공업수학

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$$A = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

1. Eigenvalue 구하기

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 3 & 3 \\ 3 & 6-\lambda & 1 \\ 3 & 1 & 6-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (4-\lambda)\big((6-\lambda)^2 - 1\big) - 3(3(6-\lambda) - 3) + 3\big(3 - 3(6-\lambda)\big) = -\lambda^3 + 16\lambda^2 - 65\lambda + 50$$

$$= -(\lambda - 10)(\lambda - 5)(\lambda - 1) = 0$$

→ $\lambda = 1, 5, 10$

$$A = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

2. Eigenvector 구하기

$i)\quad \lambda = 1$

$$A - I = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 5 & 1 \\ 3 & 1 & 5 \end{bmatrix}, [A - I|0] = \begin{bmatrix} 3 & 3 & 3 & | & 0 \\ 3 & 5 & 1 & | & 0 \\ 3 & 1 & 5 & | & 0 \end{bmatrix}$$

$$REF : \begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$ → eigenvector $v_1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

ii) $\lambda = 5$

$$A - 5I = \begin{bmatrix} -1 & 3 & 3 \\ 3 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}, [A - 5I|0] = \begin{bmatrix} -1 & 3 & 3 & | & 0 \\ 3 & 1 & 1 & | & 0 \\ 3 & 1 & 1 & | & 0 \end{bmatrix}$$

$$REF: \begin{bmatrix} -1 & 3 & 3 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow \text{eigenvector } v_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

iii) $\lambda = 10$

$A - 10I = \begin{bmatrix} -6 & 3 & 3 \\ 3 & -4 & 1 \\ 3 & 1 & -4 \end{bmatrix}, [A - 10I|0] = \left[\begin{array}{ccc|c} -6 & 3 & 3 & 0 \\ 3 & -4 & 1 & 0 \\ 3 & 1 & -4 & 0 \end{array}\right]$

$REF: \left[\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ → eigenvector $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

→Eigenvectors $v_1, v_2, v_3 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(eigenvalues가 모두 다르므로 독립이 보장됨)

→ eigen basis = $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

$$A = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

3. 대각화

$\hat{A} = P^{-1}AP$

$$P = \begin{bmatrix} -2 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[P|I] = \begin{bmatrix} -2 & 0 & 1 & | & 1 & 0 & 0 \\ 1 & -1 & 1 & | & 0 & 1 & 0 \\ 1 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & 1 & 0 & | & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & | & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = [\mathrm{I}|P^{-1}]$$

$$P^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} ,\hat{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{bmatrix} \quad \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

3. 대각화

$$\hat{A} = P^{-1}AP \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 4 & 3 & 3 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\hat{A}^{10} = P^{-1}AP * P^{-1}AP * P^{-1}AP * \cdots * P^{-1}AP = P^{-1}A^{10}P$$

$$\begin{bmatrix} 1^{10} & 0 & 0 \\ 0 & 5^{10} & 0 \\ 0 & 0 & 10^{10} \end{bmatrix} = P^{-1}AP \rightarrow P \begin{bmatrix} 1^{10} & 0 & 0 \\ 0 & 5^{10} & 0 \\ 0 & 0 & 10^{10} \end{bmatrix} P^{-1} = A^{10}$$

$$A^{10} = \begin{bmatrix} -2 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1^{10} & 0 & 0 \\ 0 & 5^{10} & 0 \\ 0 & 0 & 10^{10} \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 10^{10} \\ 1 & -5^{10} & 10^{10} \\ 1 & 5^{10} & 10^{10} \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3} + \frac{1}{3}10^{10} & -\frac{1}{3} + \frac{1}{3}10^{10} & -\frac{1}{3} + \frac{1}{3}10^{10} \\ -\frac{1}{3} + \frac{1}{3}10^{10} & \frac{1}{6} + \frac{1}{2}5^{10} + \frac{1}{3}10^{10} & \frac{1}{6} - \frac{1}{2}5^{10} + \frac{1}{3}10^{10} \\ -\frac{1}{3} + \frac{1}{3}10^{10} & \frac{1}{6} - \frac{1}{2}5^{10} + \frac{1}{3}10^{10} & \frac{1}{6} + \frac{1}{2}5^{10} + \frac{1}{3}10^{10} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

1. Eigenvalue 구하기

$$B - \lambda I = \begin{bmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & -4-\lambda \end{bmatrix}$$

$\det(B - \lambda I) = (-4-\lambda)\left((1-\lambda)^2 - 1\right) = -(\lambda+4)\lambda(\lambda-2) = 0 \rightarrow \lambda = -4, 0, 2$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

2. Eigenvector 구하기

i) $\lambda = -4$

$$B-(-4I) = \begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}, [B-(-4I)|0] = \begin{bmatrix} 5 & 1 & 0 & | & 0 \\ 1 & 5 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

REF : $\begin{bmatrix} 5 & 1 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$ → eigenvector $v_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

ii) $\lambda = 0$

$$B - 0I = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}, [B - 0I|0] = \begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 1 & 1 & 0 & | & 0 \\ 0 & 0 & -4 & | & 0 \end{bmatrix}$$

REF : $\begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & -4 & | & 0 \end{bmatrix}$ → eigenvector $v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

iii) $\lambda = 2$

$B - 2I = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -6 \end{bmatrix}, [B - 2I|0] = \begin{bmatrix} -1 & 1 & 0 & | & 0 \\ 1 & -1 & 0 & | & 0 \\ 0 & 0 & -6 & | & 0 \end{bmatrix}$

$REF : \begin{bmatrix} -1 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & -6 & | & 0 \end{bmatrix}$ → eigenvector $v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

→Eigenvectors $v_1, v_2, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(eigenvalues가 모두 다르므로 독립이 보장됨)

→ eigen basis = $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

3. 대각화

$\hat{B} = P^{-1}BP$

$$P = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$[P|I] = \begin{bmatrix} 0 & -1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 0 & 1 & 0 \\ 1 & 0 & 0 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 0 & 0 & 1 \\ 0 & 1 & 0 & | & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & | & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = [\mathrm{I}|P^{-1}]$$

$$P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \quad ,\hat{B} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \rightarrow \begin{bmatrix} -4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

1. Eigenbasis를 구하고, 대각화를 통하여 각 행렬의 10제곱을 구하시오.

3. 대각화

$$\hat{B} = P^{-1}BP \rightarrow \begin{bmatrix} -4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{B}^{10} = P^{-1}BP * P^{-1}BP * P^{-1}BP * \cdots * P^{-1}BP = P^{-1}B^{10}P$$

$$\begin{bmatrix} -4^{10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2^{10} \end{bmatrix} = P^{-1}BP \quad \rightarrow \quad P \begin{bmatrix} -4^{10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2^{10} \end{bmatrix} P^{-1} = B^{10}$$

$$B^{10} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} (-4)^{10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2^{10} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2^{10} \\ 0 & 0 & 2^{10} \\ (-4)^{10} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} 2^9 & 2^9 & 0 \\ 2^9 & 2^9 & 0 \\ 0 & 0 & 4^{10} \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & i \\ -i & 6 \end{bmatrix}$$

2. 다음 행렬은 Hermitian, skew-Hermitian, Unitary 행렬 중 어떤 것인가? 행렬의 고유값과 고유벡터를 구한 후 매트랩으로 확인하시오.

$\bar{A} = \begin{bmatrix} 6 & -i \\ i & 6 \end{bmatrix}, \bar{A}^T = \begin{bmatrix} 6 & i \\ -i & 6 \end{bmatrix} = A$ 이므로 Hermitian 이다.

$A - \lambda I = \begin{bmatrix} 6-\lambda & i \\ -i & 6-\lambda \end{bmatrix}, \det(A - \lambda I) = \lambda^2 - 12\lambda + 35 = (\lambda - 7)(\lambda - 5)$

$\rightarrow \lambda = 5, 7$

$\lambda = 5 \rightarrow \begin{bmatrix} 1 & i & | & 0 \\ -i & 1 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & i & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \rightarrow v_1 = \begin{bmatrix} 1 \\ i \end{bmatrix}$

$\lambda = 7 \rightarrow \begin{bmatrix} -1 & i & | & 0 \\ -i & -1 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & i & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \rightarrow v_2 = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

$$A = \begin{bmatrix} 6 & i \\ -i & 6 \end{bmatrix}$$

2. 다음 행렬은 Hermitian, skew-Hermitian, Unitary 행렬 중 어떤 것인가? 행렬의 고유값과 고유벡터를 구한 후 매트랩으로 확인하시오.

```
>> A = [6 i;-i 6]

A =

   6.0000 + 0.0000i   0.0000 + 1.0000i
   0.0000 - 1.0000i   6.0000 + 0.0000i

>> eig(A)

ans =

    5.0000
    7.0000

>> [V D] = eig(A)

V =

   0.0000 + 0.7071i   0.0000 + 0.7071i
  -0.7071 + 0.0000i   0.7071 + 0.0000i


D =

    5.0000         0
         0    7.0000
```

$$B = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

2. 다음 행렬은 Hermitian, skew-Hermitian, Unitary 행렬 중 어떤 것인가?
행렬의 고유값과 고유벡터를 구한 후 매트랩으로 확인하시오.

$\bar{B} = \begin{bmatrix} -i & 0 & 0 \\ 0 & 0 & -i \\ 0 & -i & 0 \end{bmatrix}, \bar{B}^T = \begin{bmatrix} -i & 0 & 0 \\ 0 & 0 & -i \\ 0 & -i & 0 \end{bmatrix} = -B$ 이므로 Skew-Hermitian 이다.

또한 $\bar{B}^T B = \begin{bmatrix} -i & 0 & 0 \\ 0 & 0 & -i \\ 0 & -i & 0 \end{bmatrix} \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 이므로 $\bar{B}^T = B^{-1}$이고 Unitary이다.

$B - \lambda I = \begin{bmatrix} i-\lambda & 0 & 0 \\ 0 & -\lambda & i \\ 0 & i & -\lambda \end{bmatrix}, \det(B - \lambda I) = (i-\lambda)(\lambda^2+1) = 0$

$$v = \begin{bmatrix} s \\ t \\ t \end{bmatrix} = s \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$\rightarrow \lambda = i, -i$

$\lambda = i \rightarrow \begin{bmatrix} 0 & 0 & 0 & | & 0 \\ 0 & -i & i & | & 0 \\ 0 & i & -i & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 & | & 0 \\ 0 & -i & i & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow x_1 = s, x_2 = t, x_3 = t \rightarrow v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\lambda = -i \rightarrow \begin{bmatrix} 2i & 0 & 0 & | & 0 \\ 0 & i & i & | & 0 \\ 0 & i & i & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2i & 0 & 0 & | & 0 \\ 0 & i & i & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow v_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$$B = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

2. 다음 행렬은 Hermitian, skew-Hermitian, Unitary 행렬 중 어떤 것인가?
행렬의 고유값과 고유벡터를 구한 후 매트랩으로 확인하시오.

```
>> B = [i 0 0;0 0 i;0 i 0]

B =

   0.0000 + 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 1.0000i
   0.0000 + 0.0000i   0.0000 + 1.0000i   0.0000 + 0.0000i

>> eig(B)

ans =

   0.0000 + 1.0000i
   0.0000 + 1.0000i
   0.0000 - 1.0000i

>> [V D] = eig(B)

V =

    1.0000         0         0
         0   -0.7071   -0.7071
         0   -0.7071    0.7071


D =

   0.0000 + 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 1.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 - 1.0000i
```

3. 두 개의 Unitary행렬의 곱도 Unitary임을 증명하고 MATLAB으로 3x3 Unitary 행렬의 예를 들어 확인하시오.

- 두 행렬 A, B가 Unitary 행렬이라고 하자.

$\bar{A}^T = A^{-1}, \bar{B}^T = B^{-1}$ 가 성립한다.

$AB(AB)^{-1} = I \ \rightarrow B(AB)^{-1} = A^{-1}$
$\rightarrow \boldsymbol{(AB)^{-1} = B^{-1}A^{-1}}$

$\overline{AB}^T = (\bar{A}\,\bar{B})^T = \bar{B}^T\bar{A}^T = B^{-1}A^{-1} = (AB)^{-1}$ 이다.

$\overline{AB}^T = (AB)^{-1}$ 이므로

두 개의 Unitary 행렬의 곱은 Unitary 행렬이다.

3. 두 개의 Unitary행렬의 곱도 Unitary임을 증명하고 MATLAB으로 3x3 Unitary 행렬의 예를 들어 확인하시오.

```
>> A = [(1+i)/2 -1/2 1/2; i/sqrt(3) 1/sqrt(3) -i/sqrt(3);(3+i)/sqrt(60) (4+3i)/sqrt(60) 5i/sqrt(60)];
>> A_hermitian = ctranspose(A);
>> A*A_hermitian

ans =

   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   1.0000 + 0.0000i  -0.0000 + 0.0000i
   0.0000 - 0.0000i  -0.0000 + 0.0000i   1.0000 + 0.0000i

>> B = [1/2 -i/2 (i-1)/2; i/2 1/2 (1+i)/2;(1+i)/2 (i-1)/2 0];
>> B_hermitian = ctranspose(B);
>> B*B_hermitian

ans =

     1     0     0
     0     1     0
     0     0     1

>> A*B*B_hermitian*A_hermitian
```

$$\boxed{(AB)(AB)^* = (AB)B^*A^*}$$

```
ans =

   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  -0.0000 + 0.0000i   1.0000 - 0.0000i  -0.0000 + 0.0000i
  -0.0000 - 0.0000i   0.0000 + 0.0000i   1.0000 - 0.0000i
```