Problem 1. Symmetry property for the Dirichlet Green's function.

Use the equations satisfied by $G_D(\vec{x_1}, \vec{x'})$ and $G_D(\vec{x_2}, \vec{x'})$ and Green's theorem with primed variables, to show that $G_D(\vec{x_1}, \vec{x_2}) = G_D(\vec{x_2}, \vec{x_1})$.

Problem 2. Images and Green functions.
An infinite conducting plane (in the xy plane) with a hemispherical boss of radius a is at zero potential. A charge q is put on the axis of the boss (which can be taken to be the z-axis) at a distance z from the plate.
(i) Find a suitable set of images for this problem. Show that the charge is attracted toward the plate with a force

$$\frac{q^2}{4z^2} + \frac{4q^2a^3z^3}{(z^4 - a^4)^2}.$$

(ii) Suppose we now have an arbitrary potential $\Phi_0(\vec{x}')$ on the two dimensional surface which is not a conductor anymore, and the original charge is removed (the potential is taken to be zero at infinity.) Write an integral expression for the potential anywhere in the z-axis ($z > a$) using a Dirichlet Green function. (you need not grind the normal derivatives of G, just find G and set up the integral.)

Problem 3 Jackson 2.8 (see next page). Do not do part (d).

2.8 A two-dimensional potential problem is defined by two straight parallel line charges separated by a distance R with equal and opposite linear charge densities λ and $-\lambda$.

(a) Show by direct construction that the surface of constant potential V is a circular cylinder (circle in the transverse dimensions) and find the coordinates of the axis of the cylinder and its radius in terms of R, λ, and V.

(b) Use the results of part a to show that the capacitance per unit length C of two right-circular cylindrical conductors, with radii a and b, separated by a distance $d > a + b$, is

$$C = \frac{2\pi\epsilon_0}{\cosh^{-1}\left(\dfrac{d^2 - a^2 - b^2}{2ab}\right)}$$

(c) Verify that the result for C agrees with the answer in Problem 1.7 in the appropriate limit and determine the next nonvanishing order correction in powers of a/d and b/d.

(d) Repeat the calculation of the capacitance per unit length for two cylinders inside each other ($d < |b - a|$). Check the result for concentric cylinders ($d = 0$).