



# Analysis of cascading failure in complex power networks under the load local preferential redistribution rule

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## ABSTRACT

In recent years several global blackouts have drawn a lot of attention to security problems in electric power transmission systems. Here we analyze the cascading failure in complex power networks based on the local preferential redistribution rule of the broken node's load, where the weight of a node is correlated with its link degree  $k$  as  $k^\beta$ . It is found that there exists a threshold  $\alpha^*$  such that cascading failure is induced and enhanced when the value of tolerance parameter is smaller than the threshold. It is also found that the larger  $\beta$  is the more robust the power network is.

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## 1. Introduction

With the development of larger-scale interconnected power systems, electricity generation and transmission have become more and more efficient. The complexity of operations in power systems has increased, meaning the security and stability of power systems are facing an unprecedented challenge. In the last decades, some of the major global power systems have suffered a reduction in stabilization, even collapse events in succession [1–3]. These disasters alarm the whole world and make people pay great attention to the reliability and security of power networks. Accurate post-mortem analyses of the causes involved in a major blackout have shown that, in most cases, large blackouts are due to the successive malfunction of a large number of components often triggered by an initial disturbance or event, such as short circuits of transmission lines, protection device mis-operation and bad weather [4]. If an initial disturbance or event causes a node to go down, its power is automatically shifted to the neighboring nodes, which in most cases are able to handle the extra load. Sometimes, however, these nodes are also overloaded and must redistribute their increased load to their neighbors. This eventually leads to a cascade of failures [5–12]: a large number of transmission lines are overloaded and malfunction at the same time. Recently, significant attention has been focused on cascading failure in complex power networks and a large number of cascading models have been presented, for instance, the dynamical flow model [13], the shortest path routing model [14], the Kuramoto-like model [15], the power flow entropy model [16], the line-node hybrid dynamics model [17,18], the hidden failure model [19], and the OPA model [20]. In these cascading failure models, the load on a node was generally estimated by its degree or betweenness and the load redistribution after failure nodes are removed from the networks are forwarded following the shortest path routing strategy, which is not practical for large power networks because there is a heavy communication cost in searching the network in real time. Therefore, to better explain complex blackouts of power systems, following the research of Wu [21] we propose a cascading model for power networks under a load local preferential redistribution rule (LLPRR) in the present work. In this rule the load on the broken node is redistributed to its neighboring

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nodes according to the preferential probability, namely, among these neighboring nodes the one with more degrees will undertake more shared load. The proposed rule has been tested on different standard IEEE test power networks as small power systems and the European power grid as a large real power system. The correlation between the robustness and the tolerance parameter of a network is also explored. In order to assess the dependence of the robustness of the whole network on the tolerance parameter  $\alpha$ , the characteristics  $S_N$  denoted as the avalanche size, i.e., the number of broken nodes has been analyzed. It is found that cascading failure is induced and enhanced when the tolerance parameter value is smaller than the critical threshold  $\alpha^*$ . It is also found that the larger  $\beta$  is the more robust the power network is.

## 2. The model

Here we consider the cascading failures triggered by removing a single node. If a node has a relatively small load, its removal will not cause major changes in the balance of loads, and subsequent overload failures are unlikely to occur. However, when the load at a node is relatively large, its removal is likely to affect significantly loads at other nodes and possibly starts a sequence of overload failures and eventually a large drop in the network performance such as those observed in real power networks. Inspired by the above process of cascading failures, we propose a cascading model for power networks based on the load local redistribution rule [21–23].

Assume the initial load of each node  $j$  in the power grid is defined as a function of its degree  $k_j$

$$F_j = k_j^\beta, \quad j = 1, 2, \dots, N \quad (1)$$

where  $N$  is the initial number of nodes;  $\beta > 0$  is a tunable parameter, which controls the strength of the initial load of the node  $j$ . This assumption is reasonable since many previous studies concerning both model power networks and real power networks have shown that the load of a node scales with its degree [13,14], namely, in the power grid a node (generator and substation) of higher degree has always a stronger ability to handle the transmission of electricity. In our model, the capacity of a node is the maximum load that the node can handle, which is severely limited by cost. Thus, it is natural to assume that the capacity  $C_j$  of node  $j$  is proportional to its initial load  $F_j$ ,

$$C_j = (1 + \alpha)F_j, \quad j = 1, 2, \dots, N \quad (2)$$

where the constant  $\alpha > 0$  is the tolerance parameter. We assume that the potential cascading failure is triggered by a small initial perturbation, e.g., unfunctioning of a single node  $i$  and the load only redistributes to its neighboring node  $j$ , in conformity to the preferential probability:

$$\Pi_j = k_j^\beta / \sum_{m \in \Omega_i} k_m^\beta \quad (3)$$

where  $m$  is the neighboring nodes of the broken node  $i$  and  $\Omega_i$  represents the set of all neighboring nodes of node  $i$ . According to the rule of (3), the additional load  $\Delta F_{ji}$  received by the node  $j$  is proportional to its initial load [24], i.e.,

$$\Delta F_{ji} = F_i k_j^\beta / \sum_{m \in \Omega_i} k_m^\beta. \quad (4)$$

Since every node has a limited capacity to handle the load, so for the node  $j$  if

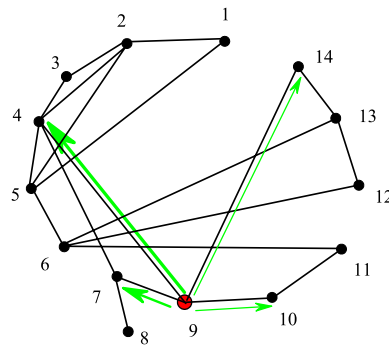
$$F_j + \Delta F_{ji} > C_j \quad (5)$$

then the node  $j$  will be broken and induce further the redistribution of the load  $F_j + \Delta F_{ji}$  and potentially further other nodes breaking. A simple power network, the IEEE 14 bus system which represents a portion of the American Electric Power System (in the Midwestern US) as of February, 1962 included 14 nodes and 20 lines [25], is used to illustrate the LLPRR procedure. The result is shown in Fig. 1, where suppose the focal node 9 is broken and its load is redistributed to its neighboring nodes 4, 7, 10 and 14. Among these neighbors, the one with higher capacity (i.e., more degrees) will undertake more shared load (denoted by the width of the links) from the failed node.

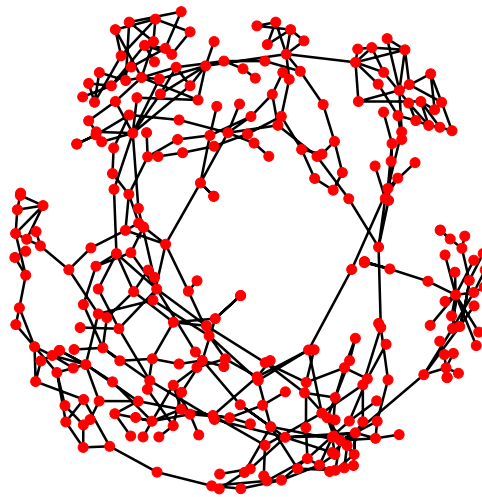
Note that since in real power networks a node of higher load always has a stronger ability to handle power flow transmission, i.e., a node's threshold is proportional to its weight, it is reasonable to preferentially reroute power flow transmission along those higher-capacity nodes to maintain normal functioning of power flow and to avoid further breakdowns [21–23].

## 3. Numerical results

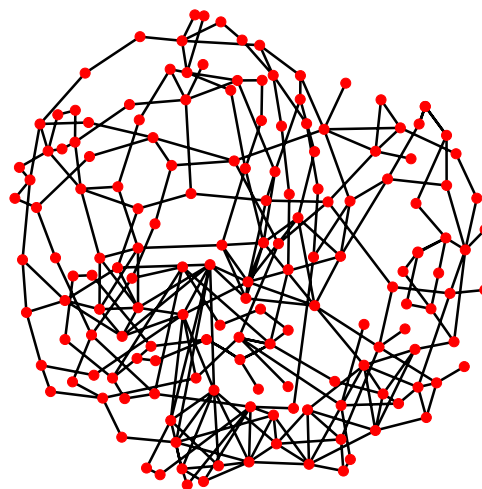
In this section, the standard IEEE 300, 162, 145, 118, 57, 30 bus test systems and the European power grid are selected as the simulation systems, whose network configurations are shown in Figs. 2–8 respectively. Note that the European power grid is nowadays the ensemble of 24 different national power grids coordinated by the Union for the Coordination of Transmission Electricity, UCTE. The distribution and location of transmission lines, plants, stations, etc., can be found in the last version (July 2007) of the UCTE Map. The map gives data from the transmission network and ignores the much



**Fig. 1.** (Color online) A simple example (The IEEE 14-bus system) to illustrate the LLPRR. Suppose the focal node 9 is broken and its load is redistributed to its neighboring nodes 4, 7, 10 and 14. Among these neighbors, the one with higher capacity (i.e., more degrees) will undertake more shared load (denoted by the width of the links) from the failed node.

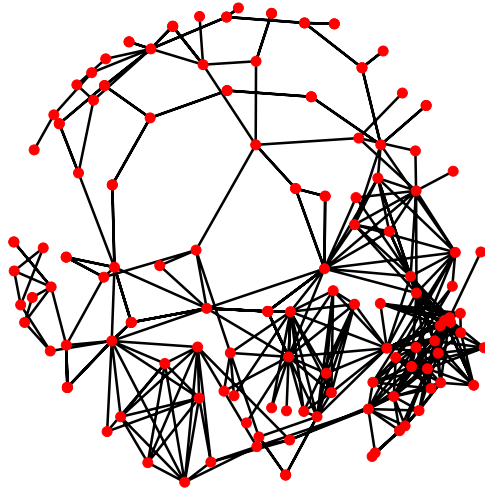


**Fig. 2.** (Color online) The topology structure of IEEE 300 nodes system.

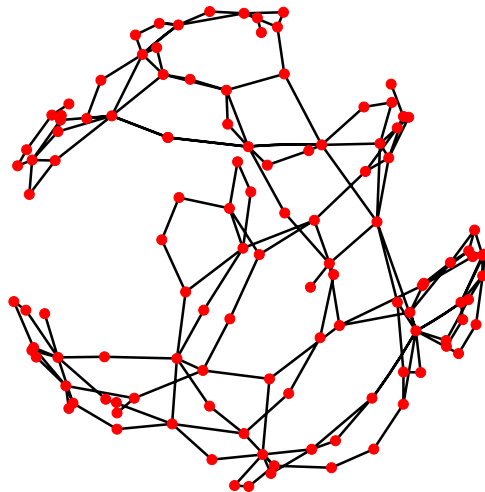


**Fig. 3.** (Color online) The topology structure of IEEE 162 bus system.

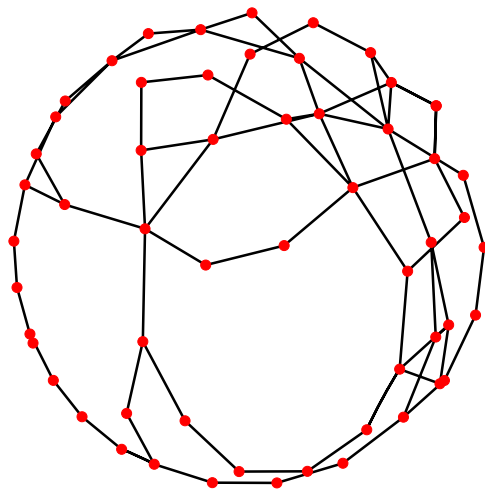
more extended distribution one. Nonetheless, it deals with more than 3000 nodes and some 200,000 km of transmission lines [26,27]. To measure the robustness degree of the whole networks against cascading failures, we remove every node in



**Fig. 4.** (Color online) The topology structure of IEEE 145 bus system.



**Fig. 5.** (Color online) The topology structure of IEEE 118 bus system.



**Fig. 6.** (Color online) The topology structure of IEEE 57 bus system.

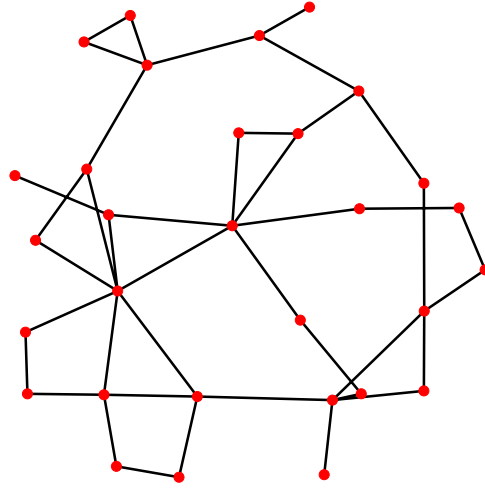


Fig. 7. (Color online) The topology structure of IEEE 30 bus system.

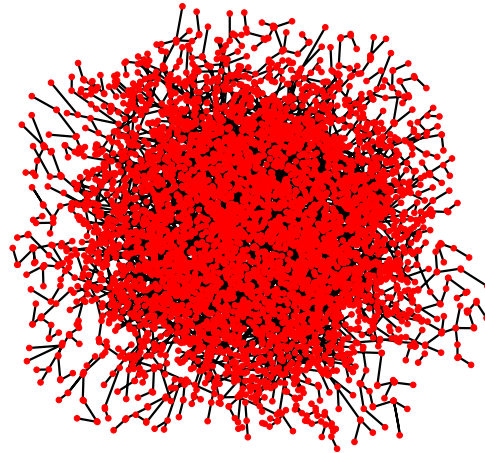
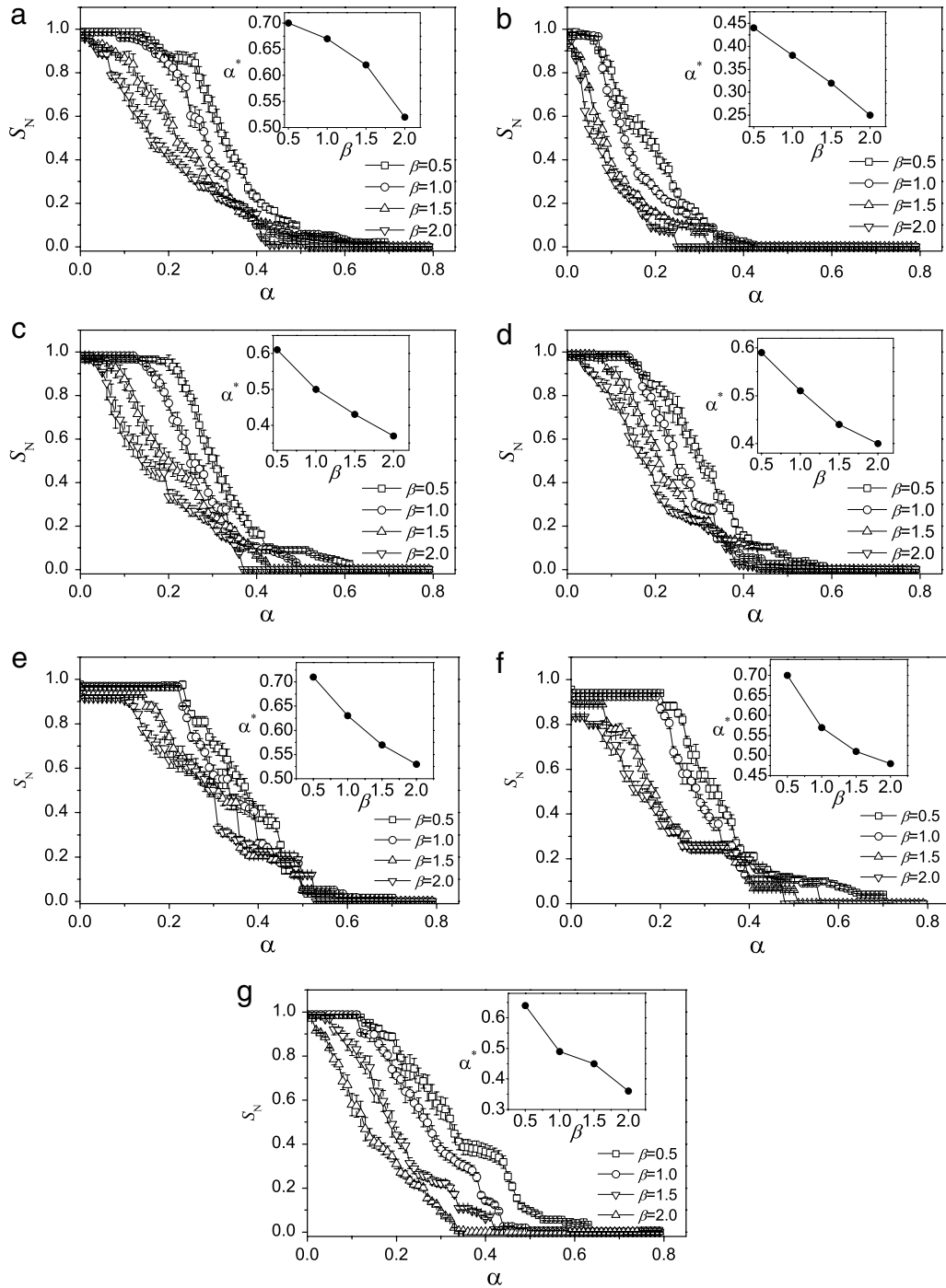


Fig. 8. (Color online) The topology structure of the European power grid.

a network one by one and calculate the corresponding results, e.g., removing the node  $i$  and calculating  $S_i$  after the cascading process is over, where  $S_i$  denotes the avalanche size, i.e., the number of broken nodes, induced by removing  $i$  [22]. To quantify the robustness of the whole network, we adopt the normalized avalanche size, i.e.,

$$S_N = \sum_{i \in N} S_i / (N(N-1)) \quad (6)$$

where the summation over all the avalanche sizes by removing each node initially at each time. The values of  $S_N$  are in the range from 0 to 1. It is clear that the more robust the network is, the smaller the parameter  $S_N$ . In our cascading model, it is evident that the  $\alpha$  will decide to the robustness level against cascading failure. Therefore, inspired by Wu et al. [21], to quantify the robustness, we propose a new measure, i.e., the critical threshold  $\alpha^*$ . The corresponding principle is that when  $\alpha > \alpha^*$  no cascading failure occurs and the system maintains its normal and efficient functioning, since every node in the network has the capacity to handle the extra load redistributed; while for  $\alpha < \alpha^*$ ,  $S_N$  suddenly increases from 0 and cascading failure emerges because the capacity of each node is limited, sweeping the whole or part of the network to stop working. Hence  $\alpha^*$  is the smallest value of protection strength to avoid cascading failure. Apparently, the lower the value of  $\alpha^*$ , the stronger the robustness of the network against cascading failure [21–23]. According to our cascading model, we investigate the relationship between some parameters and the critical threshold  $\alpha^*$ . We consider first the standard IEEE 300 bus power network. Fig. 9(a) shows the avalanche sizes  $S_N$  as a function of the tolerance parameter  $\alpha$  for several values of  $\beta$ , i.e., for  $\beta = 0.5, 1.0, 1.5, 2.0$  (with the corresponding error bars), respectively. For each curve, there exists a critical threshold  $\alpha^*$  such that when the value of  $\alpha$  is beyond this threshold, no cascading failure arises and the system maintains its normal and efficient functioning; While for the case of  $\alpha$  smaller than the threshold,  $S_N$  increases from zero and cascading failure emerges, causing the whole or part of the network to break. It is also found that for a fixed tolerance parameter  $\alpha$ , when the value of  $\beta$  is increased the parameter  $S_N$  decreases evidently. The dependences of  $\alpha^*$  on  $\beta$  are also



**Fig. 9.** Index  $S_N$  vs. tolerance parameter  $\alpha$  for  $\beta = 0.5, 1.0, 1.5, 2.0$  (with the corresponding error bars), respectively, where the insets show the dependences of  $\alpha^*$  on  $\beta$ : (a) IEEE 300 nodes system; (b) IEEE 162 nodes system; (c) IEEE 145 nodes system; (d) IEEE 118 nodes system; (e) IEEE 57 nodes system; (f) IEEE 30 nodes system; (g) The European power grid.

studied as shown in the inset to Fig. 9(a) and it is found that the value of  $\alpha^*$  shifts to a smaller value with increasing tunable parameter  $\beta$ . These phenomena show that the larger  $\beta$  is the more robust the network is in power networks. The same story could happen in other IEEE test power networks and the European power grid, which are shown in Fig. 9(b)–(g).

In previous work, Albert et al. [14] and Wu et al. [21] have also investigated the cascading failures model for complex networks. However, our work differs from the work in Refs. [14,21]: First, Ref. [14] focused on how much the performance of the North American power grid is affected by the cascading failures based on shortest path routing rule. It was found that the

power grid is robust to most perturbations, yet disturbances affecting key transmission substations greatly reduce its ability to function. In this work, we study the onset and spreading of cascading failure on different IEEE test power networks and the European power grid based on a local preferential redistribution rule. Secondly, Ref. [21] discussed cascading failures on the theoretic network, i.e., the BA network with the scale-free property. However, in this work, we implement the presented model for the practical power grid. In terms of practical utility, our result enables a possible implementation of a predicting and preventing mechanism for cascading breakdown in a real power grid.

#### 4. Conclusion

Cascading failures in complex power networks are studied based on the load local preferential redistribution rule, where the weight of a node is correlated with its link degree  $k$  as  $k^\beta$ . Different standard IEEE test power networks and the European power grid are employed to illustrate our results. In order to assess the effect of tolerance parameter  $\alpha$  and tunable parameter  $\beta$  upon the robustness of the whole network, we have analyzed the following characteristics:  $S_N$  denoted as the avalanche size, i.e., the number of broken nodes. It is found that cascading failure is induced and enhanced when the tolerance parameter value is smaller than the threshold  $\alpha^*$ . It is also found that the larger  $\beta$  is the more robust the power grid is.

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