We will show that the contrapositive of this statement is correct, i.e. that if n is not a multiple of 3, then n^2 is not a multiple of 3. So, as we are assuming that n is not a multiple of 3, we know that $n = 3k+1$ or $n = 3k+2$ for some $k \in \mathbb{Z}$. We will consider each case in turn. Suppose $n = 3k + 1$, then $n^2 = (3k + 1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$, which is not a multiple of 3. Now we consider the second case: $n^2 = (3k + 2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1$, which is not a multiple of 3. So, in both cases n^2 is not a multiple of 3, and we have established the contrapositive, which is equivalent to the original statement.

Theorem. Let $f(x)$ be a real polynomial of degree $n \geq 2$ with only real roots, such that $f(x) > 0$ for $-1 < x < 1$ and $f(-1) = f(1) = 0$. Let $A = \int_{-1}^{1} f(x)\,dx$, and let T be the area of the tangential triangle given by $f(x)$ (see figure). Then $\frac{2}{3} \cdot T \leq A$.

Here is an argument about the claim:

Since $f(x)$ has only real roots, and none of them in the open interval $(-1,1)$, it can be written — apart from a constant positive factor which cancels out in the end — in the form

$$f(x) = (1-x^2)\prod_i(\alpha_i - x)\prod_j(\beta_j + x) \qquad (1)$$

with $\alpha_i \geq 1, \beta_j \geq 1$. Hence

$$A = \int_{-1}^{1}(1-x^2)\prod_i(\alpha_i - x)\prod_j(\beta_j + x)\,dx.$$

By making the substitution $x \longmapsto -x$, we find that also

$$A = \int_{-1}^{1}(1-x^2)\prod_i(\alpha_i + x)\prod_j(\beta_j - x)\,dx.$$

and hence by the inequality of the arithmetic and geometric mean (note that all factors are ≥ 0)

$$\begin{aligned}
A &= \int_{-1}^{1}\frac{1}{2}[(1-x^2)\prod_i(\alpha_i - x)\prod_j(\beta_j + x) + (1-x^2)\prod_i(\alpha_i + x)\prod_j(\beta_j - x)]\,dx \\
&\geq \int_{-1}^{1}(1-x^2)\left(\prod_i(\alpha_i^2 - x^2)\prod_j(\beta_j^2 - x^2)\right)^{\frac{1}{2}}\,dx \\
&\geq \int_{-1}^{1}(1-x^2)\left(\prod_i(\alpha_i^2 - 1)\prod_j(\beta_j^2 - 1)\right)^{\frac{1}{2}}\,dx \\
&= \frac{4}{3}\left(\prod_i(\alpha_i^2 - 1)\prod_j(\beta_j^2 - 1)\right)^{\frac{1}{2}}
\end{aligned}$$

Let us compute $f'(1)$ and $f'(-1)$. (We may assume $f'(-1), f'(1) \neq 0$, since otherwise $T = 0$ and the inequality $\frac{2}{3}\cdot T \leq A$ becomes trivial). By (1) above we see

$$f'(1) = -2\prod_i(\alpha_i - 1)\prod_j(\beta_j + 1),$$

and similarity

$$f'(-1) = 2\prod_i(\alpha_i + 1)\prod_j(\beta_j - 1).$$

Hence we conclude

$$A > \frac{2}{3}\left(-f'(1)f'(-1)\right)^{\frac{1}{2}}.$$

Applying now the inequality of the harmonic and the geometric mean to $-f'(1)$ and $f'(1)$, we arrive at the conclusion

$$A \geq \frac{2}{3}\cdot\frac{2}{\frac{1}{-f'(1)} + \frac{1}{f'(-1)}} = \frac{4}{3}\cdot\frac{f'(1)f'(-1)}{f'(1) - f'(-1)} = \frac{2}{3}\cdot T$$

For any positive integer n, if n^2 is divisible by 3, then n is divisible by 3.

Let n be an integer such that $n^2 = 3x$, where x is any integer. Then $3 \mid n^2$. Since $n^2 = 3x$, $n \cdot n = 3x$. Thus, $3 \mid n$. Therefore if n^2 is a multiple of 3, then n is a multiple of 3.