Problem 1. Use the expansion of a function $f(\rho)$ with $0 \le \rho \le a$ vanishing at $\rho = a$ in terms of Bessel functions of the type $J_m(\frac{x_{mn}\rho}{a})$ to derive a completeness relation of the form

$$\frac{\delta(\rho-\rho')}{\rho} = \sum_n A_n J_m(\frac{x_{mn}\rho}{a}) J_m(\frac{x_{mn}\rho'}{a})$$

Similarly, consider the expansion of a function $f(z)$ for $0 \le z \le L$ and vanishing both at $z = 0$ and $z = L$. Derive a completeness relation for $\delta(z-z')$ in terms of a sum of double products of sin functions.

Problem 2. Jackson 3.23. Obtain only the first two expressions given in the problem. Hint: for each case identify two regions in the cylinder, separated by a surface containing the point charge. Write expressions for the potential in each of the regions and match them across the boundary using the appropriate (dis)continuity conditions.

3.23 A point charge q is located at the point (ρ', ϕ', z') inside a grounded cylindrical box defined by the surfaces $z = 0$, $z = L$, $\rho = a$. Show that the potential inside the box can be expressed in the following alternative forms:

$$\Phi(\mathbf{x}, \mathbf{x}') = \frac{q}{\pi\epsilon_0 a} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} \frac{e^{im(\phi-\phi')} J_m\left(\frac{x_{mn}\rho}{a}\right) J_m\left(\frac{x_{mn}\rho'}{a}\right)}{x_{mn} J_{m+1}^2(x_{mn}) \sinh\left(\frac{x_{mn}L}{a}\right)} \times \sinh\left[\frac{x_{mn}}{a} z_<\right] \sinh\left[\frac{x_{mn}}{a}(L - z_>)\right]$$

$$\phi(\mathbf{x}, \mathbf{x}') = \frac{q}{\pi\epsilon_0 L} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} e^{im(\phi-\phi')} \sin\left(\frac{n\pi z}{L}\right) \sin\left(\frac{n\pi z'}{L}\right) \frac{I_m\left(\frac{n\pi\rho_<}{L}\right)}{I_m\left(\frac{n\pi a}{L}\right)} \times \left[I_m\left(\frac{n\pi a}{L}\right) K_m\left(\frac{n\pi\rho_>}{L}\right) - K_m\left(\frac{n\pi a}{L}\right) I_m\left(\frac{n\pi\rho_>}{L}\right)\right]$$

$$\Phi(\mathbf{x}, \mathbf{x}') = \frac{2q}{\pi\epsilon_0 L a^2} \sum_{m=-\infty}^{\infty} \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{e^{im(\phi-\phi')} \sin\left(\frac{k\pi z}{L}\right) \sin\left(\frac{k\pi z'}{L}\right) J_m\left(\frac{x_{mn}\rho}{a}\right) J_m\left(\frac{x_{mn}\rho'}{a}\right)}{\left[\left(\frac{x_{mn}}{a}\right)^2 + \left(\frac{k\pi}{L}\right)^2\right] J_{m+1}^2(x_{mn})}$$

Discuss the relation of the last expansion (with its extra summation) to the other two.