The Sequel to the Lemmings' Story

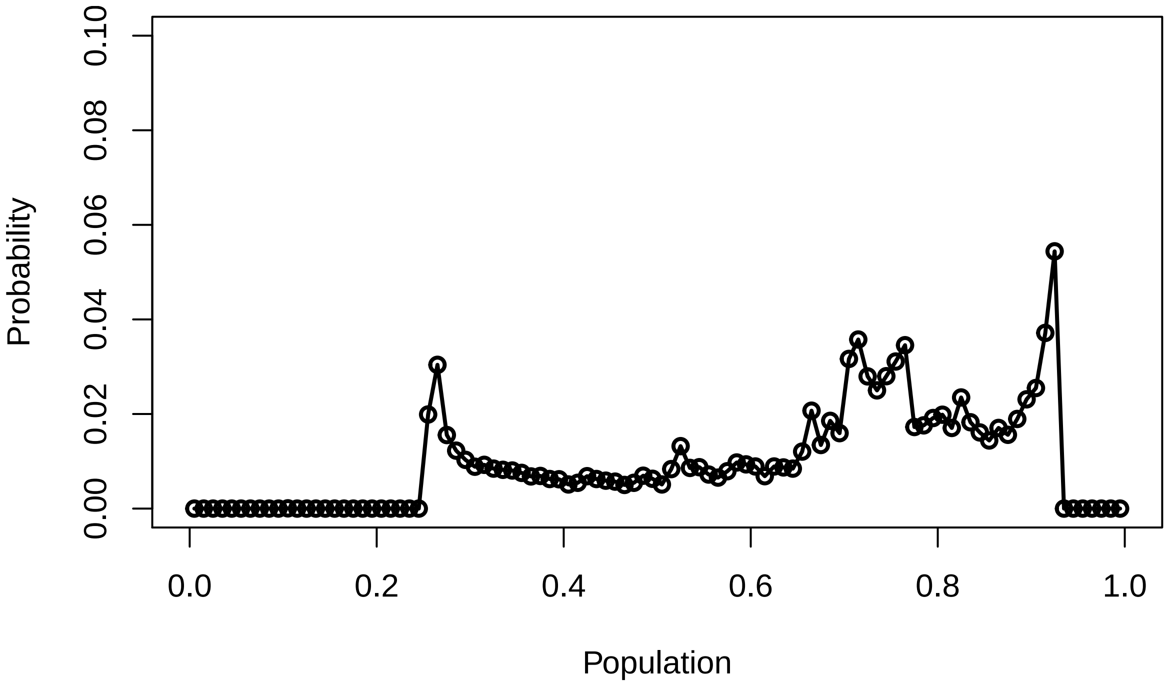
From our discussions on Monday, we have learned that the lemming population exhibits chaotic behavior when $r > 3.6$. In this activity, we try to see whether there is any order in the chaos.

Exercise 1: Generating the probability distribution of X

Your task is to calculate the probability distribution of X in time (i.e. what is the proportion of time X_n spends within the window $0 < X < 0.01$, $0.01 < X < 0.02$ etc.) by filling in the following code. To properly sample the probability distribution, you will have to calculate X_n till $n = 20000$.

```
# function for the population iteration
lemming_population <- function(r,X_0,n){
  # Fill in your function written on Monday for the lemming population
  Ls <- vector(length=n+1)
  # initialize the initial population
  Ls[1] <- X0
  for (i in 2:(n + 1)) {
    Ls[i] <- r * Ls[i - 1] * (1 - Ls[i - 1])
  }
  # output should be an array of n + 1 elements
  return(Ls)
}

NumSteps <- 20000
r <- 3.7                    # pick a value of r (r>3.6)
X0 <- 0.1                  # pick a value of initial value X_0
XVec <- lemming_population(r, X0, NumSteps) # compute the lemming population

BinNum <- 100          # number of bins between 0 and 1
BinSize <- 1/BinNum    # compute the bin size
BinCounts <-matrix(0,1,BinNum) #initialize bin counts to zero

for (i in 1:BinNum){   #

upper_limit <- BinSize * i
lower_limit <- upper_limit - BinSize
logicalvector <- (XVec >= lower_limit & XVec < upper_limit)
BinCounts[i] <-  sum(as.integer(logicalvector))

  # TO FILL IN: This counts the number of X values in each bin.
  # You may use as.integer on logicalvector, and use
  #'sum' on the resulting object.

}

BinCentres <- seq(BinNum)*BinSize - BinSize/2 # find the centers of the bins
Probability_density <- BinCounts/length(XVec) # compute probability density
plot(BinCentres,Probability_density,
```

```r
     xlim = c(0, 1), ylim = c(0, 0.1),
     xlab="Population", ylab="Probability",
     type="o",lwd=2)   # plot histogram
```

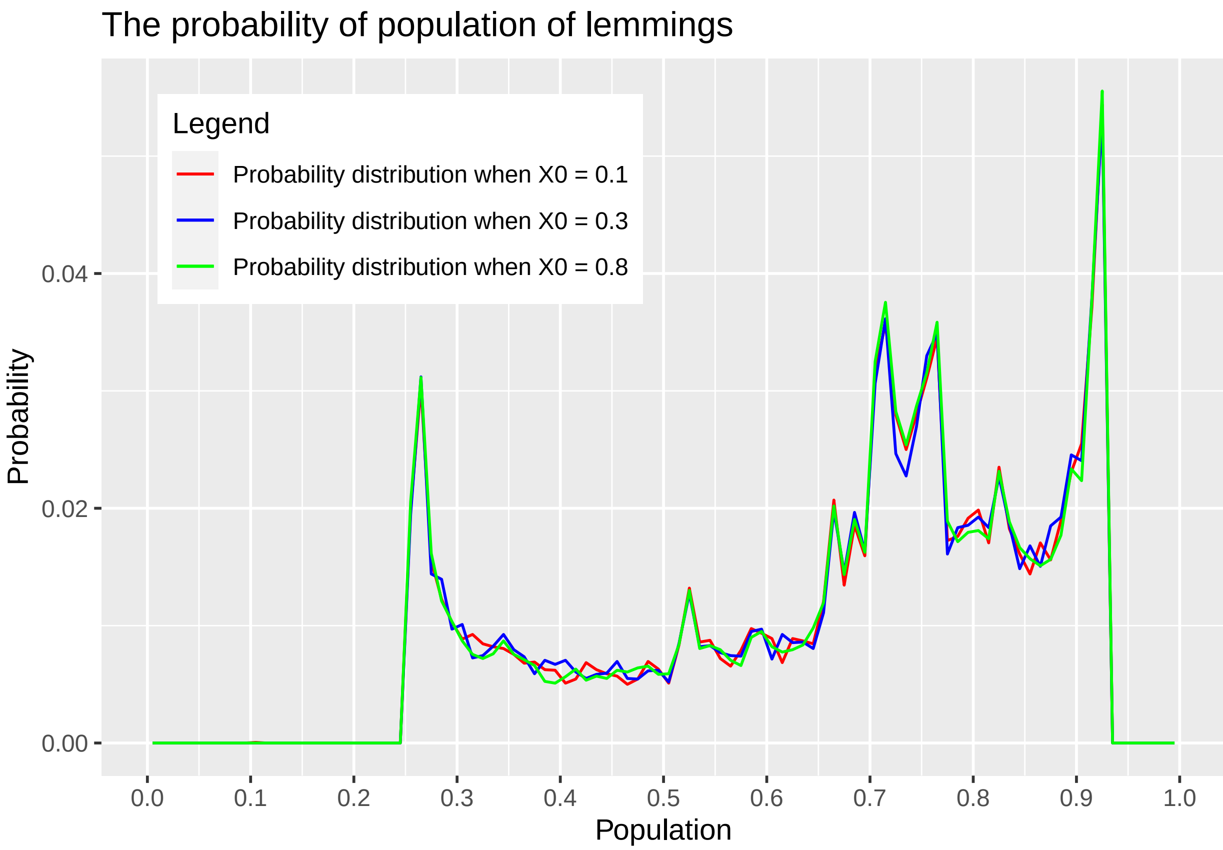


Exercise 2: Sensitivity of the probability distribution to the initial conditions

Choosing r in the chaotic regime $r > 3.6$, plot the probability distribution of X for or 3 different initial conditions X_0 on the same plot. Does the probability distribution (and thus the statistical quantities) depend on the values of the initial conditions?

```r
# writing a function to get the probability distribution
binCount <- function(lemming_pop, BinNum) {
  BinSize <- 1/BinNum    # compute the bin size
  BinCounts <-matrix(0,1,BinNum) #initialize bin counts to zero

  for (i in 1:BinNum){
    upper_limit <- BinSize * i
    lower_limit <- upper_limit - BinSize
    logicalvector <- (lemming_pop >= lower_limit & lemming_pop < upper_limit)
    BinCounts[i] <-  sum(as.integer(logicalvector))
  }
  Probability_density <- BinCounts/length(lemming_pop)
  return(Probability_density)
}

NumSteps <- 20000
r <- 3.7
BinNum <- 100

X0 <- 0.1
lemming_pop_1 <- lemming_population(r, X0, NumSteps)
prob_dist_1 <- binCount(lemming_pop_1, BinNum)
```

```r
X0 <- 0.3
lemming_pop_2 <- lemming_population(r, X0, NumSteps)
prob_dist_2 <- binCount(lemming_pop_2, BinNum)

X0 <- 0.8
lemming_pop_3 <- lemming_population(r, X0, NumSteps)
prob_dist_3 <- binCount(lemming_pop_3, BinNum)


BinCentres <- seq(BinNum)*BinSize - BinSize/2 # find the centers of the bins

data_pop <- data.frame(BinCentres = BinCentres,
                       prob_dist_1 = prob_dist_1,
                       prob_dist_2 = prob_dist_2,
                       prob_dist_3 = prob_dist_3)

data_pop %>% ggplot(aes(x = BinCentres)) +
  geom_line(aes(y = prob_dist_1,
                color = "Probability distribution when X0 = 0.1")) +
  geom_line(aes(y = prob_dist_2,
                color = "Probability distribution when X0 = 0.3")) +
  geom_line(aes(y = prob_dist_3,
                color = "Probability distribution when X0 = 0.8")) +
  scale_color_manual("Legend", values = c("red", "blue", "green")) +
  labs(x = "Population",
       y = "Probability") +
  ggtitle("The probability of population of lemmings") +
  theme(legend.position = c(.05, .95),
        legend.justification = c("left", "top")) +
  scale_x_continuous(breaks = seq(0, 1, 0.1))
```

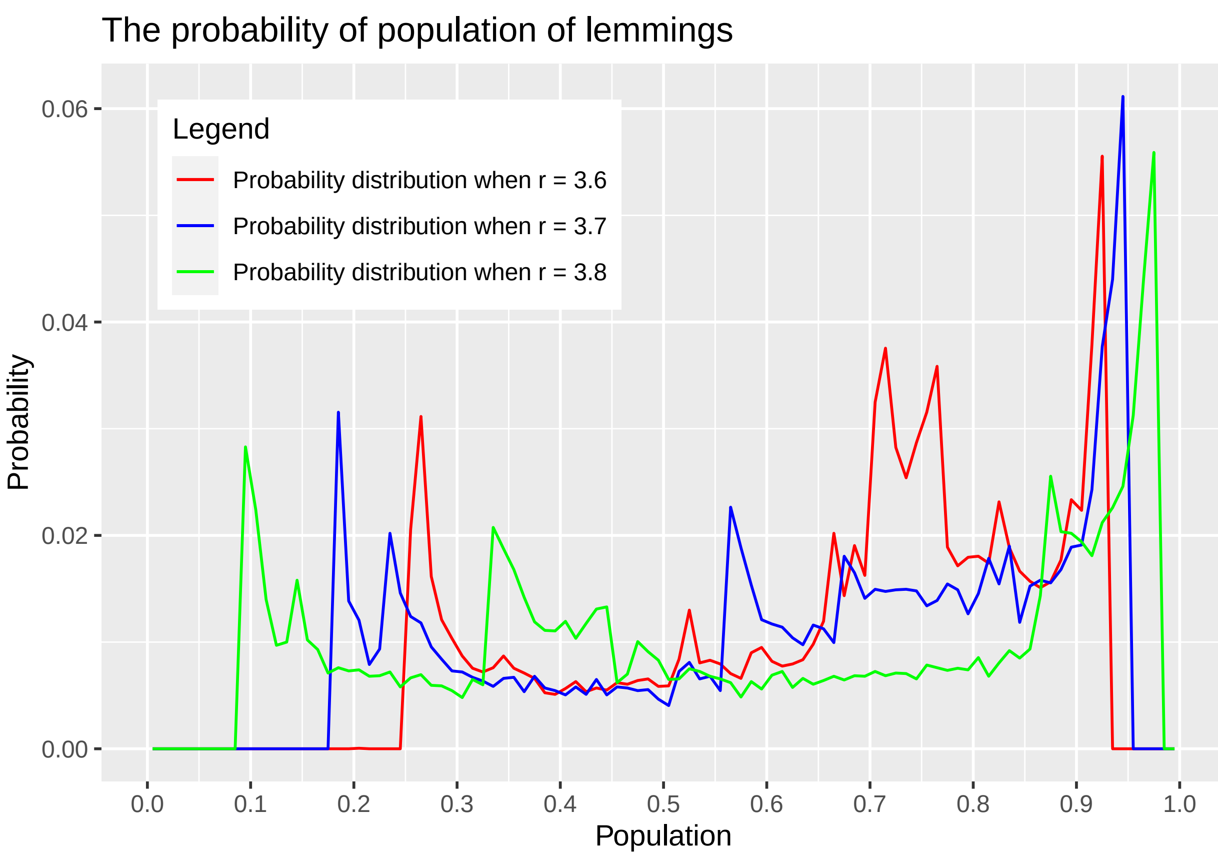


Exercise 3: Sensitivity of the probability distribution to the value of r

Choose a value of r slightly different from the one you chose in Exercise 2. Plot the probability distribution of X for the two different values of r, starting from the same initial X_0. Is the probability distribution (and thus the statistical quantities) sensitive to the value of r?

```
NumSteps <- 20000
X0 <- 0.2
BinNum <- 100

r <- 3.7
lemming_pop_1 <- lemming_population(r, X0, NumSteps)
prob_dist_1 <- binCount(lemming_pop_1, BinNum)

r <- 3.8
lemming_pop_2 <- lemming_population(r, X0, NumSteps)
prob_dist_2 <- binCount(lemming_pop_2, BinNum)

r <- 3.9
lemming_pop_3 <- lemming_population(r, X0, NumSteps)
prob_dist_3 <- binCount(lemming_pop_3, BinNum)


BinCentres <- seq(BinNum)*BinSize - BinSize/2 # find the centers of the bins

data_pop <- data.frame(BinCentres = BinCentres,
                       prob_dist_1 = prob_dist_1,
```

```
                        prob_dist_2 = prob_dist_2,
                        prob_dist_3 = prob_dist_3)

data_pop %>% ggplot(aes(x = BinCentres)) +
  geom_line(aes(y = prob_dist_1,
                color = "Probability distribution when r = 3.6")) +
  geom_line(aes(y = prob_dist_2,
                color = "Probability distribution when r = 3.7")) +
  geom_line(aes(y = prob_dist_3,
                color = "Probability distribution when r = 3.8")) +
  scale_color_manual("Legend", values = c("red", "blue", "green")) +
  labs(x = "Population",
       y = "Probability") +
  ggtitle("The probability of population of lemmings") +
  theme(legend.position = c(.05, .95),
        legend.justification = c("left", "top")) +
  scale_x_continuous(breaks = seq(0, 1, 0.1))
```



Exercise 4: Work out the long-time average of X

On Monday, we saw there were different behaviors of X in the limit of long time-settling to stable fixed points, period-2 and period-4 cycles, and finally non-periodic chaotic motion.

How does the average value of X over long times varies? Plot the long-time average of X vs.r. (i.e. take the values $X_{101}, \cdots, X_{200}$ for each r and work out their average, and plot these averages against r). See if the long-time average of X varies smoothly with each r.