theta})$, + value $\vec{\theta}$ are currently.

- So if $\varepsilon_i \overset{iid}{\sim} N(0,\sigma_\varepsilon^2)$

$$\Rightarrow \quad P(\vec{y}\,|\,\vec{\theta}) = \frac{1}{(2\pi\sigma_\varepsilon^2)^{N_y/2}} \exp\left(-\frac{1}{2\sigma_\varepsilon^2}\sum_{i=1}^{N_y}\left(y_i - f(\underline{x}_i;\vec{\theta})\right)^2\right)$$

where $i=1,\dots,N_y$ is number of observations.

How to get $\displaystyle\int_\Omega P(\vec{y}\,|\,\theta)\,\pi_0(\vec{\theta})\,d\vec{\theta}$?

i) Quadrature 

ii) Markov Chain Monte Carlo (MCMC)