

Chapter 4&5. Data Cube: Cube Computation

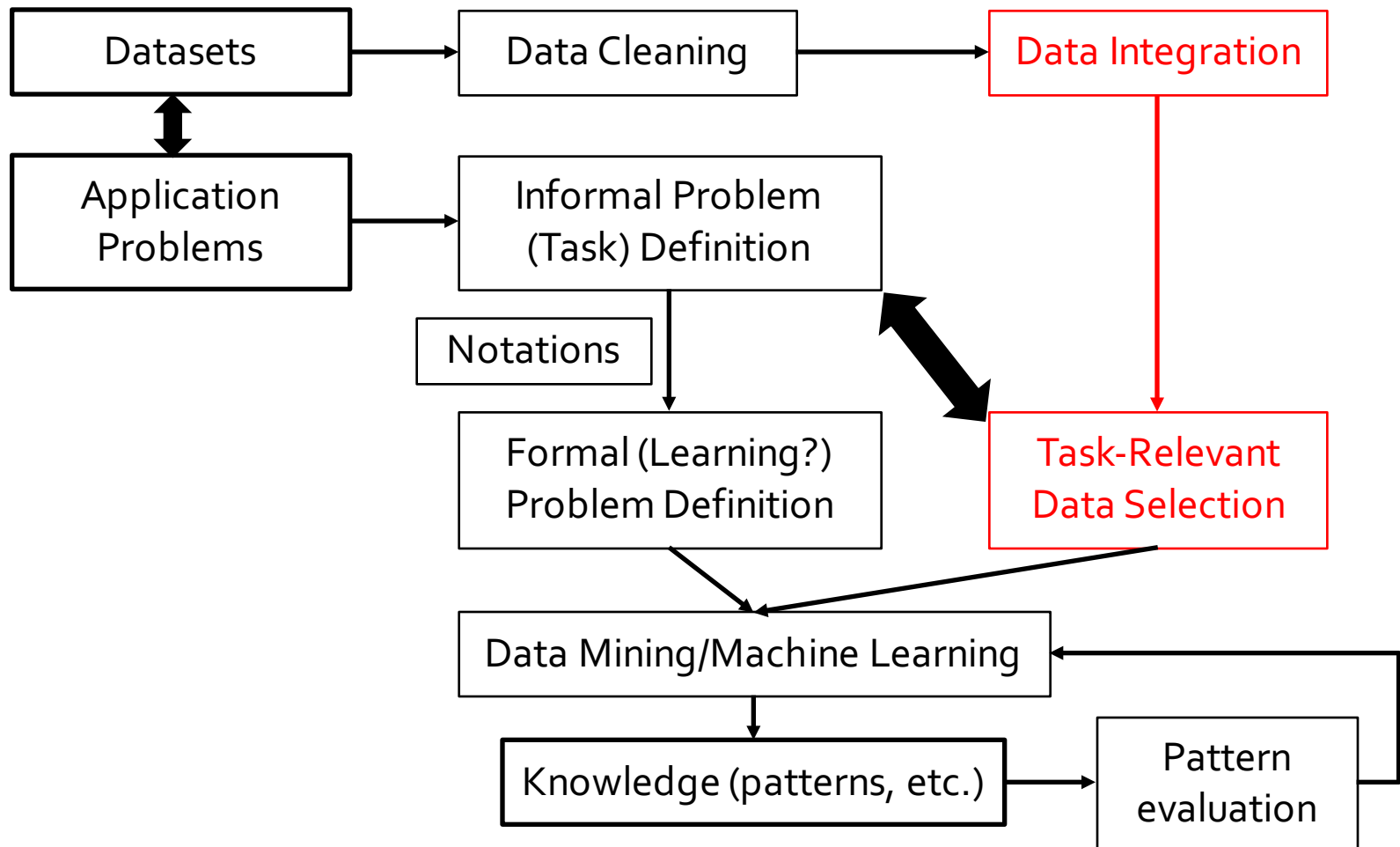
Meng Jiang

CSE 40647/60647 Data Science Fall 2017

Introduction to Data Mining

Previously on Data Science ...

- Chapter 1. Introduction.



Previously on Data Science ...

- Chapter 2. Get to Know Your Data.
 - Data Objects and Attribute Types
 - Basic Statistical Descriptions
 - Central tendency (mean, median, mode, etc.)
 - Outlierness (variance, standard deviation, z-score, etc.)
 - Data Visualization
 - Box plot, Histogram, Bar chart, Q plot, Q-Q plot, Scatter plot, etc.
 - Measuring Data Similarity and Dissimilarity
 - Minkowski distances
 - Jaccard/cosine similarity
 - KL divergence

Previously on Data Science ...

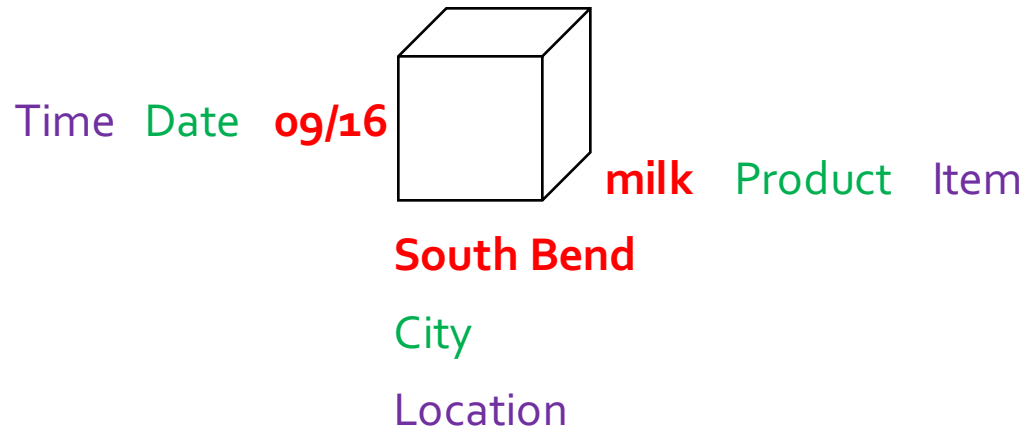
- Chapter 3. Data Processing.
 - Data cleaning: Missing data, Noisy data
 - Data integration: Redundant data
 - Correlation analysis: Chi-square test, Covariance
 - Data reduction
 - Regression analysis: Linear, non-Linear
 - Histogram, Clustering, Sampling
 - Normalization: Min-max, Z-score, Decimal scaling
 - Dimensionality reduction
 - Feature selection
 - Feature extraction: PCA (eigenvectors), etc.

Concrete Learning Goals

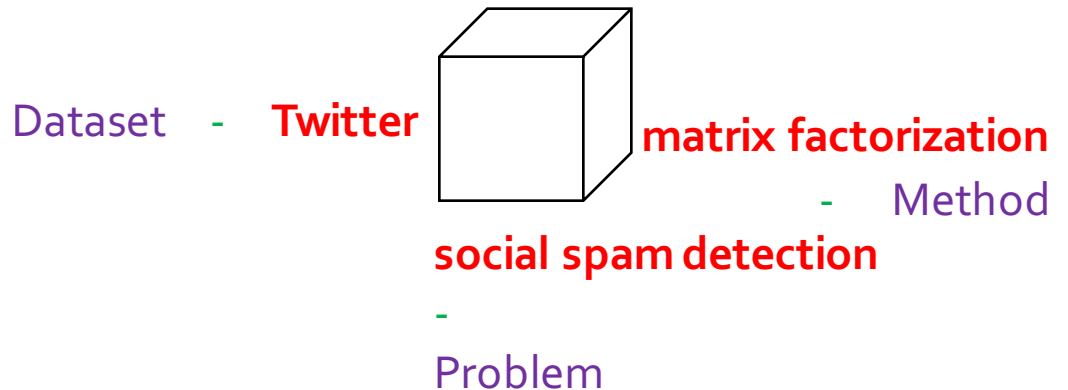
- Can process raw data: data cleaning, data integration, data reduction, dimension reduction
- Can describe data warehouse, OLAP, data cube concepts and technology that work on multi-dimensional datasets
- Can use Apriori and FP-Growth for frequent pattern mining
- Can describe diverse patterns, sequential patterns, graph patterns
- Can use Decision Tree, Naïve Bayes, Ensembles for classification
- Can describe SVMs and Neural Networks for classification
- Can use K-Partitioning Methods (K-Means, etc.) for clustering
- Can describe Kernel-based Clustering and Density-based Clustering
- Can use appropriate measures to evaluate results of different functionalities

Cells: Dimension, Dimension Level and Dimension Value

A cell of transactions:

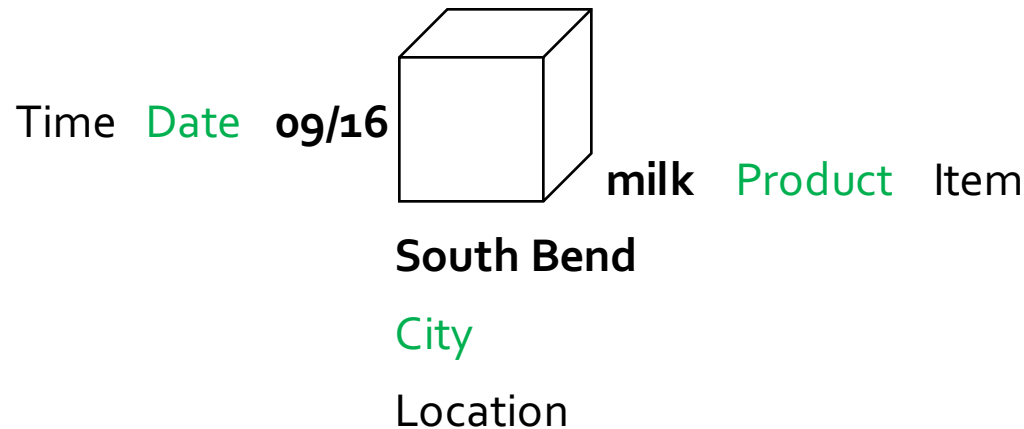


A cell of papers:



Cells: Dimension Level and Concept Hierarchy

A cell of transactions:

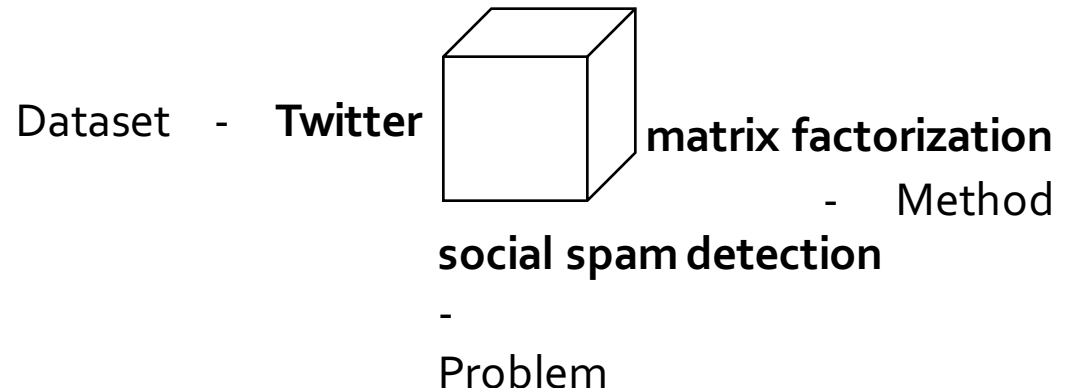


Time: Year-Quarter-Month-Week-Day

Location: Country-State-City-Street

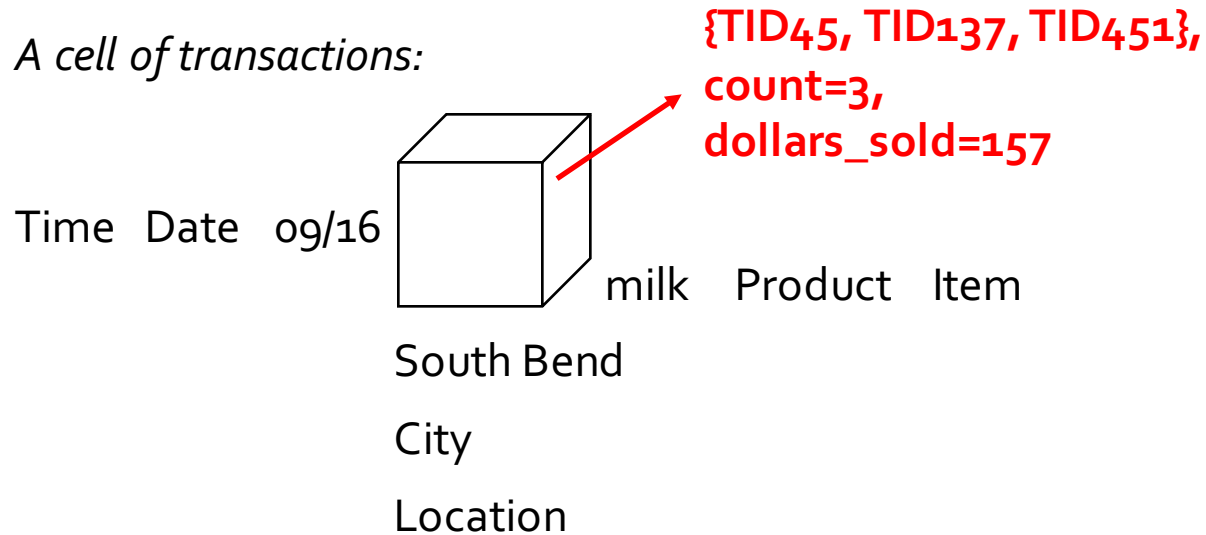
Item: Department-Product-Model

A cell of papers:

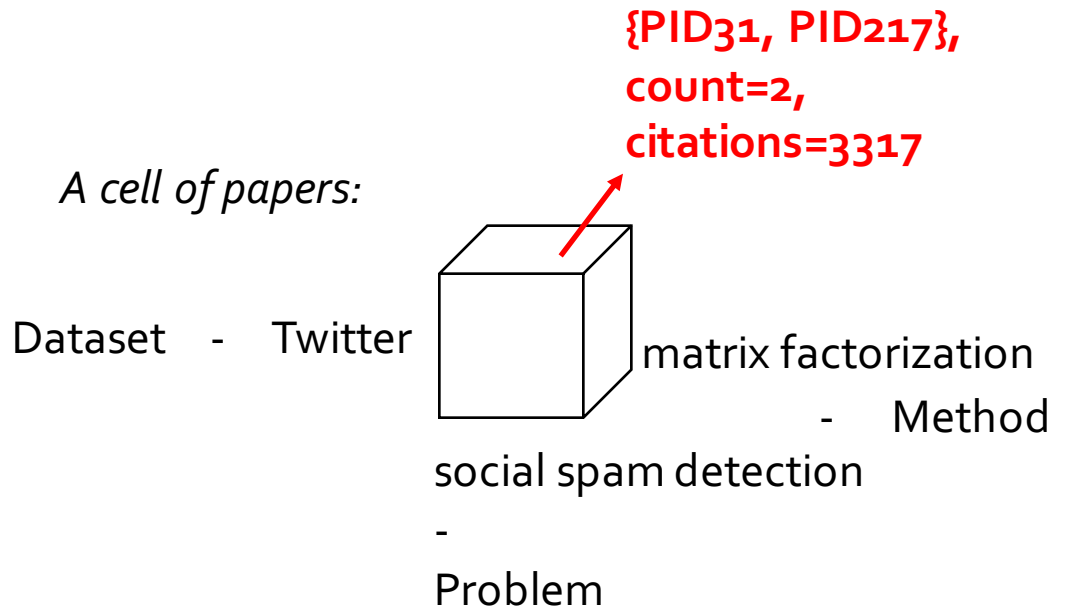


Cells: Facts or Measures

A cell of transactions:

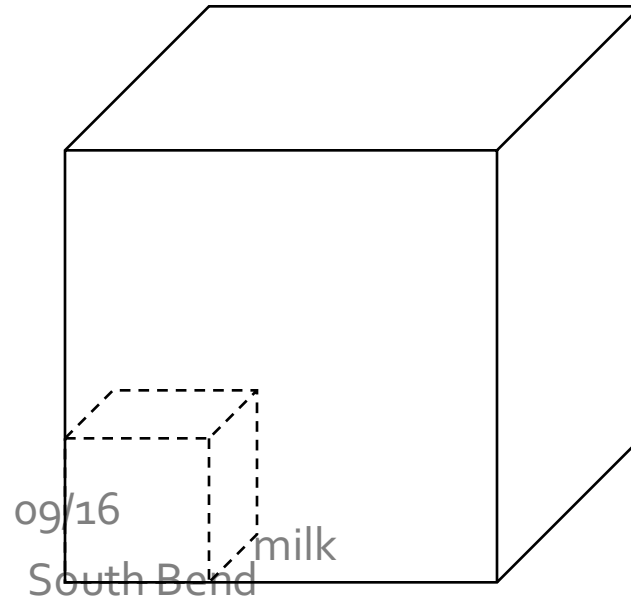


A cell of papers:



Cuboids: Dimension, Dimension Level

Time Date



Product Item

City

Location

Base Cells and Aggregate Cells

- Suppose a cuboid has 3 dimensions (time, location, item) at specific dimension levels (date, city, product).
- Base cells
 - (09/16, South Bend, milk)
- Aggregate cells
 - (*, South Bend, milk)
 - (09/16, *, milk)
 - (09/16, South Bend, *)
 - (*, *, milk)
 - (*, South Bend, *)
 - (09/16, *, *)
 - (*, *, *), called the **Apex cell**

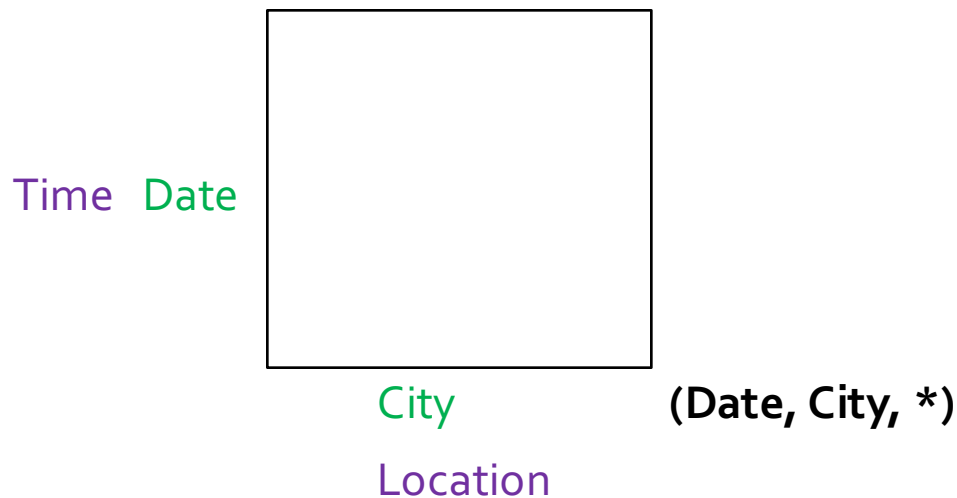
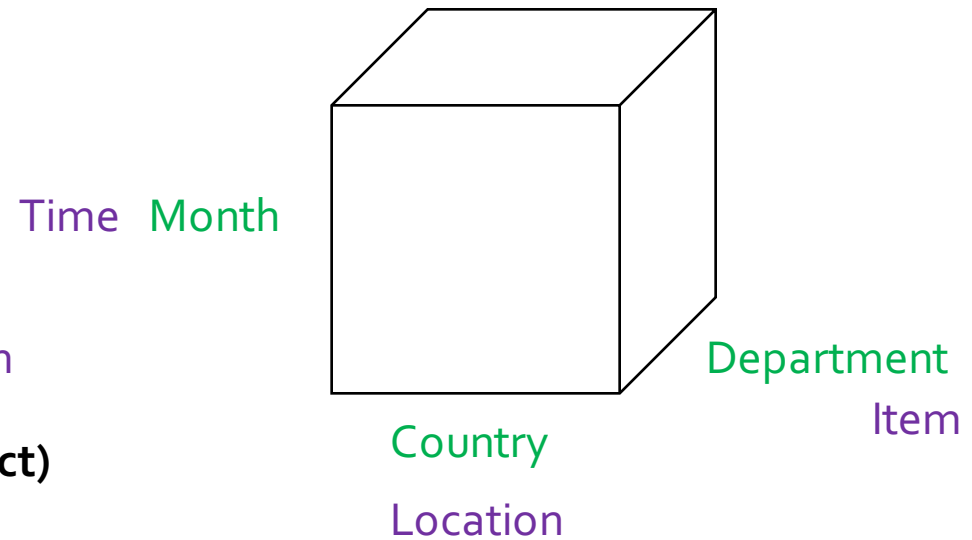
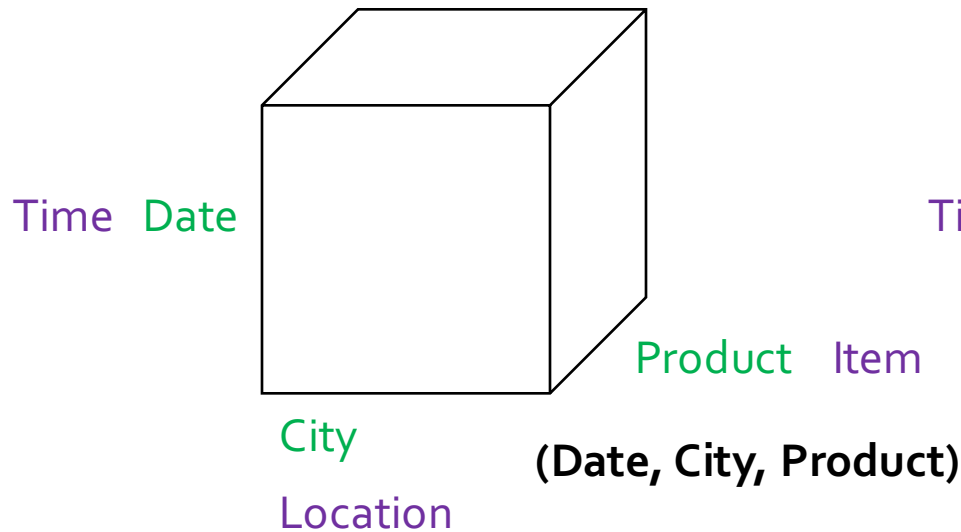
parent vs child cells

ancestor vs descendant cells

sibling cell:

(09/16, Mishawaka, milk)

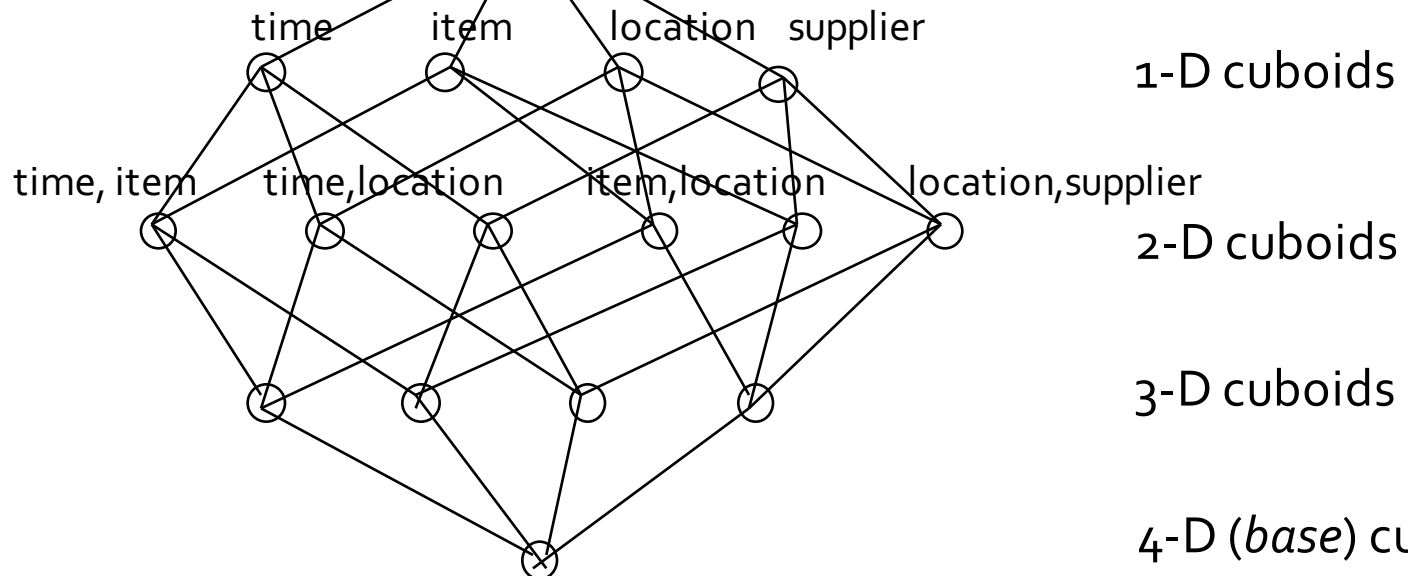
Base Cuboids and Aggregate Cuboids



Apex cuboid: (*, *, *)

(N-Dimensional) Data Cube

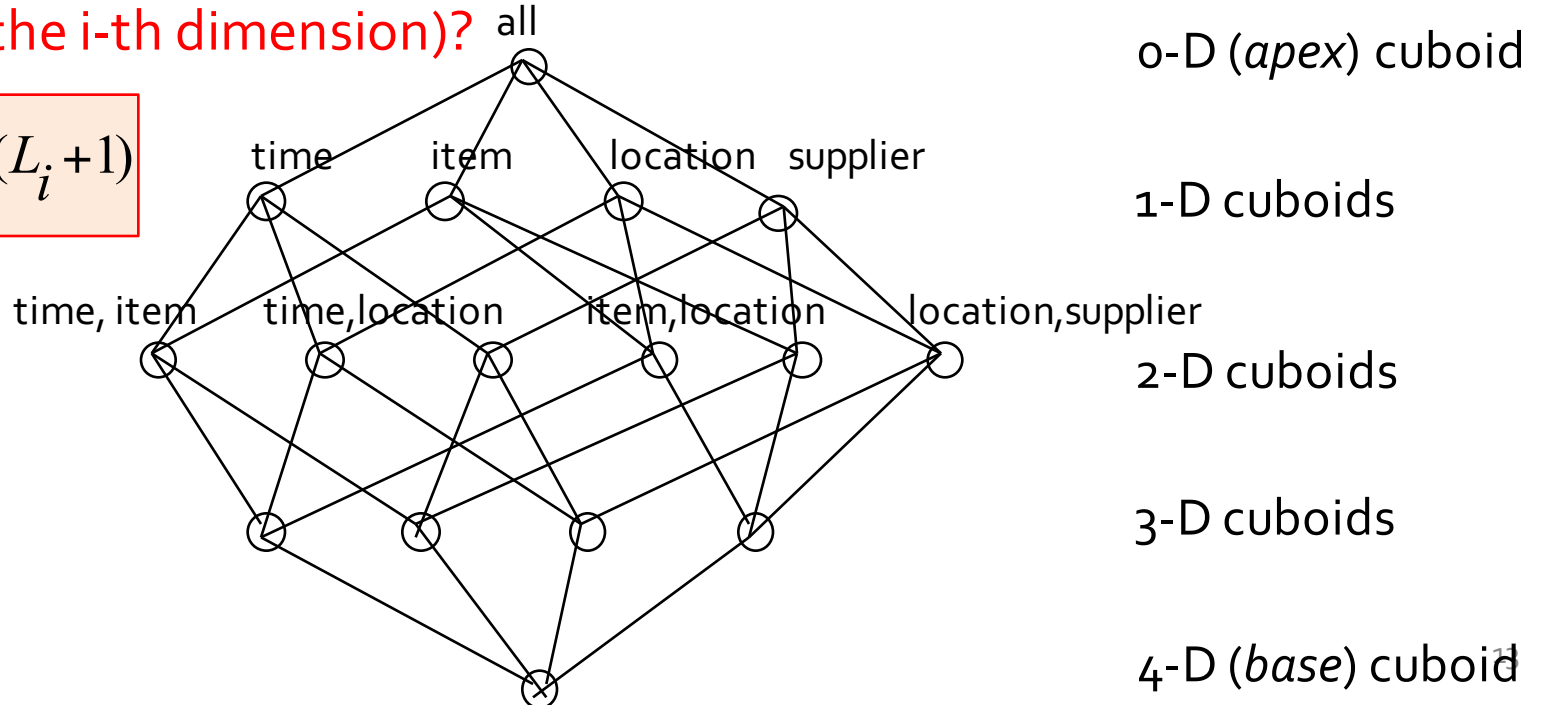
- Data cube can be viewed as a lattice of cuboids
 - The bottom-most cuboid is the base cuboid
 - The top-most cuboid (apex) contains only one cell
 - How many cuboids in an n-dimensional cube with L_i levels (at the i-th dimension)?
- all
- 0-D (*apex*) cuboid



(N-Dimensional) Data Cube

- Data cube can be viewed as a lattice of cuboids
 - The bottom-most cuboid is the base cuboid
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 - How many cuboids in an n-dimensional cube with L_i levels (at the i-th dimension)?

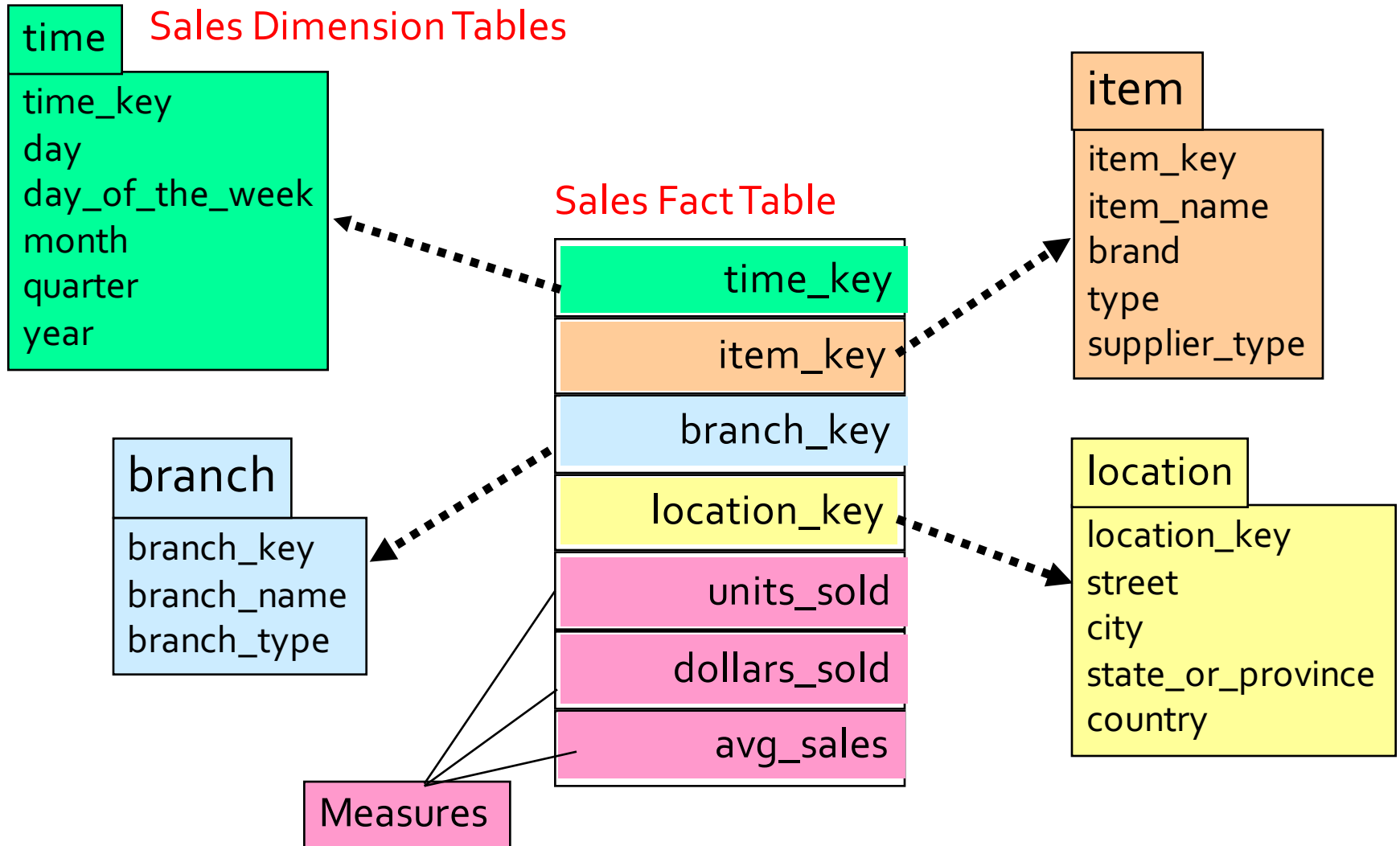
$$T = \prod_{i=1}^n (L_i + 1)$$



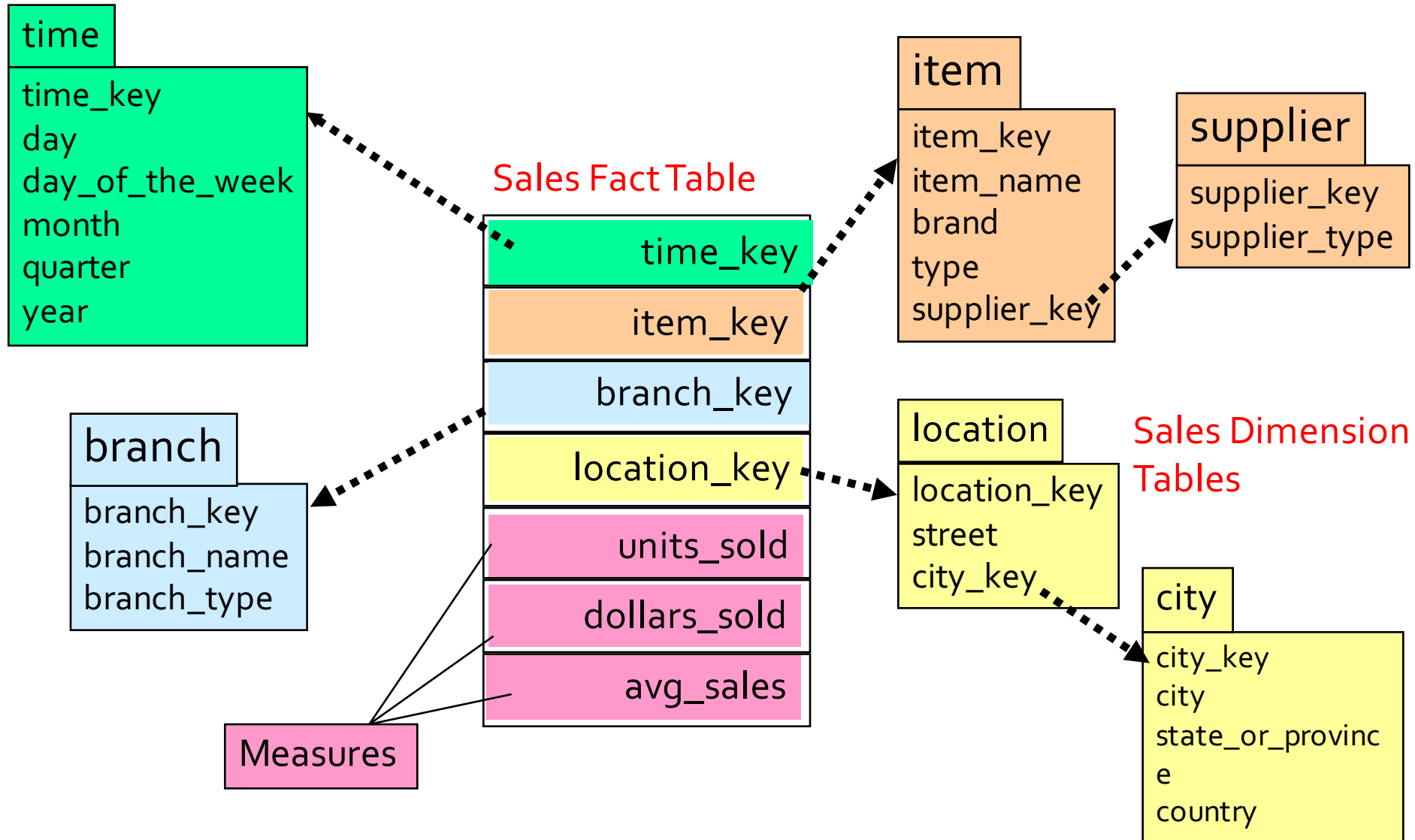
Data Cube: Definition

- **Data cube:** A lattice of cuboids
 - In data warehousing literature, an **n-D base cube** is called a **base cuboid**
 - The top most **o-D cuboid**, which holds the highest-level of summarization, is called the **apex cuboid**
 - The lattice of cuboids forms a **data cube**
- A data cube, such as sales, allows data to be modeled and viewed in multiple dimensions
 - **Dimension tables**, such as item (item_name, brand, type), or time (day, week, month, quarter, year)
 - **Fact table** contains **measures** (such as dollars_sold) and keys to each of the related dimension tables
 - Schemas: Dimension tables and Fact tables

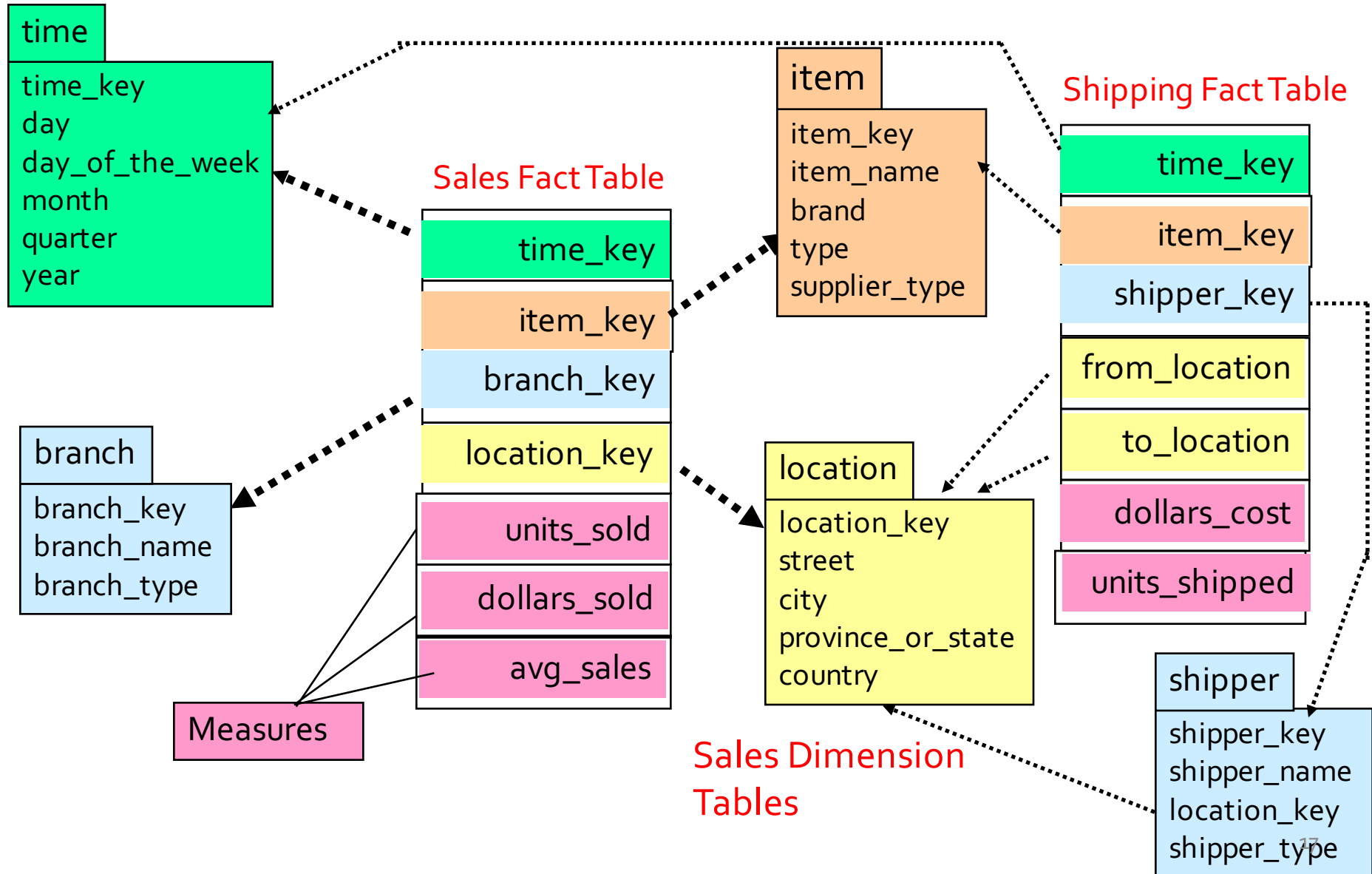
Star Schema



Snowflake Schema



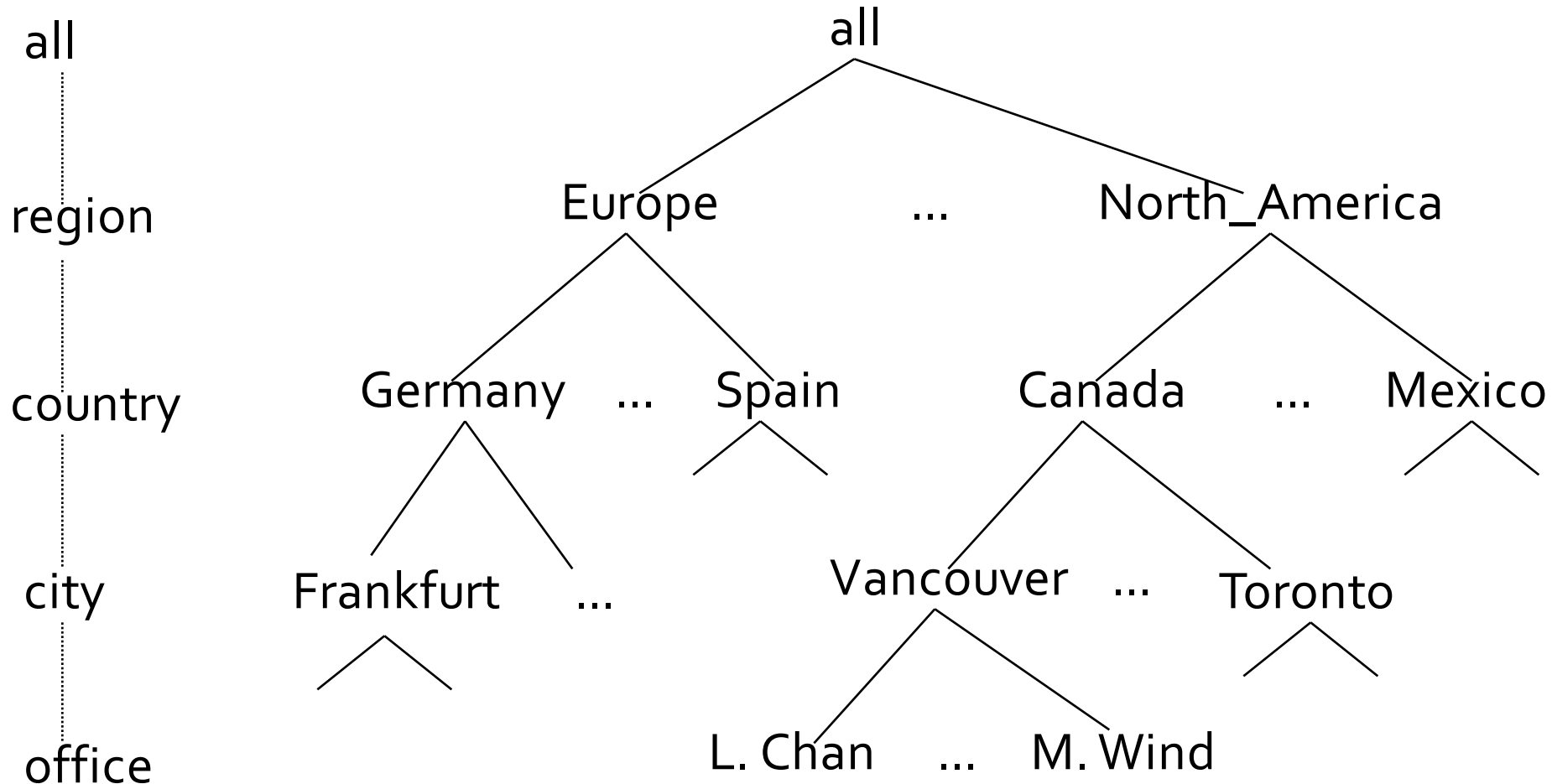
Fact Constellation



Modeling of Data Cubes

- Modeling data cubes: dimensions & measures
 - **Star schema:** A **fact table** in the middle connected to a set of **dimension tables**
 - **Snowflake schema:** A refinement of star schema where some dimensional hierarchy is normalized into **a set of smaller dimension tables**, forming a shape similar to snowflake
 - **Fact constellations:** **Multiple fact tables** share dimension tables, viewed as a collection of stars, therefore called galaxy schema or fact constellation

Concept Hierarchy: Dimension Level and Dimension Value

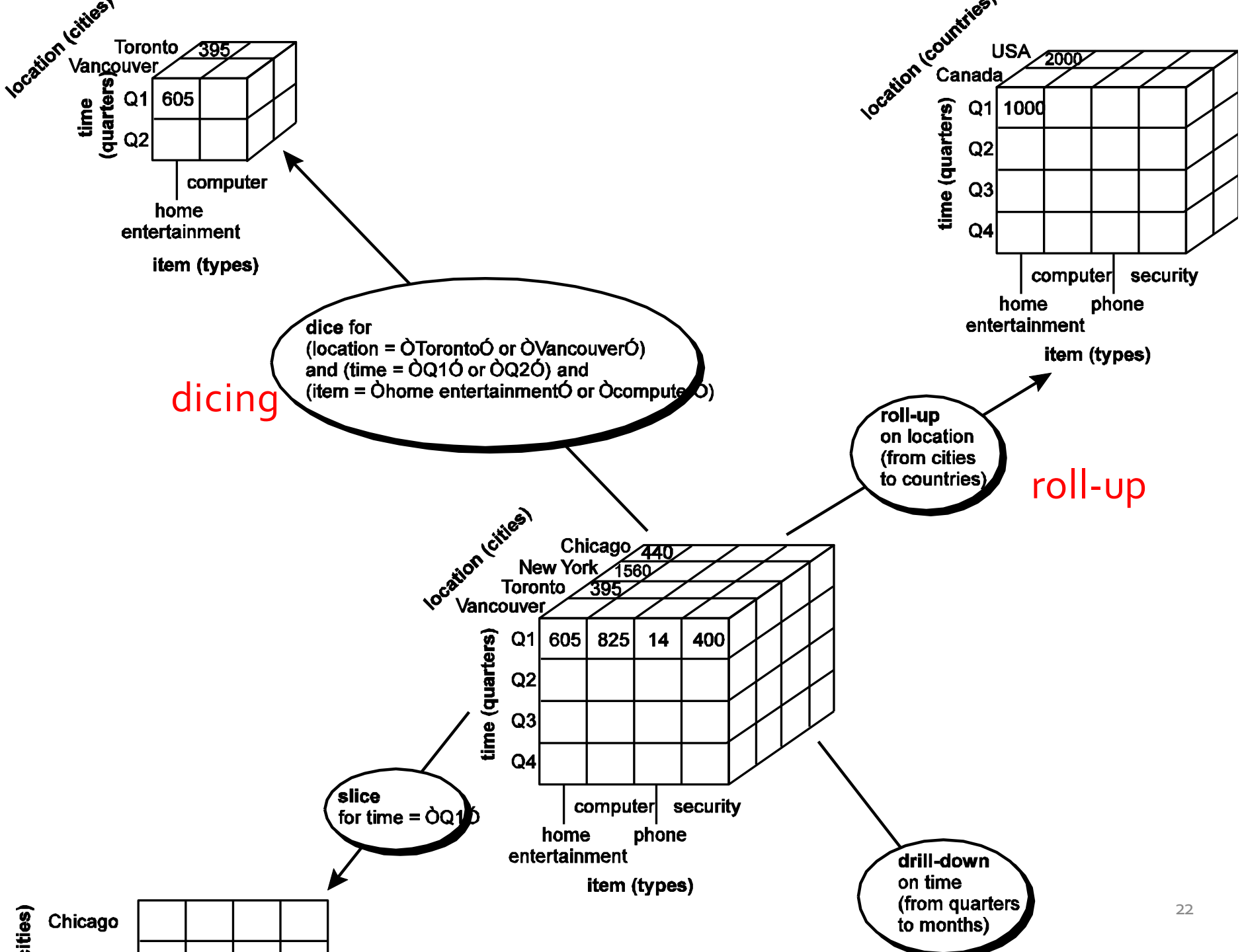


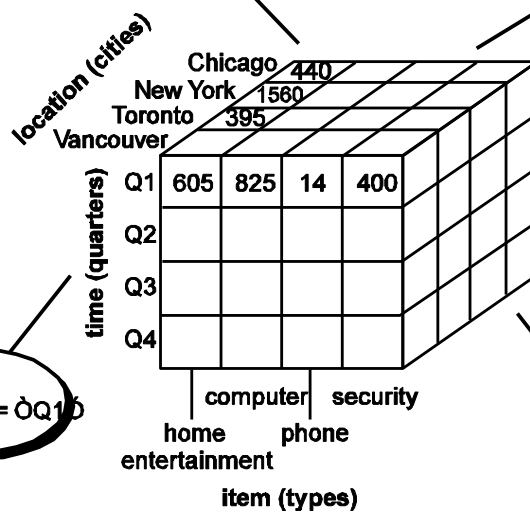
Data Cube Measures: Three Categories

- **Distributive**: if the result derived by applying the function to n aggregate values is the same as that derived by applying the function on all the data without partitioning
 - E.g., `count()`, `sum()`, `min()`, `max()`
- **Algebraic**: if it can be computed by an **algebraic function** with M arguments (where M is a bounded integer), each of which is obtained by applying a **distributive aggregate function**
 - $\text{avg}(x) = \text{sum}(x) / \text{count}(x)$
- **Holistic**: if there is no constant bound on the storage size needed to describe a sub-aggregate.
 - E.g., `median()`, `mode()`, `rank()`
- Q: How about `standard_deviation()`, `Q1()`, `Q3()`?

Typical Data Cube Operations

- **Roll up (drill up):** summarize data
 - *by climbing up hierarchy or by dimension reduction*
- **Drill down (roll down):** reverse of roll-up
 - *from higher level summary to lower level summary or detailed data, or introducing new dimensions*
- **Slice and dice:** *project and select*
- **Pivot (rotate):** *reorient the cube, visualization*





slice
for time = Q1

location (cities)

Chicago				
New York				
Toronto				
Vancouver	605	825	14	400

computer security
home phone
entertainment

item (types)

slicing

pivot

item (types)

home entertainment				605
computer				825
phone				14
security				400

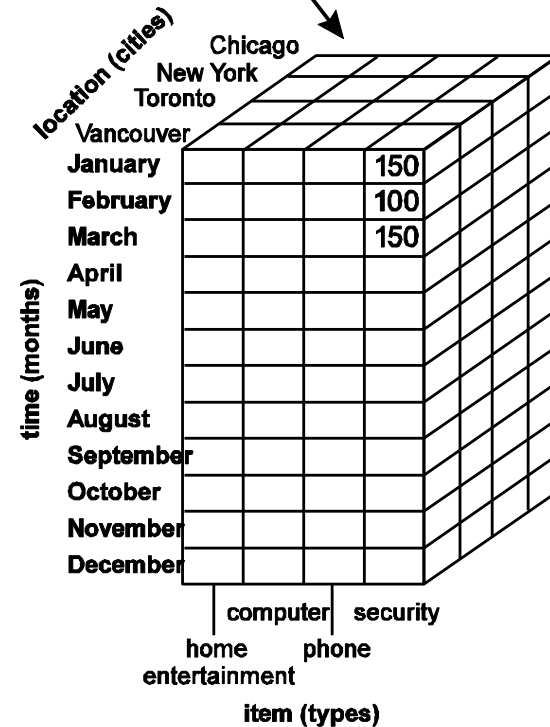
New York Vancouver
Chicago Toronto

location (cities)

pivot

drill-down
on time
(from quarters
to months)

drill-down

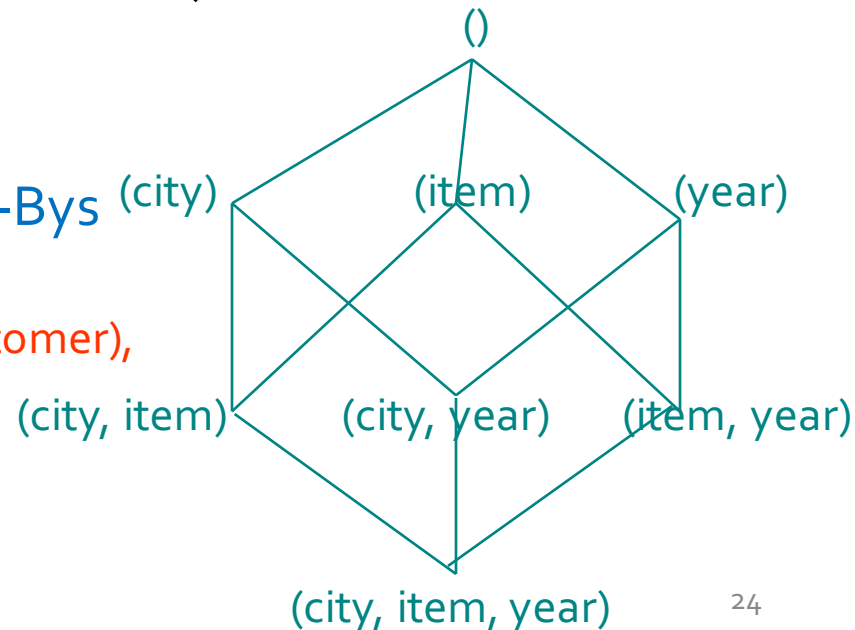


The “Compute Cube” Operator

- Cube definition and computation

```
define cube sales [item, city, year]: sum (sales_in_dollars)
compute cube sales
```
- Transform it into a SQL-like language (with a new operator **cube by**, introduced by **Gray et al.'97**)

```
SELECT item, city, year, SUM (amount)
FROM SALES
CUBE BY item, city, year
```
- Need compute the following **Group-Bys**
(year, product, customer),
(year, product), (year, customer), (product, customer),
(year), (product), (customer)
()



Data Cube History

Data cube: A relational aggregation operator generalizing group-by, cross-tab, and sub-totals

2981

1997

J Gray, S Chaudhuri, A Bosworth, A Layman, D Reichart, M Venkatrao, ...
Data Mining and Knowledge Discovery 1 (1), 29-53

Data Cube: A Relational Aggregation Operator Generalizing Group-By, Cross-Tab, and Sub-Totals

Jim Gray
Surajit Chaudhuri
Adam Bosworth
Andrew Layman
Don Reichart
Murali Venkatrao
Frank Pellow
Hamid Pirahesh¹

May 1997

Technical Report
MSR-TR-97-32

Microsoft Research
Microsoft Corporation
One Microsoft Way
Redmond, WA 98052

*Surajit Chaudhuri is a computer scientist best known for his contributions to database management systems. He is currently a **distinguished scientist at Microsoft Research**, where he leads the Data Management, Exploration and Mining group.*

*Adam Bosworth is a former **Vice President of Product Management at Google Inc.** from 2004–2007; prior to that, he was senior VP Engineering and Chief Software Architect at BEA Systems responsible for ...*

*Hamid Pirahesh, Ph.D., is an **IBM fellow, ACM Fellow and a senior manager responsible for the exploratory database department at IBM Research** - Almaden in San Jose, California. Dr. Hamid Pirahesh is the senior manager at IBM Almaden Research Center in San Jose, California.*

Jim Gray Summary Home Page

[Microsoft eScience Group](#)

As you may be aware, Jim Gray has [gone missing](#).

We (his colleagues in Microsoft Research) have heard from many of his collaborators about projects and collaborations that he had underway with them and who are unsure how to proceed. If you find yourself in this situation, please email grayproj@microsoft.com and we will follow up with you to find the best way forward.

Jim Gray is a researcher and manager of Microsoft Research's [eScience Group](#). His primary research interests are in databases and transaction processing systems -- with particular focus on using computers to make scientists more productive. He and his group are working in the areas of astronomy, geography, hydrology, oceanography, biology, and health care. He continues a long-standing interest on building supercomputers with commodity components, thereby reducing the cost of storage, processing, and networking by factors of 10x to 1000x over low-volume solutions. This includes work on building fast networks, on building huge web servers with *CyberBricks*, and building very inexpensive and very high-performance storage servers.

Jim also is working with the astronomy community to build the [world-wide telescope](#) and has been active in building online databases like <http://terraService.Net> and <http://skyserver.sdss.org>. When the entire world's astronomy data is on the Internet and is accessible as a single distributed database, the Internet will be the world's best telescope. This is part of the larger agenda of getting all information online and easily accessible (digital libraries, digital government, online science ...). More generally, he is working with the science community (Oceanography, Hydrology, environmental monitoring, ..) to build the world-wide digital library that integrates all the world's scientific literature and the data in one easily-accessible collection. He is active in the research community, is an ACM, NAE, NAS, and AAAS Fellow, and received the ACM Turing Award for his work on transaction processing. He also edits of a series of books on data management.



James Nicholas "Jim" Gray (born January 12, 1944; presumed lost at sea January 28, 2007; declared deceased May 16, 2012^[4]) was an [American computer scientist](#) who received the [Turing Award](#)^[5] in 1998 "for seminal contributions to [database](#) and [transaction processing](#) research and technical leadership in system implementation."

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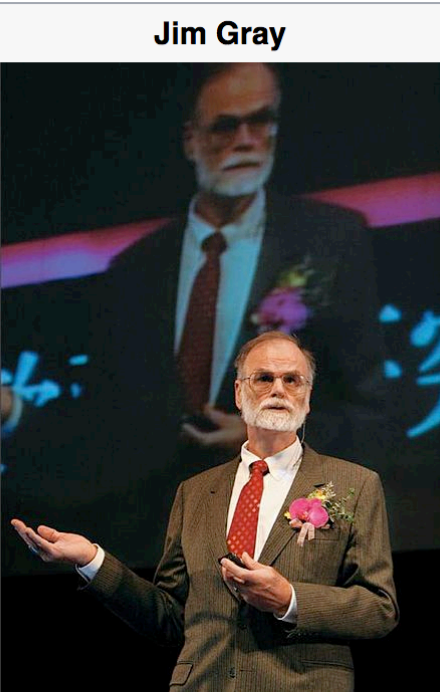
[References](#)

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Early years [edit]

Gray was born in [San Francisco, California](#), the second child of a mother who was a teacher and a father in the U.S. Army; the family moved to [Rome](#) where Gray spent most of the first three years of his life, learning to speak Italian before English.^[2] The family then moved to [Virginia](#), spending about four years there, until Gray's parents divorced, after which he returned to San Francisco with



Gray in 2006

Born	James Nicholas Gray <div>January 12, 1944^[1]</div> <div>San Francisco, California^[2]</div>
Disappeared	January 28, 2007 (aged 63) <div>Waters near San Francisco</div>
Status	Dead in absentia , May 16, 2012 (aged 68)
Nationality	American
Alma mater	University of California, Berkeley (Ph.D)
Occupation	Computer scientist
Employer	IBM Tandem Computers DEC Microsoft

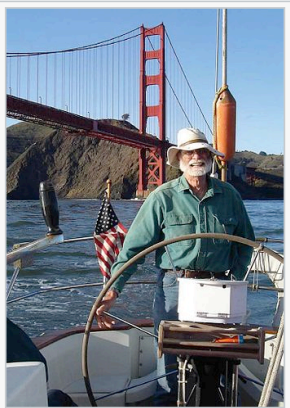
On Sunday, January 28, 2007, during a short solo [sailing](#) trip to the [Farallon Islands](#) near [San Francisco](#) to scatter his mother's ashes, Gray and his 40-foot yacht, *Tenacious*, were reported missing by his wife, Donna Carnes. The [Coast Guard](#) searched for four days using a [C-130](#) plane, helicopters, and patrol boats but found no sign of the vessel.^{[21][22][23][24]}

Gray's boat was equipped with an automatically deployable EPIRB ([Emergency Position-Indicating Radio Beacon](#)), which should have deployed and begun transmitting the instant his vessel sank. The area around the Farallon Islands where Gray was sailing is well north of the East-West ship channel used by freighters entering and leaving [San Francisco Bay](#). The weather was clear that day and no ships reported striking his boat, nor were any distress radio transmissions reported.

On February 1, 2007, the [DigitalGlobe](#) satellite did a scan of the area, generating thousands of images.^[25] The images were posted to [Amazon Mechanical Turk](#) in order to distribute the work of searching through them, in hopes of spotting his boat.

In the immediate aftermath of the disappearance, many theories were put forward on how Gray disappeared.^[26]

After being missing for five years, Gray was legally assumed to have died at sea on January 28, 2012.^{[4][33]}



Jim Gray on the *Tenacious* in January 2006

Efficient Data Cube Computation

- Data cube can be viewed as a lattice of cuboids
 - The bottom-most cuboid is the base cuboid
 - The top-most cuboid (apex) contains only one cell
 - How many cuboids in an n-dimensional cube with L_i levels?
- Materialization of data cube
 - **Full materialization:** Materialize every (cuboid)
 - **No materialization:** Materialize none (cuboid)
 - **Partial materialization:** Materialize some cuboids
 - Which cuboids to materialize?
 - Selection based on size, sharing, access frequency, etc.

$$T = \prod_{i=1}^n (L_i + 1)$$

Review: Data Cube

- Concepts
 - Cell, Cuboid, Cube
 - Dimension, Dimension Level, Dimension Value
 - Base/Aggregate Cell/Cuboid
- Components
 - Dimension tables and Fact tables
 - Concept hierarchy and Measures
 - Schemas
- Operations
- Materialization **Partial materialization:**
Which cuboids to materialize?

Q: What do they hate the most?



Iceberg



Cube Materialization: Full Cube vs. Iceberg Cube

- Full cube vs. iceberg cube
compute cube sales **iceberg** as
select date, product, city, department, count(*)
from salesInfo
cube by date, product, city
having count(*) >= min support
- Compute *only* the **cells** whose **measure** satisfies
the **iceberg condition**
- Only a small portion of cells may be “above the
water” in a **sparse cube**
- Ex.: Show only those cells whose **count** is no less
than 100



Why Iceberg Cube?

- Advantages of computing iceberg cubes
 - No need to save nor show those cells whose value is below the threshold (iceberg condition)
 - Efficient methods may even avoid computing the un-needed, intermediate cells
 - Avoid explosive growth
- Example: A cube with 100 dimensions
 - Suppose it contains only 2 base cells and the count of each cell is 1:
 - $\{(a_1, a_2, a_3, \dots, a_{100}) : 1, (a_1, a_2, b_3, \dots, b_{100}) : 1\}$
 - How many **aggregate cells** if “having count ≥ 1 ” (**non-empty**)?
 - What are the **iceberg cells** with condition “having count ≥ 2 ”?

Suppose it contains only 2 base cells:

$$\{(a_1, a_2, a_3, \dots, a_{100}), (a_1, a_2, b_3, \dots, b_{100})\}$$

How many non-empty aggregate cells?

For $\{(a_1, a_2, a_3, \dots, a_{100}), (a_1, a_2, b_3, \dots, b_{100})\}$, the total # of non-base cells should be $2 * (2^{\{100\}} - 1) - 4$.

This is calculated as follows:

- $(a_1, a_2, a_3, \dots, a_{100})$ will generate $2^{\{100\}} - 1$ non-base cells
- $(a_1, a_2, b_3, \dots, b_{100})$ will generate $2^{\{100\}} - 1$ non-base cells

Among these, 4 cells are overlapped and thus minus 4 so we get:

$$2 * 2^{\{100\}} - 2 - 4$$

These 4 cells are:

- $(a_1, a_2, *, \dots, *)$: 2
- $(a_1, *, *, \dots, *)$: 2
- $(*, a_2, *, \dots, *)$: 2
- $(*, *, *, \dots, *)$: 2

Is Iceberg Cube Good Enough?

Closed Cube & Cube Shell

- Let cube P have only 2 base cells: $\{(a_1, a_2, a_3 \dots, a_{100}):10, (a_1, a_2, b_3, \dots, b_{100}):10\}$
 - How many cells will the iceberg cube contain if “having $\text{count}(\ast) \geq 10$ ”?
 - Answer: $2^{101} - 4$ (base+aggregate; still too big!)
- **Close cube:**
 - A cell c is **closed** if there **exists no cell** d , such that d is a **descendant** of c , and d has the **same measure** value as c
 - Ex. The same cube P has only 3 closed cells:
 - $\{(a_1, a_2, \ast, \dots, \ast): 20, (a_1, a_2, a_3 \dots, a_{100}):10, (a_1, a_2, b_3, \dots, b_{100}):10\}$
 - A **closed cube** is a cube consisting of only closed cells

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