Chapter 7. Advanced Frequent Pattern Mining

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Introduction to Data Mining

Research on Pattern Mining: A Road Map

- Research on Pattern Mining
  - Kinds of patterns and rules
    - Basic Patterns
      - frequent pattern
      - association rule
      - closed/max pattern
      - generator
    - Multilevel & Multidimensional Patterns
      - multilevel (uniform, varied, or itemset-based support)
      - multidimensional pattern (incl. high-dimensional pattern)
      - continuous data (discretization-based, or statistical)
    - Extended Patterns
      - approximate pattern
      - uncertain pattern
      - compressed pattern
      - rare pattern/negative pattern
      - high-dimensional and colossal patterns
  - Mining Methods
    - Basic Mining Methods
      - candidate generation (Apriori, partitioning, sampling, ...)
      - Pattern growth (FPgrowth, HMine, FPMax, Closet+, ...)
      - vertical format (EClat, CHARM, ...)
    - Mining Interesting Patterns
      - interestingness (subjective vs. objective)
      - constraint-based mining
      - correlation rules
      - exception rules
    - Distributed, parallel & incremental
      - distributed/parallel mining
      - incremental mining
      - stream pattern
  - Extensions & Application
    - Extended Data Types
      - sequential ad time-series patterns
      - structural (e.g., tree, lattice, graph) patterns
      - spatial (e.g., co-location) pattern
      - temporal (evolutionary, periodic)
      - image, video and multimedia patterns
      - network patterns
    - Applications
      - pattern-based classification
      - pattern-based clustering
      - pattern-based semantic annotation
      - collaborative filtering
      - privacy-preserving

Advanced Frequent Pattern Mining

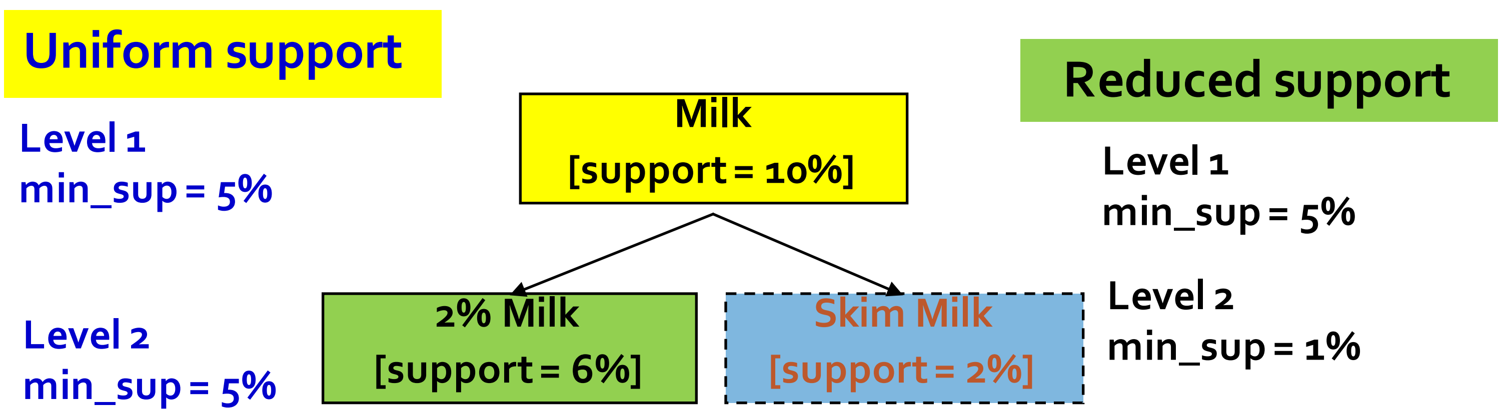
- **Mining Diverse Patterns**
- Constraint-Based Frequent Pattern Mining
- Mining High-Dimensional Data and Colossal Patterns
- Sequential Pattern Mining
- Graph Pattern Mining

Mining Diverse Patterns

- Mining Multiple-Level Associations
- Mining Multi-Dimensional Associations
- Mining Quantitative Associations
- Mining Negative Correlations
- Mining Compressed and Redundancy-Aware Patterns

Mining Multiple-Level Frequent Patterns

- Items often form hierarchies
  - Ex.: Dairyland 2% milk; Wonder wheat bread
- How to set min-support thresholds?
  - Uniform min-support across multiple levels (reasonable?)
  - Level-reduced min-support: Items at the lower level are expected to have lower support
- Efficient mining: Shared multi-level mining
  - Use the lowest min-support to pass down the set of candidates



Redundancy Filtering at Mining Multi-Level Associations

- Multi-level association mining may generate many redundant rules
- Redundancy filtering: Some rules may be redundant due to "ancestor" relationships between items

  (Suppose the 2% milk sold is about ¼ of milk sold in gallons)

  - milk $\Rightarrow$ wheat bread [support = 8%, confidence = 70%] (1)
  - 2% milk $\Rightarrow$ wheat bread [support = 2%, confidence = 72%] (2)
- A rule is *redundant* if its support is close to the "expected" value, according to its "ancestor" rule, and it has a similar confidence as its "ancestor"
  - Rule (1) is an ancestor of rule (2), which one to prune?

Customized Min-Supports for Different Kinds of Items

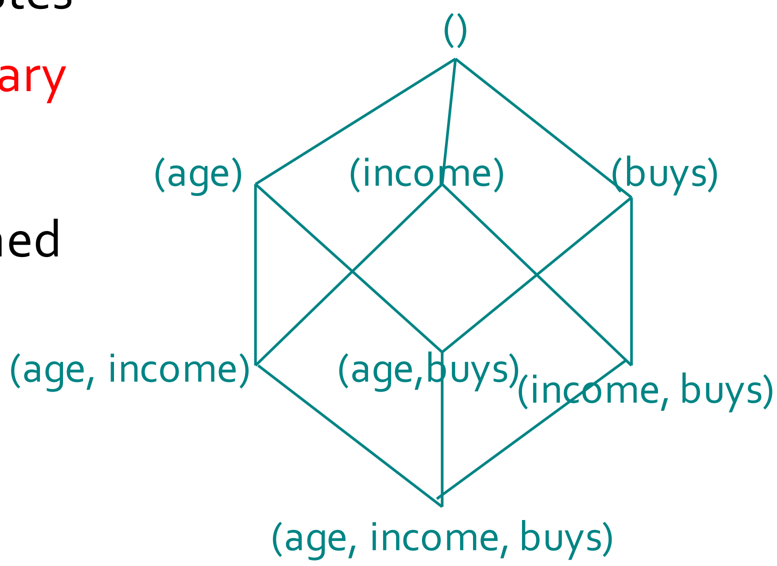
- We have used the same min-support threshold for all the items or item sets to be mined in each association mining
- In reality, some items (e.g., diamond, watch, …) are valuable but less frequent
- It is necessary to have customized min-support settings for different kinds of items
- One Method: Use group-based "individualized" min-support
  - E.g., {diamond, watch}: 0.05%; {bread, milk}: 5%; …
  - How to mine such rules efficiently?
    - Existing scalable mining algorithms can be easily extended to cover such cases

Mining Multi-Dimensional Associations

- Single-dimensional rules (e.g., items are all in "product" dimension)
  - buys(X, "milk") ⇒ buys(X, "bread")
- Multi-dimensional rules (i.e., items in ≥ 2 dimensions or predicates)
  - Inter-dimension association rules (*no repeated predicates*)
    - age(X, "18-25") ∧ occupation(X, "student") ⇒ buys(X, "coke")
  - Hybrid-dimension association rules (*repeated predicates*)
    - age(X, "18-25") ∧ buys(X, "popcorn") ⇒ buys(X, "coke")
- Attributes can be categorical or numerical
  - Categorical Attributes (e.g., *profession*, *product*: no ordering among values): Data cube for inter-dimension association
  - Quantitative Attributes: Numeric, implicit ordering among values—discretization, clustering, and gradient approaches

Mining Quantitative Associations

- Mining associations with numerical attributes
  - Ex.: Numerical attributes: age and salary
- Methods
  - Static discretization based on predefined concept hierarchies
    - Data cube-based aggregation
  - Dynamic discretization based on data distribution
  - Clustering: Distance-based association
    - First one-dimensional clustering, then association
  - Deviation analysis:
    - Gender = female ⇒ Wage: mean=$7/hr (overall mean = $9)



Mining Extraordinary Phenomena in Quantitative Association Mining

- Mining extraordinary (i.e., interesting) phenomena
  - Ex.: Gender = female $\Rightarrow$ Wage: mean=$7/hr (overall mean = $9)
  - LHS: a subset of the population
  - RHS: an extraordinary behavior of this subset
- The rule is accepted only if a statistical test (e.g., Z-test) confirms the inference with high confidence
- Subrule: Highlights the extraordinary behavior of a subset of the population of the super rule
  - Ex.: (Gender = female) ^ (South = yes) $\Rightarrow$ mean wage = $6.3/hr
- Rule condition can be categorical or numerical (quantitative rules)
  - Ex.: Education in [14-18] (yrs) $\Rightarrow$ mean wage = $11.64/hr

– Efficient methods have been developed for mining such extraordinary rules (e.g., Aumann and Lindell@KDD'99)

Rare Patterns vs. Negative Patterns

- Rare patterns
  - Very low support but interesting (e.g., buying Rolex watches)
  - How to mine them? Setting individualized, group-based min-support thresholds for different groups of items
- Negative patterns
  - Negatively correlated: Unlikely to happen together
  - Ex.: Since it is unlikely that the same customer buys both a Ford Expedition (an SUV car) and a Ford Fusion (a hybrid car), buying a Ford Expedition and buying a Ford Fusion are likely negatively correlated patterns
  - How to define negative patterns?

Defining Negative Correlated Patterns

- A support-based definition
  - If itemsets A and B are both frequent but rarely occur together, i.e., sup(A U B) << sup (A) × sup(B)
  - Then A and B are negatively correlated

Does this remind you the definition of *lift*?

- Is this a good definition for large transaction datasets?
- Ex.: Suppose a store sold two needle packages A and B 100 times each, but only one transaction contained both A and B
  - When there are in total 200 transactions, we have
    - s(A U B) = 0.005, s(A) × s(B) = 0.25, s(A U B) << s(A) × s(B)
  - But when there are 10^5 transactions, we have
    - s(A U B) = $1/10^5$, s(A) × s(B) = $1/10^3 \times 1/10^3$, s(A U B) > s(A) × s(B)
  - What is the problem? — Null transactions: The support-based definition is not null-invariant!

Defining Negative Correlation: Need Null-Invariance in Definition

- A good definition on negative correlation should take care of the null-invariance problem
  - Whether two itemsets A and B are negatively correlated should not be influenced by the number of null-transactions
- A Kulczynski measure-based definition
  - If itemsets A and B are frequent but $(P(A|B) + P(B|A))/2 < \epsilon$, where ϵ is a negative pattern threshold, then A and B are negatively correlated
- For the same needle package problem:
  - No matter there are in total 200 or 10^5 transactions
  - If $\epsilon = 0.01$, we have $(P(A|B) + P(B|A))/2 = (0.01 + 0.01)/2 < \epsilon$

Mining Compressed Patterns

- Why mining compressed patterns?
  - Too many scattered patterns but not so meaningful
- Pattern distance measure

$$Dist(P_1, P_2) = 1 - \frac{|T(P_1) \cap T(P_2)|}{|T(P_1) \cup T(P_2)|}$$

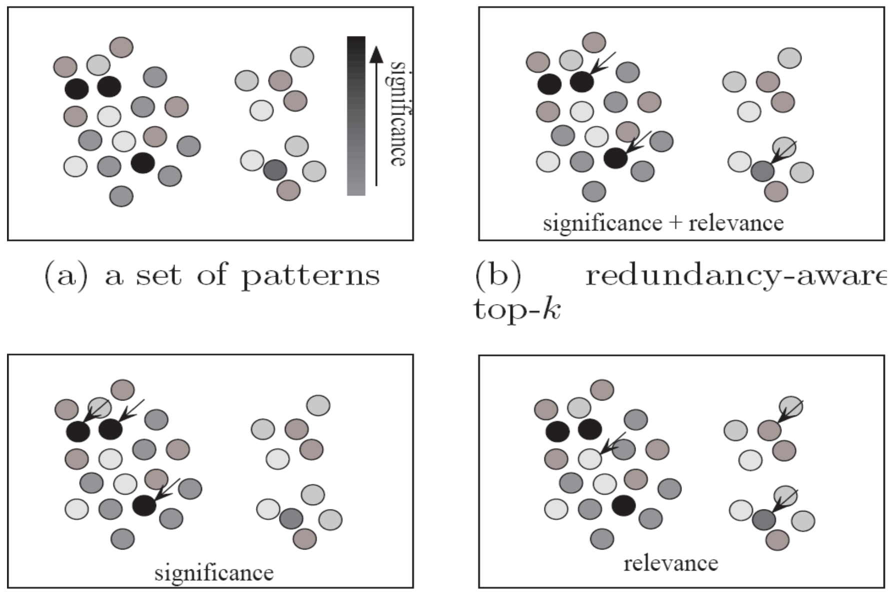
- δ-clustering: For each pattern P, find all patterns which can be expressed by P and whose distance to P is within δ (δ-cover)
- All patterns in the cluster can be represented by P
- Method for efficient, direct mining of compressed frequent patterns (e.g., D. Xin, J. Han, X. Yan, H. Cheng, "On Compressing Frequent Patterns", Knowledge and Data Engineering, 60:5-29, 2007)

Pat-ID	Item-Sets	Support
P1	{38,16,18,12}	205227
P2	{38,16,18,12,17}	205211
P3	{39,38,16,18,12,17}	101758
P4	{39,16,18,12,17}	161563
P5	{39,16,18,12}	161576

- Closed patterns
  - P1, P2, P3, P4, P5
  - Emphasizes too much on support
  - There is no compression
- Max-patterns
  - P3: information loss
- Desired output (a good balance):
  - P2, P3, P4

Redundancy-Aware Top-k Patterns

- Desired patterns: high significance & low redundancy



(a) a set of patterns (b) redundancy-aware top-k

(c) traditional top-k (d) summarization

- Method: Use MMS (Maximal Marginal Significance) for measuring the combined significance of a pattern set
- Xin et al., Extracting Redundancy-Aware Top-K Patterns, KDD'06

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- **Constraint-Based Frequent Pattern Mining**
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Why Constraint-Based Mining?

- Finding all the patterns in a dataset autonomously? — unrealistic!
  - Too many patterns but not necessarily user-interested!
- Pattern mining should be an interactive process
  - User directs what to be mined using a data mining query language (or a graphical user interface)
- Constraint-based mining
  - User flexibility: provides constraints on what to be mined
  - Optimization: explores such constraints for efficient mining
    - Constraint-based mining: Constraint-pushing, similar to push selection first in DB query processing

Constraints in General Data Mining

A data mining query can be in the form of a meta-rule or with the following language primitives.

- Knowledge type constraint:
  - Ex.: classification, association, clustering, outlier finding, ...
- Data constraint — using SQL-like queries
  - Ex.: find products sold together in NY stores this year
- Dimension/level constraint
  - Ex.: in relevance to region, price, brand, customer category
- Rule (or pattern) constraint
  - Ex.: small sales (price < $10) triggers big sales (sum > $200)
- Interestingness constraint
  - Ex.: strong rules: $min\_sup \geq 0.02$, $min\_conf \geq 0.6$, $min\_correlation \geq 0.7$

Meta-Rule Guided Mining

- A meta-rule can contain partially instantiated predicates & constants
  - $P_1(X, Y) \wedge P_2(X, W) \Rightarrow buys(X, \text{"iPad"})$
- The resulting mined rule can be
  - age(X, "15-25") ^ profession(X, "student") ⇒ buys(X, "iPad")
- In general, (meta) rules can be in the form of
  - $P_1 \wedge P_2 \wedge \ldots \wedge P_l \Rightarrow Q_1 \wedge Q_2 \wedge \ldots \wedge Q_r$
- Method to find meta-rules
  - Find frequent (l + r) predicates (based on *min-support*)
  - Push constants deeply when possible into the mining process
    - Using constraint-push techniques introduced in this lecture
  - Also, push min_conf, min_correlation, and other measures as early as possible (measures acting as constraints)

Different Kinds of Constraints Lead to Different Pruning Strategies

- Constraints can be categorized as
  - Pattern space pruning constraints vs. data space pruning constraints
- Pattern space pruning constraints
  - Anti-monotonic: If constraint c is violated, its further mining can be terminated
  - Monotonic: If c is satisfied, no need to check c again
  - Succinct: if the constraint c can be enforced by directly manipulating the data
  - Convertible: c can be converted to monotonic or anti-monotonic if items can be properly ordered in processing
- Data space pruning constraints
  - Data succinct: Data space can be pruned at the initial pattern mining process
  - Data anti-monotonic: If a transaction t does not satisfy c, then t can be pruned to reduce data processing effort

Pattern Space Pruning with Pattern Anti-Monotonicity

- Constraint c is anti-monotone
  - If an itemset S violates constraint c, so does any of its superset
  - That is, mining on itemset S can be terminated
- Ex. 1: c1: sum(S.price) ≤ v is anti-monotone
- Ex. 2: c2: range(S.profit) ≤ 15 is anti-monotone
  - Itemset ab violates c2 (range(ab) = 40)
  - So does every superset of ab
- Ex. 3. c3: sum(S.Price) ≥ v is not anti-monotone
- Ex. 4. Is c4: support(S) ≥ σ anti-monotone?
  - Yes! Apriori pruning is essentially pruning with an anti-monotonic constraint!

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

price(item)>0

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	5

Pattern Monotonicity and Its Roles

- A constraint c is monotone: if an itemset S satisfies the constraint c, so does any of its superset
  - That is, we do not need to check c in subsequent mining
- Ex. 1: c1: sum(S.Price) ≥ v is monotone
- Ex. 2: c2: min(S.Price) ≤ v is monotone
- Ex. 3: c3: range(S.profit) ≥ 15 is monotone
  - Itemset ab satisfies c3
  - So does every superset of ab

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

price(item)>0

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	5

Data Space Pruning with Data Anti-Monotonicity

- A constraint c is *data anti-monotone*: In the mining process, if a data entry *t* cannot satisfy a pattern *p* under *c*, *t* cannot satisfy *p*'s superset either
  - Data space pruning: Data entry *t* can be pruned
- Ex. 1: c_1: *sum(S.Profit) ≥ v* is data anti-monotone
  - Let constraint c_1 be: sum{S.Profit} ≥ 25
    - T_{30}: {b, c, d, f, g} can be removed since none of their combinations can make an S whose sum of the profit is ≥ 25
- Ex. 2: c_2: *min(S.Price) ≤ v* is data anti-monotone
    - Consider *v* = 5 but every item in transaction T_{50} has a price higher than 10
- Ex. 3: c_3: *range(S.Profit) ≥ 25* is data anti-monotone

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

price(item)>0

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	5

Data Space Pruning Should Be Explored Recursively

Example. c_3: *range(S.Profit) > 25*

- We check b's projected database
  - But item "a" is infrequent (sup = 1)
- After removing "a (40)" from T_{10}
  - T_{10} cannot satisfy c_3 any more
    - since "b (0)" and "c (-20), d (-15), f (-10), h (5)"
  - By removing T_{10}, we can also prune "h" in T_{20}

b's-proj. DB

TID	Transaction
10	a, c, d, f, h
20	c, d, f, g, h
30	c, d, f, g

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	5

min_sup = 2

price(item) > 0

Constraint:
range{S.profit} > 25

b's-proj. DB

TID	Transaction
~~10~~	~~a, c, d, f, h~~
20	c, d, f, g, ~~h~~
30	c, d, f, g

Recursive Data Pruning

b's FP-tree

single branch: cdfg: 2

Only a single branch "cdfg: 2" to be mined in b's projected DB

- Note: c_3 prunes T_{10} effectively only after "a" is pruned (by min-sup) in b's projected DB

Succinctness: Pruning Both Data and Pattern Spaces

- Succinctness: if the constraint c can be enforced by directly manipulating the data
- Ex. 1: To find those patterns without item i
  - Remove i from DB and then mine (pattern space pruning)
- Ex. 2: To find those patterns containing item i
  - Mine only i-projected DB (data space pruning)
- Ex. 3: c_3: min(S.Price) ≤ v is succinct
  - Start with only items whose price ≤ v (pattern space pruning) and remove transactions with high-price items only (data space pruning)
- Ex. 4: c_4: sum(S.Price) ≥ v is not succinct
  - It cannot be determined beforehand since sum of the price of itemset S keeps increasing

Convertible Constraints: Ordering Data in Transactions

- Convert tough constraints into (anti-)monotone by proper ordering of items in transactions
- Examine c1: avg(S.profit) > 20
  - Order items in value-descending order
    - <a, g, f, h, b, d, c, e>
  - An itemset ab violates c1 (avg(ab) = 20)
    - So does ab* (i.e., ab-projected DB)
    - C1: anti-monotone if patterns grow in the right order!
- Can item-reordering work for Apriori?
  - Does not work for level-wise candidate generation!
  - avg(agf) = 23.3 > 20, but avg(gf) = 15 < 20

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	10
g	20
h	5

min_sup = 2

price(item)>0

TID	Transaction
10	a, b, c, d, f, h
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How to Handle Multiple Constraints?

- It is beneficial to use multiple constraints in pattern mining
- But different constraints may require potentially conflicting item-ordering
  - If there exists an order R making both c1 and c2 convertible, try to sort items in the order that benefits pruning most
  - If there exists conflict ordering between c1 and c2
    - Try to sort data and enforce one constraint first (which one?)
    - Then enforce the other when mining the projected databases
- Ex. c_1: avg(S.profit) > 20, and c_2: avg(S.price) < 50
  - Sorted in profit descending order and use c1 first (assuming c_1 has more pruning power)
  - For each project DB, sort trans. in price ascending order and use c_2 at mining

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Mining Long Patterns: Challenges

- Mining long patterns is needed in bioinformatics, social network analysis, software engineering, …
  - But the methods introduced so far mine only short patterns (e.g., length < 10)
- Challenges of mining long patterns
  - The curse of "downward closure" property of frequent patterns
    - Any sub-pattern of a frequent pattern is frequent
    - If $\{a_1, a_2, \ldots, a_{100}\}$ is frequent, then $\{a_1\}, \{a_2\}, \ldots, \{a_{100}\}, \{a_1, a_2\}, \{a_1, a_3\}, \ldots, \{a_1, a_{100}\}, \{a_1, a_2, a_3\}, \ldots$ are all frequent! There are about 2^{100} such frequent itemsets!
  - No matter searching in breadth-first (e.g., Apriori) or depth-first (e.g., FPgrowth), if we still adopt the "small to large" step-by-step growing paradigm, we have to examine so many patterns, which leads to combinatorial explosion!

Colossal Patterns: A Motivating Example

- Let min-support $\sigma = 20$
- # of closed/maximal patterns of size 20: about $\binom{40}{20}$
- But there is only one pattern with size close to 40 (*i.e.*, *long* or *colossal*)
  - $\alpha = \{41, 42, \ldots, 79\}$ of size 39
- Q: How to find it without generating an exponential number of size-20 patterns?

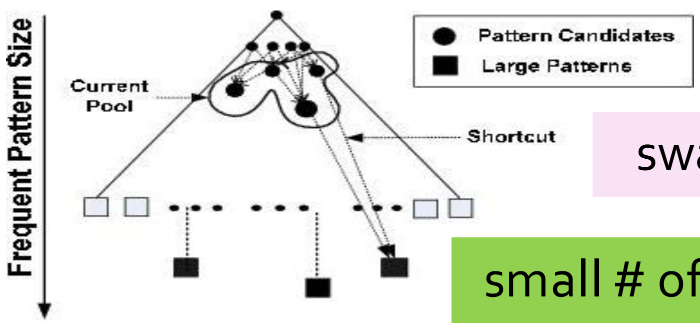
T_1 = 2 3 4 ….. 39 40
T_2 = 1 3 4 ….. 39 40
: .
: .
T_{40}=1 2 3 4 …… 39
T_{41}= 41 42 43 ….. 79
T_{42}= 41 42 43 ….. 79
: .
: .
T_{60}= 41 42 43 … 79

The existing fastest mining algorithms (*e.g.*, FPClose, LCM) fail to complete running

A new algorithm, *Pattern-Fusion*, outputs this colossal pattern in seconds

What Is Pattern-Fusion?

- Not strive for completeness (why?)
- Jump out of the swamp of the mid-sized intermediate "results"
- Strive for mining almost complete and representative colossal patterns: identify "short-cuts" and take "leaps"
- Key observation
  - The larger the pattern or the more distinct the pattern, the greater chance it will be generated from small ones
- Philosophy: Collection of small patterns hints at the larger patterns
- Pattern fusion strategy ("not crawl but jump"): Fuse small patterns together in one step to generate new pattern candidates of significant sizes



swamp of mid-sized patterns

small # of colossal patterns

Observation: Colossal Patterns and Core Patterns

- Suppose dataset D contains 4 colossal patterns (below) plus many small patterns
  - $\{a_1, a_2, \ldots, a_{50}\}$: 40, $\{a_3, a_6, \ldots, a_{99}\}$: 60, $\{a_5, a_{10}, \ldots, a_{95}\}$: 80, $\{a_{10}, a_{20}, \ldots, a_{100}\}$: 100
- If you check the pattern pool of size-3, you may likely find
  - $\{a_2, a_4, a_{45}\}$: ~40; $\{a_3, a_{34}, a_{39}\}$: ~40; …, $\{a_5, a_{15}, a_{85}\}$: ~80, …, $\{a_{20}, a_{40}, a_{85}\}$: ~80, …
- If you merge the patterns with similar support, you may obtain candidates of much bigger size and easily validate whether they are true patterns
- *Core patterns* of a colossal pattern α: A set of subpatterns of α that cluster around α by sharing a similar support
- A colossal pattern has far more core patterns than a small-sized pattern
- A random draw from a complete set of pattern of size c would be more likely to pick a core pattern (or its descendant) of a colossal pattern
- A colossal pattern can be generated by merging a set of core patterns

Robustness of Colossal Patterns

- Core Patterns: For a frequent pattern α, a subpattern β is a τ-core pattern of α if β shares a similar support set with α, i.e.,

  $$\frac{|D_\alpha|}{|D_\beta|} \geq \tau \qquad 0 < \tau \leq 1$$

  where τ is called the core ratio
- (d, τ)-robustness: A pattern α is *(d, τ)-robust* if *d* is the maximum number of items that can be removed from α for the resulting pattern to remain a τ-core pattern of α
- For a (d, τ)-robust pattern α, it has $\Omega(2^d)$ core patterns
- Robustness of Colossal Patterns: A colossal pattern tends to have much more core patterns than small patterns
- Such core patterns can be clustered together to form "dense balls" based on pattern distance defined by

A random draw in the pattern space will hit somewhere in the ball with high probability

$$Dist(\alpha, \beta) = 1 - \frac{|D_\alpha \cap D_\beta|}{|D_\alpha \cup D_\beta|}$$

The Pattern-Fusion Algorithm

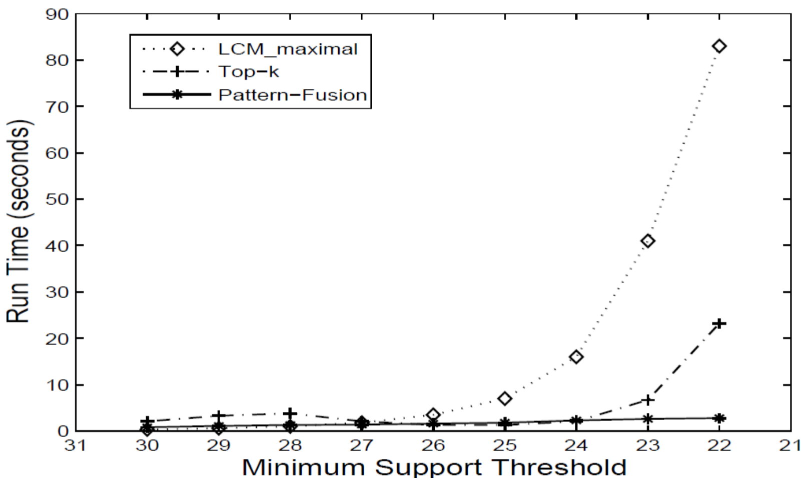
- Initialization (Creating initial pool): Use an existing algorithm to mine all frequent patterns up to a small size, e.g., 3
- Iteration (Iterative Pattern Fusion):
  - At each iteration, K seed patterns are randomly picked from the current pattern pool
  - For each seed pattern thus picked, we find all the patterns within a bounding ball centered at the seed pattern
  - All these patterns found are fused together to generate a set of super-patterns
  - All the super-patterns thus generated form a new pool for the next iteration
- Termination: when the current pool contains no more than K patterns at the beginning of an iteration

Experimental Results on Data Set: ALL

- ALL: A popular gene expression clinical data set on ALL-AML leukemia, with 38 transactions, each with 866 columns. There are 1,736 items in total.
  - When minimum support is high (e.g., 30), Pattern-Fusion gets all the largest colossal patterns with size greater than 85

Pattern Size	110	107	102	91	86	84	83
The complete set	1	1	1	1	1	2	6
Pattern-Fusion	1	1	1	1	1	1	4
Pattern Size	82	77	76	75	74	73	71
The complete set	1	2	1	1	1	2	1
Pattern-Fusion	0	2	0	1	1	1	1

Mining colossal patterns on a Leukemia dataset



Algorithm runtime comparison on another dataset

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Sequence Databases and Sequential Patterns

- Sequential pattern mining has broad applications
  - Customer shopping sequences
    - Purchase a laptop first, then a digital camera, and then a smartphone, within 6 months
  - Medical treatments, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, ...
  - Weblog click streams, calling patterns, ...
  - Software engineering: Program execution sequences, ...
  - Biological sequences: DNA, protein, ...
- Transaction DB, sequence DB vs. time-series DB
- Gapped vs. non-gapped sequential patterns
  - Shopping sequences, clicking streams vs. biological sequences

Sequential Pattern and Sequential Pattern Mining

- Sequential pattern mining: Given a set of sequences, find the complete set of frequent subsequences (i.e., satisfying the min_sup threshold)

A *sequence database*

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

A *sequence*: < (ef) (ab) (df) c b >

- An element may contain a set of items (also called events)
- Items within an element are unordered and we list them alphabetically

<a(bc)dc> is a *subsequence* of <a(abc)(ac)d(cf)>

- Given support threshold min_sup = 2, <(ab)c> is a sequential pattern

Sequential Pattern Mining Algorithms

- Algorithm requirement: Efficient, scalable, finding complete set, incorporating various kinds of user-specific constraints
- The Apriori property still holds: If a subsequence s_1 is infrequent, none of s_1's super-sequences can be frequent
- Representative algorithms
  - GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)
  - Vertical format-based mining: SPADE (Zaki@Machine Leanining'00)
  - Pattern-growth methods: PrefixSpan (Pei, et al. @TKDE'04)
- Mining closed sequential patterns: CloSpan (Yan, et al. @SDM'03)
- Constraint-based sequential pattern mining

GSP: Apriori-Based Sequential Pattern Mining

- Initial candidates: All singleton sequences
  - <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>
- Scan DB once, count support for each candidate
- Generate length-2 candidate sequences

SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

min_sup = 2

Cand.	sup
<a>	3
<b>	5
<c>	4
<d>	3
<e>	3
<f>	2
<g>	1
<h>	1

	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
<e>	<ea>	<eb>	<ec>	<ed>	<ee>	<ef>
<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

Length-2 candidates:
36 + 15= 51
Without Apriori pruning:
8*8+8*7/2=92 candidates

GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)

GSP Mining and Pruning

- Repeat (for each level (i.e., length-k))
  - Scan DB to find length-k frequent sequences
  - Generate length-(k+1) candidate sequences from length-k frequent sequences using Apriori
  - set k = k+1
- Until no frequent sequence or no candidate can be found

Sequential Pattern Mining in Vertical Data Format: The SPADE Algorithm

- A sequence database is mapped to: <SID, EID>
- Grow the subsequences (patterns) one item at a time by Apriori candidate generation

SID	Sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

min_sup = 2

Ref: SPADE (Sequential PAttern Discovery using Equivalent Class) [M. Zaki 2001]

SID	EID	Items
1	1	a
1	2	abc
1	3	ac
1	4	d
1	5	cf
2	1	ad
2	2	c
2	3	bc
2	4	ae
3	1	ef
3	2	ab
3	3	df
3	4	c
3	5	b
4	1	e
4	2	g
4	3	af
4	4	c
4	5	b
4	6	c

a		b		...
SID	EID	SID	EID	...
1	1	1	2	
1	2	2	3	
1	3	3	2	
2	1	3	5	
2	4	4	5	
3	2			
4	3			

ab			ba			...
SID	EID (a)	EID(b)	SID	EID (b)	EID(a)	...
1	1	2	1	2	3	
2	1	3	2	3	4	
3	2	5				
4	3	5				

aba				...
SID	EID (a)	EID(b)	EID(a)	...
1	1	2	3	
2	1	3	4	

PrefixSpan: A Pattern-Growth Approach

- Prefix and suffix
  - Given <a(abc)(ac)d(cf)>
  - Prefixes: <a>, <aa>, <a(ab)>, <a(abc)>, …
  - Suffix: Prefixes-based projection
- PrefixSpan Mining: Prefix Projections
  - Step 1: Find length-1 sequential patterns
    - <a>, <b>, <c>, <d>, <e>, <f>
  - Step 2: Divide search space and mine each projected DB
    - <a>-projected DB,
    - <b>-projected DB,
    - …
    - <f>-projected DB, …

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

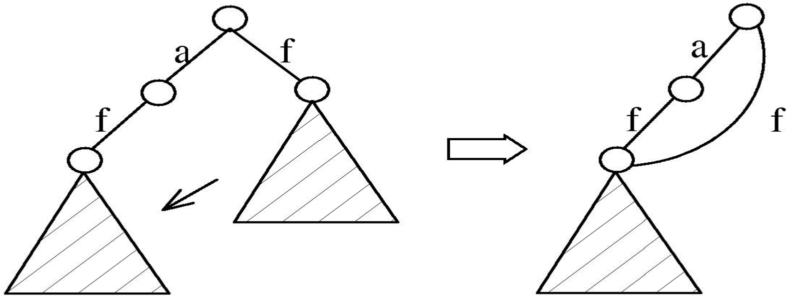
Prefix	Suffix (Projection)
<a>	<(abc)(ac)d(cf)>
<aa>	<(_bc)(ac)d(cf)>
<ab>	<(_c)(ac)d(cf)>

PrefixSpan (Prefix-projected Sequential pattern mining) Pei, et al. @TKDE'04

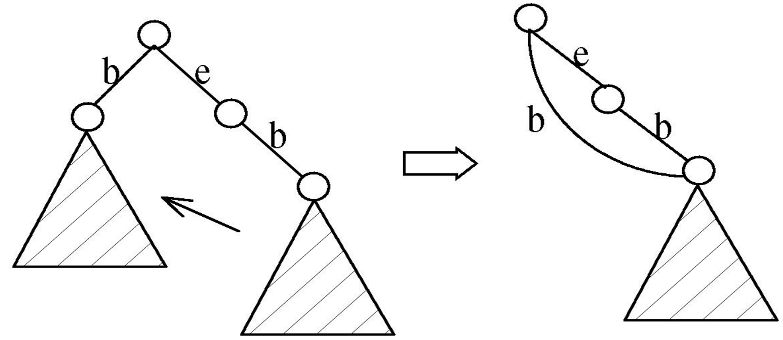
CloSpan: Mining Closed Sequential Patterns

- A closed sequential pattern *s*: There exists no superpattern *s′* such that *s′* ⊃ *s*, and *s′* and *s* have the same support
- Which ones are closed? <abc>: 20, <abcd>:20, <abcde>: 15
- Why directly mine closed sequential patterns?
  - Reduce # of (redundant) patterns
  - Attain the same expressive power
- Property P_1: If $s \supset s_1$, s is closed iff two project DBs have the same size
- Explore *Backward Subpattern* and *Backward Superpattern* pruning to prune redundant search space
- Greatly enhances efficiency (Yan, et al., SDM'03)

Backward subpattern pruning



Backward superpattern pruning



Constraint-Based Sequential-Pattern Mining

- Share many similarities with constraint-based itemset mining
- Anti-monotonic: If S violates *c*, the super-sequences of S also violate *c*
  - sum(S.price) < 150; min(S.value) > 10
- Monotonic: If S satisfies *c*, the super-sequences of S also do so
  - element_count (S) > 5; S $\supseteq$ {PC, digital_camera}
- Data anti-monotonic: If a sequence s_1 with respect to S violates c_3, s_1 can be removed
  - c_3: sum(S.price) ≥ v
- Succinct: Enforce constraint c by explicitly manipulating data
  - S $\supseteq$ {i-phone, MacAir}
- Convertible: Projection based on the sorted value not sequence order
  - value_avg(S) < 25; profit_sum (S) > 160
  - max(S)/avg(S) < 2; median(S) – min(S) > 5

Timing-Based Constraints in Seq.-Pattern Mining

- Order constraint: Some items must happen before the other
  - {algebra, geometry} → {calculus} (where "→" indicates ordering)
  - Anti-monotonic: Constraint-violating sub-patterns pruned
- Min-gap/max-gap constraint: Confines two elements in a pattern
  - E.g., mingap = 1, maxgap = 4
  - Succinct: Enforced directly during pattern growth
- Max-span constraint: Maximum allowed time difference between the 1st and the last elements in the pattern
  - E.g., maxspan (S) = 60 (days)
  - Succinct: Enforced directly when the 1st element is determined
- Window size constraint: Events in an element do not have to occur at the same time: Enforce max allowed time difference
  - E.g., window-size = 2: Various ways to merge events into elements

Episodes and Episode Pattern Mining

- Episodes and regular expressions: Alternative to seq. patterns
  - Serial episodes: A → B
  - Parallel episodes: A | B   Indicating partial order relationships
  - Regular expressions: (A|B)C*(D → E)
- Methods for episode pattern mining
  - Variations of Apriori/GSP-like algorithms
  - Projection-based pattern growth
    - Q_1: Can you work out the details?
  - Q_2: What are the differences between mining episodes and constraint-based pattern mining?

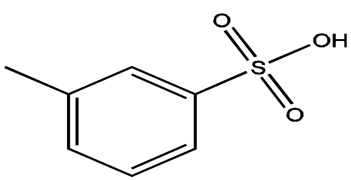
Advanced Frequent Pattern Mining

- Mining Diverse Patterns
- Constraint-Based Frequent Pattern Mining
- Mining High-Dimensional Data and Colossal Patterns
- Sequential Pattern Mining
- **Graph Pattern Mining**

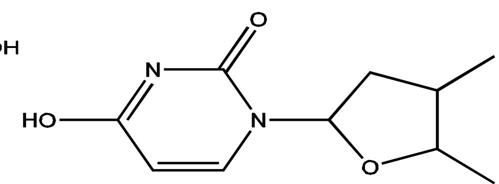
Frequent (Sub)Graph Patterns

- Given a labeled graph dataset D = {G_1, G_2, …, G_n), the supporting graph set of a subgraph *g* is $D_g = \{G_i \mid g \subseteq G_i, G_i \in D\}$.
  - support(*g*) = $|D_g| / |D|$
- A (sub)graph *g* is ***frequent*** if *support*(*g*) ≥ *min_sup* Ex.: Chemical structures
- Alternative:
  - Mining frequent subgraph patterns from a single large graph or network

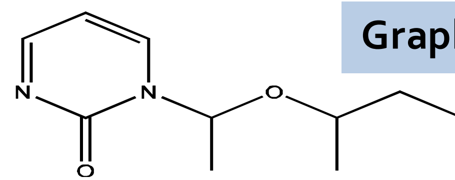
Graph Dataset



(A)



(B)

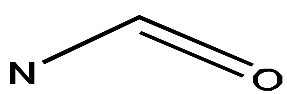


(C)

min_sup = 2

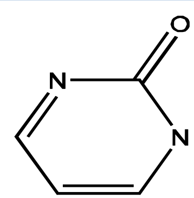
Frequent Graph Patterns

(1)



support = 67%

(2)



Applications of Graph Pattern Mining

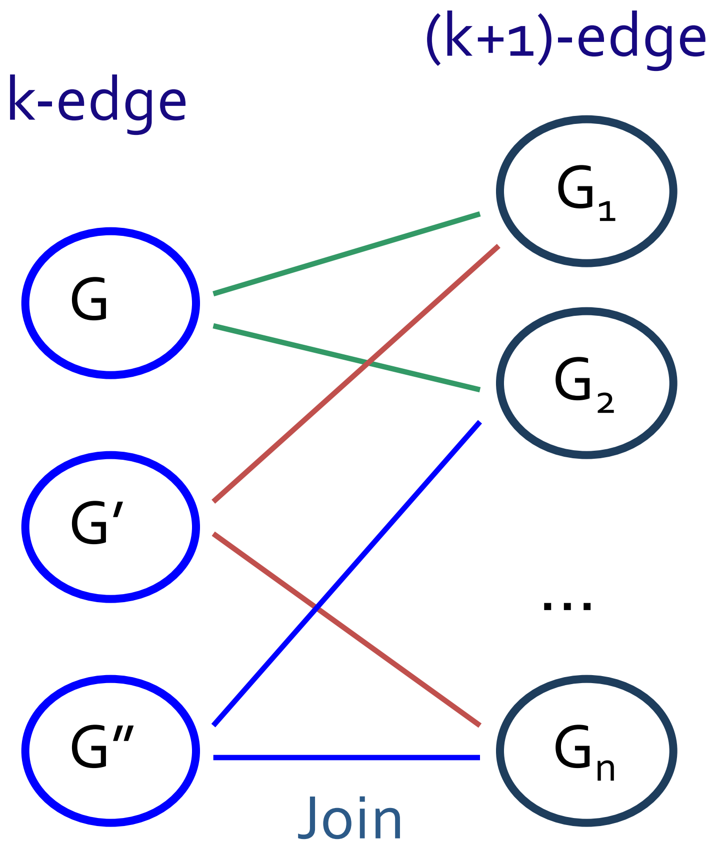
- Bioinformatics
  - Gene networks, protein interactions, metabolic pathways
- Chem-informatics: Mining chemical compound structures
- Social networks, web communities, tweets, …
- Cell phone networks, computer networks, …
- Web graphs, XML structures, semantic Web, information networks
- Software engineering: program execution flow analysis
- Building blocks for graph classification, clustering, compression, comparison, and correlation analysis
- Graph indexing and graph similarity search

Graph Pattern Mining Algorithms: Different Methodologies

- Generation of candidate subgraphs
  - Apriori vs. pattern growth (e.g., FSG vs. gSpan)
- Search order
  - Breadth vs. depth
- Elimination of duplicate subgraphs
  - Passive vs. active (e.g., gSpan (Yan&Han'02))
- Support calculation
  - Store embeddings (e.g., GASTON (Nijssen&Kok'04, FFSM (Huan, et al.'03), MoFa (Borgelt and Berthold ICDM'02))
- Order of pattern discovery
  - Path → tree → graph (e.g., GASTON (Nijssen&Kok'04)

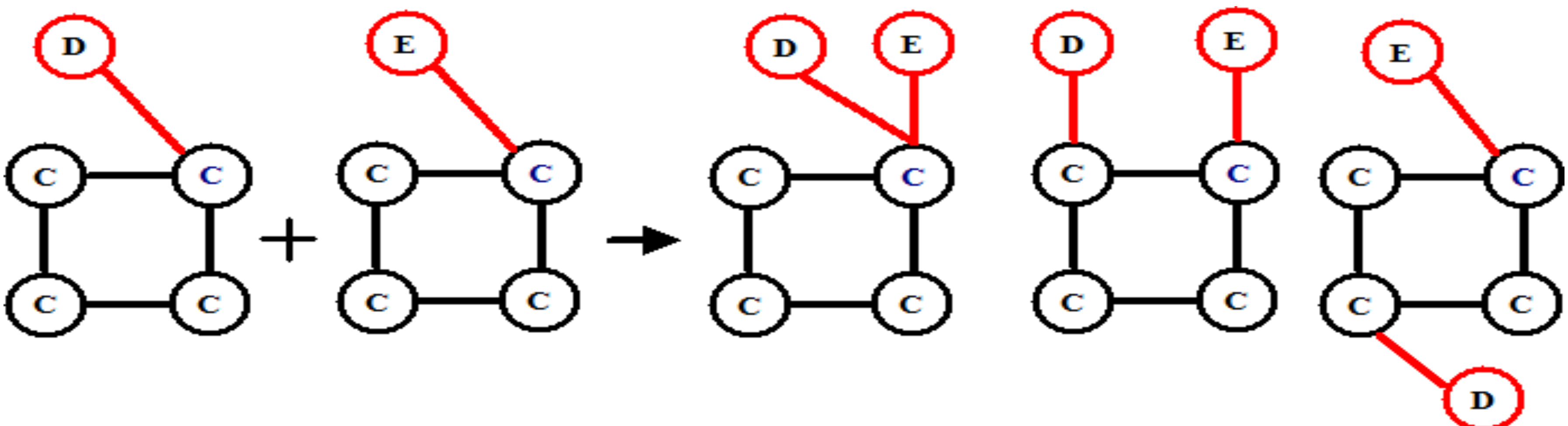
Apriori-Based Approach

- The Apriori property (anti-monotonicity): A size-*k* subgraph is frequent if and only if all of its subgraphs are frequent
- A candidate size-(*k*+1) edge/vertex subgraph is generated if its corresponding two *k*-edge/vertex subgraphs are frequent
- Iterative mining process:
  - Candidate-generation → candidate pruning → support counting → candidate elimination



Candidate Generation: Vertex Growing vs. Edge Growing

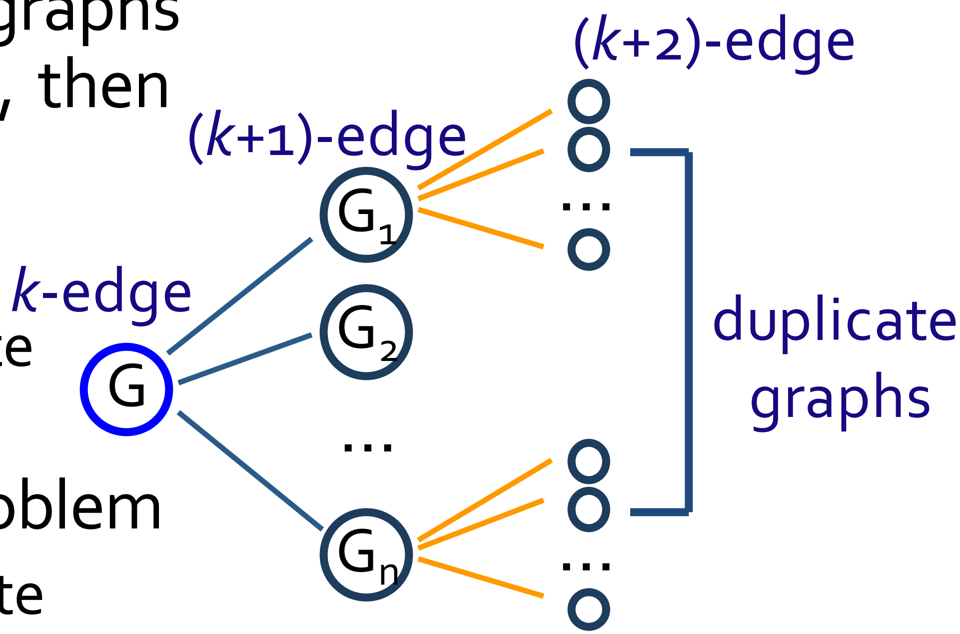
- Methodology: breadth-search, Apriori joining two size-k graphs
  - Many possibilities at generating size-(k+1) candidate graphs



- Generating new graphs with one more vertex
  - AGM (Inokuchi, et al., PKDD'00)
- Generating new graphs with one more edge
  - FSG (Kuramochi and Karypis, ICDM'01)
- Performance shows via edge growing is more efficient

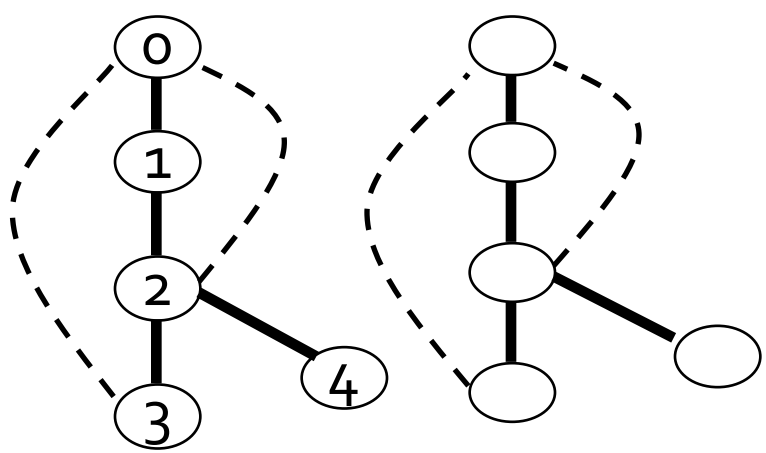
Pattern-Growth Approach

- Depth-first growth of subgraphs from *k*-edge to (*k*+1)-edge, then (*k*+2)-edge subgraphs
- Major challenge
  - Generating many duplicate subgraphs
- Major idea to solve the problem
  - Define an order to generate subgraphs
  - DFS spanning tree: Flatten a graph into a sequence using depth-first search
  - gSpan (Yan & Han: ICDM'02)



gSPAN: Graph Pattern Growth in Order

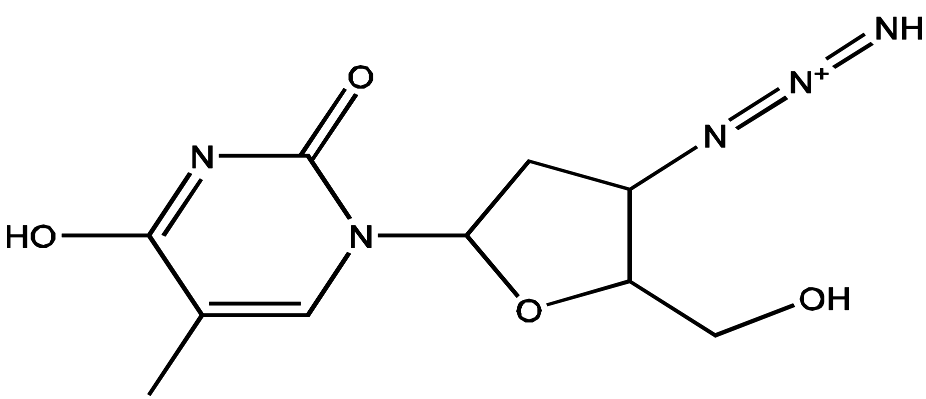
- **Right-most path extension** in subgraph pattern growth
  - Right-most path: The path from root to the right-most leaf (choose the vertex w. the smallest index at each step)
  - Reduce generation of duplicate subgraphs
- **Completeness:** The Enumeration of graphs using right-most path extension is complete
- DFS Code: Flatten a graph into a sequence using depth-first search



e_0: (0,1)
e_1: (1,2)
e_2: (2,0)
e_3: (2,3)
e_4: (3,0)
e_5: (2,4)

Why Mining Closed Graph Patterns?

- Challenge: An **n**-edge frequent graph may have 2^n subgraphs
- Motivation: Explore *closed frequent subgraphs* to handle graph pattern explosion problem
- A frequent graph G is *closed* if there exists no supergraph of G that carries the same support as G

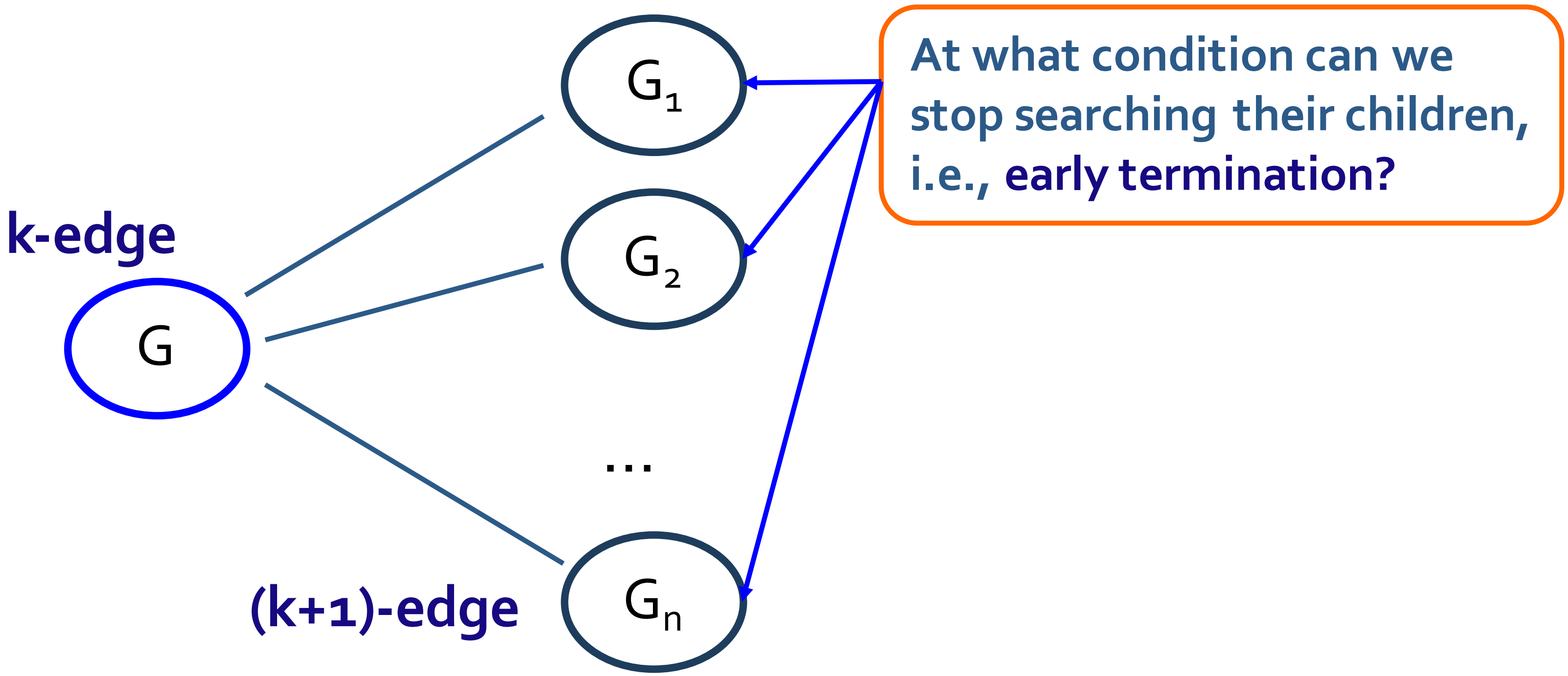


If this subgraph is *closed* in the graph dataset, it implies that none of its frequent super-graphs carries the same support

- *Lossless compression:* Does not contain non-closed graphs, but still ensures that the mining result is complete
- Algorithm CloseGraph: Mines closed graph patterns directly

CloseGraph: Directly Mining Closed Graph Patterns

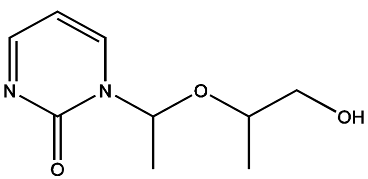
- CloseGraph: Mining closed graph patterns by extending gSpan
- Suppose G and G_1 are frequent, and G is a subgraph of G_1
- If **in any part of the graph in the dataset where G occurs, G_1 also occurs**, then we need not grow G (except some special, subtle cases), since none of G's children will be closed except those of G_1



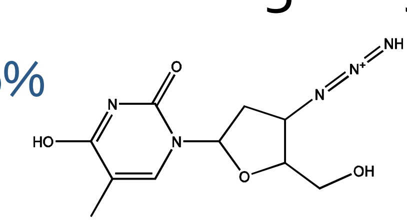
Experiment and Performance Comparison

- The AIDS antiviral screen compound dataset from NCI/NIH
- The dataset contains 43,905 chemical compounds
- Discovered Patterns: The smaller minimum support, the bigger and more interesting subgraph patterns discovered

20%



10%



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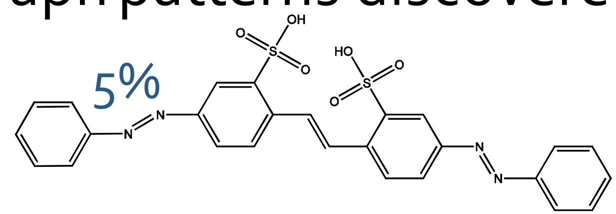
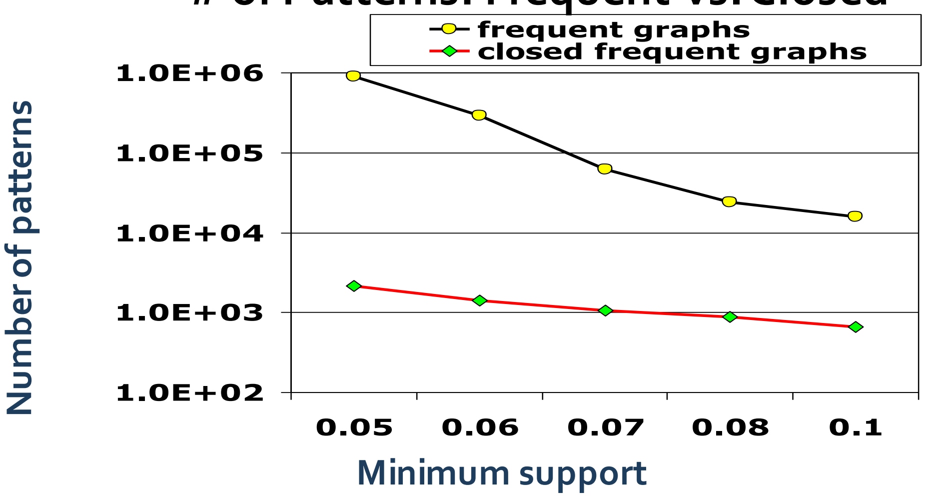
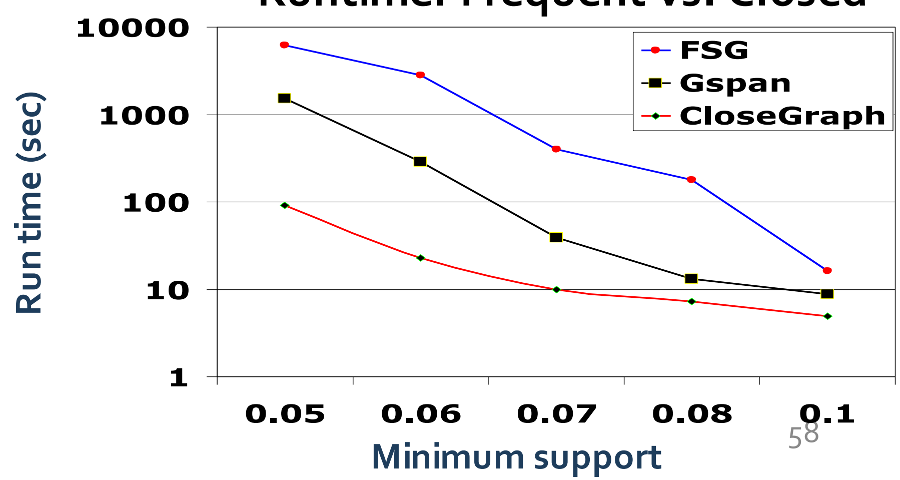


# of Patterns: Frequent vs. Closed



Runtime: Frequent vs. Closed



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