Convex Optimization for the Birdbath Function.
Thomas B. Kinsman, Ph.D., Nov. 2, 2021

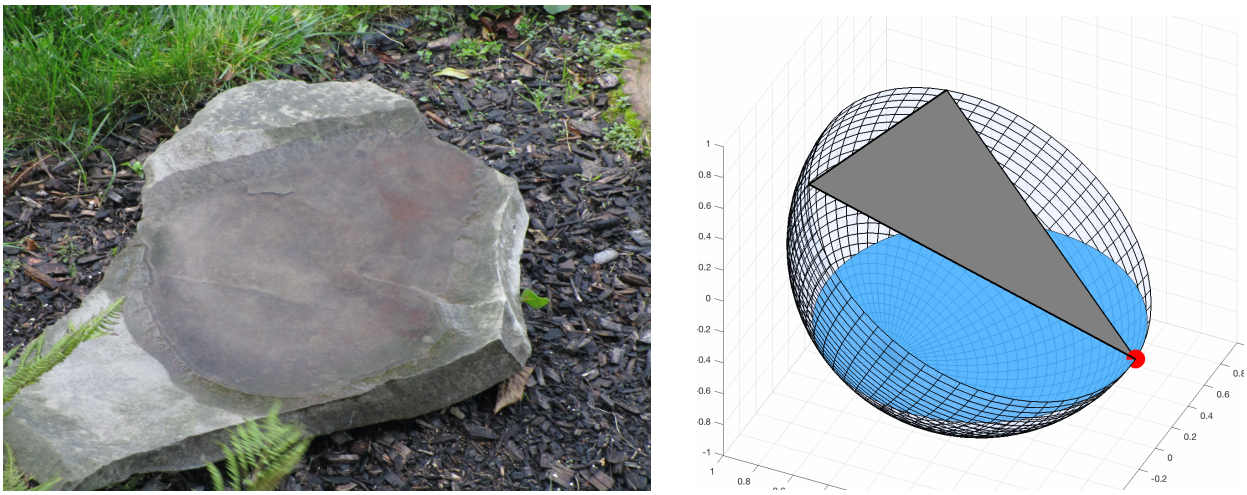

Figure 1 – The Bird Bath (left). The birdbath is a three-cornered section of a sphere. The student must adjust the roll, tilt, and twist to maximize the amount of water that the birdbath will hold. The Bird Bath is modeled as three points on a hemi-sphere (right). The lowest point determines where the water leaks out of, and thus the maximum amount of water the birdbath will hold. Given a function that approximates the triangle on a hemi-sphere, the student's job is to use convex optimization to adjust the roll, tilt, and twist to maximize the amount of water the birdbath will hold.

Problem Statement:

1. You are provided with a function that you cannot understand.

   DR KINSMAN's code is intentionally obtuse to avoid you reverse-engineering the answer.
   DO NOT WORRY ABOUT BEING ABLE TO UNDERSTAND IT.
   IT IS AN EXAMPLE OF HORRIBLE CODE.

   This is exactly the same as trying to adjust the parameters in an artificial neural network to find the best results from the neural network. You adjust the parameters to maximize the accuracy of the neural network.

   Each of you was assigned a different function. Do not copy your final parameter answers off of your neighbor.

2. The only thing you know about the birdbath function you are given:
   a. The working parameter ranges for roll, tilt, and twist are all in the range [-90, 90]
      The range of [-30, 30] is a good set of limits to use for practical purposes initially, because who would want to use a birdbath on its side?

   b. The birdbathfunc_...( ) function returns the fraction of a sphere that is held in the birdbath. The maximum possible theoretical value is ½, or 0.500000000. However, no computer should be able to get this value unless it is rounding up. You should be able to get close.

3. You need to write a search function that finds the parameters which maximize the function you were given. You and your partner might use the same search function, but you will have different answers for the best parameters.

(continued)

4. You need to find: [roll,tilt,twist] = arg max{ birdbathNNN(roll, tilt, twist) }.

5. You are going to write a special search function to find the best parameters.
   The search strategy is a form of either:
   a. Axially aligned grid search / gradient descent (change only one parameter at a time),
   b. Gradient descent (which can change several parameters at once),
   c. Grid search (trying a range of values for all parameters),
   d. Something else (some honors students are attempting to use Genetic algorithms.)

6. Remember that you will need to run your search several different times, starting at random values each time. Otherwise, you can end up in a local extrema point.

7. In theory, some theory professors think they will have an exact function that they can take the derivative of. In reality, nobody can really take the derivative of a neural network.
   ["In theory, there is no difference between theory and practice, but in practice, there is." -Dr. K]

   An ANN is an auto-differentiator. The point here is that the ANN does special things inside that even people who use them do not understand. The ANN is automatically finding the derivatives, and adjusting the parameters to optimize some function.

8. There are hundreds of methods for convex optimization.
   Here are three that were described in the lecture:
   a. Axially aligned gradient descent says:
      If you can only change one of the parameters, which parameter should you change?
      You can only modify either:
      i. the roll,
      ii. the tilt, or
      iii. the twist

   b. Gradient descent is trying to estimate the local multi-dimensional derivative (change in the function), depending on the changes in the parameters. Then make a change in the parameters to improve your local function. However, you might find that changing too many parameters at once causes the computer to fail.

   c. Grid search here is trying a range of (roll, tilt, and twist), then decrease the size of the grid to refine the answer.

9. Write the routine which finds the parameters to maximize the function you were given.

10. Print out:
    a. The maximum that the function found, to at least six digits past the decimal point.
    b. The parameters used to get that maximum value.

Questions to answer in your write up. Please <u>Copy</u> the questions into the write up.

1. Who was on the team?
2. For Person LastName1: (2)
   a. Which function were you assigned? What was the name of your function?
   b. What were the set of best parameters your program finds?
      roll = nnn in degrees
      tilt = mmm in degrees
      twist = zzz in degrees.
   c. To at least six digits past the decimal point, what was the fraction of the water you could fit into the birdbath using your numbers?
3. For Person LastName2: (2)
   a. Which function were you assigned? What was the name of your function?
   b. What were the set of best parameters your program finds?
      roll = nnn in degrees
      tilt = mmm in degrees
      twist = zzz in degrees.
   c. To at least six digits past the decimal point, what was the fraction of the water you could fit into the birdbath using your numbers?
4. What approach or approaches did you use? (2)
   Did you use two different methods?
   Did you try other methods?
   Did you have any complications along the way?
5. How did the work breakdown structure go with the team? (1)
   Did you take the opportunity to meet someone new?
   How did you assure that both of you contributed?
6. Is the idea of a team homework idea helping to reduce stress? (1)
7. Rosenbrock's Banana**:** (1)
   Look up the function called Rosenbrock's Banana. Write three paragraphs about it. Why is it famous? How does it apply to this homework assignment? What makes it difficult?
   Can you generate a plot of this (optional).
8. Conclusion and Summary: (1)
   What else did you learn along the way here?
   What else can you conclude?
   What did you learn about?
   What did each of you learn?
   Dr. K expects at least two solid paragraphs here.

9. ~~**Bonus B: (1)**~~
   ~~Implement a convex optimization program to find the minima of Rosenbrock's Banana function, from methods described in class.~~
   ~~Work with the function $f(x,y) = (1-x)^2 + 100(y-x^2)^2$~~

What to hand in to the dropbox:

Dr. Kinsman's code is intentionally obtuse so that you cannot easily read it and solve for the answer. However, *your* code must be readable.

1. Create a Directory with your two names:
   a. HW_04_LastName1_LastName2

2. In that directory put:
   a. Your team write-up, named HW_04_LastName1_LastName2.pdf, with the answers to questions and comments.

   b. Your well-commented code files.
   Submit one file per student to make your lives simple.
      i. HW_04_LastName1_Program_832.py ← STUDENT NUMBER 1.
      ii. HW_04_LastName2_Program_428.py ← STUDENT NUMBER 2.

   c. Add verbose comments to your code so that when read, we know what your approach was, and significant steps. Dr. Kinsman should be able to read it and understand your strategy. YOUR CODE, SHOULD BE EASY TO READ.

3. Then zip up your entire directory and submit it to the dropbox.

You will be sent an individual function to maximize. Each student is assigned a different program.

The graders will run your program(s) like this:
(These are table stakes. The programs must work.)

```
python3     HW_NN_BirdBathFunc_Qz_cls420.py
```

(Use python3 not python2)
Your program must:

i. Print out the solution found.
ii. Print out the highest fraction of water you were able to get into the birdbath.
iii. Again, do not use any single letter variables names. Points off for using single letter variable names. Your variable names should convey the meaning in the context of the code. (A Microsoft requirement.)

Dr. Kinsman's code is intentionally obtuse so that you cannot easily read it and solve for the answer. However, *your* code must be readable.