# Chapter 7. Advanced Frequent Pattern Mining: Diverse Patterns

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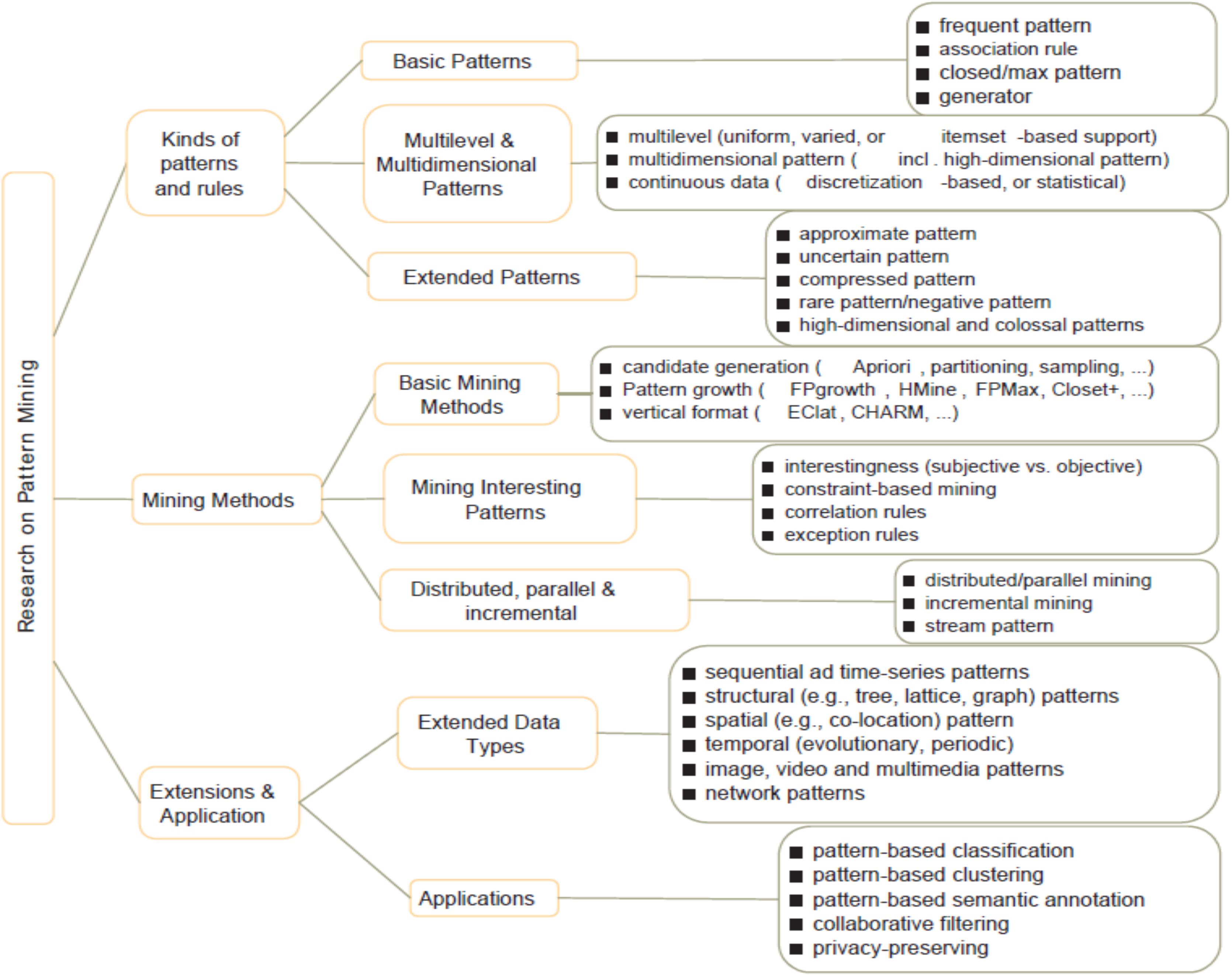
Data Science

# Research on Pattern Mining: A Road Map



# Advanced Frequent Pattern Mining

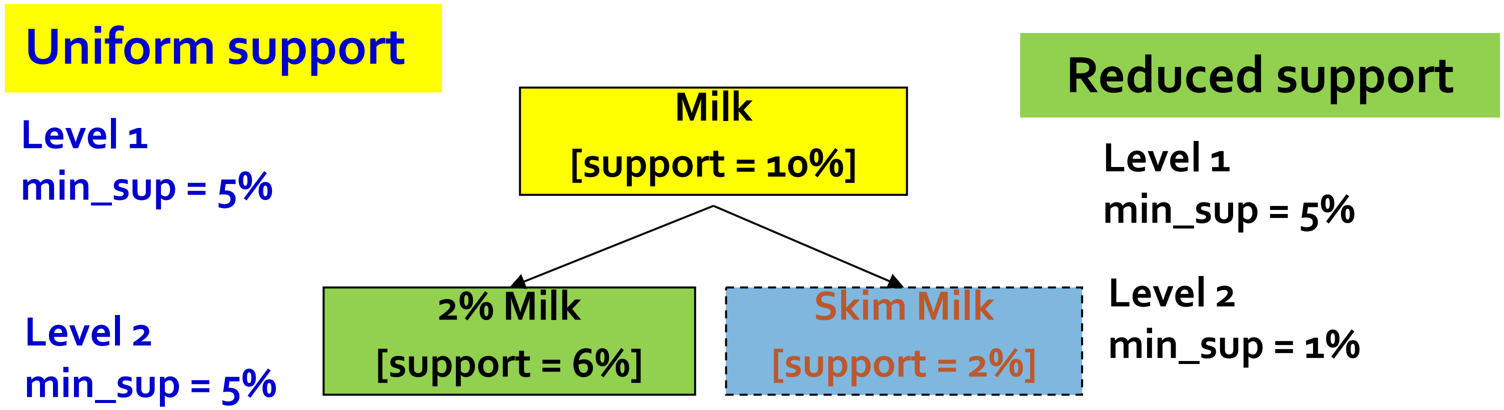
- **Mining Diverse Patterns**
- Constraint-Based Frequent Pattern Mining
- Sequential Pattern Mining
- Graph Pattern Mining

# Mining Diverse Patterns

- Mining Multiple-Level Associations
- Mining Multi-Dimensional Associations
- Mining Quantitative Associations
- Mining Negative Correlations

# Mining Multiple-Level Frequent Patterns

- Items often form hierarchies
  - Ex.: Dairyland 2% milk; Wonder wheat bread
- How to set min-support thresholds?
  - Uniform min-support across multiple levels (reasonable?)
  - Level-reduced min-support: Items at the lower level are expected to have lower support



# Redundancy Filtering at Mining Multi-Level Associations

- Multi-level association mining may generate many redundant rules
- Redundancy filtering: Some rules may be redundant due to "ancestor" relationships between items

  (Suppose the 2% milk sold is about ¼ of milk sold in gallons)

  – milk $\Rightarrow$ wheat bread [support = 8%, confidence = 70%] (1)
  – 2% milk $\Rightarrow$ wheat bread [support = 2%, confidence = 72%] (2)
- A rule is *redundant* if its support is close to the "expected" value, according to its "ancestor" rule, and it has a similar confidence as its "ancestor"
  – Rule (1) is an ancestor of rule (2), which one to prune?

# Customized Min-Supports for Different Kinds of Items

- We have used the same min-support threshold for all the items or item sets to be mined in each association mining
- In reality, some items (e.g., diamond, watch, …) are valuable but less frequent
- It is necessary to have customized min-support settings for different kinds of items
- One Method: Use group-based "individualized" min-support
  - E.g., {diamond, watch}: 0.05%; {bread, milk}: 5%; …

# Mining Multi-Dimensional Associations

- Single-dimensional rules (e.g., items are all in "product" dimension)
  - buys(X, "milk") $\Rightarrow$ buys(X, "bread")
- Multi-dimensional rules (i.e., items in $\geq$ 2 dimensions or predicates)
  - Inter-dimension association rules (*no repeated predicates*)
    - age(X, "18-25") $\wedge$ occupation(X, "student") $\Rightarrow$ buys(X, "coke")
  - Hybrid-dimension association rules (*repeated predicates*)
    - age(X, "18-25") $\wedge$ buys(X, "popcorn") $\Rightarrow$ buys(X, "coke")

# Mining Quantitative Associations

- Mining quantitative associations
  - Ex.: Gender = female $\Rightarrow$ Wage: mean=$7/hr (overall mean = $9)
  - LHS: a subset of the population
  - RHS: an ***extraordinary*** behavior of this subset
- Rule condition can be categorical or numerical
  - Ex.: (Gender = female) ^ (South = yes) $\Rightarrow$ mean wage = $6.3/hr
  - Ex.: Education in [14-18] (yrs) $\Rightarrow$ mean wage = $11.64/hr
- Data cube technology?

# Rare Patterns vs. Negative Patterns

- Rare patterns
  - Very low support but interesting (e.g., buying Rolex watches)
- Negative patterns
  - Negatively correlated: Unlikely to happen together
  - Ex.: Since it is unlikely that the same customer buys both a Ford Expedition (an SUV car) and a Ford Fusion (a hybrid car), buying a Ford Expedition and buying a Ford Fusion are likely negatively correlated patterns
  - How to define negative patterns?

# Defining Negative Correlated Patterns

- A support-based definition
  - If itemsets A and B are both frequent but rarely occur together, i.e., $sup(A \cup B) \ll sup(A) \times sup(B)$
  - Then A and B are negatively correlated

Does this remind you the definition of *lift*?

- Is this a good definition for large transaction datasets?
- Ex.: Suppose a store sold two needle packages A and B 100 times each, but only one transaction contained both A and B
  - When there are in total 200 transactions, we have
    - $s(A \cup B) = 0.005$, $s(A) \times s(B) = 0.25$, $s(A \cup B) \ll s(A) \times s(B)$
  - But when there are $10^5$ transactions, we have
    - $s(A \cup B) = 1/10^5$, $s(A) \times s(B) = 1/10^3 \times 1/10^3$, $s(A \cup B) > s(A) \times s(B)$
  - What is the problem? — Null transactions: The support-based definition is not null-invariant!

# Defining Negative Correlation: Need Null-Invariance in Definition

- A good definition on negative correlation should take care of the null-invariance problem
  - Whether two itemsets A and B are negatively correlated should not be influenced by the number of null-transactions
- A Kulczynski measure-based definition
  - If itemsets A and B are frequent but $(P(A|B) + P(B|A))/2 < \epsilon$, where $\epsilon$ is a negative pattern threshold, then A and B are negatively correlated
- For the same needle package problem:
  - No matter there are in total 200 or $10^5$ transactions
  - If $\epsilon = 0.01$, we have $(P(A|B) + P(B|A))/2 = (0.01 + 0.01)/2 < \epsilon$

# Advanced Frequent Pattern Mining

- Mining Diverse Patterns
- Constraint-Based Frequent Pattern Mining
- **Sequential Pattern Mining**
- Graph Pattern Mining

# Pattern Mining Methods

| Pattern | Closed Pattern (Concepts) | Idea 1: Pattern candidate generation and pruning | Idea 2: Pattern growth |
|---|---|---|---|
| **Frequent pattern (itemset)** | ? | ? | ? |
| **Sequential pattern** | ? | ? | ? |
| **Graph pattern** | ? | ? | ? |

# Pattern Mining Methods

| Pattern | Closed Pattern (Concepts) | Idea 1: Pattern candidate generation and pruning | Idea 2: Pattern growth |
|---|---|---|---|
| **Frequent pattern (itemset)** | Closed frequent itemset | Apriori (1994) | FP-Growth (2000) |
| **Sequential pattern** | Closed seq. pattern | GSP (1996) | PrefixSpan (2004) |
| **Graph pattern** | Closed graph pattern | FSG (2000-2001) | gSpan (2002) |

# Sequential Patterns: Applications

- Sequential pattern mining has broad applications
  - Customer shopping sequences
    - Purchase a laptop first, then a digital camera, and then a smartphone, within 6 months
  - Medical treatments, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, …
  - Weblog click streams, calling patterns, …
  - Software engineering: Program execution sequences, …
  - Biological sequences: DNA, protein, …

# Sequential Pattern and Sequential Pattern Mining

- Sequential pattern mining: Given a set of sequences, find the complete set of frequent subsequences (i.e., satisfying the min_sup threshold)

A *sequence database*

| SID | Sequence |
|---|---|
| 10 | <a(abc)(ac)d(cf)> |
| 20 | <(ad)c(bc)(ae)> |
| 30 | <(ef)(ab)(df)cb> |
| 40 | <eg(af)cbc> |

A *sequence*: < (ef) (ab) (df) c b >

- An element may contain a set of items (also called events)
- Items within an element are unordered and we list them alphabetically

<a(bc)dc> is a *subsequence* of <a(abc)(ac)d(cf)>

- Given support threshold min_sup = 2, <(ab)c> is a sequential pattern

http://hanj.cs.illinois.edu/pdf/span01.pdf

# Sequence vs Element/Itemset/Event vs Item/Instance

Let $I = \{i_1, i_2, \ldots, i_n\}$ be a set of all **items**. An **itemset** is a subset of items. A **sequence** is an ordered list of itemsets. A sequence $s$ is denoted by $\langle s_1 s_2 \cdots s_l \rangle$, where $s_j$ is an itemset, i.e., $s_j \subseteq I$ for $1 \le j \le l$. $s_j$ is also called an **element** of the sequence, and denoted as $(x_1 x_2 \cdots x_m)$, where $x_k$ is an item, i.e., $x_k \in I$ for $1 \le k \le m$. For brevity, the brackets are omitted if an element has only one item. That is, element $(x)$ is written as $x$. An item can occur at most once in an element of a sequence, but can occur multiple times in different elements of a sequence. The

# Sequential Pattern Mining Algorithms

- Algorithm requirement: Efficient, scalable, finding complete set, incorporating various kinds of user-specific constraints
- The Apriori property still holds: If a subsequence $s_1$ is infrequent, none of $s_1$'s super-sequences can be frequent
- Representative algorithms
  - Apriori-based Generalized Sequential Patterns: GSP (Srikant & Agrawal @ EDBT'96)
  - Pattern-growth methods: PrefixSpan (Pei, et al. @TKDE'04)
- Mining **closed** sequential patterns: CloSpan (Yan, et al. @SDM'03)
- Constraint-based sequential pattern mining

# GSP: Apriori-Based Sequential Pattern Mining

| SID | Sequence |
|---|---|
| 10 | <(bd)cb(ac)> |
| 20 | <(bf)(ce)b(fg)> |
| 30 | <(ah)(bf)abf> |
| 40 | <(be)(ce)d> |
| 50 | <a(bd)bcb(ade)> |

- Initial candidates: All singleton sequences
  - <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>
- Scan DB once, count support for each candidate
- Generate length-2 candidate sequences

*min_sup* = 2

| Cand. | sup |
|---|---|
| <a> | 3 |
| <b> | 5 |
| <c> | 4 |
| <d> | 3 |
| <e> | 3 |
| <f> | 2 |
| <g> | 1 |
| <h> | 1 |

| | <a> | <b> | <c> | <d> | <e> | <f> |
|---|---|---|---|---|---|---|
| <a> | <aa> | <ab> | <ac> | <ad> | <ae> | <af> |
| <b> | <ba> | <bb> | <bc> | <bd> | <be> | <bf> |
| <c> | <ca> | <cb> | <cc> | <cd> | <ce> | <cf> |
| <d> | <da> | <db> | <dc> | <dd> | <de> | <df> |
| <e> | <ea> | <eb> | <ec> | <ed> | <ee> | <ef> |
| <f> | <fa> | <fb> | <fc> | <fd> | <fe> | <ff> |

Length-2 candidates:
$36 + 15 = 51$
Without Apriori pruning:
$8*8+8*7/2=92$ candidates

| | <a> | <b> | <c> | <d> | <e> | <f> |
|---|---|---|---|---|---|---|
| <a> | | <(ab)> | <(ac)> | <(ad)> | <(ae)> | <(af)> |
| <b> | | | <(bc)> | <(bd)> | <(be)> | <(bf)> |
| <c> | | | | <(cd)> | <(ce)> | <(cf)> |
| <d> | | | | | <(de)> | <(df)> |
| <e> | | | | | | <(ef)> |
| <f> | | | | | | |

GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)

# GSP Mining and Pruning

- Repeat (for each level (i.e., length-k))
  - Scan DB to find length-k frequent sequences
  - Generate length-(k+1) candidate sequences from length-k frequent sequences using Apriori
  - set k = k+1
- Until no frequent sequence or no candidate can be found

# PrefixSpan: A Pattern-Growth Approach

- Prefix and suffix
  - Given <a(abc)(ac)d(cf)>
  - Prefixes: <a>, <aa>, <a(ab)>, <a(abc)>, …
  - Prefixes-based projection
- PrefixSpan Mining: Prefix Projections
  - Step 1: Find length-1 sequential patterns
    - <a>, <b>, <c>, <d>, <e>, <f>
  - Step 2: Divide search space and mine each projected DB
    - <a>-projected DB,
    - <b>-projected DB,
    - …
    - <f>-projected DB, …

| SID | Sequence |
|---|---|
| 10 | <a(abc)(ac)d(cf)> |
| 20 | <(ad)c(bc)(ae)> |
| 30 | <(ef)(ab)(df)cb> |
| 40 | <eg(af)cbc> |

| Prefix | Suffix (Projection) |
|---|---|
| <a> | <(abc)(ac)d(cf)> |
| <aa> | <(_bc)(ac)d(cf)> |
| <ab> | <(_c)(ac)d(cf)> |

PrefixSpan (Prefix-projected Sequential pattern mining) Pei, et al. @TKDE'04

http://hanj.cs.illinois.edu/pdf/span01.pdf

# Advanced Frequent Pattern Mining

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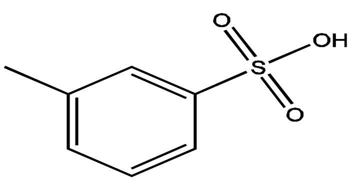
- Sequential Pattern Mining
- **Graph Pattern Mining**

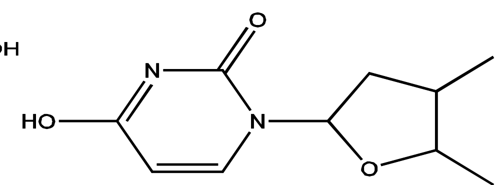
# Frequent (Sub)Graph Patterns

- Given a labeled graph dataset $D = \{G_1, G_2, \ldots, G_n\}$, the supporting graph set of a subgraph $g$ is $D_g = \{G_i \mid g \subseteq G_i, G_i \in D\}$.
  - support($g$) = $|D_g| / |D|$
- A (sub)graph $g$ is ***frequent*** if *support*($g$) ≥ *min_sup* Ex.: Chemical structures
- Alternative:
  - Mining frequent subgraph patterns from a single large graph or network

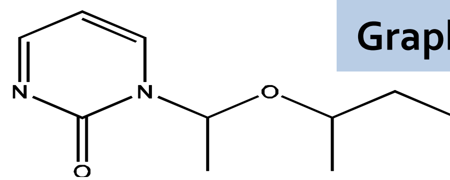
**Graph Dataset**




(A)




(B)

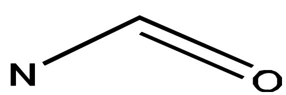



(C)

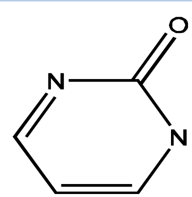
min_sup = 2

**Frequent Graph Patterns**

(1)




(2)

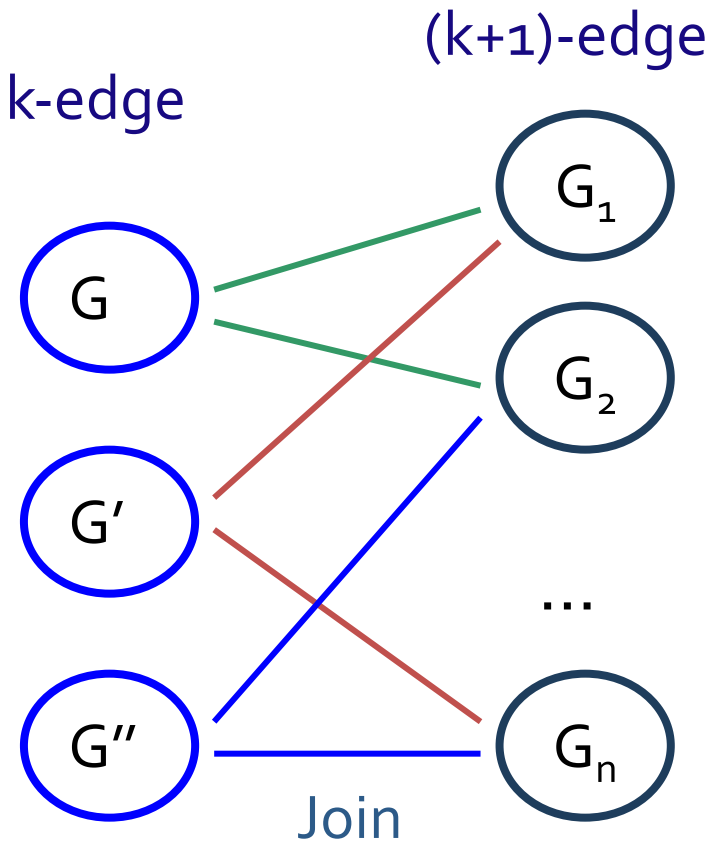


# Graph Pattern Mining: Applications

- Bioinformatics
  - Gene networks, protein interactions, metabolic pathways
- Chem-informatics: Mining chemical compound structures
- Social networks, web communities, tweets, …
- Cell phone networks, computer networks, …
- Web graphs, XML structures, semantic Web, information networks
- Software engineering: program execution flow analysis
- Building blocks for graph classification, clustering, compression, comparison, and correlation analysis
- Graph indexing and graph similarity search

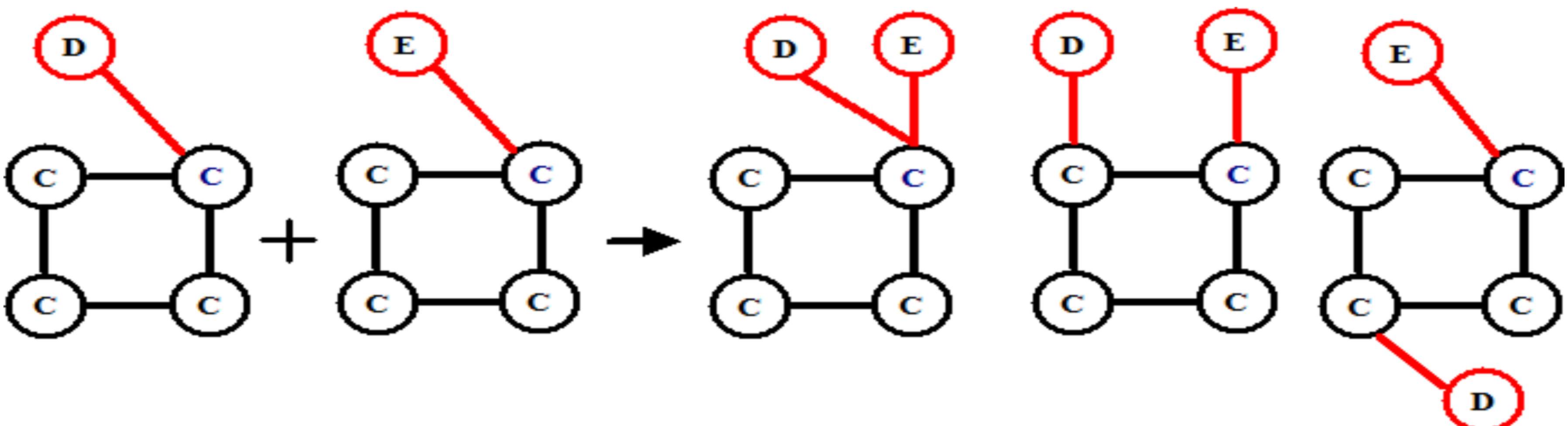
# Apriori-Based Approach

- The Apriori property (anti-monotonicity): A size-*k* subgraph is frequent if and only if all of its subgraphs are frequent
- A candidate size-(*k*+1) edge/vertex subgraph is generated if its corresponding two *k*-edge/vertex subgraphs are frequent
- Iterative mining process:
  - Candidate-generation → candidate pruning → support counting → candidate elimination



# Candidate Generation: Vertex Growing vs. Edge Growing

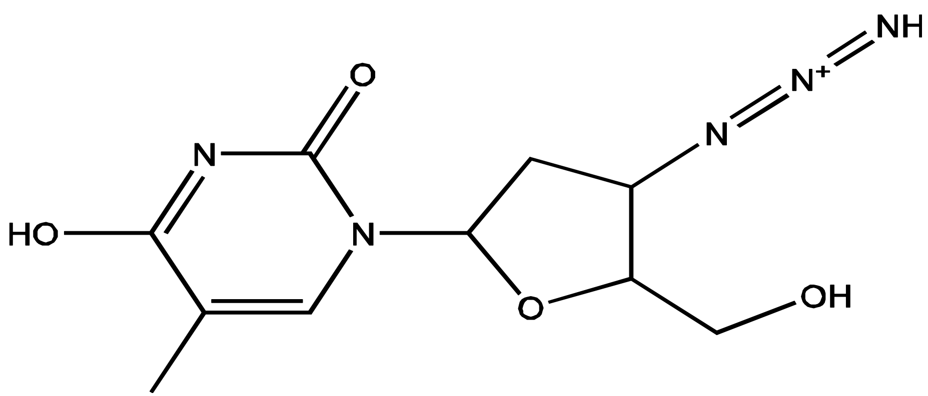
- Methodology: breadth-search, Apriori joining two size-k graphs
  - Many possibilities at generating size-(k+1) candidate graphs




- Generating new graphs with one more vertex
  - AGM (Inokuchi, et al., PKDD'00)
- Generating new graphs with one more edge
  - FSG (Kuramochi and Karypis, ICDM'01)
- Performance shows via edge growing is more efficient

# Why Mining Closed Graph Patterns?

- Challenge: An **n**-edge frequent graph may have $2^n$ subgraphs
- Motivation: Explore *closed frequent subgraphs* to handle graph pattern explosion problem
- A frequent graph G is *closed* if there exists no supergraph of G that carries the same support as G



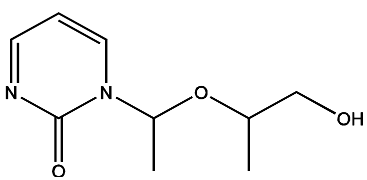

If this subgraph is *closed* in the graph dataset, it implies that none of its frequent super-graphs carries the same support

- *Lossless compression:* Does not contain non-closed graphs, but still ensures that the mining result is complete
- Algorithm CloseGraph: Mines closed graph patterns directly

# Experiment and Performance Comparison

- The AIDS antiviral screen compound dataset from NCI/NIH
- The dataset contains 43,905 chemical compounds
- Discovered Patterns: The smaller minimum support, the bigger and more interesting subgraph patterns discovered

20%



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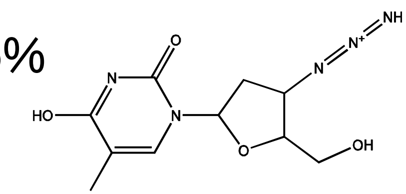

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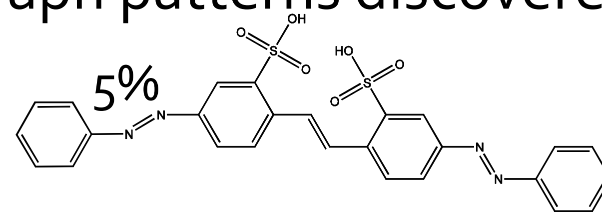
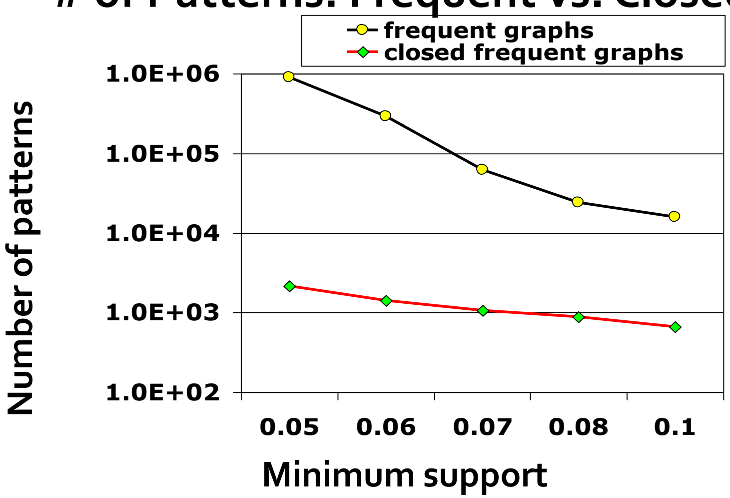
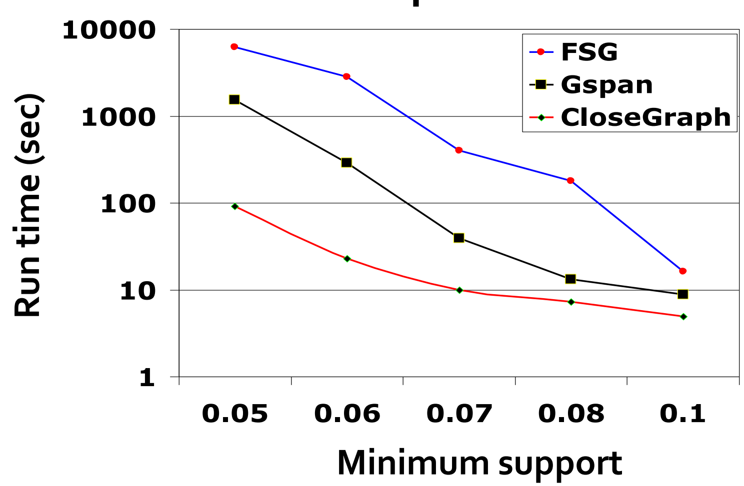


**# of Patterns: Frequent vs. Closed**




**Runtime: Frequent vs. Closed**



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