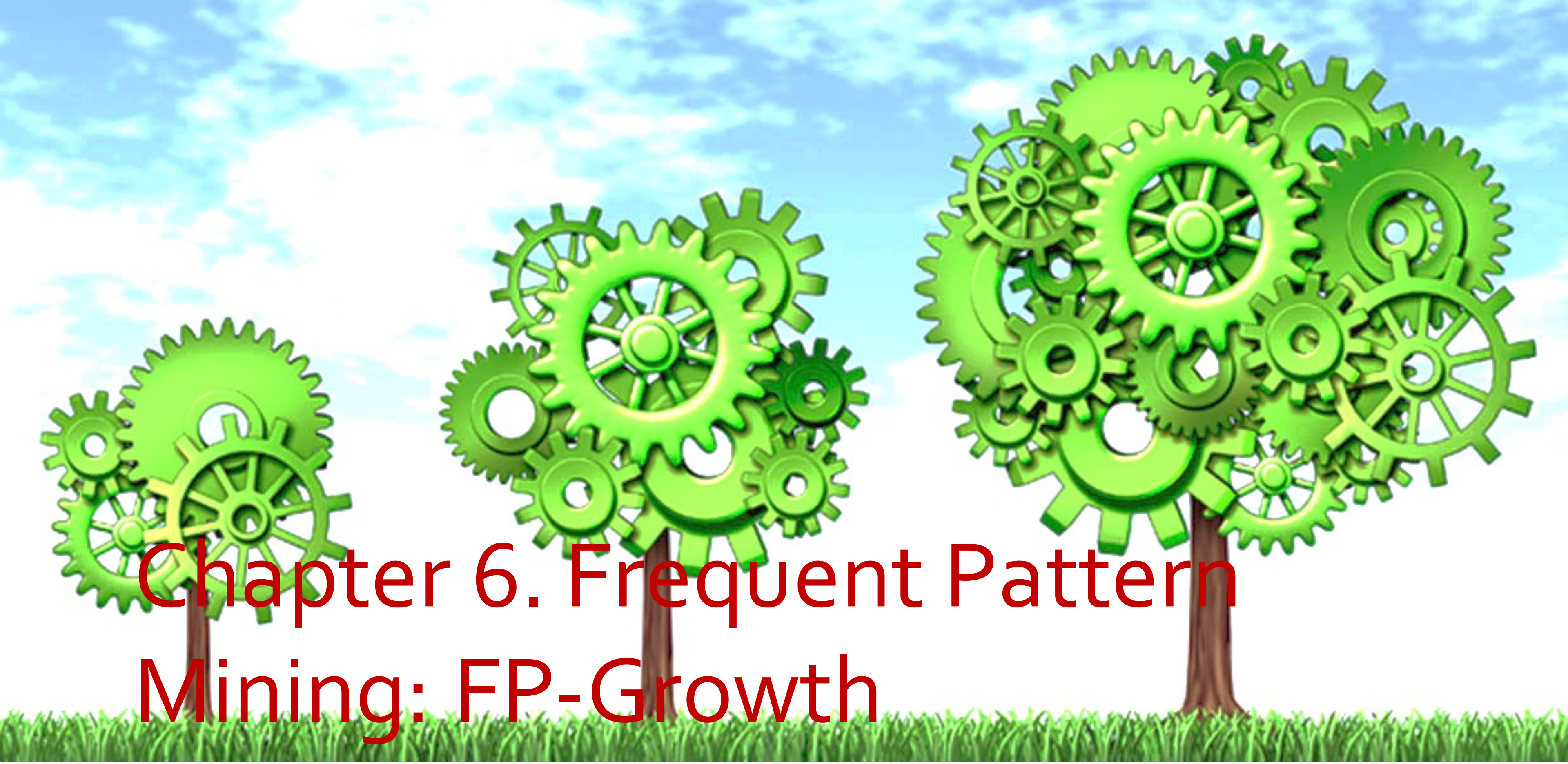

Chapter 6. Frequent Pattern Mining: FP-Growth

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CSE 40647/60647 Data Science Fall 2017

Introduction to Data Mining


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Pattern Discovery: Definition

- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets
- Pattern discovery: Uncovering patterns from massive data
- Motivation examples:
  - **What products were often purchased together?**
  - What are the subsequent purchases after buying an iPad?
  - What code segments likely contain copy-and-paste bugs?
  - What word sequences likely form phrases in this corpus?

Frequent Patterns (Itemsets)

- Itemset: A set of one or more items
- k-itemset: $X = \{x_1, \ldots, x_k\}$
- *(absolute) support (count)* of X: Frequency or the number of occurrences of an itemset X
- *(relative) support*, *s:* The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is *frequent* if the support of X is no less than a *minsup* threshold

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

Let *minsup = 50%*

Freq. 1-itemsets:

Beer: 3 (60%); Nuts: 3 (60%)

Diaper: 4 (80%); Eggs: 3 (60%)

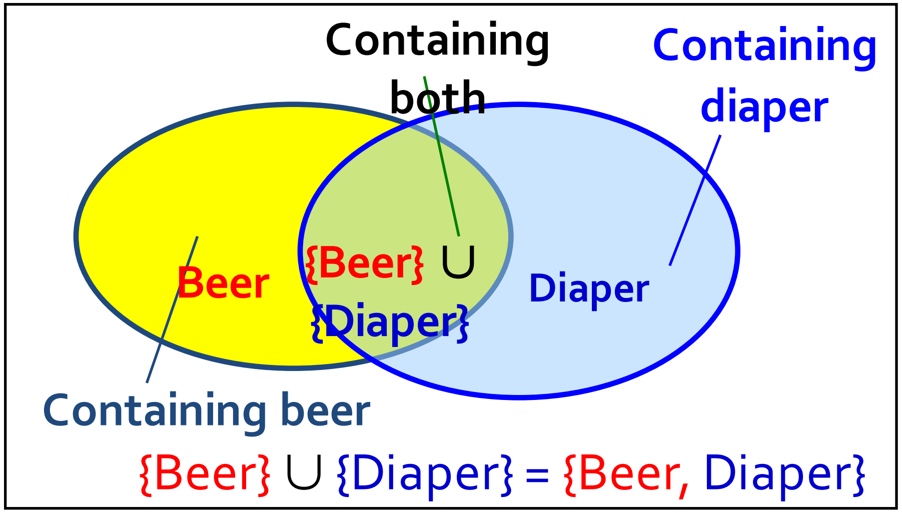
Freq. 2-itemsets:

{Beer, Diaper}: 3 (60%)

From Frequent Itemsets to Association Rules

- Association rules: $X \rightarrow Y$ (s, c)
  - Support, *s*: The probability that a transaction contains $X \cup Y$
  - Confidence, *c: The* conditional probability that a transaction containing X also contains *Y*
  - $c = sup(X \cup Y) / sup(X)$
- **Association rule mining**: Find all of the rules, $X \rightarrow Y$, with minimum support and confidence
- Frequent itemsets: Let *minsup = 50%*
  - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
  - Freq. 2-itemsets: {Beer, Diaper}: 3
- Association rules: Let *minconf* = 50%
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



Note: Itemset: $X \cup Y$, a subtle notation!

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB_1 contain?
  - TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Assuming (absolute) *minsup* = 1
  - Let's have a try

1-itemsets: $\{a_1\}$: 2, $\{a_2\}$: 2, …, $\{a_{50}\}$: 2, $\{a_{51}\}$: 1, …, $\{a_{100}\}$: 1,

2-itemsets: $\{a_1, a_2\}$: 2, …, $\{a_1, a_{50}\}$: 2, $\{a_1, a_{51}\}$: 1 …, …, $\{a_{99}, a_{100}\}$: 1, …

99-itemsets: $\{a_1, a_2, \ldots, a_{99}\}$: 1, …, $\{a_2, a_3, \ldots, a_{100}\}$: 1

100-itemset: $\{a_1, a_2, \ldots, a_{100}\}$: 1

- In total: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$ sub-patterns!

A too huge set for any computer to compute or store!

Expressing Patterns in Compressed Form: Closed Patterns

- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is *frequent*, and there exists *no super-pattern* Y ⊃ X, ***with the same support as X***
  - Let Transaction DB TDB_1: T_1: {a_1, …, a_{50}}; T_2: {a_1, …, a_{100}}
  - Suppose *minsup* = 1. How many closed patterns does TDB_1 contain?
    - Two: P_1: "{a_1, …, a_{50}}: 2"; P_2: "{a_1, …, a_{100}}: 1"
- Closed pattern is a lossless compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - You will still be able to say: "{a_2, …, a_{40}}: 2", "{a_5, a_{51}}: 1"

Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern Y ⊃ X, ~~with the same support as X~~
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB_1: T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - Suppose *minsup* = 1. How many max-patterns does TDB_1 contain?
    - One: P: "$\{a_1, \ldots, a_{100}\}$: 1"
- Max-pattern is a lossy compression!
  - We only know $\{a_1, \ldots, a_{40}\}$ is frequent
  - But we do not know the real support of $\{a_1, \ldots, a_{40}\}$, …, any more!
- Thus in many applications, mining closed-patterns is more desirable than mining max-patterns

The Downward Closure Property of Frequent Patterns: Apriori

- Observation: From TDB$_{1:}$ T$_1$: {a_1, …, a_{50}}; T$_2$: {a_1, …, a_{100}}
  - We get a frequent itemset: {a_1, …, a_{50}}
  - Also, its subsets are all frequent: {a_1}, {a_2}, …, {a_{50}}, {a_1, a_2}, …, {a_1, …, a_{49}}, …
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - **Apriori: Any subset of a frequent itemset must be frequent**
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?

A sharp knife for pruning!

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find **frequent** (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm: An Example

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

minsup = 2

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan

F_3

Itemset	sup
{B, C, E}	2

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k
F_k : Frequent itemset of size k

K := 1;
F_k := {frequent items}; // frequent 1-itemset

While (F_k != ∅) **do {** // when F_k is non-empty
  C_{k+1} := candidates generated from F_k; // candidate generation
  Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;
  k := k + 1
}

return $\cup_k F_k$ // return F_k generated at each level

FPGrowth: Mining Frequent Patterns by Pattern Growth

- Idea: Frequent pattern growth (FPGrowth)
  - Find frequent single items and partition the database based on each such item
  - Recursively grow frequent patterns by doing the above for each partitioned database (also called *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed
- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

YouTube: https://www.youtube.com/watch?v=LXx1xKF9oDg

Example: Construct FP-tree from a Transactional DB

TID	Items in the Transaction	Ordered, frequent items
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

Answer:
f:4, a:3, c:4, b:3, m:3, p:3;
fm: 3, cm: 3, am: 3, cp:3;
fcm: 3, fam:3, cam: 3;
fcam: 3.

1. Scan DB once, find single item frequent pattern:

Let min_support = 3

f:4, a:3, c:4, b:3, m:3, p:3

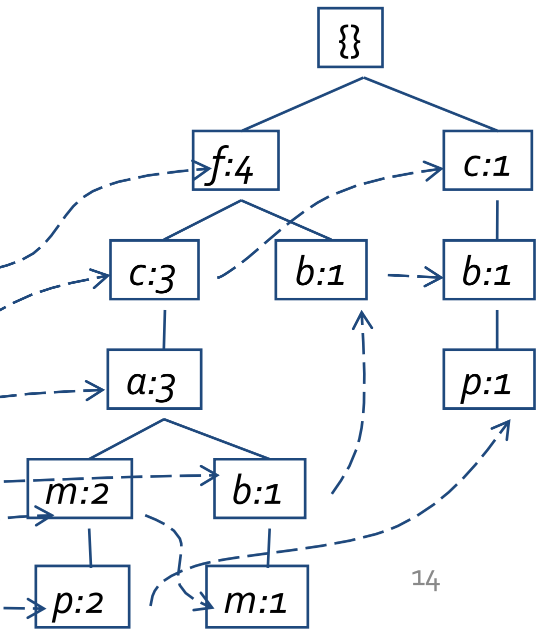
2. Sort frequent items in frequency descending order, f-list

F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

Header Table

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

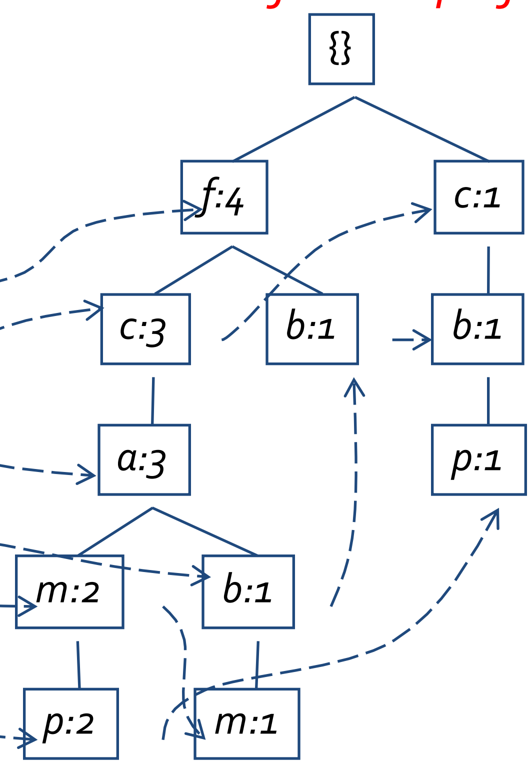


Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - Patterns containing p: p's conditional database: *fcam:2, cb:1*
  - Patterns having m but no p: m's conditional database: *fca:2, fcab:1*
  - …… ……
- *p*'s conditional pattern base: *transformed prefix paths* of item *p*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



***Conditional* pattern bases**

Item	*Conditional pattern base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

Mine Each Conditional Pattern-Base Recursively

***Conditional* pattern bases**

item	***cond. pattern base***
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

min_support = 3

For each conditional pattern-base

- Mine single-item patterns
- Construct its **cond. FP-tree** & mine it

p-conditional PB: ***fcam:2, cb:1 → c: 3***

m-conditional PB: ***fca:2, fcab:1 → fca: 3***

b-conditional PB: ***fca:1, f:1, c:1 → ϕ***

a-conditional PB: ***fc:3 → fc:3***

c-conditional PB: ***f:3 → f:3***

Mine Each Conditional Pattern-Base Recursively

Conditional **pattern bases**

item	*cond. pattern base*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

min_support = 3

For each conditional pattern-base

- Mine single-item patterns
- Construct its **cond. FP-tree** & **mine** it

p-conditional PB: ***fcam:2, cb:1 → c: 3***

m-conditional PB: ***fca:2, fcab:1 → fca: 3***

b-conditional PB: ***fca:1, f:1, c:1 → ϕ***

a-conditional PB: ***fc:3 → fc:3***

c-conditional PB: ***f:3 → f:3***

{} – *f:3* – *c:3* – *a:3*
m-cond. FP-tree

{} – *f:3* – *c:3*
am-cond. FP-tree

{} – *f:3*
cm-cond. FP-tree

{} – *f:3*
cam-cond. FP-tree

mine(<f:3, c:3, a:3>|m)

→ (am:3) + mine(<f:3, c:3>|am)

→ (cam:3) + (fam:3) + mine (<f:3>|cam)

→ (fcam:3)

→ (cm:3) + mine(<f:3>|cm)

→ (fcm:3)

→ (fm:3)

Mine Each Conditional Pattern-Base Recursively

Conditional pattern bases

item	*cond. pattern base*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

min_support = 3

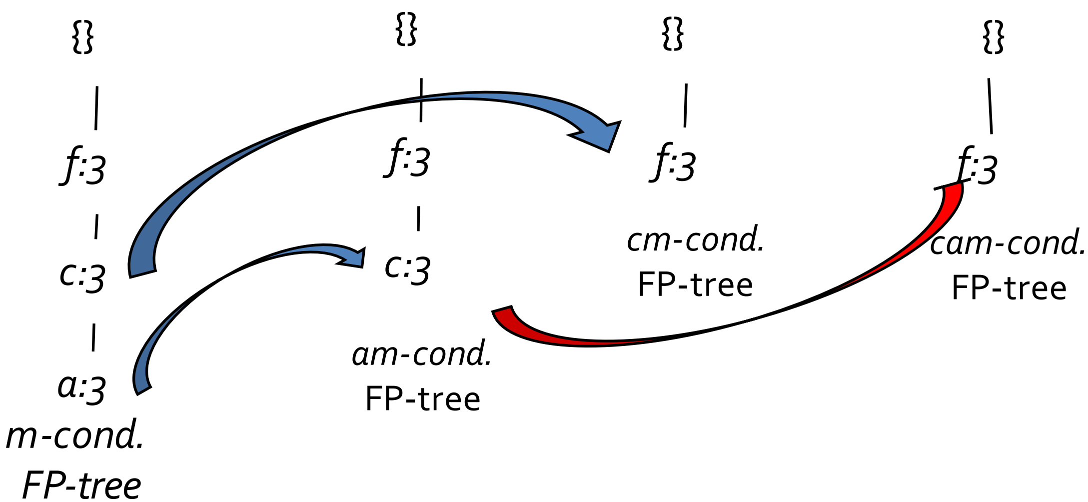
For each conditional pattern-base

- Mine single-item patterns
- Construct its cond. FP-tree & mine it

p-conditional PB: ***fcam:2, cb:1 → c: 3***

m-conditional PB: ***fca:2, fcab:1 → fca: 3***

b-conditional PB: ***fca:1, f:1, c:1 → ϕ***



Actually, for single branch FP-tree, all frequent patterns can be generated in one shot

m: 3
fm: 3, cm: 3, am: 3
fcm: 3, fam:3, cam: 3
fcam: 3

Try FP-Growth?

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

minsup = 2

1^{st} scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3^{rd} scan

F_3

Itemset	sup
{B, C, E}	2

Try WikiBooks' Example

- https://en.wikibooks.org/wiki/Data_Mining_Algorithms_In_R/Frequent_Pattern_Mining/The_FP-Growth_Algorithm

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