- We call $\underset{\sim}{F} = \frac{1}{\sigma^2} \underset{\sim}{S}^T \underset{\sim}{S}$ the Fisher information matrix (FIM).
  - Note that if $\underset{\sim}{S}$ is one-to-one, the $F = \frac{1}{\sigma^2} \underset{\sim}{S}^T \underset{\sim}{S}$ will have full rank.
  - $\underset{\sim}{F}$ is $p \times p$, where p is the number of parameters.
  - If $\underset{\sim}{F}$ is full rank, all parameters are identifiable

- Note that if $\varepsilon \sim N(0, \underset{\sim}{\Sigma})$, where $\underset{\sim}{\Sigma}$ is the covariance

$$\underset{\sim}{F} = \underset{\sim}{S}^T \underset{\sim}{\Sigma} \underset{\sim}{S},$$ where $\underset{\sim}{\Sigma}$ contains $Var(\varepsilon_i) = \sigma_i^2$ on the diagonal

$\boxed{\text{Sensitivity Identifiability}}$

- Let $y_i = f(x_i ; \vec{\theta})$, $i = 1, \ldots, N_y$ + assume $\vec{\theta}^*$ minimizes

$$J(\vec{\theta}) = \frac{1}{N_y} \sum_{i=1}^{N_y} \left(\underline{y_i} - \underline{f(x_i ; \vec{\theta})} \right)^2$$

- If f is continuously differentiable w.r.t. $\vec{\theta}$, then

$$\nabla_\theta f \Big|_{\theta^*} = \left[\frac{\partial f}{\partial \theta_1}, \ldots, \frac{\partial f}{\partial \theta_p} \right] \Bigg|_{\vec{\theta} = \vec{\theta}^*}$$

- A linear approximation from $\vec{\theta}^* \to \vec{\theta}$ is given by

$$f(x ; \vec{\theta}) \approx f(x ; \vec{\theta}^*) + \nabla_\theta f \Big|_{\theta^*} \cdot \Delta \vec{\theta}, \quad \Delta \vec{\theta} = \vec{\theta}^* - \vec{\theta}$$

- Then we write

$$\Delta y = f(x ; \vec{\theta}^*) - f(x ; \vec{\theta}) \approx \underset{\sim}{S} \, \Delta \vec{\theta}$$

Def | Sensitivity Identifiability

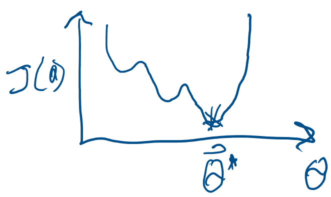
- the parameters $\vec{\theta} \in \mathbb{R}^p$, are sensitivity identifiable if $\underset{\sim}{S}\Big|_{\vec{\theta}=\vec{\theta}^*}$ is a one-to-one mapping.
  - Note: the Null space of $\underset{\sim}{S}$ is the nonidentifiable subspace of $\vec{\theta}$

- this gives the following relationship

$$\nabla_\theta^2 LL(\vec{\theta}^* | \vec{y}) = -\frac{1}{\sigma^2} \underset{\sim}{S}^T(\vec{\theta}^*) \underset{\sim}{S}(\vec{\theta}^*) \approx -\underset{\sim}{F}(\vec{\theta}^*)$$

  - $\underset{\sim}{F}(\theta)$ is full rank $\Rightarrow$ $\underset{\sim}{S}$ is one-to-one $\Rightarrow$ MLE (or OLS) is unique.

Ex)



$\boxed{\text{UQ Crime}}$ correlation $\not\Rightarrow$ identifiability issues.

- A correlation $= \pm 1$ does imply non-identifiability, but correlation $\neq \pm 1$ does not.

Parameter Influence

- A parameter, θ_i, is deemed functionally non-influential if there is a manifold, $NI(\vec{\theta})$, where $|f(\vec{\theta}, \theta_i) - f(\vec{\theta}, \theta_i + \Delta\theta_i)| < \eta$, $\eta > 0$ for all $\theta_i \in NI(\vec{\theta})$.

Local Sensitivity (Chapter 8)

- Sensitivity analysis (SA) is the process of quantifying the relative contributions of inputs $(\vec{\theta})$ to responses, $\vec{y}$.
  - we call $\vec{y}$ the "quantity of interest" QoI

the relative contributions of inputs (θ) to responses, y.

. we call $\vec{y}$ the "quantity of interest" QoI.