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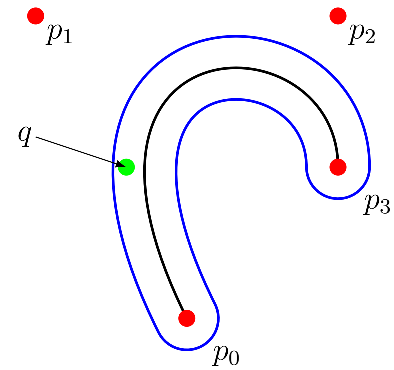
Procedurally Generated Handwriting



To model a handwritten language we assume the hand draws curves with thickness ϵ which may change based on physical attributes of the curve. For a fountain pen, the thickness of the stroke depends on the minimum and maximum widths of the nib, the angle of the nib to the paper, and tangent of the stroke with the angle of the nib. For brushes, the amount of ink on the bristles and the force with which the brush is pressed also have an effect. Of course, there are many other factors so in general we define a function T which returns a value in $[0, \infty)$. We denote ϵ the output of this function. The T used for in [Figure 1] is a constant function.

To model each stroke (or part of a stroke), we define a Bézier curve $B(t)$ for $t \in [0, 1]$ over four control points p_0, p_1, p_2, $p_3 \in \mathbb{R}^2$. For our purposes, the control points are in $[0, 1]^2$. To clarify notation, $B(t) = \begin{bmatrix} B_u(t) & B_v(t) \end{bmatrix}^T$

To determine whether this point lies inside or outside the region, we must first express $B(t)$ implicitly; for $t \in [0, 1]$,

$$B(t) = (1-t)^3 p_0 + 3(1-t)^2 t p_1 + 3(1-t)t^2 p_2 + t^3 p_3$$

where $p_i = (u_i, v_i)$, the coordinates of each control point.

Next, we determine if a given point $q = (u, v)$ lies inside of the blue region. One way to accomplish this is to find the nearest point to q on B, denoted $\hat{q} = (\hat{u}, \hat{v})$. Since B is a parametrized curve there exists a $\hat{t}$ such that $B(\hat{t}) = \hat{q}$. Then, q is contained in the stroke if and only if $||q - \hat{q}||_2 < \epsilon$ for some $\hat{t} \in [0, 1]$. Notice that both the $\hat{q}$ and $\hat{t}$ may not be unique. This is unimportant since every $\hat{q}$ is the same distance from q. We find this $\hat{q}$ by observing the vector $\hat{q} - q$ is orthogonal to $\mathrm{d}B/\,\mathrm{d}t$. Notice there may exist multiple $\hat{t} \in [0, 1]$ such that $B(\hat{t}) = \hat{q}$. Of these, we choose the $\hat{t}$ such that $||q - \hat{q}||_2$ is minimized. So we solve for t in

$$\begin{aligned} 0 &= \hat{q} \cdot \frac{\mathrm{d}B}{\mathrm{d}t} \\ &= B_u(t)\frac{\mathrm{d}B_u}{\mathrm{d}t}(t) + B_v(t)\frac{\mathrm{d}B_v}{\mathrm{d}t}(t) \end{aligned}$$

which amounts to finding the real roots of a fifth-order polynomial. For the exact formula of this polynomial see the attached Python code.