Problem 1

Exercises with δ-functions

(i) A charge Q is spread uniformly over a spherical shell of radius R. Express the charge density using a delta function in spherical coordinates. Repeat for a ring of radius R with charge Q lying in the xy plane.

(ii) Express a three dimensional delta function in cylindrical coordinates (ρ, ϕ and z).

(iii) A charge λ per unit length is distributed uniformly over a cylindrical surface of radius b. Give the charge density using a delta function in cylindrical coordinates

(iv) What is $\nabla^2 \ln r$ in two dimensions ?

Problem 2

Exercises with Gauss's and Stokes' theorems. For parts (i) and (ii) V is the volume enclosed by the closed surface S. Show that

(i) $\int_V \nabla\psi \, d^3x = \int_S \psi \, d\vec{a}$, and that as a consequence of this one has that $\int_S d\vec{a} = 0$ for a closed surface S.

(ii) $\int_V \nabla \times \vec{A} \, d^3x = \int_S d\vec{a} \times \vec{A}$.

(iii) $\int_S d\vec{a} \times \nabla\psi = \oint_\Gamma \psi \, d\vec{\ell}$ where Γ is the boundary of the surface S.

Problem 3. A uniform electric field is produced by fixed charges located at a large distance from a particular spherical region in the field. Find a distribution of charge density and dipole layer density on the surface of the spherical region that will make the field zero in the interior of this region without changing it in the exterior. For definiteness, choose an electric field of the form $\vec{E} = E_0\,\vec{k}$ along the z-direction and let the spherical region be centered at $\vec{x} = \vec{0}$ and have radius a.
Note that the specification of the dipole layer strength requires the number D and an orientation, that is, a unit vector. In our case, state your orientation (into the sphere or out of the sphere). A $D > 0$ and an orientation $\vec{n}$, for example, means that the dipole points in the direction of $\vec{n}$.
Illustrate in two separate sketches the values of σ and D on the surface of the sphere by using $+$'s or $-$'s to indicate charges, and little arrows to represent the direction of the dipoles in the dipole layer.
Calculate the electric field that the surface distribution of σ produces – all by itself. (Hints: it produces a constant electric field inside and the field of an electric dipole outside).

Problem 4 Jackson 1.5.