DEPARTMENT OF PHYSICS
AND ASTRONOMY
*Faculty of Science
and Engineering*

MACQUARIE
University

Trigonometric Parallax

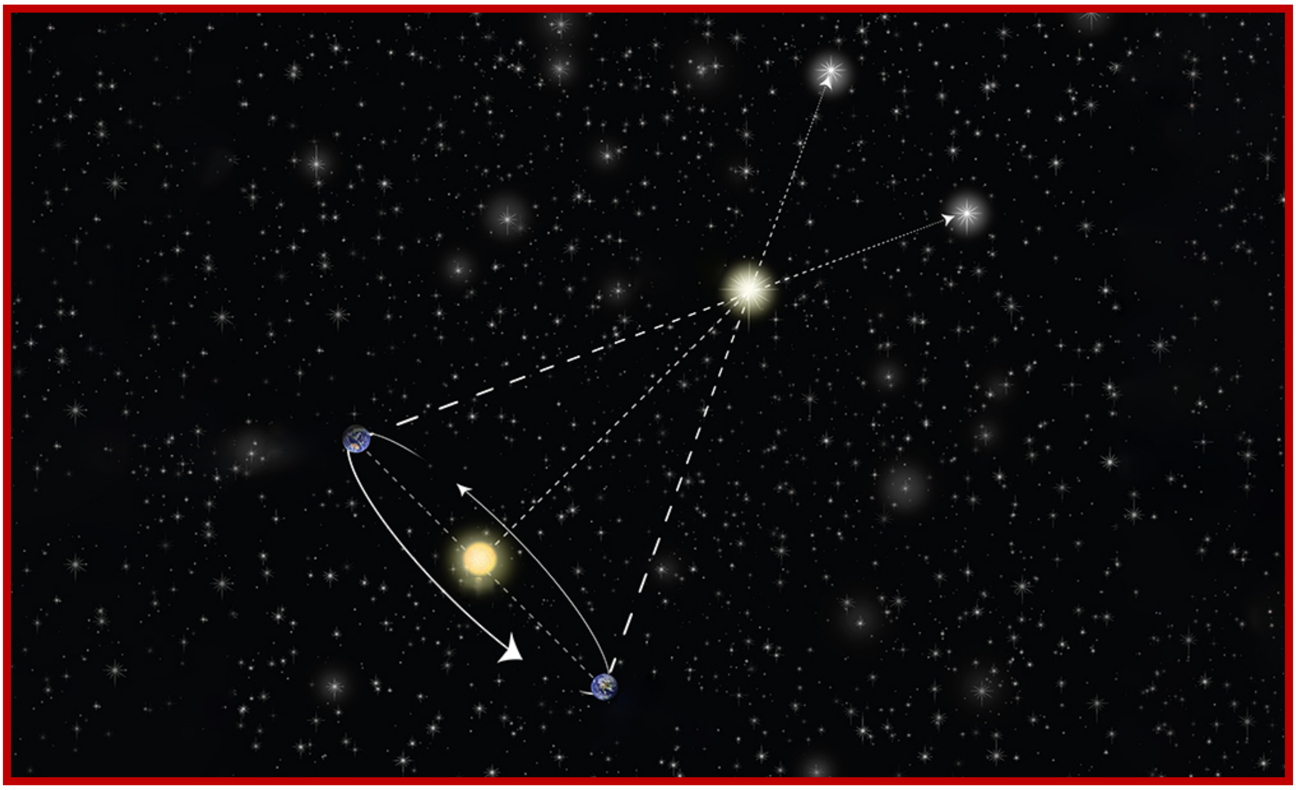

Figure 1: Estimating the distance to a nearby star using trigonometric parallax (Image Credit: ESA/Hubble)

Introduction

When staring up at the stars, have you ever wondered how far away they are? What do you think you'd have to do to measure that distance? Unfortunately, you can't stretch a measuring tape between yourself and the stars, so we need another approach. While astronomers have developed several techniques, one of the most popular methods is known as trigonometric parallax, which relies on a star's apparent movement against the background of more distant stars as Earth revolves around the sun.

Let's test parallax. First, close your right eye, and place your thumb over a distant object. Now, switch eyes, so that your left is closed, and your right is open. Your thumb will appear to shift slightly against the background. By measuring this movement and knowing the distance between your eyes, you can calculate the distance to your thumb.

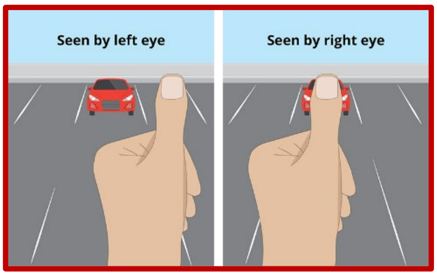

Figure 2: Parallax with your thumb (Image credit: www.blog.cupix.com)

When performing this technique on a star, first a photograph of a star is taken six months apart so that the diameter of the Earth's orbit around the Sun forms the baseline of a surveying triangle. The distance to a star is given by:

$$\boxed{d = \frac{b/2}{\sin(\theta/2)}}$$

In this case, d is the distance to the star and b is the length of the baseline, which is the known diameter of the Earth's orbit around the sun (approximately 300 million kilometres). The total angular shift of the star against the backdrop of more distance stars is θ. The parallax angle is designated as half of the total shift (i.e. $\theta/2$).

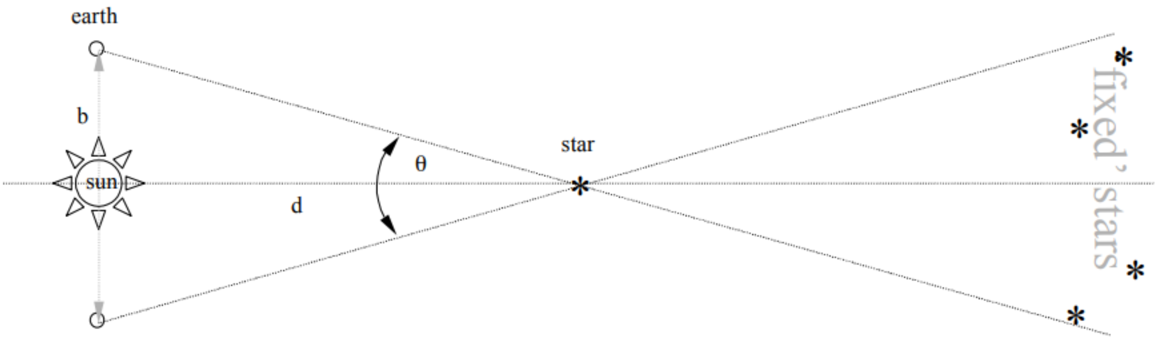

Figure 3: Measuring distances using the trigonometric parallax method.

In this experiment, you will perform a measurement using the trigonometric parallax method. The experiment is analogous to the method used in astronomy to measure distances to the local stars, but rather than waiting six months for the Earth to reposition, you will use two tripods with a camera, and a much closer object as a target. As for the backdrop of fixed stars, it's recommended you use a distant building or mountain.

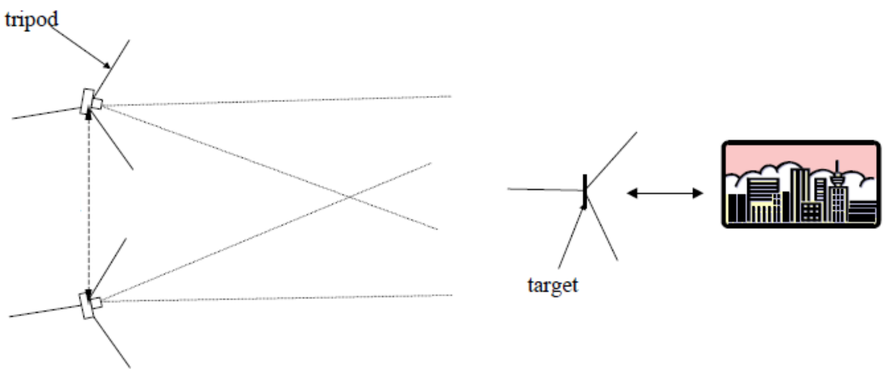

Figure 4: Trigonometric parallax experiment setup.

Instructions

1. Start by taking the equipment out to an open location where you have a clear view of a distant building or mountain (i.e. the 'fixed' stars).
2. Mount the camera on one of the tripods and attach the metre ruler and the target (i.e. our "target" star) to another tripod using blue tack or tape.
3. Place the camera in auto mode and take a photograph of the metre ruler at 5 metres. Ensure that the ruler is orthogonal to the camera when taking the photograph (as in Figure 5 below). This photograph, which we'll call the "calibration photo", is important and will be used later to calculate the physical size of the CCD pixels.
4. Next, place two of the tripods 1.0 m apart and carefully measure the distance between the top centres of each tripod.
5. Place the third tripod, with the ruler and target, about 5 m away from the midpoint between the other two tripods ensuring that the ruler is orthogonal to the midpoint.
6. Using the camera, ensure that the target and your background object are both in focus (i.e. the lens is focussed at infinity) and then take a photograph of the target from both tripods. Be careful not to move the tripods during this step.
7. Moving the target back at 2 m intervals (e.g. 5, 7, 9, 11, 13 m), repeat step 6 for each. Ensure someone is recording which photograph corresponds to which interval. The camera does not have to be pointing in the same direction for each interval, but make sure your background object remains in the photograph.
8. When all photographs have been taken, pass the camera over to the experiment supervisor so they can copy them over to a computer for analysis.
9. Complete the analysis detailed on the next page.

Figure 5: Photo of the metre ruler and target attached to a tripod.

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Analysis

1) On your computer, open the software, ImageJ and then open your calibration photo.

2) Next, we'll use this photo to calculate the physical size of the camera's CCD pixel (i.e. the pixel size). This can be completed using the figure, two equations, and steps outlined below:

$\frac{x}{f} = \frac{r}{d} \rightarrow x = \frac{rf}{d}$ (equation 1)	$Pixel\ Size = \frac{x}{n}$ (equation 2)

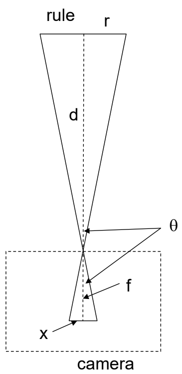

Figure 6: pixel size

f is the focal length of the lens (this can be found in **`Show Info`** in ImageJ)

x is the physical distance of a point on the image from the centre line

d is the distance along the centre line to the ruler

r is the distance from the centre line to a point on the ruler

n is the number of pixels from the centre line

i) First, let's assign x as the distance from the middle of the ruler to the edge of the ruler in the calibration photo. We know the physical distance of this (r) is 50cm. Using equation 1 above, calculate the value for x in mm.

Value for x in millimetres.
The ruler was photographed at 3.38 m with a camera focal length of 21.3 mm
x = rf/d = 500 mm x 21.3 mm / 3380 mm = 3.15 mm

ii) Second, using equation 2 and your result for x above, calculate the pixel size in mm. Recall that n is the number of pixels from the centre line to the edge of the ruler. While zooming into the photo and counting the number of pixels will net you a result, try using the "Straight" command in ImageJ (circled below) to perform the action instantly.

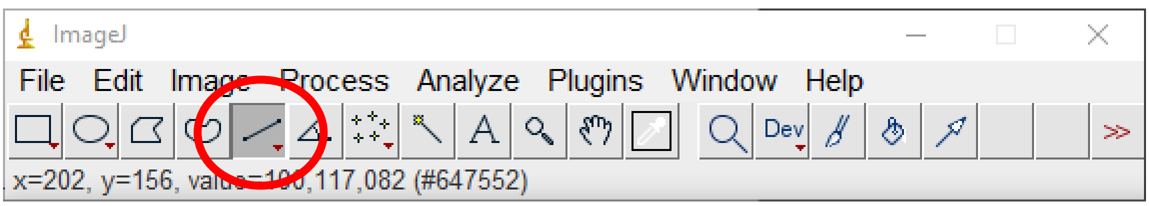

Figure 7: Straight command in ImageJ

Value for the *Pixel Size* in millimetres and nanometres.
Using ImageJ, the number of pixels along 50 cm of ruler was found to be 980.
Pixel size = x/n = 3.15/980 = 0.003214 mm = 3.214 μm

3) For each parallax image, calculate the distance in pixels and millimetres between the centre of the target and your reference point along the x-axis using ImageJ. Refer to Figure 7 and use the table below.

   a) Starting with the first photo, at the shortest distance (e.g. 5 m from camera to target), hold your cursor over the middle of the "target" marker and record the x-coordinate (displayed in ImageJ) in the table below.

   b) Ensure you stay along the x-axis (y value should not change), hold your cursor over the middle of the "reference" marker and record the x-coordinate in the table below.

   c) Calculate the difference in pixels between the above values (Δx). Refer to the table below for an example.

   d) Repeat steps a to c for the second photo.

   e) The sum of the two Δx values is the x value in pixels. Calculate this and record it in the table.

   f) Convert x from pixels to millimetres by multiplying your x (pixel) value by the $Pixel\ Size$ you calculated earlier and record the value for x(mm) in the table.

   g) Calculate the parallax angle ϕ using the trigonometric relation $\phi = \theta/2 = \tan^{-1}(x/f)$. Recall from Figure 3 that the parallax angle is designated as half of the total shift, $\theta/2$. Record the value for ϕ (radians) in the table.

   h) Recalling the tripods were placed 1.0 m apart, calculate the parallax distance using equation 1.

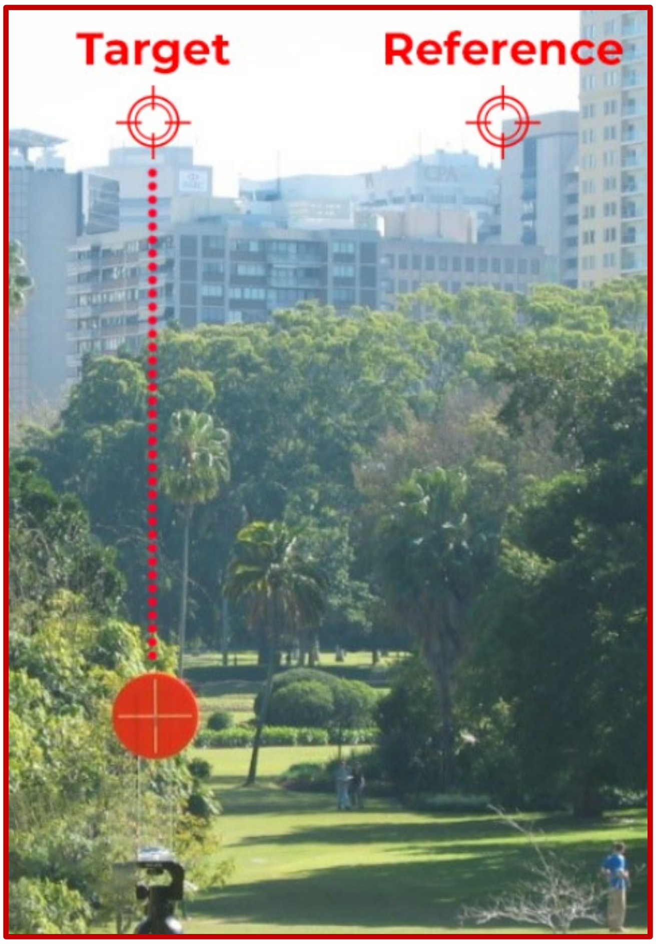

Figure 8: An example of measuring the distance between the target and the reference

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Example table:

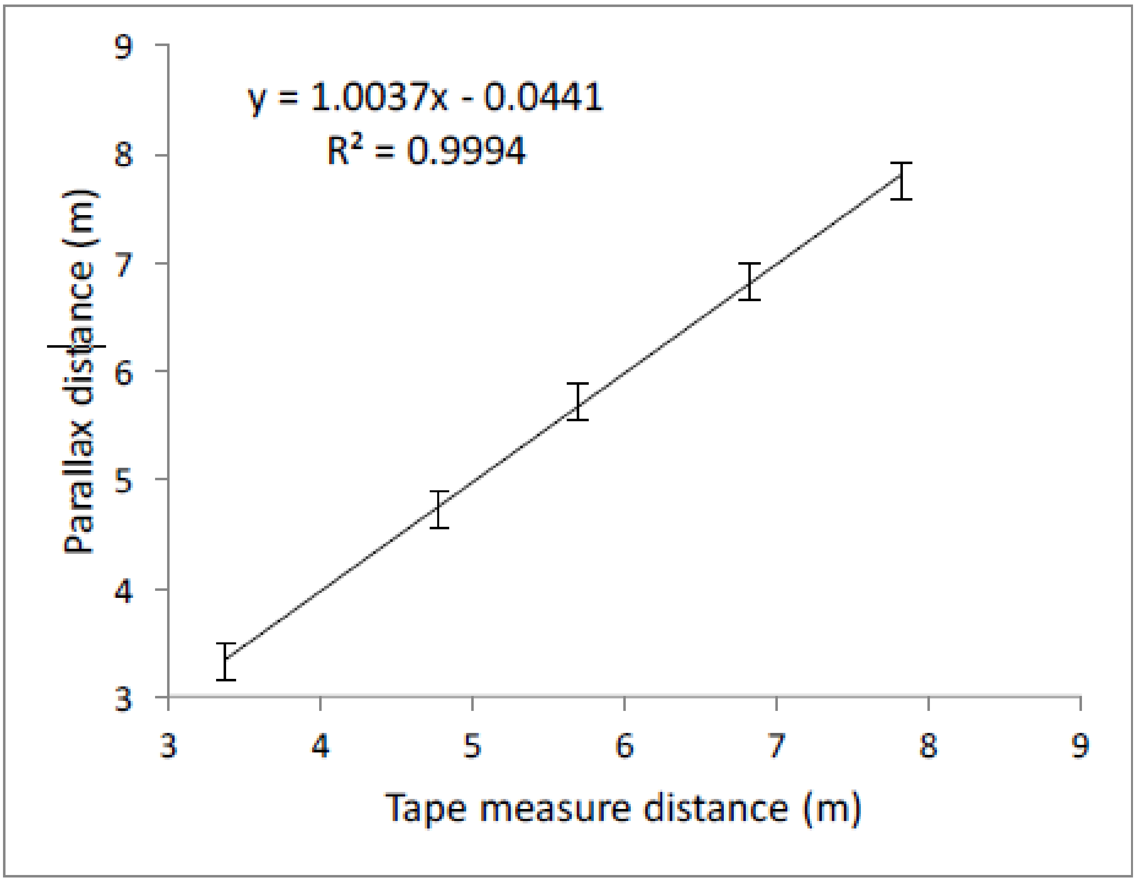
Tape (m)		Photo 1	Photo 2	x (pixels)	x (mm)	ϕ (radians)	Parallax (m)
6.6	Target x co-ord	1778	1766				
	Reference x co-ord	1272	1760				
	Δx (pixels)	506	6	512	1.65	0.077	**6.5**

Example Table

Record your data here:

Tape (m)		Photo 1	Photo 2	x (pixels)	x (mm)	ϕ (radians)	Parallax (m)
7.82	Target x co-ord	1272	1766				
	Reference x co-ord	1778	1760				
	Δx (pixels)	506	6	512	1.65	0.077	7.76
6.82	Target x co-ord	1668	1218				
	Reference x co-ord	1646	1778				
	Δx (pixels)	22	560	582	1.87	0.088	6.83
5.69	Target x co-ord	916	1732				
	Reference x co-ord	1524	1646				
	Δx (pixels)	608	86	694	2.23	0.105	5.73
4.78	Target x co-ord	1950	826				
	Reference x co-ord	1818	1536				
	Δx (pixels)	132	710	842	2.71	0.127	4.72
3.38	Target x co-ord	894	1939				
	Reference x co-ord	1976	1828				
	Δx (pixels)	1082	111	1193	3.84	0.180	3.33

4) Finally, plot the parallax distance verses the distance measured using a tape measure. Also include the best fit line and calculate the goodness-of-fit measure.



Acknowledgements

The experiment is based on the work by Hughes, Powell, Carroll, and Cowley. 2015.