$$E(X) = \mu = \int_{\mathbb{R}} x f_X(x)\, dx$$

$$Var(X) = \sigma^2 = E\left[(X-\mu)^2\right] = \int_{\mathbb{R}} (x-\mu)^2 f(x)\, dx$$

- Note σ^2 quatifies the pdf's variability or width

$\boxed{\text{Properties}}$

- $E[aX] = aE[X]$, $a \in \mathbb{R}$
- $Var[aX] = a^2 Var[X]$
- Note: $(Var[X])^{1/2} = \sqrt{\sigma^2} = \sigma \equiv$ standard deviation

- Note:

$$Var[X] = E\left[(X-\mu)^2\right] = E\left[X^2 - 2X\mu + \mu^2\right]$$

$$= E\left[X^2\right] - 2\mu E[X] + E\left[\mu^2\right]$$

$$= E\left[X^2\right] - 2\mu^2 + \mu^2 = E\left[X^2\right] - \mu^2$$

$$Var[X] = E\left[X^2\right] - \left(E[X]\right)^2$$

$$\Rightarrow \mathrm{Var}[X] = E[X^2] - \left(E[X]\right)^2$$

- The third + fourth moments are skewness + kurtosis

Multivariate Statistics

Def Let X_i be a R.V. + $\vec{X} = [X_1, X_2, X_3, \dots, X_n]$ be a random vector. The joint cdf + pdf of $\vec{X}$ are:

cdf: $$F_{\vec{X}}(x_1, x_2, \dots, x_n) = P\{X_1 \le x_1, X_2 \le x_2, \dots, X_n \le x_n\}$$

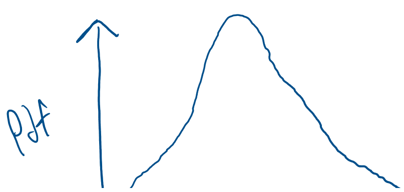
in 2D: $$F_X(x_1, x_2) = \int_{-\infty}^{x_1}\int_{-\infty}^{x_2} f_X(s_1, s_2)\, ds_1\, ds_2$$

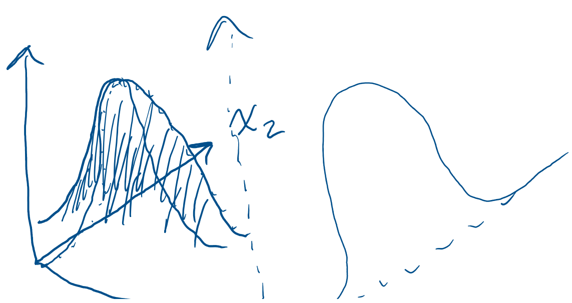
where $f_X = \dfrac{\partial^N}{\partial x_1 \partial x_2 \cdots \partial x_N} F_X$ is the joint pdf.

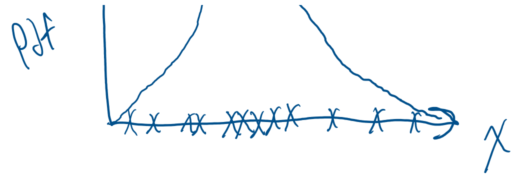
Def For two R.V.s, X_1 + X_2, the marginal pdf is the pdf for one variable over the entire space for another:

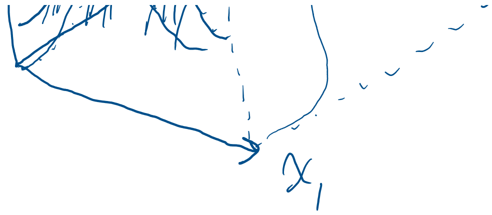
$$\Rightarrow f_{X_1}(x_1) = \int_{\mathbb{R}} f_X(x_1, x_2)\, dx_2$$

- Ex)









Def

- Let X_1 + X_2 be continuous R.V.s, with joint pdf $f_X(x_1, x_2)$.

Then the pdf of X_1 conditioned on $X_2 = x_2$ is

$$f(x_1 | x_2) = \frac{f_X(x_1, x_2)}{f_{X_2}(x_2)}$$

Important Concepts

Independence

- Given $X_1, X_2, \ldots, X_N$ R.V.s, we say they are "independent"

if

$$f_X(x_1, x_2, x_3, \ldots, x_N) = \prod_{i=1}^{N} f_{X_i}(x_i)$$

Covariance

- Two R.V.s, X + Y, have covariance

$$\mathrm{Cov}(X,Y) = E\left[\left(X - E[X]\right)\left(Y - E[Y]\right)\right]$$

$$= E\left[XY - X E[Y] - E[X] Y + E[X]E[Y]\right]$$

$$= E[XY] - E[X]E[Y] - E[X]E[Y] + E[X]E[Y]$$

$$= E[XY] - E[X]E[Y]$$

Note $E[E[X]] = E[X]$ since $E[X]$ is constant

$= E[XY] - E[X]E[Y]$

- Note: if X + Y are independent, then

$$E[XY] = E[X]E[Y] \Rightarrow Cov(X,Y) = 0$$

<u>Def</u>

- The <u>Pearson Correlation</u> is $\rho_{XY} = \dfrac{Cov(X,Y)}{\sigma_X \sigma_Y}$

(for X + Y R.V.s) where $\rho_{XY} \in [-1, 1]$.

- Note: $\rho_{XY} = -1$ or $\rho_{XY} = 1$, we have a linear relationship between X + Y
- Uncorrelated variables, $\rho_{XY} \approx 0$, does <u>NOT</u> imply independent