공업수학

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1. 다음 행렬의 행렬식을 구하시오.

- 1-1

$$A = \begin{bmatrix} 4 & -1 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{bmatrix}, \det(A) = \begin{vmatrix} 4 & -1 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{vmatrix} = 4 \times 2 \times 5 = 40$$ (triangular matrix)

1. 다음 행렬의 행렬식을 구하시오.

- 1-2

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 16 \end{bmatrix} = (-1)^{1+1} \times 1 \times \begin{vmatrix} 4 & 2 & 0 \\ 2 & 9 & 2 \\ 0 & 2 & 16 \end{vmatrix} + (-1)^{1+2} \times 2 \times \begin{vmatrix} 2 & 2 & 0 \\ 0 & 9 & 2 \\ 0 & 2 & 16 \end{vmatrix} + 0 + 0$$

$$= 4 \begin{vmatrix} 9 & 2 \\ 2 & 16 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 0 & 16 \end{vmatrix} - 2 \left\{ 2 \begin{vmatrix} 9 & 2 \\ 2 & 16 \end{vmatrix} - 2 \begin{vmatrix} 0 & 2 \\ 0 & 16 \end{vmatrix} \right\}$$

$$= -64$$

2. Cramer's rule을 이용하여 다음 연립방정식의 해를 구하시오.

$$\begin{aligned} 3x \; - 2y \; + z &= \;\; 13 \\ -2x \; + \;\; y \; + 4z &= \;\; 11 \\ x \; + 4y - 5z &= -31 \end{aligned}$$

$$A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & 4 \\ 1 & 4 & -5 \end{bmatrix}$$

$$\det(A) = 3\begin{vmatrix} 1 & 4 \\ 4 & -5 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 4 \\ 1 & -5 \end{vmatrix} + 1\begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} = -60$$

$$x = \frac{\begin{vmatrix} 13 & -2 & 1 \\ 11 & 1 & 4 \\ -31 & 4 & -5 \end{vmatrix}}{\det(A)}, y = \frac{\begin{vmatrix} 3 & 13 & 1 \\ -2 & 11 & 4 \\ 1 & -31 & -5 \end{vmatrix}}{\det(A)}, z = \frac{\begin{vmatrix} 3 & -2 & 13 \\ -2 & 1 & 11 \\ 1 & 4 & -31 \end{vmatrix}}{\det(A)}$$

2. Cramer's rule을 이용하여 다음 연립방정식의 해를 구하시오.

$$\begin{vmatrix} 13 & -2 & 1 \\ 11 & 1 & 4 \\ -31 & 4 & -5 \end{vmatrix} = 13\begin{vmatrix} 1 & 4 \\ 4 & -5 \end{vmatrix} - (-2)\begin{vmatrix} 11 & 4 \\ -31 & -5 \end{vmatrix} + \begin{vmatrix} 11 & 1 \\ -31 & 4 \end{vmatrix} = -60$$

- $x = \dfrac{\begin{vmatrix} 13 & -2 & 1 \\ 11 & 1 & 4 \\ -31 & 4 & -5 \end{vmatrix}}{\det(A)} = \dfrac{\begin{vmatrix} 13 & -2 & 1 \\ 11 & 1 & 4 \\ -31 & 4 & -5 \end{vmatrix}}{-60} = \dfrac{-60}{-60} = 1$

$$\begin{vmatrix} 3 & 13 & 1 \\ -2 & 11 & 4 \\ 1 & -31 & -5 \end{vmatrix} = 3\begin{vmatrix} 11 & 4 \\ -31 & -5 \end{vmatrix} - 13\begin{vmatrix} -2 & 4 \\ 1 & -5 \end{vmatrix} + \begin{vmatrix} -2 & 11 \\ 1 & -31 \end{vmatrix} = 180$$

- $y = \dfrac{\begin{vmatrix} 3 & 13 & 1 \\ -2 & 11 & 4 \\ 1 & -31 & -5 \end{vmatrix}}{\det(A)} = \dfrac{\begin{vmatrix} 3 & 13 & 1 \\ -2 & 11 & 4 \\ 1 & -31 & -5 \end{vmatrix}}{-60} = \dfrac{180}{-60} = -3$

2. Cramer’s rule을 이용하여 다음 연립방정식의 해를 구하시오.

$$\begin{vmatrix} 3 & -2 & 13 \\ -2 & 1 & 11 \\ 1 & 4 & -31 \end{vmatrix} = 3\begin{vmatrix} 1 & 11 \\ 4 & -31 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 11 \\ 1 & -31 \end{vmatrix} + 13\begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} = -240$$

- $z = \dfrac{\begin{vmatrix} 3 & -2 & 13 \\ -2 & 1 & 11 \\ 1 & 4 & -31 \end{vmatrix}}{\det(A)} = \dfrac{\begin{vmatrix} 3 & -2 & 13 \\ -2 & 1 & 11 \\ 1 & 4 & -31 \end{vmatrix}}{-60} = \dfrac{-240}{-60} = 4$

$$\therefore x = 1, y = -3, z = 4$$

3. 다음 행렬의 inverse matrix를 구하시오.

- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \det(A) = 1(45-48) - 2(36-42) + 3(32-35) = 0$

$\det(A) = 0$ 이므로 Singular matrix이다. → 역행렬 존재하지 않음

3. 다음 행렬의 inverse matrix를 구하시오.

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$, $\det(A) = 1$ 이므로 invertible 이다.

$[A|I] = \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 \\ 2 & 1 & 0 & | & 0 & 1 & 0 \\ 5 & 4 & 1 & | & 0 & 0 & 1 \end{bmatrix}$ 을 $[I|A^{-1}]$로 변형하면 A^{-1}를 구할 수 있다.

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 \\ 2 & 1 & 0 & | & 0 & 1 & 0 \\ 5 & 4 & 1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow[-5R_1 + R_3 \to R_3]{-2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 4 & 1 & | & -5 & 0 & 1 \end{bmatrix} \xrightarrow{-4R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 3 & -4 & 1 \end{bmatrix}$$

$$AA^{-1} = I \quad \to \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}$$

4. 3-2의 행렬을 이용해서 $(A^T)^{-1} = (A^{-1})^T$가 됨을 확인하시오.

- $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}$, $(A^{-1})^T = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

$[A^T|I] = \begin{bmatrix} 1 & 2 & 5 & | & 1 & 0 & 0 \\ 0 & 1 & 4 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$을 $[I|(A^T)^{-1}]$로 변형하면 $(A^T)^{-1}$를 구할 수 있다.

$$\begin{bmatrix} 1 & 2 & 5 & | & 1 & 0 & 0 \\ 0 & 1 & 4 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-2R_2 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & -3 & | & 1 & -2 & 0 \\ 0 & 1 & 4 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow[-4R_3 + R_2 \to R_2]{3R_3 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 & | & 1 & -2 & 3 \\ 0 & 1 & 0 & | & 0 & 1 & -4 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\therefore (A^T)^{-1} = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = (A^{-1})^T}$$

#5. MATLAB

#1-1

```
>> %1-1
>> A = [4 -1 8; 0 2 3; 0 0 5]

A =

     4    -1     8
     0     2     3
     0     0     5

>> det(A)

ans =

    40
```

#1-2

```
>> %1-2
>> A = [1 2 0 0; 2 4 2 0; 0 2 9 2; 0 0 2 16]

A =

     1     2     0     0
     2     4     2     0
     0     2     9     2
     0     0     2    16

>> det(A)

ans =

   -64
```

#5. MATLAB

#3-1

```
>> %3
>> A = [1 2 3; 4 5 6; 7 8 9]

A =

     1     2     3
     4     5     6
     7     8     9

>> det(A)

ans =

  -9.5162e-16

>> inv(A)
경고: 행렬이 특이 행렬에 가깝거나 준특이 행렬(badly scaled)일 수 있습니다. 결과값이 부정확할 수 있습니다.
RCOND =  2.202823e-18.


ans =

   1.0e+16 *

    0.3153   -0.6305    0.3153
   -0.6305    1.2610   -0.6305
    0.3153   -0.6305    0.3153
```

#3-2

```
>> %3
>> A = [1 0 0; 2 1 0;5 4 1]

A =

     1     0     0
     2     1     0
     5     4     1

>> det(A)

ans =

     1

>> inv(A)

ans =

    1.0000         0         0
   -2.0000    1.0000         0
    3.0000   -4.0000    1.0000
```

#5. MATLAB

#4

```
>> %4
>> A = [1 0 0; 2 1 0;5 4 1]

A =

     1     0     0
     2     1     0
     5     4     1

>> inv(transpose(A))

ans =

     1    -2     3
     0     1    -4
     0     0     1

>> transpose(inv(A))

ans =

    1.0000   -2.0000    3.0000
         0    1.0000   -4.0000
         0         0    1.0000
```