Linear logic

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December 5, 2016

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta}$$

$$\frac{\Gamma \vdash \Delta \qquad \Gamma \vdash \Delta}{\Gamma \vdash \Delta}$$

Classical linear logic

Double-sided form

Structural rules

Weakening

$$\frac{\Gamma \vdash \Delta}{\Gamma, !A \vdash \Delta}$$

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, ?A}$$

Identity

$$\frac{}{A \vdash A}$$

Logical rules

$\perp$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, A^{\perp} \vdash \Delta}$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash A^{\perp}, \Delta}$$

&

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \,\&\, B \vdash \Delta}$$

$$\frac{\Gamma, B \vdash \Delta}{\Gamma, A \,\&\, B \vdash \Delta}$$

$$\frac{\Gamma \vdash \Delta, A \qquad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \,\&\, B}$$

$\oplus$

$$\frac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, A \oplus B}$$

$$\frac{\Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \oplus B}$$

$$\frac{\Gamma, A \vdash \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma, A \oplus B \vdash \Delta}$$

$\otimes$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \otimes B \vdash \Delta}$$

$$\frac{\Gamma_1 \vdash \Delta_1, A \qquad \Gamma_2 \vdash \Delta_2, B}{\Gamma_1, \Gamma_2 \vdash \Delta_1, \Delta_2, A \otimes B}$$

$\parr$

$$\frac{\Gamma \vdash \Delta, A, B}{\Gamma \vdash \Delta, A \parr B}$$

$$\frac{\Gamma_1, A \vdash \Delta_1 \qquad \Gamma_2, B \vdash \Delta_2}{\Gamma_1, \Gamma_2, A \parr B \vdash \Delta_1, \Delta_2}$$