To begin with, we know (by the expected form of the Taylor Series) that:

$$\sin(x) = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 ...$$

Examining $x = 0$:

$$\sin(0) = 0 = A$$

Deriving the above:

$$\cos(x) = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + ...$$

Examining $x = 0$:

$$\cos(0) = 1 = B$$

Deriving the above:

$$-\sin(x) = 2C + 6Dx + 12Ex^2 + 20Fx^3 + ...$$

Examining $x = 0$:

$$-\sin(0) = 0 = 2C \Rightarrow C = 0$$

Deriving the above:

$$-\cos(x) = 6D + 24Ex + 60Fx^2 + ...$$

Examining $x = 0$:

$$-\cos(0) = -1 = 6D \Rightarrow D = -\frac{1}{6}$$

Deriving the above:

$$\sin(x) = 24E + 120Fx + ...$$

Examining $x = 0$:

$$\sin(0) = 0 = 24E \Rightarrow E = 0$$

Deriving the above:

$$\cos(x) = 120F + ...$$

Examining $x = 0$:

$$\cos(0) = 1 = 120F \Rightarrow F = \frac{1}{120}$$

So, finally by the Taylor Series equation at the top of this derivation we have:

$$\sin(x) = 0 + (1)\,x + (0)\,x^2 + \left(-\frac{1}{6}\right)x^3 + (0)\,x^4 + \left(\frac{1}{120}\right)x^5 ...$$

Simplified:

$$\sin(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5...$$

Looking at these 3 terms, we can see that the power on x is simply iterating odd numbers, so we need something like:

$$x^{2i-1}\, \forall i \in \{1, 2, ...\}$$

to produce the x values to odd exponents.

We see that the denominators of the terms is the factorial of the exponent on x, so:

$$(2i-1)!\, \forall i \in \{1, 2, ...\}$$

should produce the denominators.

Next we observe that the signs are alternating, so the following will produce them:

$$(-1)^{i-1}\ \forall i \in \{1, 2, ...\}$$

Putting this all together we see that the sin function can be represented as an infinite sum that looks like:

$$\sin(x) = \sum_{i=1}^{\infty} \frac{x^{2i-1}}{(2i-1)!} (-1)^{i-1}$$