

# Minimal Work: A Grasp Quality Metric for Deformable Hollow Objects

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**Abstract**—Robot grasping of deformable hollow objects such as plastic bottles and cups is challenging as the grasp should resist disturbances while minimally deforming the object so as not to damage it or dislodge liquids. We propose minimal work as a novel grasp quality metric that combines wrench resistance and the object deformation. We introduce an efficient algorithm to compute required work to resist an external wrench for a manipulation task by solving a linear program. The algorithm first computes the minimum required grasp force and an estimation of the gripper jaw displacements based on the object’s deformability at different locations measured with physical experiments. The work done by the jaws is the product of the grasp force and the displacements. The grasp quality metric is computed based on the required work under perturbations of grasp poses to address uncertainties in actuation. We collect 460 physical grasps with a UR5 robot and a Robotiq gripper. Physical experiments suggest the minimal work quality metric reaches 74.2% balanced accuracy and is up to 24.2% higher than classical wrench-based quality metrics, where the balanced accuracy is the raw accuracy normalized by the number of successful and failed real-world grasps.

## I. INTRODUCTION

For rigid objects, wrench-based quality metrics [1], [2] are widely used to optimize grasp placements and to estimate grasp success [3], [4] since they quantify grasps and are suitable for both general and task-oriented grasps.

Grasping deformable objects is more challenging. In addition to resisting external disturbances, grasps should minimize the deformation of the object to avoid damage or dislodging liquids e.g. when grasping plastic cups and bottles. Existing grasp planning for deformable objects focus either on holding deformable planar objects [5]–[7] or lifting 3D objects with a pre-selected grasp placement [8], [9].

We propose the *minimal work* quality metric, a novel quality metric that considers both wrench resistance and object deformation. Fig. 1 shows an example of planned grasps for a plastic cup with the proposed metric. To compute the quality of a grasp, we first estimate the minimum required grasp force to resist an external wrench with the un-deformed object’s shape. We use the Robust Efficient Area Contact Hypothesis (REACH) model [10] to estimate the contact area and the pressure distribution by means of the constructive solid geometry intersection of the extruded polygon of the jaw with the object. The friction wrenches of the non-planar area contacts are then modeled with a 6D ellipsoidal limit surface [11]. We formulate an optimization problem to solve the minimum grasp force subject to the friction constraints. The required work of the jaws to complete the task provides



Fig. 1: Plastic cup example. Left: stiffness of cup, red indicates a low stiffness. Middle: three planned grasps, red indicates high work. Right: minimal work grasp in physical experiment.

the metric. The object’s deformation is considered as an approximation of the jaw displacement, which are estimated from the minimum grasp force. We decouple the wrench analysis and the computation for object’s deformation, so that the grasp quality can be efficiently computed without Finite Element Method (FEM) simulation for each grasp or repeatedly determining wrench resistance throughout the grasp.

This paper provides the following contributions:

- 1) A novel minimal work grasp quality metric for 3D deformable hollow objects that considers both wrench resistance and object deformation.
- 2) An efficient algorithm to compute minimal required work to resist a 6D external wrench by solving a linear program.
- 3) Physical experiments that suggest grasps planned with the minimal work quality metric lead to 74.2% balanced accuracy, 24.2% and 12.7 % higher than the grasp reliability metric and the minimal force metric, respectively.

## II. RELATED WORK

We summarize related work in wrench-based grasp quality metrics for rigid objects and grasp planning for deformable objects. Excellent surveys for contact modeling can be found in [12]–[15], for grasp quality metrics in [16], and for deformable object manipulation in [17].

### A. Grasping rigid objects

A common grasp quality metric, force closure [18], evaluates a grasp by whether it can resist any 6D disturbance wrenches with an arbitrarily large grasp force. The volume of a grasp wrench space (GWS) [2] and the  $\epsilon$ -metric [1]

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are also widely used to quantify grasp quality. While the volume reflects the quality of the entire grasp, the  $\epsilon$ -metric identifies the weakest point of a grasp, as  $\epsilon$  is the shortest distance between the origin and any facet of the GWS. The GWS used to evaluate grasps is typically constructed with a bounded sum-magnitude of grasp forces for computational efficiency. Krug et al. [19] suggest that such a construction is over-conservative for fully actuated grippers, as the force of each jaw is limited independently.

External disturbance wrenches can be estimated for specific tasks. Task-oriented grasp quality metrics are, therefore, better suitable to estimate wrench resistance for such tasks. A task wrench space (TWS) describes expected disturbance wrenches during a manipulation task. The TWS is typically modeled with possible wrenches that can be imposed on an object [3], [20], or an 6D ellipsoid [2]. The quality of a grasp is the scale of the TWS, so that it just fits into the GWS. In addition, the quality can be measured with the minimal required force for a task [21] or minimal coefficient of friction [4]. Lin and Yu [22] observe that some disturbance wrenches happen more often than others during a task execution and, therefore, select the grasp whose GWS covers most frequent disturbances. They further select the optimal grasp in [23], which minimizes the required motion effort of the end effector to fulfill a certain task.

### B. Grasping deformable objects

Manipulating deformable objects is an active area, with applications such as food handling [24], fabric manipulation [25], [26], and elastic rod manipulation [27]. When a frictionless grasp immobilizes a rigid object, it is defined as form closure. Gopalakrishnan and Goldberg [5] generalize this concept to holding deformable objects with frictionless contacts, where a grasp is defined as deform closure, when positive work is required to release the object. Wakamatsu et al. [28] introduce the bounded force closure metric, which guarantees a force closure grasp under a maximal allowable external force. Delgado et al. [29] reduce object deformation for a holding task by computing the maximum allowed force to be exerted on an object. Jia et al. [6] propose a grasping strategy to squeeze holding deformable planar objects based on work performed by the jaws. When two jaws squeeze and immobilize an object, and a third jaw tries to break the grasp by pushing the object, the translations of the two pushing jaws that minimize the required work to balance the object is selected. Since the metric targets planar objects, the 3D geometry or the gravity is not considered and a point contact model is used for friction analysis. Lin et al. [8], [30] address the problem of lifting a deformable object based on an object mesh model and jaw positions. An FEM formulation computes the object deformation based on the jaw displacements. The object will be lifted if the majority of the contact points are sticking. Similarly, Zaidi et al. [9] use FEM simulation to manipulate objects with large deformations, such as objects made of foam or rubber. Alt et al [31] use FEM simulation and heuristics to plan grasps for deformable thin-walled objects.

Inspired by [5] and [6], the proposed minimal work quality metric optimizes grasp placements to manipulate 3D deformable hollow objects. Furthermore, the metric is suitable for many tasks that can be modeled as target wrenches to be resisted.

## III. PROBLEM STATEMENT

### A. Overview

We consider the problem of grasp planning for 3D deformable hollow objects with compliant jaw pads based on the ability of a grasp to resist target wrenches and the deformability of the object at the grasp location.

### B. Assumptions

We make the following assumptions:

- 1) The geometry and the deformability are known for the objects to be grasped.
- 2) Quasi-static analysis and Coulomb friction with a known coefficient of friction.
- 3) A linear elastic model of soft jaw pads and objects.

### C. Notation

- $w \in \mathbb{R}^6$ : wrench of a contact, which is concatenated by a 3D force and a 3D torque.
- $C_i$ : the constraint set that limits the maximum possible friction wrench and the wrench impressed by the normal pressure of the  $i$ -th contact.
- $W$ : the work performed by the gripper jaws
- $t \in \mathbb{R}^6$ : a target wrench to be resisted with a grasp

### D. Metrics

We compare the proposed minimal work grasp quality metric with two widely used metrics. Each grasp is computed under  $K$  perturbations of grasp poses to address uncertainties in actuation.

- 1) Grasp reliability metric  $q_r$ :  $r = 1$  if the grasp is able to resist the target wrench  $t \in \mathbb{R}^6$  without exceeding the maximum closing force and  $r = 0$  otherwise:

$$q_r = \frac{1}{K} \sum_{k=1}^K r_k.$$

- 2) Minimal force metric  $q_f$ : The minimal required grasp force to resist  $t$ :

$$q_f = \frac{1}{K} \sum_{k=1}^K \left( 1 - \frac{F_k}{F_{\max}} \right).$$

where  $F_{\max}$  is the maximal grasp force used in the experiments.

### E. Task modeling

Two manipulation tasks are considered: 1) vertical lifting and 2) lifting and  $90^\circ$  rotation. We model each task with a 6D gravity wrench to be resisted under multiple object poses obtained by discretizing the trajectory of the task. A single pose is considered for the lifting task, since the gravity wrench remains unchanged during the manipulation, while three poses are considered for the lifting and rotation task.

### F. Objective

We use the balanced accuracy score to evaluate the prediction accuracy of each quality metric by comparing them with real-world grasps. The predicted grasp success is binary and is true if the metric is higher than a threshold. The balanced accuracy is suitable for imbalanced datasets and is computed with the raw accuracy, where each sample is weighted with the inverse prevalence of its true class.

### IV. MINIMAL WORK GRASP QUALITY METRIC

To evaluate a grasp candidate, we compute the minimal work of the gripper jaws required to complete a manipulation task. We first model the frictional contacts and compute the minimal grasp force by formulating Equation (1) as a linear program (LP). We then estimate the deformation based on the force and object stiffness at the contact locations. The work of each gripper jaw is the product of grasp force and jaw displacement. The sum of the work of each jaw forms the work of the grasp.

The proposed algorithm to compute the minimal work can use different contact models and object's stiffness acquisition methods. We use the REACH model [10] and a 6D ellipsoidal limit surface to describe a contact. The stiffness is collected with physical robots. Details can be found in Sec. V and VI-A.

For a grasp with  $N$  contacts, denote  $\mathbf{G} \in \mathbb{R}^{6 \times 6N}$  as the grasp matrix,  $\mathbf{w}_i \in \mathbb{R}^6$  as the wrench applied at the  $i$ -th contact, and  $\mathbf{F} = [F_1, \dots, F_N]^T$  as a vector of grasp forces, where  $F_i$  is the grasp force at the  $i$ -th contact. The minimal required grasp force to resist a target wrench  $\mathbf{t}$  is computed by:

$$\begin{aligned} & \underset{\mathbf{F}, \{\mathbf{w}_1, \dots, \mathbf{w}_N\}}{\text{minimize}} \quad \mathbf{F} \cdot \mathbf{1}_N \\ & \text{subject to} \quad \mathbf{G} \begin{bmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_N \end{bmatrix} = \mathbf{t}, \\ & \quad \mathbf{w}_i \in \mathcal{C}_i, \forall i. \end{aligned} \quad (1)$$

Based on Hooke's law, the work  $W$  is computed by:

$$W = \sum_{i=1}^N F_i \cdot d_i, \text{ with } d_i = \frac{F_i}{s_{o_i}} + \epsilon, \quad (2)$$

where  $d_i$  is the displacement of the  $i$ -th jaw and  $s_{o_i}$  is the object stiffness at contact  $i$ .  $\epsilon$  is a small number, such that the minimal work quality metric is applicable to rigid objects or objects contain a rigid part. The displacement  $d_i = \epsilon$  and the minimal work quality reduces to comparing minimal force between grasps.

Finally, the quality  $q_w$  of a grasp under  $K$  perturbations of grasp pose is computed by:

$$q_w = \frac{1}{K} \sum_{k=1}^K \left( 1 - \frac{W_k}{W_{\max}} \right), \quad (3)$$

where  $W_{\max}$  is used for normalization and is the the product of maximal grasp force and object's deformation of collected data.

### V. ALGORITHM

To compute the minimal required grasp force, classical wrench-based grasp analysis first model the possible friction and normal wrench of each contact and then estimate the total wrench that a grasp can exert on an object. The estimation of friction and normal wrench highly depend on the contact details. We first describe the method used in this work to estimate contact area and pressure distribution.

#### A. REACH: contact geometry

Danielczuk et al. [10] proposed the Robust Efficient Area Contact Hypothesis (REACH) model for contact profile estimation between soft jaw pads and rigid objects. Given an object's geometry modeled as a triangular mesh, the contact area is computed as the constructive solid geometry intersection of the extruded polygon of the jaw with the object. The intersection estimates the deformation of the soft pad around the object at each point on the contact and the pressure distribution linearly scales with the gripper pad deformation. The REACH model provides contact area consists of triangles and the normal pressure of each triangle.

We apply the REACH model to estimate contact information between compliant jaw pads and deformable hollow objects due to its highly computational efficiency compare to e.g. the Finite Element Method. We note that the obtained contact details may not be accurate for objects with high deformability.

#### B. 6D ellipsoidal limit surface

Grasps with soft jaws may result in a non-planar contact area, where the friction wrench applied at the contact is six-dimensional (6D). This work uses the 6D ellipsoid proposed in [11] as the limit surface model. Note that other friction models are applicable to the proposed work computation algorithm as well.

Friction depends on relative body motion. For a given contact area and pressure distribution, possible friction wrenches are obtained by sampling the instantaneous relative motion, defined as body twist in screw theory [32]. The direction of frictional force of each triangle is obtained by projecting the velocity onto the triangle plane, while the magnitude is the product of the coefficient of friction and the normal force applied at the triangle. The frictional torque is computed with respect to the friction-weighted center of pressure. By summing up the friction contribution of each triangle, we obtain a friction wrench of the contact for each sampled twist.

We fit the friction wrenches to a 6D ellipsoid by solving a convex optimization. The friction wrench is constrained to be in the interior of the ellipsoid. The surface of the fitted ellipsoid is then evenly sampled to acquire linear constraints.

We use a 6D ellipsoidal limit surface to model the friction wrenches of each contact. Given the ellipsoid matrix  $\mathbf{A} \in \mathbb{R}^{6 \times 6}$ , the friction wrenches  $\mathbf{f} \in \mathbb{R}^6$  are constrained by:

$$\mathbf{f}^T \mathbf{A} \mathbf{f} \leq 1. \quad (4)$$

For computational efficiency, we compute the linearized constraints  $\mathcal{C}_i$  of the  $i$ -th contact by evenly sample the ellipsoid with  $M$  points  $\mathbf{p} \in \mathbb{R}^{6 \times M}$  on the surface of  $\mathbf{A}$ . Each point and the outward normal of  $\mathbf{A}$  at the point form a hyperplane. The friction wrenches are constrained in the interior of the  $M$  hyperplanes:

$$\mathcal{C}_i = \{\mathbf{w}_i \in \mathbb{R}^6 \mid \mathbf{n} \cdot (\mathbf{w}_i - \mathbf{f}_\perp) \leq \mathbf{n} \cdot \mathbf{p}\}, \quad (5)$$

with  $\mathbf{n} = \mathbf{A}\mathbf{p}$ ,

where  $\mathbf{f}_\perp \in \mathbb{R}^6$  is the wrench impressed by the normal pressure,  $\mathbf{n}$  are the normals of  $\mathbf{A}$  at  $\mathbf{p}$ .

## VI. EXPERIMENTS AND RESULTS

### A. Acquisition of object stiffness

The object stiffness is required to compute the required work of gripper jaws for a specific task. We estimate the object stiffness by means of its 3D mesh and physical experiments. One can use e.g. the Finite Element Analysis to compute the object deformation with a closing force of the gripper. However, the deformability of e.g. plastic bottles and cups highly depends on the wall thickness, the geometry, and material of the object, which are non-trivial to obtain. Therefore, we use a physical robot to collect object's stiffness at different locations in this work.

We estimate the object's stiffness based on 1) a known gripper closing force  $F_c$ , 2) the gripper opening  $L_s$  when its just in contact with the object, and 3) the opening  $L_e$  when  $F_c$  is reached. We first plan antipodal grasps in simulation for each object.  $L_s$  is estimated by checking the minimal distance between the 3D object mesh and the gripper mesh at the grasp location.

At each planned grasp location, the Robotiq 2F-85 gripper closes with a minimal possible force  $F_c = 20N$ . The object's stiffness  $s_{o_i}$  at the location  $i$ , obtained by the two intersection points of the grasp axes and the object surface is computed by:

$$s_{o_i} = \frac{F_c}{L_{s_i} - L_{e_i}}.$$

We repeat each grasp 5 times and the median of the collected gripper opening after reaching the closing force is selected as  $L_e$ .

We note that the actual grasp force of the Robotiq gripper depends on the object's material and the gripper closing speed. Fig. 2 shows experiment results of the repeatability of the gripper. An object with an open lid is grasped at five locations, where the object has different deformability at each location. The gripper closes repeatedly with the minimal force and multiple speeds, ranges from 0 to 255, where 0 is the lowest speed. For each grasp location and each closing speed, the gripper closes 15 times and the gripper opening (cm) is recorded after the force is reached. We restart the gripper program every three closes.

Fig. 2 shows the mean and standard deviation of the gripper opening at each grasp location with a closing speed. Results show the the gripper opening has a higher error at grasp pose 1-3 than pose 4-5. This observation suggests when

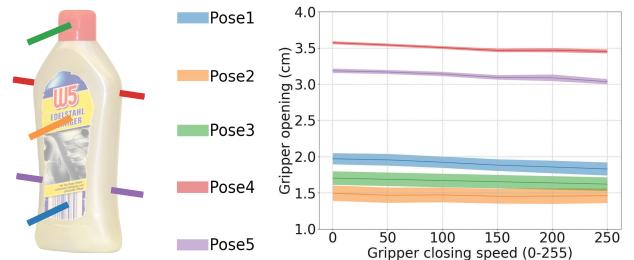


Fig. 2: Robotiq gripper repeatability test.

TABLE I: Balanced accuracy of three quality metrics: grasp reliability (GR), minimal force (MF), and the proposed minimal work (MW).

Object	Vertical lifting			Lifting and 90° rotation		
	GR	MF	MW	GR	MF	MW
1	0.500	0.767	<b>0.833</b>	0.575	0.785	<b>0.875</b>
3	0.500	0.687	<b>0.851</b>	0.516	0.714	<b>0.813</b>
5	0.500	0.392	<b>0.542</b>	<b>0.680</b>	0.630	0.450
All	0.500	0.615	<b>0.742</b>	0.590	0.710	<b>0.713</b>

the gripper opening is small, e.g. when the object is thin or has large deformation, the gripper repeatability is lower than with a large gripper opening.

Although the absolute gripper opening and the collected object's stiffness may not be highly accurate, the relative object's stiffness is reasonable, which is essential to compute optimal grasp placements.

### B. Simulation results: Planned grasps with three quality metrics

We plan grasps on five objects based on their 3D meshes and the interpolated stiffness maps, as shown in Fig. 3(a) and (b), where two objects are 3D printed using NinjaFlex TPU flexible filament since we want to experiment with objects of various shapes and materials. Note that some inaccuracies of the measured stiffness exist, such as the rigid cap of the object 1 and the neck of object 2.

We sample antipodal grasps candidates and compute grasp quality for 1) vertical lifting and 2) lifting and 90° rotation task. Three quality metrics are compared: grasp reliability, minimal force, and minimal work, as shown in Fig. 3. The two tasks are modeled with a 6D gravity wrench to be resisted under one and three object poses obtained by discretizing the manipulation trajectory, respectively, since the gravity wrench remains the same for the vertical lifting task. The metric of each grasp is computed under 20 perturbations of grasp poses for each object pose of the task. The lowest quality value of a grasp among all object poses is selected as the value for each metric.

Fig. 3 suggests the planned grasps with the proposed minimal work quality metric avoid causing large deformations of the object, while are able to resist the gravitational disturbances of the manipulation tasks.

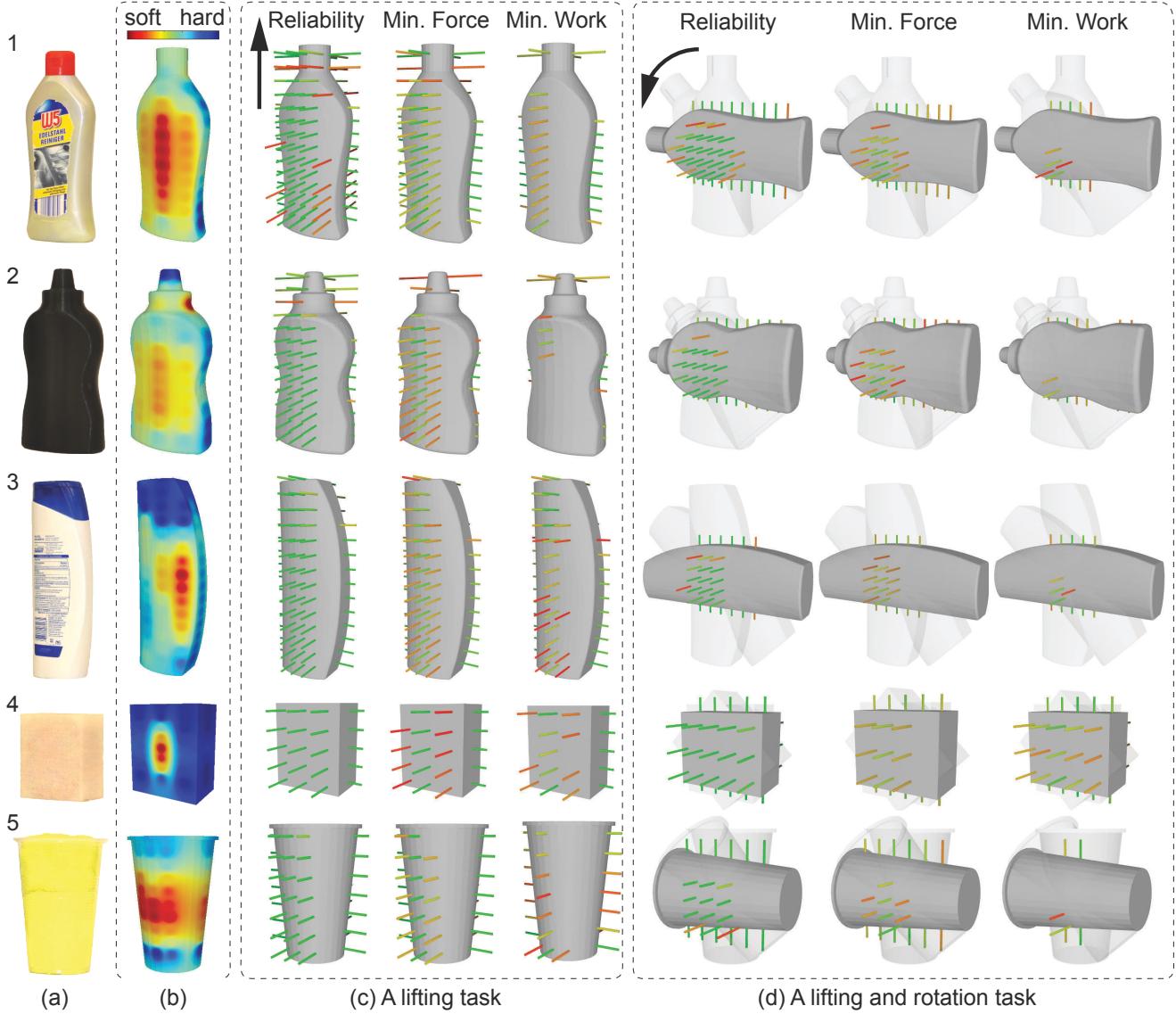


Fig. 3: Planned grasps for five physical objects. Object 2 and 4 are 3D printed using NinjaFlex TPU flexible filament. (b): The interpolated stiffness map. Grasp qualities with three metrics for a lifting task, and (c) a lifting and  $90^\circ$  rotation task (d). The proposed minimal work metric computes grasps that resist gravitational disturbances without causing large deformation.

### C. Physical experiments

We evaluate the planned grasps for three representative objects (object 1,3,5) with physical experiments for the two manipulation tasks. We select 46 grasp poses in total for three objects that cover different regions of object and each grasp is repeated five times for each task, which sums up to 460 grasps in total. We consider a grasp is successful if the task is completed while the object is returned into its original shape when the grasp force is released and the content is not dislodged during the grasp.

Objects are filled with wet towels to imitate the weight filled with liquid without changing the object's deformability or damage the electrical devices. Object 1 and 3 are sealed with a balloon to infer the content spillage. By measuring

the balloon's inflation amount before and after the grasp, the content is considered spilled if the inflation difference is larger than a threshold.

We use the balanced accuracy score to evaluate the prediction accuracy, which is the raw accuracy balanced by the number of successful and failed grasps in the collected data. Table I shows the balanced accuracy of the three grasp quality metrics for the two manipulation tasks and Fig. 4 shows examples of correct and wrong prediction of the planned minimal work grasps. The proposed metric reaches 74.2% and 71.3% balanced accuracy for the two tasks, respectively, and are up to 24.2% higher than the grasp reliability and the minimal force metric. However, we note that the prediction accuracy is relatively low for object 5

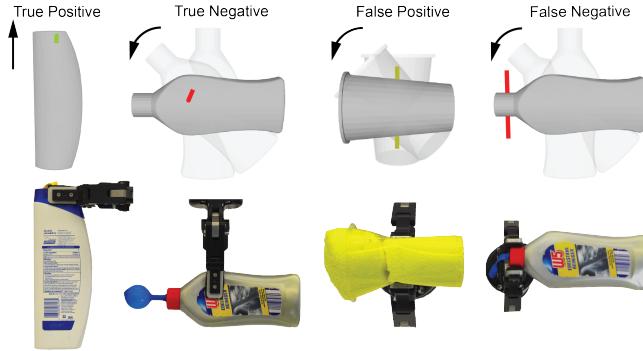


Fig. 4: Examples of grasp success predictions with the minimal work quality metric. A grasp is considered successful if the manipulation task succeeded while the content is not dislodged. An inflated balloon suggests liquids in the container might dislodge.

(a plastic cup) compared to other objects. This suggests the proposed algorithm for minimal work computation may not perform well for objects with large deformations. Furthermore, the possible minimum grasp force with the Robotiq Gripper is much larger than the planned force and causes large object deformations and false positives.

## VII. DISCUSSION AND FUTURE WORK

We propose a minimal work quality metric to plan grasps for 3D deformable hollow objects. We evaluate the proposed metric with real-world grasps for a vertical lifting and for a lifting and 90° rotation task. Physical experiments suggest that 74.2% and 71.3% balanced accuracy can be achieved for the two tasks, respectively, and up to 24.2% higher than the classical wrench-based quality metrics.

### A. Limitations

We note that the proposed method may not perform well for objects that have large deformations. A Finite Element Method can be used to compute the deformation of such objects and the pressure distribution of the contact. The grasp quality can then be computed based on objects's deformed shape. Furthermore, one reason that leads to false positives is that the actual possible minimal grasp force of the Robotiq gripper is higher than specified in the planned grasps. We plan to use e.g. a Schunk gripper mounted with force sensors to plan grasps with computed minimal grasp force.

### B. Future work

We note that the minimal work grasp is applicable to grasping rigid objects, when the jaw pads are deformable. We intend to further investigate this duality and plan grasps for both rigid and deformable objects.

## VIII. ACKNOWLEDGMENTS

This research was performed at the AUTOLAB at UC Berkeley in affiliation with the Berkeley AI Research (BAIR) Lab, Berkeley Deep Drive (BDD), the Real-Time Intelligent Secure Execution (RISE) Lab, and the CITRIS “People and Robots” (CPAR) Initiative. The authors were supported in part by the Scalable Collaborative Human-Robot Learning (SCHooL)

Project, NSF National Robotics Initiative Award 1734633 and by donations from Google, Siemens, Amazon Robotics, Toyota Research Institute, Autodesk, ABB, Samsung, Knapp, Loccioni, Honda, Intel, Comcast, Cisco, Hewlett-Packard and by equipment grants from PhotoNeo and NVIDIA. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Sponsors. We thank our colleagues who helped with experiments and provided helpful feedback and suggestions, especially William Wong, Priya Sundaresan, Harry Zhang, Jackson Chui, Kate Sanders, and Andrew Lee.

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