공업수학

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#1 다음 함수는 even, odd function 중 어느 것인가? 혹은 아무것도 아닌가?

- $\sin^2 x$

$\sin^2 x = \sin x \times \sin x$: odd × odd = even

$(\sin^2(-x) = \sin^2(x))$

- e^x

$e^{-x} \neq e^x$

$e^{-x} \neq -e^x$

→ 아무것도 아님.

#2 다음 함수의 Fourier Series를 구하시오.

- $f(x) = x^2 \qquad (-1 < x < 1)\,, period\; p = 2 = 2L,\; L = 1$

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x)$$

$$a_0 = \frac{1}{2}\int_{-1}^{1} x^2\, dx = \int_0^1 x^2\, dx = \frac{1}{3}$$

$$a_n = \int_{-1}^{1} x^2 \cos(n\pi x)\, dx = 2\int_0^1 x^2 \cos(n\pi x)\, dx = \frac{4\cos n\pi}{(n\pi)^2}$$

$$b_n = \int_{-1}^{1} x^2 \sin(n\pi x)\; dx = 0 \;\; (odd)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)cos(\frac{n\pi x}{L})dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)sin(\frac{n\pi x}{L})dx$$

#2 다음 함수의 Fourier Series를 구하시오.

- $a_n = 2\int_0^1 x^2 \cos(n\pi x)\, dx$

$$\int_0^1 x^2 \cos(n\pi x)\, dx = \cancelto{0}{\left[x^2 \frac{1}{n\pi} \sin(n\pi x)\right]_0^1} - \int_0^1 2x \frac{1}{n\pi} (\sin n\pi x)\, dx$$

$$= -\int_0^1 2x \frac{1}{n\pi} (\sin n\pi x)\, dx = -\left[2x \frac{1}{(n\pi)^2} (-\cos n\pi x)\right]_0^1 + \int_0^1 2 \frac{1}{(n\pi)^2} (-\cos(n\pi x))\, dx$$

$$= \frac{2}{(n\pi)^2} \cos(n\pi) + \cancelto{0}{\left[\frac{-2}{(n\pi)^3} \sin(n\pi x)\right]_0^1}$$

$$= \frac{2}{(n\pi)^2} \cos(n\pi)$$

$\rightarrow a_n = \dfrac{4}{(n\pi)^2} \cos(n\pi)$

#2 다음 함수의 Fourier Series를 구하시오.

- $x^2 = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\pi x + b_n \sin n\pi x)$

  $= \frac{1}{3} + \sum_{n=1}^{\infty}\left(\frac{4}{(n\pi)^2}\cos(n\pi)\cos(n\pi x)\right)$

- $\cos n\pi = \begin{cases} -1, & n = odd \\ 1, & n = even \end{cases}$

$\therefore x^2 = \frac{1}{3} - \frac{4}{\pi^2}\cos \pi x + \frac{4}{4\pi^2}\cos 2\pi x - \frac{4}{9\pi^2}\cos 3\pi x + \cdots$

#3 2번 문제를 이용하여 $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots$값을 구하시오.

$$x^2=\frac{1}{3}-\frac{4}{\pi^2}\cos\pi x+\frac{4}{4\pi^2}\cos 2\pi x-\frac{4}{9\pi^2}\cos 3\pi x+\cdots$$

$$x=1$$

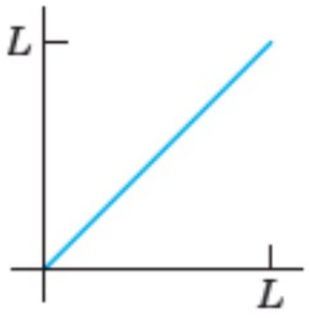
$$1=\frac{1}{3}+\frac{4}{\pi^2}+\frac{4}{4\pi^2}+\frac{4}{9\pi^2}+\frac{4}{16\pi^2}+\cdots$$

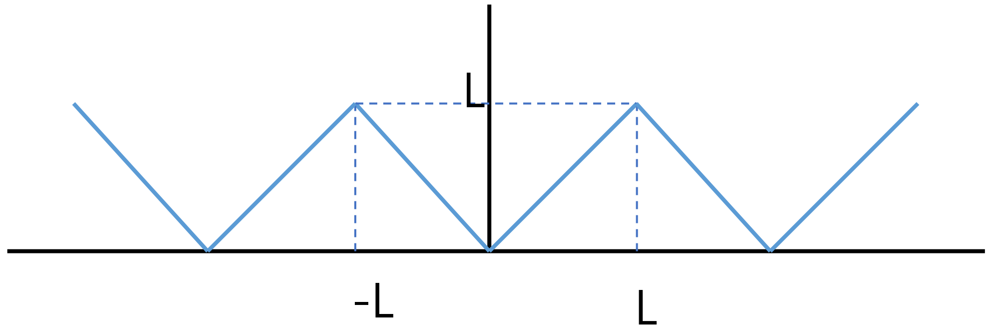
$$1-\frac{1}{3}=\frac{4}{\pi^2}\left(1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots\right)$$

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots=\frac{2}{3}\frac{\pi^2}{4}=\frac{\pi^2}{6}$$

#4 다음 함수의 Fourier cosine series와 Fourier sine series를 구하시오.

- Even periodic extension





$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{L}{2}$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx = \frac{1}{L}\int_{-L}^{0} -x\cos\frac{n\pi x}{L}dx + \frac{1}{L}\int_{0}^{L} x\cos\frac{n\pi x}{L}dx = \frac{2}{L}\int_{0}^{L} x\cos\frac{n\pi x}{L}dx \quad \left(x\cos\frac{(n\pi x)}{L} : odd\right)$$

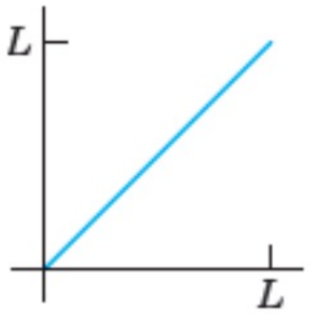
$$= \frac{2L}{n\pi}\sin(n\pi) + \frac{2L}{(n\pi)^2}\cos(n\pi) - 2\frac{L}{(n\pi)^2} = \frac{2L}{(n\pi)^2}\{n\pi\sin(n\pi) + \cos(n\pi) - 1\} = \frac{2L}{(n\pi)^2}(\cos(n\pi) - 1) \quad \because \sin(n\pi) = 0$$

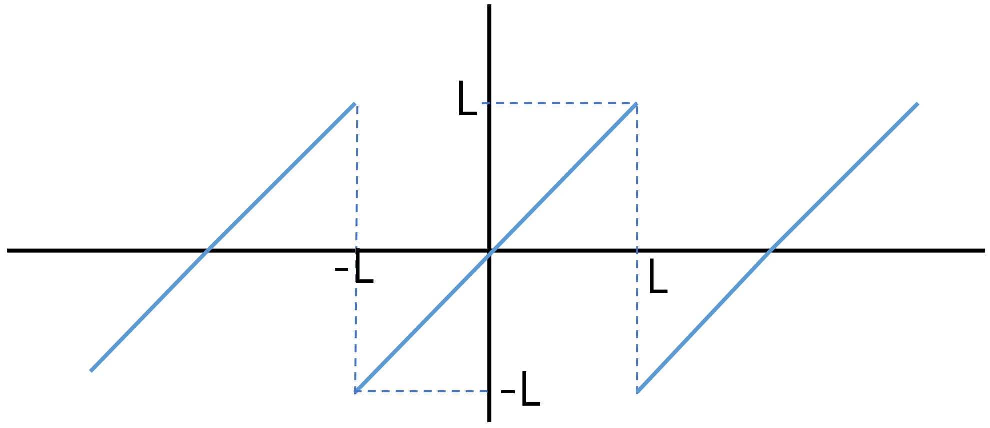
$$a_n = \begin{cases} \frac{-4L}{(n\pi)^2}, & n = odd \\ 0, & n = even \end{cases}$$

$$f(x) = \frac{L}{2} + \sum_{n=1}^{\infty} \frac{2L}{(n\pi)^2}(\cos(n\pi) - 1)\cos\frac{n\pi x}{L} \quad = \frac{L}{2} + \left(-\frac{4L}{\pi^2}\right)\left\{\cos\frac{\pi x}{L} + \frac{\cos 3\pi x}{9L} + \frac{\cos 5\pi x}{25L} + \cdots\right\}$$

#4 다음 함수의 Fourier cosine series와 Fourier sine series를 구하시오.

- odd periodic extension





$f(x) = \Sigma_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

$$b_n = \frac{1}{L}\int_{-L}^{L} x \sin\left(\frac{n\pi x}{L}\right)\, dx = \frac{2}{L}\int_{0}^{L} x \sin\left(\frac{n\pi x}{L}\right)\, dx = \frac{2L}{n\pi}(-\cos n\pi)$$

$$b_n = \begin{cases} \frac{2L}{n\pi}, & n = odd \\ -\frac{2L}{n\pi}, & n = even \end{cases}$$

$$f(x) = \Sigma_{n=1}^{\infty} \frac{2L}{n\pi}(-\cos n\pi) \sin \frac{n\pi x}{L} = \frac{2L}{\pi}\left(\sin\frac{\pi x}{L} - \frac{1}{2}\sin\frac{2\pi x}{L} + \frac{1}{3}\sin\frac{3\pi x}{L} - \frac{1}{4}\sin\frac{4\pi x}{L} + \cdots\right)$$