Two-Factor Between-Participants Designs

PSYC214: Statistics For Group Comparisons

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Week 7

Learning Objectives

- How to calculate F ratios for two-factor between-participants designs
- How to calculate simple main effects, if the interaction is significant

Two-Factor Between-Participants Designs

- The simplest two-factor between-participants design is a 2 $\times$ 2 factorial design:
  - there are two factors, each with two levels, yielding a total of four cells or conditions
  - each participant contributes a single score to one condition only
- We can ask whether either of the **main effects** is significant
- We can also ask whether the **interaction** is significant
  - an interaction is interpreted in terms of the **simple main effects**

A Typical Between-Participants 2 × 2 Design

	A_1	A_2	
B_1	P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9	P_{10} P_{11} P_{12} P_{13} P_{14} P_{15} P_{16} P_{17} P_{18}	Mean B_1
B_2	P_{19} P_{20} P_{21} P_{22} P_{23} P_{24} P_{25} P_{26} P_{27}	P_{28} P_{29} P_{30} P_{31} P_{32} P_{33} P_{34} P_{35} P_{36}	Mean B_2
	Mean A_1	Mean A_2	

A typical between-participants 2x2 design. Each participant only performs one of the four possible combinations of conditions

Main Effects

A_1	A_2
P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9 P_{19} P_{20} P_{21} P_{22} P_{23} P_{24} P_{25} P_{26} P_{27}	P_{10} P_{11} P_{12} P_{13} P_{14} P_{15} P_{16} P_{17} P_{18} P_{28} P_{29} P_{30} P_{31} P_{32} P_{33} P_{34} P_{35} P_{36}
Mean A_1	Mean A_2

B_1	P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9	P_{10} P_{11} P_{12} P_{13} P_{14} P_{15} P_{16} P_{17} P_{18}	Mean B_1
B_2	P_{19} P_{20} P_{21} P_{22} P_{23} P_{24} P_{25} P_{26} P_{27}	P_{28} P_{29} P_{30} P_{31} P_{32} P_{33} P_{34} P_{35} P_{36}	Mean B_2

Main effect of A: Is the difference between means of A_1 and A_2 significant (ignoring factor B)?

Main effect of B: Is the difference between means of B_1 and B_2 significant (ignoring factor A)?

Simple Main Effects of Factor A

	A_1	A_2	
B_1	P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9 Mean A_1 (at B_1)	P_{10} P_{11} P_{12} P_{13} P_{14} P_{15} P_{16} P_{17} P_{18} Mean A_2 (at B_1)	**Simple main effect of A at B_1:** Is the difference between means of A_1 and A_2 significant at B_1 of factor B?
B_2	P_{19} P_{20} P_{21} P_{22} P_{23} P_{24} P_{25} P_{26} P_{27} Mean A_1 (at B_2)	P_{28} P_{29} P_{30} P_{31} P_{32} P_{33} P_{34} P_{35} P_{36} Mean A_2 (at B_2)	**Simple main effect of A at B_2:** Is the difference between means of A_1 and A_2 significant at B_2 of factor B?

Simple Main Effects of Factor B

	A_1		A_2	
B_1	P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9	Mean B_1 (at A_1)	P_{10} P_{11} P_{12} P_{13} P_{14} P_{15} P_{16} P_{17} P_{18}	Mean B_1 (at A_2)
B_2	P_{19} P_{20} P_{21} P_{22} P_{23} P_{24} P_{25} P_{26} P_{27}	Mean B_2 (at A_1)	P_{28} P_{29} P_{30} P_{31} P_{32} P_{33} P_{34} P_{35} P_{36}	Mean B_2 (at A_2)

Simple main effect of B at A_1: Is the difference between means of B_1 and B_2 significant at A_1 of factor A?

Simple main effect of B at A_2: Is the difference between means of B_1 and B_2 significant at A_2 of factor A?

Simple Main Effects

- There are two ways a pair of simple main effects may differ in their trends:
  1. one of a pair has a significant difference but not the other. For example, the mean of A_1 differs from the mean of A_2 at level B_2 *but not* at level B_1
  2. both simple main effects are significant, but in the opposite direction. For example, the mean of A_1 is greater than the mean of A_2 at level B_1, but the pattern is reversed at level B_2

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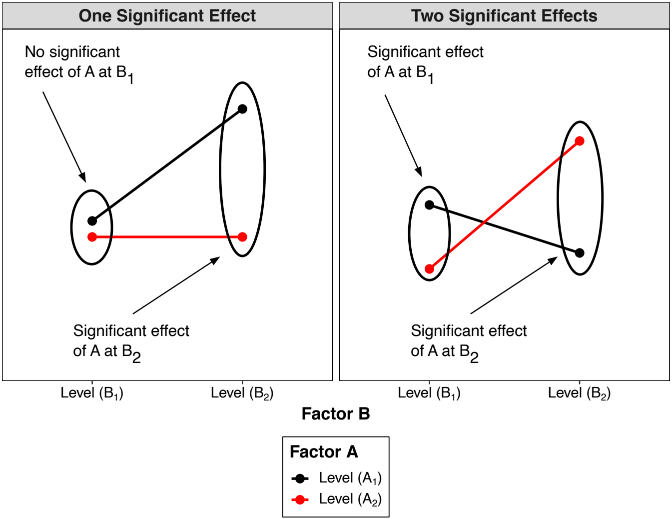

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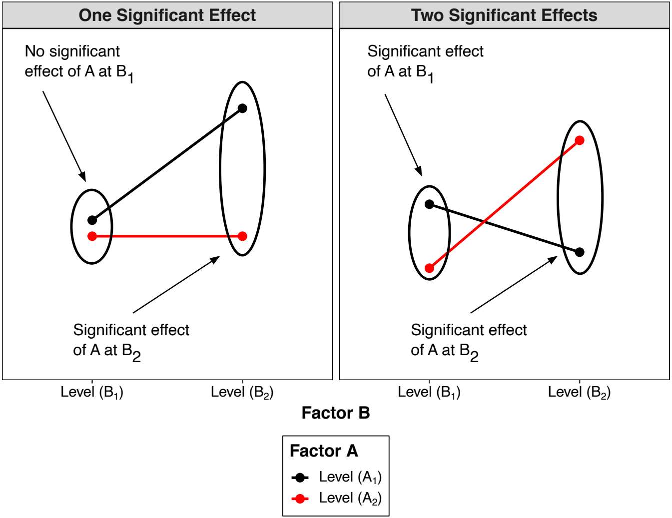
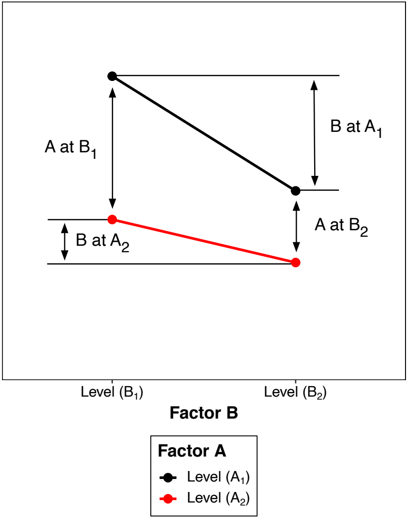


Simple Main Effects

- Sometimes, all of the simple main effects may be significant
- But, their levels of significance and effect sizes will vary
- In these instances, when reporting the interaction we must draw attention to the different sizes of the trends
- This makes it harder to report, as a certain degree of subjective interpretation is required

Simple Main Effects



Analysis a 2 × 2 Between-Participants Factorial Design

- The first stage of analysis seeks to uncover which of the two main effects and interactions are significant
- If the interaction is significant, then in a second stage we perform a simple main effects analysis
- Although a second factor has been added, the F ratio remains the same:

$$F = \frac{\text{treatment effects + experimental error}}{\text{experimental error}}$$

- As this is a between-participants design:

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

Analysis a 2 × 2 Between-Participants Factorial Design

- The main difference is that there are now three F ratios, one for each of the three effects

Hypothetical Data For COVID-19 Study

		Factor A: Fear	
		Level A_1 no fear appeal	Level A_2 fear appeal
Factor B: Efficacy	Level B_1 no efficacy message	P_1 5	P_{13} 6
		P_2 4	P_{14} 4
		P_3 6	P_{15} 4
		P_4 4	P_{16} 5
		P_5 5	P_{17} 8
		P_6 6	P_{18} 3
	Level B_2 efficacy message	P_7 6	P_{19} 10
		P_8 6	P_{20} 9
		P_9 5	P_{21} 6
		P_{10} 3	P_{22} 9
		P_{11} 8	P_{23} 8
		P_{12} 3	P_{24} 7

Hypothetical Data For COVID-19 Study

		Factor A: Fear		
		Level A_1	*Level A_2*	
		no fear appeal	*fear appeal*	*Overall*
Factor *B*:	Level B_1 no efficacy message	5.00	5.00	5.00
Efficacy	Level B_2 efficacy message	5.17	8.17	6.67
	Overall	5.08	6.58	5.83

Notation

$$SS_{BETWEEN} = \frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A} - \frac{(\sum Y)^2}{N}$$

$$SS_{WITHIN} = \sum Y^2 - \frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A}$$

$$SS_{TOTAL} = \sum Y^2 - \frac{(\sum Y)^2}{N}$$

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Notation

$$\frac{(\sum Y)^2}{N} \text{ is } \frac{(\text{grand total})^2}{\text{the number of scores that make up the grand total}}$$

$$\frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A} \text{ is } \frac{(\text{level total of } A_1)^2 + (\text{level total of } A_2)^2}{\text{the number of scores that make up each level}}$$

$$\sum Y^2 \text{ is } \frac{(\text{score}_1)^2 + (\text{score}_2)^2 + (\text{score}_3)^2 (\text{and so on})}{1 \text{ (only one number makes up each individual score)}}$$

Basic Ratios

$$[T] : \text{basic ratio of the grand total, } \frac{(\sum Y)^2}{N}$$

$$[A] : \text{basic ratio of the level totals, } \frac{(\sum A_1)^2 + (\sum A_2)^2}{N_A}$$

$$[Y] : \text{basic ratio of the individual scores, } \sum Y^2$$

Basic Ratios

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- To compute the components of a factorial between-participants ANOVA, two additional ratios are required
- $[B]$ is the basic ratio of the level totals of factor B. If there are two levels in factor B, then $[B]$ =

$$\frac{(\text{level total of } B_1)^2 + (\text{level total of } B_2)^2}{\text{the number of scores that make up each level}} = \frac{(\sum B_1)^2 + (\sum B_2)^2}{N_B}$$

Basic Ratios

- $[AB]$ is the basic ratio of the cell totals, where a cell total is the total of all the scores in any one of the cells. For a 2 × 2 design, $[AB]$ =

$$\frac{\left(\text{cell total of } A_1B_1\right)^2 + \left(\text{cell total of } A_1B_2\right)^2 + \left(\text{cell total of } A_2B_1\right)^2 + \left(\text{cell total of } A_2B_2\right)^2}{\text{the number of scores in each cell}}$$

$$= \left(\sum A_1B_1\right)^2 + \left(\sum A_1B_2\right)^2 + \left(\sum A_2B_1\right)^2 + \left(\sum A_2B_2\right)^2$$

Calculating Basic Ratios For The Hypothetical Data

		Factor A: Fear			
		Level A_1 no fear appeal	Level A_2 fear appeal		
Factor B Efficacy	Level B_1 no efficacy message	Total A_1B_1 = 30	Total A_2B_1 = 30	Total B_1 = 30 + 30 = 60	$[B] = \frac{60^2 + 80^2}{12} = \frac{3600 + 6400}{12} = 833.3333$
	Level B_2 efficacy message	Total A_1B_2 = 31	Total A_2B_2 = 49	Total B_2 = 31 + 49 = 80	
		Total A_1 = 30 + 31 = 61	Total A_2 = 30 + 49 = 79	$[Y] = 910$	
		$[A] = \frac{61^2 + 79^2}{12} = \frac{3721 + 6241}{12} = \frac{9962}{12} = 830.1667$		$[T] = \frac{140}{24} = \frac{19600}{24} = 816.6667$	

$$[AB] = \frac{30^2 + 30^2 + 31^2 + 49^2}{6} = \frac{900 + 900 + 961 + 2401}{6} = \frac{5162}{6} = 860.3333$$

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Calculating The Sum of Squares For The Error Term

- Within-group variance is a measure of the extent to which people within each of the groups behave differently, despite being treated alike
- For a 2×2 between-participants design, people have been treated exactly alike *only* within each of the four cells
- To calculate the error term, we compute and combine the Sums of Squares and degrees of freedom using the smallest unit of identically treated participants—the four cells
- This gives a single measure of experimental error that can be used for calculating the *F*s for all the effects

Calculating The Sum of Squares For The Error Term

- We calculate the error term, SS_{WITHIN}, as follows:

$$SS_{WITHIN} = [Y] - [AB] \quad SS_{WITHIN} \text{will be designated } SS_{S/AB}$$

- This produces the error term that will be used to calculate all the *F*s
- This is the overall measure of the extent to which participants behaved differently despite being treated alike

Between-Group Sum of Squares

- We also need to calculate the total between-group Sum of Squares for the four cells
- This is a measure of the variability due to the various experimental treatments
- It is a measure of how distant each of the four cell means is from the grand mean
- It tells us the overall extent to which the treatments caused scores to differ
- The between-group Sum of Squares is calculated as:

$$SS_{BETWEEN} = [AB] - [T] \quad SS_{BETWEEN} \text{ will be designated } SS_{AB}$$

Total Sum of Squares

- We also need to calculate the total Sum of Squares
- This is a measure of total variability for the entire data set *irrespective* of experimental treatments
- It is calculated as:

$$SS_{TOTAL} = [Y] - [T]$$

Calculating The Sums of Squares For The Two Main Effects

- Two between-group sums of squares are required, one for each of the main effects
- Each main effect is treated as being completely independent from the other
    - e.g., when calculating the main effect of factor A, the fact participants were treated in different ways at factor B is ignored
- The Sums of Squares for the two main effects are calculated as:

$$\text{for the between-group sums of squares for factor } A, SS_A = [A] - [T]$$
$$\text{for the between-group sums of squares for factor } B, SS_B = [B] - [T]$$

Calculating The Sums of Squares For The Two Main Effects

- To test the significance of the interaction, a final Sums of Squares is required
- This is calculated as:

$$SS_{INTERACTION}, SS_{A \times B} = [AB] - [A] - [B] + [T]$$

- *This is the variability in thee group means not accounted for by the main effects*
- It is the variability caused by the interaction between factor A and factor B

Calculating The Sums of Squares Discussed So Far

Within-group Sum of Squares: $SS_{S/AB} = [Y] - [AB]$

$$= 910 - 860.3333 = 49.67$$

Total between-group Sum of Squares: $SS_{AB} = [AB] - [T]$

$$= 860.3333 - 816.6667 = 43.67$$

Total Sum of Squares: $SS_{TOTAL} = [Y] - [T]$

$$= 910 - 816.6667 = 93.33$$

Calculating The Sums of Squares Discussed So Far

Between-group Sum of Squares for factor A: $SS_A = [A] - [T]$

$$= 830.1667 - 816.667 = 13.50$$

Between-group Sum of Squares for factor B: $SS_B = [B] - [T]$

$$= 833.3333 - 816.6667 = 16.67$$

Sum of Squares for interaction: $SS_{A \times B} = [AB] - [A] - [B] + [T]$

$$= 860.3333 - 830.1667 - 833.3333 + 816.6667 = 13.50$$

Degrees of Freedom

- For the main effects:

$$df_A = (\text{number of levels in factor } A - 1) = (a - 1)$$
$$(a \text{ is the number of levels in factor } A)$$

$$df_B = (\text{number of levels in factor } B - 1) = (b - 1)$$
$$(b \text{ is the number of levels in factor } B)$$

- For the interaction:

$$df_{A \times B} = df_A \times df_B = (a - 1)(b - 1)$$

Degrees of Freedom

- For the within-group variance (the error term):

$$\begin{aligned} df_{S/AB} &= [(\text{number of cells}) \times (\text{number of scores in cell} - 1)] \\ &= ab(s-1) \\ &(s \text{ is the number of scores in a cell}) \end{aligned}$$

- For the total degrees of freedom:

$$df_{TOTAL} = (\text{total number of scores } - 1) = (abs) - 1$$

Degrees of Freedom

- The various degrees of freedom should add up so that:

$$df_{TOTAL} = df_A + df_B + df_{A \times B} + df_{S/AB}$$

Calculating The Degrees of Freedom Discussed So Far

$$df_A = (a-1) = 2-1 = 1 (\text{factor } A \text{ has two levels})$$

$$df_B = (b-1) = 2-1 = 1 (\text{factor } B \text{ has two levels})$$

$$df_{A\times B} = (a-1)(b-1) = 1 \times 1 = 1$$

$$df_{S/AB} = ab(s-1) = 2 \times 2(6-1) = 20\,(\text{six participants per cell})$$

$$df_{TOTAL} = (abs) - 1 = (2 \times 2 \times 6) - 1 = 23$$

Summary ANOVA Table By Components

Source	Sum of Squares	Degrees of freedom	Mean Square	F	p
A	$[A] - [T]$	$(a - 1)$	$\frac{[A] - [T]}{(a - 1)}$	$\frac{\text{Mean Square}_A}{\text{Mean Square}_{S/AB}}$	tables
B	$[B] - [T]$	$(b - 1)$	$\frac{[B] - [T]}{(b - 1)}$	$\frac{\text{Mean Square}_B}{\text{Mean Square}_{S/AB}}$	tables
$A \times B$	$[AB] - [A] - [B] + [T]$	$(a - 1)(b - 1)$	$\frac{[AB] - [A] - [B] + [T]}{(a - 1)(b - 1)}$	$\frac{\text{Mean Square}_{A \times B}}{\text{Mean Square}_{S/AB}}$	tables
S/AB	$[Y] - [AB]$	$ab(s - 1)$	$\frac{[Y] - [AB]}{ab(s - 1)}$		
$TOTAL$	$[Y] - [T]$	$(abs) - 1$			

ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A	13.50	1			
B	16.67	1			
$A \times B$	13.50	1			
S/AB	49.67	20			
TOTAL	93.33	23			

ANOVA Table For Hypothetical Data

Research Methods I

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2 × 2 Factorial Design
Structure
Main Effects
Simple Main Effects

Analysis a 2 × 2 Design
Data
Basic Ratios
SS WITHIN, BETWEEN, & TOTAL
SS Main Effects
SS Interaction
DF
ANOVA Table

Simple Main Effects
Between-Group SS & DF
Simple Main Effects Table

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A	13.50	1	13.50		
B	16.67	1	16.67		
$A \times B$	13.50	1	13.50		
S/AB	49.67	20	2.48		
TOTAL	93.33	23	4.06		

ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A	13.50	1	13.50	5.44	
B	16.67	1	16.67	6.72	
$A \times B$	13.50	1	13.50	5.44	
S/AB	49.67	20	2.48		
$TOTAL$	93.33	23	4.06		

ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A	13.50	1	13.50	5.44	$< .05$
B	16.67	1	16.67	6.72	$< .05$
$A \times B$	13.50	1	13.50	5.44	$< .05$
S/AB	49.67	20	2.48		
TOTAL	93.33	23	4.06		

ANOVA Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
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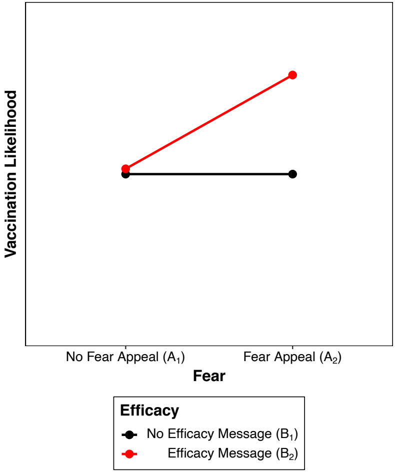
$A \times B$	13.50	1	13.50	5.44	$< .05$
S/AB	49.67	20	2.48		
$TOTAL$	93.33	23	4.06		

Interaction Plot

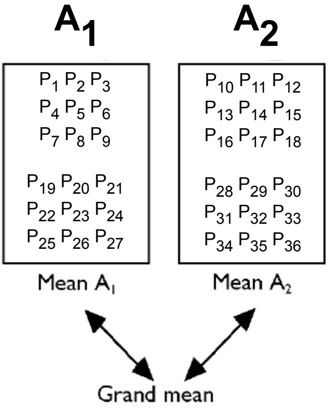


Simple Main Effects

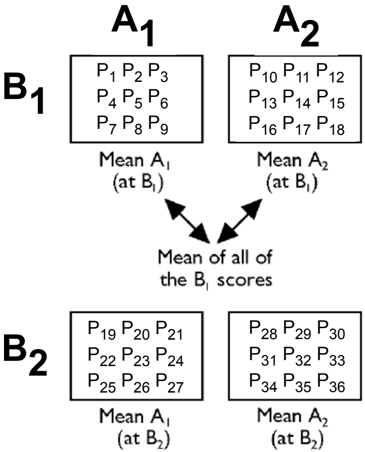
- If the interaction is significant, then we interpret it by analysing the simple main effects
- In a 2 $\times$ 2 design, these are simply pairwise comparisons, analogous to using four *t*-tests
- This involves calculating the between-group variance for each simple main effect, before dividing each variance by the error term (*S/AB*) from the original ANOVA
- Thus, the significance of the simple main effects is evaluated using the same error term used to test the significance of the main effects and interaction

Simple Main Effects

Main effect of A: To find out whether the main effect of A is significant, calculate the between-group variance of the means of A_1 and A_2 in relation to the grand mean (ignoring factor B). The bigger thee variance, the bigger the difference between thee means and the more likely that the difference is significant.



Simple Main Effects



Simple main effect of A at B_1: To find out whether this simple main effect is significant, calculate the between-group variance for means A_1 and A_2 in relation to the overall mean of all the B_1 scores. The bigger the variance, the bigger the difference between the means and the more likely that the difference is significant.

Ignore B_2 when testing the significance of the simple main effect of A at B_1

Calculating Between-Group Sum of Squares

- The formula for calculating a between-group Sum of Squares is the basic ratio of the group totals of interest, minus the basic ratio of the total of these totals [T]
- For example, the formula for calculating the between-group variance for the main effect of factor A is [A] - [T]
- The basic ratios used to calculate the between-group variances for the simple main effects are analogous to these

Calculating Between-Group Sum of Squares

- For example:

- $[A_{B_1}]$ is the basic ratio of factor *A*, but *only* for the B_1 scores: square the total for A_1B_1, square the total for A_2B_1, add the squares together and divide by the number of scores that make up each cell.

- $[T_{B_1}]$ is the basic ratio of the total of the scores at level B_1 of factor *B*: take the total of all the scores in level B_1 and square the total, divide the square by the number of scores making up this total.

- *Eight basic ratios are required to test the four simple main effects ...*

Calculating Between-Group Sum of Squares

Sum of Squares between groups of factor A at level B_1 ($SS_{A\ at\ B_1}$) :

$$[A_{B_1}] - [T_{B_1}]$$

Sum of Squares between groups of factor A at level B_2 ($SS_{A\ at\ B_2}$) :

$$[A_{B_2}] - [T_{B_2}]$$

Sum of Squares between groups of factor B at level A_1 ($SS_{B\ at\ A_1}$) :

$$[B_{A_1}] - [T_{A_1}]$$

Sum of Squares between groups of factor B at level A_2 ($SS_{B\ at\ A_2}$) :

$$[B_{A_2}] - [T_{A_2}]$$

Calculating Between-Group Degrees Of Freedom

- All degrees of freedom are equal to the number of ([number of levels in each simple main effect]) - 1
- For the two simple main effects of *A*, the degrees of freedom are given by (a - 1), where a is the number of levels in factor *A*
- For the two simple main effects of *B*, the degrees of freedom are given by (b - 1), where b is the number of levels in factor *B*

Calculating Between-Group Sum of Squares

		Factor A: Fear		
		Level A_1 no fear appeal	Level A_2 fear appeal	
Factor B Efficacy	Level B_1 no efficacy message	Total A_1B_1 = 30	Total A_2B_1 = 30	Total B_1 = 30 + 30 = 60
	Level B_2 efficacy message	Total A_1B_2 = 31	Total A_2B_2 = 49	Total B_2 = 31 + 49 = 80
		Total A_1 = 30 + 31 = 61	Total A_2 = 30 + 49 = 79	

Calculating Between-Group Sum of Squares

- Fear (no fear appeal *vs.* fear appeal) for no efficacy message (*A* at B_1)

$$[A_{B_1}] = \frac{30^2 + 30^2}{6} = 300 \quad [T_{B_1}] = \frac{60^2}{12} = 300 \quad [A_{B_1}] - [T_{B_1}] = 0$$

- Fear (no fear appeal *vs.* fear appeal) for efficacy message (*A* at B_2)

$$[A_{B_2}] = \frac{31^2 + 49^2}{6} = 560.33 \quad [T_{B_2}] = \frac{80^2}{12} = 533.33 \quad [A_{B_2}] - [T_{B_2}] = 27$$

Calculating Between-Group Sum of Squares

- Fear (no fear appeal *vs.* fear appeal) for no efficacy message (*A* at B_1)

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Calculating Between-Group Sum of Squares

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Calculating Between-Group Sum of Squares

- Efficacy (no efficacy message *vs.* efficacy message) for no fear appeal (*B* at A_1)

$$[B_{A_1}] = \frac{30^2 + 31^2}{6} = 310.17 \;\; [T_{A_1}] = \frac{61^2}{12} = 310.08 \;\; [B_{A_1}] - [T_{A_1}] = .09$$

- Efficacy (no efficacy message *vs.* efficacy message) for fear appeal (*B* at A_2)

$$[B_{A_2}] = \frac{30^2 + 49^2}{6} = 550.17 \;\; [T_{A_2}] = \frac{79^2}{12} = 520.08 \;\; [B_{A_2}] - [T_{A_2}] = 30.09$$

Calculating Between-Group Sum of Squares

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Calculating Between-Group Sum of Squares

Research Methods I
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$$[B_{A_2}] = \frac{30^2 + 49^2}{6} = 550.17 \quad [T_{A_2}] = \frac{79^2}{12} = 520.08 \quad [B_{A_2}] - [T_{A_2}] = 30.09$$

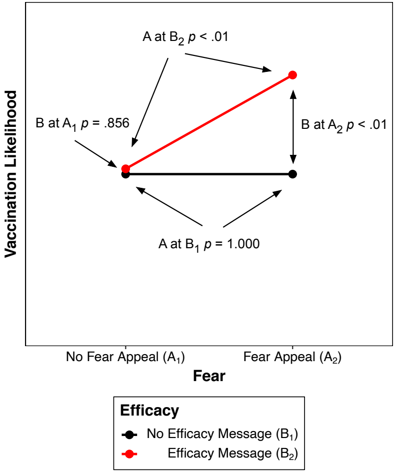
Summary Simple Main Effects Table By Components

SOURCE	Sum of Squares	Degrees of freedom	Mean Square	F	p
A at B_1	$[A_{B_1}] - [T_{B_1}]$	$(a - 1)$	$\frac{[A_{B_1}] - [T_{B_1}]}{(a - 1)}$	$\frac{\text{Mean Square}_{A \text{ at } B_1}}{\text{Mean Square}_{S/AB}}$	tables
A at B_2	$[A_{B_2}] - [T_{B_2}]$	$(a - 1)$	$\frac{[A_{B_2}] - [T_{B_2}]}{(a - 1)}$	$\frac{\text{Mean Square}_{A \text{ at } B_2}}{\text{Mean Square}_{S/AB}}$	tables
B at A_1	$[B_{A_1}] - [T_{A_1}]$	$(b - 1)$	$\frac{[B_{A_1}] - [T_{A_1}]}{(b - 1)}$	$\frac{\text{Mean Square}_{B \text{ at } A_1}}{\text{Mean Square}_{S/AB}}$	tables
B at A_2	$[B_{A_2}] - [T_{A_2}]$	$(b - 1)$	$\frac{[B_{A_2}] - [T_{A_2}]}{(b - 1)}$	$\frac{\text{Mean Square}_{B \text{ at } A_2}}{\text{Mean Square}_{S/AB}}$	tables
S/AB	$[Y] - [AB]$	$ab(s - 1)$	$\frac{[Y] - [AB]}{ab(s - 1)}$		

Simple Main Effects Table For Hypothetical Data

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A at B_1	0.00	1	0.00	0.00	$> .05$
A at B_2	27.00	1	27.00	10.89	$< .01$
B at A_1	0.09	1	0.09	0.04	$> .05$
B at A_2	30.09	1	30.09	12.13	$< .01$
S/AB (error)	49.67	20	2.48		

Interaction Plot



Additional Resources

- The R code for all plots generated in this lecture (minus annotations) has been uploaded with these slides to the Week 6 lecture folder (R Plots For Lecture 7.R)

In Next Week's Lab ...

- Running a 2 × 2 (and 2 × 3) between-participants ANOVA in R
- Calculating and interpreting simple main effects
- Follow-up tests for factors with more than two levels

References

Roberts, M. J., & Russo, R. (1999, Chapter 9–10). *A student's guide to Analysis of Variance*. Routledge: London.