Adjoint sensitivity approach to determine optimal set of stations for tsunami source inversion

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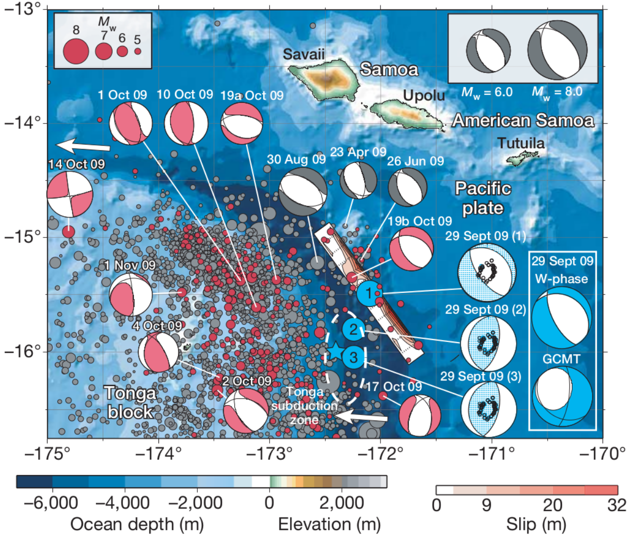
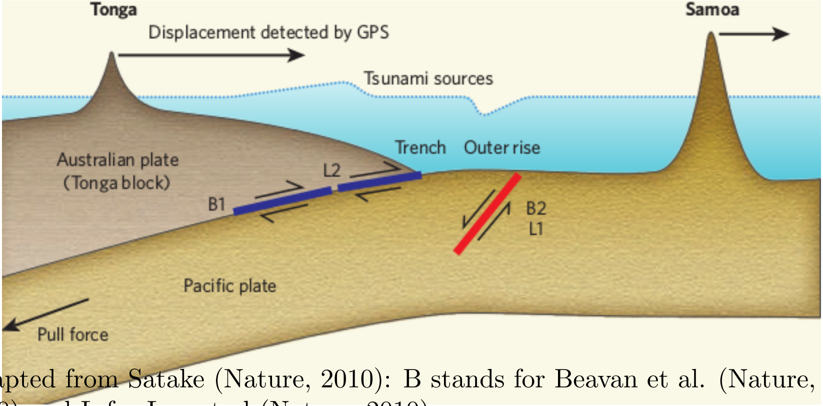
AGU Fall Meeting 2017, New Orleans

The 2009 Samoa earthquake: A doublet earthquake



Adapted from Lay et. al 2010 (nature)

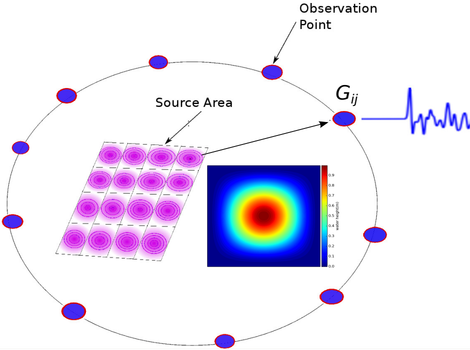
- The 2009 Samoa earthquake is a doublet where both normal and thrust faulting occurs.
- There is a disagreement regarding the order of occurrence and the time duration between two faulting.



Adapted from Satake (Nature, 2010): B stands for Beavan et al. (Nature, 2010) and L for Lay et al (Nature, 2010)

First Step (Green's function computation):

- Determine the source area and divide it into subregions.
- Create initial source over each subregion and compute Green's function G_{ij} from each of the source patches where G_{ij} is the GF from ith source to jth station.



Box function with cosine tapering:

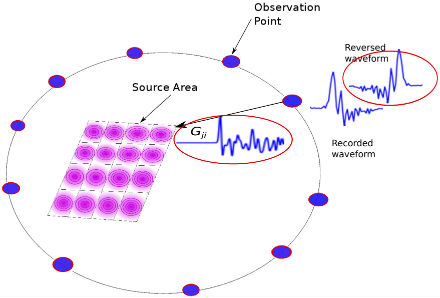
$\eta_i = 0.25 \times [1 + \cos(\pi x_i/L)] \times [1 + \cos(\pi y_i/L)]; \quad -L \leq x_i, y_i \leq L$
where x_i, y_i measure distance parallel and perpendicular to the trench axis, respectively and L is the dimension of the source patch.

Second Step (amplitude estimation):

- Convolve Green's function G_{ji} (because of reciprocity) with reversed observed waveforms.

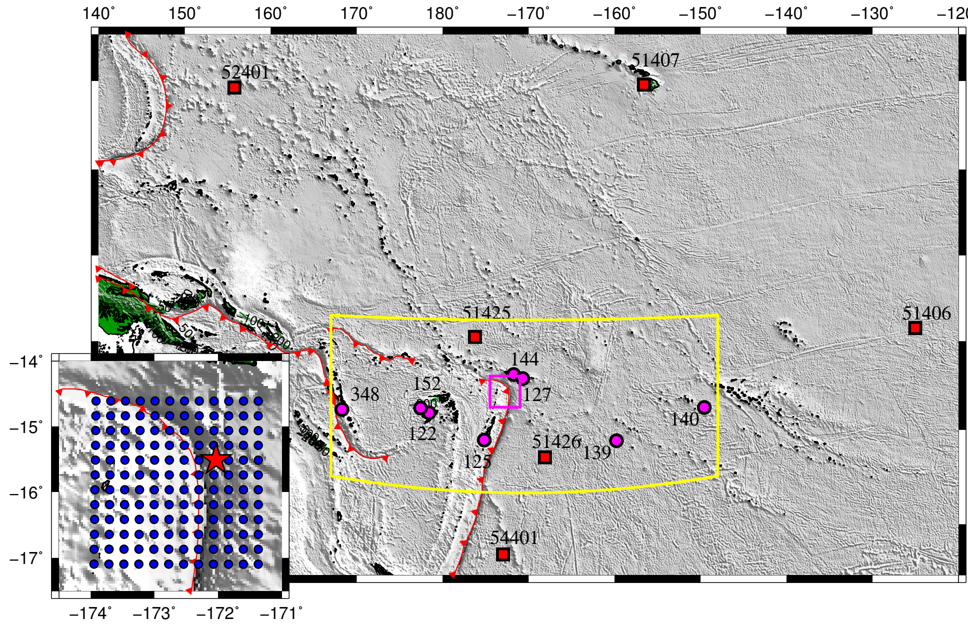
$$TR_j(\mathbf{s}_i, t) = G_i(\mathbf{x}_j, t) * d_j(T - t)$$

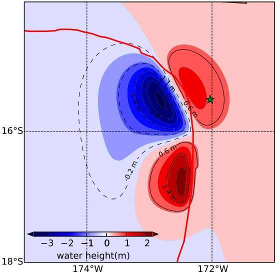
- Estimate amplitude at each subregion: Scaled wave height at the final time.

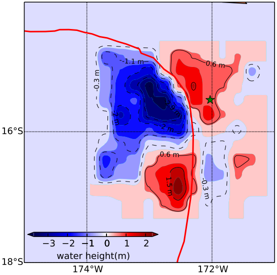


$$a_i^{TR}(t) = \frac{1}{p} \sum_{j=1}^{p} \frac{1}{|G_{ij}|^2} TR_j(\mathbf{s}_i, T - t - \delta t_i)$$

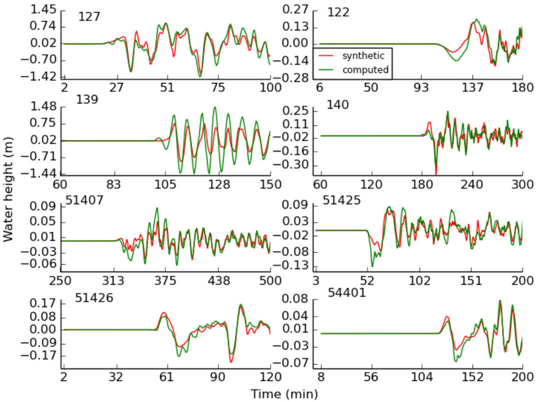
Computational domain with observations







Original(Top);
Reconstructed(lower)



Waveform agreement between predicted and synthetic observed waveforms.

Source model with 14 stations

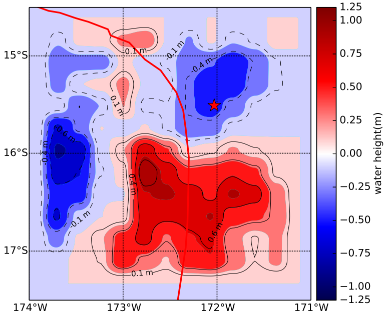
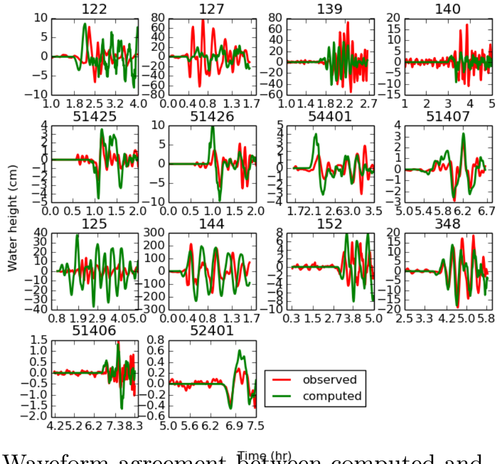


Figure : Recovered source model using 14 stations: 122, 127, 125, 139, 140, 144, 152, 348, 51406, 51407, 51425, 51426, 54401, 52401



Waveform agreement between computed and observed waveforms.

- Unrealistic source model because of uplift right to the trench line.

Adjoint Sensitivity approach: Determine optimal set of stations

The spatial distribution of error in the model space is

$$\delta a_{ij} = \frac{1}{|G_{ij}|^2} G_i(\mathbf{x}_j, t) * R_j(-t)|_\tau \tag{1}$$

where $*$ denotes convolution and $R_j(t) = \psi_j(t) - d_j(t)$.
The error contributed by the jth station as:

$$\delta E_j = \sqrt{\frac{1}{n} \sum_{i=1}^{n} |\delta a_{ij}|^2} \tag{2}$$

The station contributing maximum error are removed from the set for the next iteration.
The cumulative error contributed by the all stations in each inversion:

$$\delta E = \frac{1}{p} \sum_{j=1}^{p} \delta E_j. \tag{3}$$

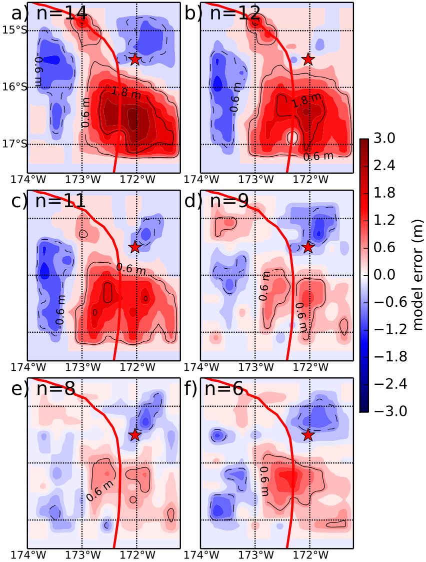
This is stored in order to determine optimal set of stations.

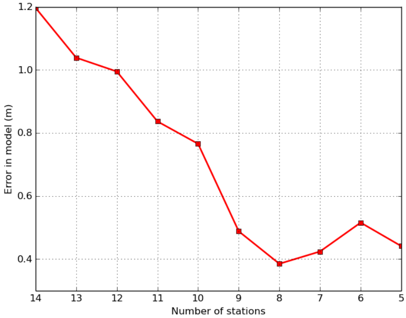
Determine optimal set of stations

Table : The optimal set of observations is selected by adjoint model.

No of stations	list of stations	model error	removal
14	122, 125, 127, 139, 140, 144, 152, 348, 51406, 51407, 51425, 51426, 54401, 52401	1.197	125
13	122, 127, 139, 140, 144, 152, 348, 51406, 51407, 51425, 51426, 54401, 52401	1.042	144
12	122, 127, 139, 140, 152, 348, 51406, 51407, 51425, 51426, 54401, 52401	0.994	152
11	122, 127, 139, 140, 348, 51406, 51407, 51425, 51426, 54401, 52401	0.836	51406
10	122, 127, 139, 140, 348, 51407, 51425, 51426, 54401, 52401	0.765	52401
9	122, 127, 139, 140, 348, 51407, 51425, 51426, 54401	0.488	348
8	122, 127, 139, 140, 51407, 51425, 51426, 54401	0.385	122
7	127, 139, 140,51407, 51425, 51426, 54401	0.424	139
6	127, 140,51407, 51425, 51426, 54401	0.515	51407
5	127, 140, 51425, 51426, 54401	0.441	51426

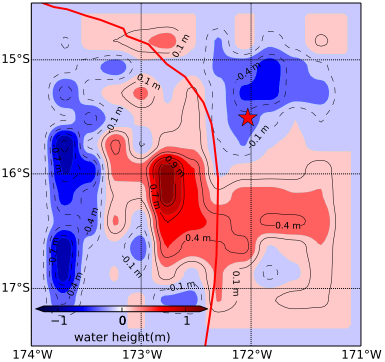
Determine optimal set of stations





Error in the source model estimated by using adjoint method with different set of stations.

Source model with optimal set of stations



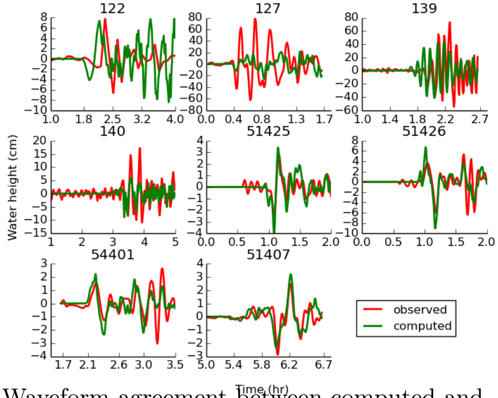
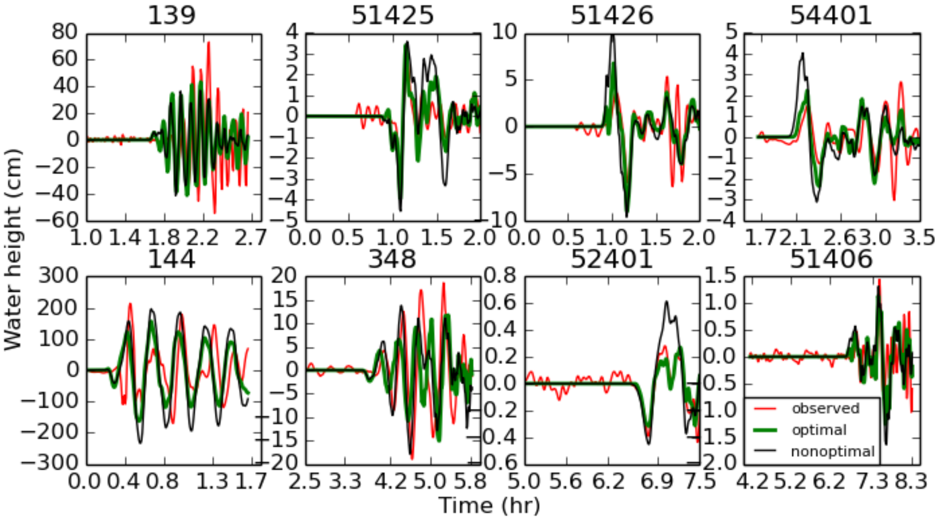


Figure : Recovered source model using 8 stations.

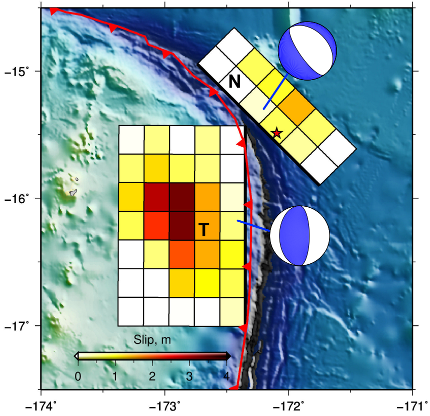
Waveform agreement between computed and observed waveforms.

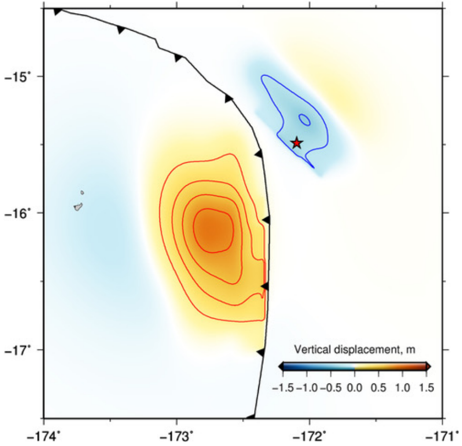
- Source model is more realistic as maximum uplift for thrust faulting occurs to the west of the Tonga trench.
- Waveform fittings improved when optimal set is used in source inversion



Comparison of the waveforms computed by using source model with optimal and non-optimal set of stations. Lower row shows the stations which are discarded by AS method.

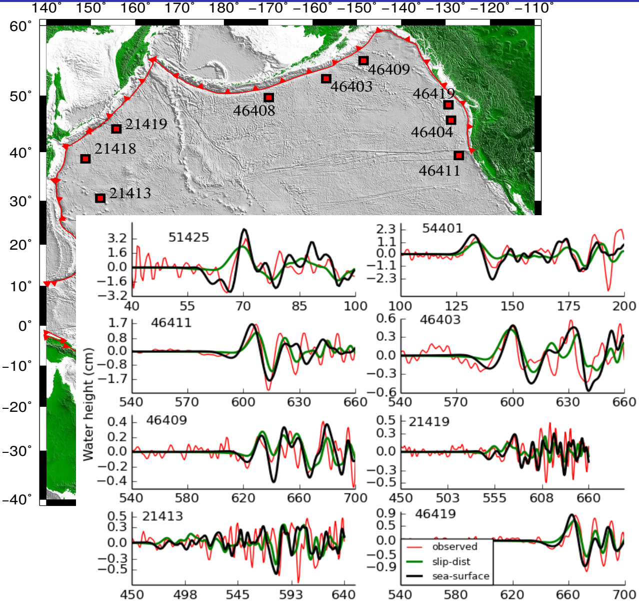
Slip distribution corresponding to the source model with optimal stations





- Our result suggests that the normal fault plan dips toward the north-east.

Waveform fitting at far-field observations, not used in source inversion



Conclusion

- The GFTRI with adjoint sensitivity method is implement in the 2009 Samoa earthquake which is believed to be a doublet. Our result shows that the method is capable of recovering source model of a complex earthquake with more details.

- The optimal set of stations determined by AS method improves the inversion result significantly. It suggests that judicious choice of observation in inversion is important for producing a reliable source model.

- Slip distribution is estimated from the reconstructed source model. The waveform comparison at near- and far-field stations shows that two-step inversion from recorded data to slip distribution of a fault via sea-surface displacement is feasible.