Chapter 6. Frequent Pattern Mining: Concepts and Apriori

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Introduction to Data Mining

Pattern Discovery: Definition

- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets
- Pattern discovery: Uncovering patterns from massive data
- Motivation examples:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?
  - What code segments likely contain copy-and-paste bugs?
  - What word sequences likely form phrases in this corpus?

Pattern Discovery: Why Is It Important?

- Finding inherent regularities in a data set
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: Discriminative pattern-based analysis
  - Cluster analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis

Frequent Patterns (Itemsets)

- Itemset: A set of one or more items
- k-itemset: $X = \{x_1, \ldots, x_k\}$
- (*absolute*) *support* (*count*) of X: Frequency or the number of occurrences of an itemset X
- (*relative*) *support*, *s:* The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is *frequent* if the support of X is no less than a *minsup* threshold

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

Let *minsup = 50%*

Freq. 1-itemsets:

Beer: 3 (60%); Nuts: 3 (60%)

Diaper: 4 (80%); Eggs: 3 (60%)

Freq. 2-itemsets:

{Beer, Diaper}: 3 (60%)

From Frequent Itemsets to Association Rules

- Association rules: $X \rightarrow Y$ (s, c)
  - Support, *s*: The probability that a transaction contains $X \cup Y$
  - Confidence, *c: The* conditional probability that a transaction containing X also contains *Y*
  - $c = sup(X \cup Y) / sup(X)$
- **Association rule mining**: Find all of the rules, $X \rightarrow Y$, with minimum support and confidence
- Frequent itemsets: Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Association rules: Let *minconf* = 50%
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

Tid	Items bought
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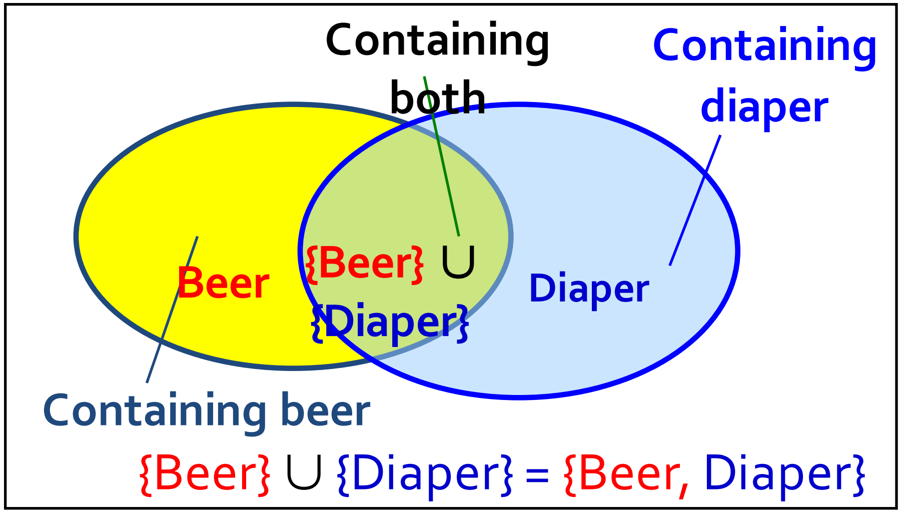


Note: Itemset: $X \cup Y$, a subtle notation!

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain?
  - TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Assuming (absolute) *minsup* = 1
  - Let's have a try

1-itemsets: $\{a_1\}$: 2, $\{a_2\}$: 2, …, $\{a_{50}\}$: 2, $\{a_{51}\}$: 1, …, $\{a_{100}\}$: 1,

2-itemsets: $\{a_1, a_2\}$: 2, …, $\{a_1, a_{50}\}$: 2, $\{a_1, a_{51}\}$: 1 …, …, $\{a_{99}, a_{100}\}$: 1, …

99-itemsets: $\{a_1, a_2, \ldots, a_{99}\}$: 1, …, $\{a_2, a_3, \ldots, a_{100}\}$: 1

100-itemset: $\{a_1, a_2, \ldots, a_{100}\}$: 1

- In total: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$ sub-patterns!

A too huge set for any computer to compute or store!

Expressing Patterns in Compressed Form: Closed Patterns

- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *frequent*, and there exists *no super-pattern* Y ⊃ X, ***with the same support as X***
  - Let Transaction DB TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "$\{a_1, \ldots, a_{50}\}$: 2"; P_2: "$\{a_1, \ldots, a_{100}\}$: 1"
- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - You will still be able to say: "$\{a_2, \ldots, a_{40}\}$: 2", "$\{a_5, a_{51}\}$: 1"

Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern $Y \supset X$, ~~with the same support as X~~
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "$\{a_1, \ldots, a_{100}\}$: 1"
- Max-pattern is a lossy compression!
  - We only know $\{a_1, \ldots, a_{40}\}$ is frequent
  - But we do not know the real support of $\{a_1, \ldots, a_{40}\}$, …, any more!
- Thus in many applications, mining closed-patterns is more desirable than mining max-patterns

The Downward Closure Property of Frequent Patterns: Apriori

- Observation: From TDB$_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - We get a frequent itemset: $\{a_1, \ldots, a_{50}\}$
  - Also, its subsets are all frequent: $\{a_1\}$, $\{a_2\}$, …, $\{a_{50}\}$, $\{a_1, a_2\}$, …, $\{a_1, \ldots, a_{49}\}$, …
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - **Apriori: Any subset of a frequent itemset must be frequent**
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?

A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - **Level-wise, join-based approach: Apriori** (Agrawal & Srikant@VLDB'94)
  - **Vertical data format approach: Eclat** (Zaki, Parthasarathy, Ogihara, Li@KDD'97)
  - **Frequent pattern projection and growth: FPgrowth** (Han, Pei, Yin @SIGMOD'00)

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find **frequent** (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k
F_k : Frequent itemset of size k

K := 1;
F_k := {frequent items}; // frequent 1-itemset

While (F_k != $\emptyset$) **do {** // when F_k is non-empty
  C_{k+1} := candidates generated from F_k; // candidate generation
  Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;
  k := k + 1
}

return $\cup_k F_k$ // return F_k generated at each level

The Apriori Algorithm: An Example

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

minsup = 2

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

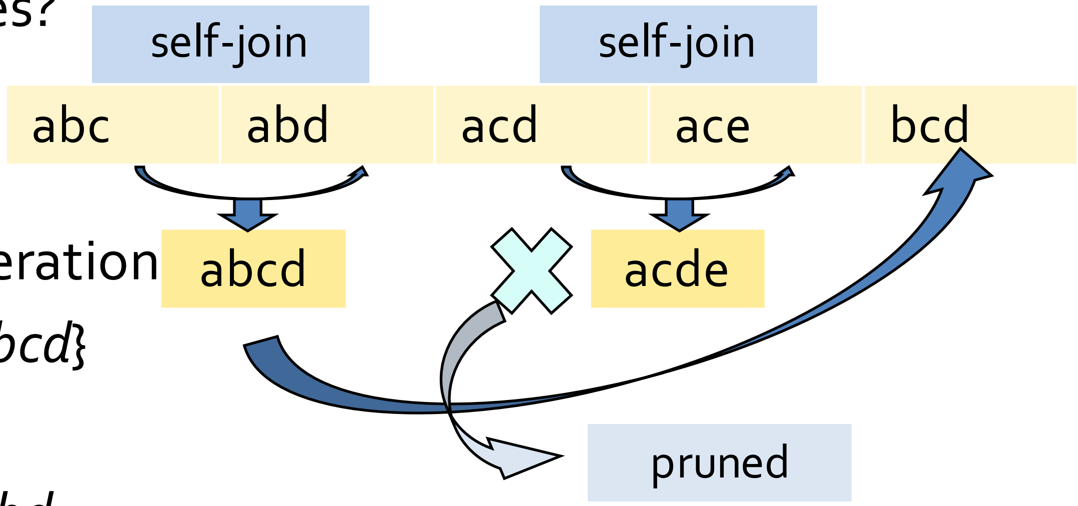
3rd scan

F_3

Itemset	sup
{B, C, E}	2

Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining F_k
  - Step 2: pruning
- Example of candidate-generation
  - $F_3 = \{abc, abd, acd, ace, bcd\}$
  - Self-joining: $F_3 * F_3$
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in F_3
  - $C_4 = \{abcd\}$



Candidate Generation: An SQL Implementation

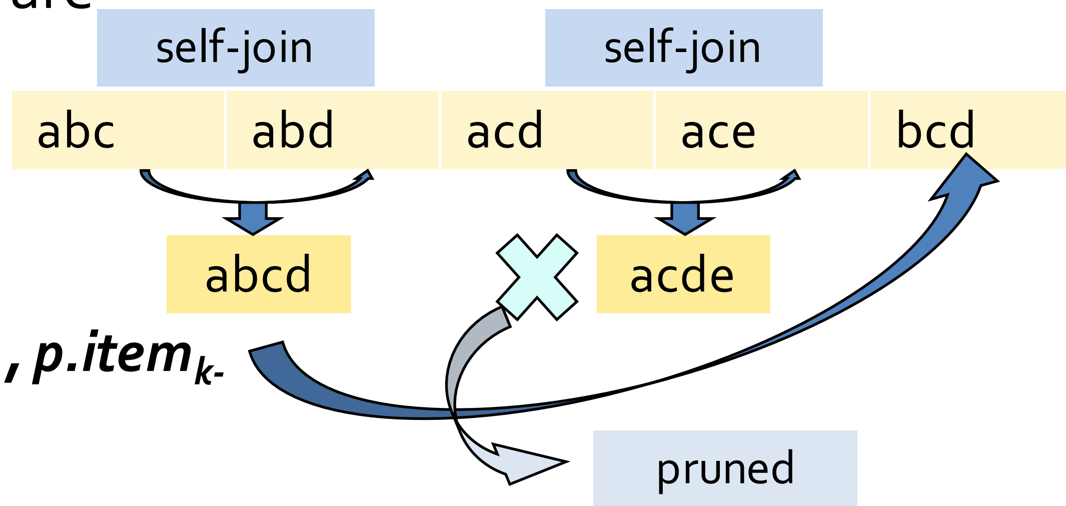
- Suppose the items in F_{k-1} are listed in an order
- Step 1: self-joining F_{k-1}

  insert into $\boldsymbol{C_k}$

  select $\boldsymbol{p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}}$

  from $\boldsymbol{F_{k-1}}$ ***as p***, $\boldsymbol{F_{k-1}}$ ***as q***

  where $\boldsymbol{p.item_1 = q.item_1, \ldots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}}$



- Step 2: pruning

  for all ***itemsets c in*** $\boldsymbol{C_k}$ do

  for all ***(k-1)-subsets s of c*** do

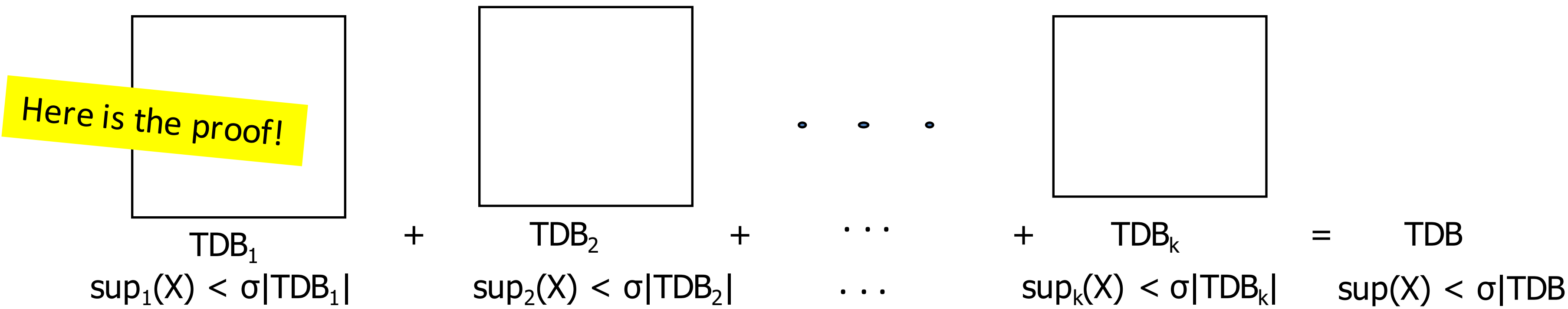
  if *(s is not in* F_{k-1}*)* **then delete** *c* **from** C_k

Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
  - Partitioning (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)
- Shrink the number of candidates
  - Hashing (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)
- Exploring special data structures
  - Tree projection (Agarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypecube decomposition (e.g., LCM: Uno, et al., 2004)

Partitioning for Parallelization

- Theorem: Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB



- Method: (A. Savasere, E. Omiecinski and S. Navathe, VLDB'95)
  - Scan 1: Partition database and find local frequent patterns
  - Scan 2: Consolidate global frequent patterns

Discussion

- How do you define frequent patterns in scientific knowledge discovery and technology exploration?
  - {“social spam detection”, “matrix factorization”}
  - {“social spam detection”, “Twitter”}
  - …

- Do you believe in the association: Diapers → Beer?

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