공업수학

20181796 김민준

#1 Let

- $A = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{bmatrix}, C = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix},$

$D = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}, E = \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}, u = \begin{bmatrix} 1.5 \\ 0 \\ -3.0 \end{bmatrix}, v = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, w = \begin{bmatrix} -5 \\ -30 \\ 10 \end{bmatrix}$

#1

- 1. $A + 2B$

$$A + 2B = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix} + 2\begin{bmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix} + \begin{bmatrix} 0 & 10 & 4 \\ 10 & 6 & 8 \\ -4 & 8 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 12 & 8 \\ 16 & 11 & 13 \\ -3 & 8 & -7 \end{bmatrix}$$

$$\therefore A + 2B = \begin{bmatrix} 0 & 12 & 8 \\ 16 & 11 & 13 \\ -3 & 8 & -7 \end{bmatrix}$$

- 2. $C + 2A$

$$C + 2A = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix}$$

→ C : 3by2, A : 3by3 matrix 열의 수가 다르기 때문에 연산 불가능

#1

- 3. $(D + 2E) + C$

$$(D + 2E) + C = D + 2E + C = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix} + 2\begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 6 & 8 \\ 6 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 9 & 12 \\ 9 & -3 \end{bmatrix}$$

$$\therefore (D + 2E) + C = \begin{bmatrix} 1 & 7 \\ 9 & 12 \\ 9 & -3 \end{bmatrix}$$

- 4. $u + (3v - w)$

$$u + (3v - w) = u + 3v - w = \begin{bmatrix} 1.5 \\ 0 \\ -3.0 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} -5 \\ -30 \\ 10 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \\ -3.0 \end{bmatrix} + \begin{bmatrix} -3 \\ 9 \\ 6 \end{bmatrix} + \begin{bmatrix} 5 \\ 30 \\ -10 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 39 \\ -7 \end{bmatrix}$$

$$\therefore u + (3v - w) = \begin{bmatrix} 3.5 \\ 39 \\ -7 \end{bmatrix}$$

#2~5 Let

- $A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix}$
- $a = [1 \quad -2 \quad 0], b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$

#2

- $b^T Ab$

$$b^T Ab = \begin{bmatrix}3 & 1 & -1\end{bmatrix}\begin{bmatrix}4 & -2 & 3\\ -2 & 1 & 6\\ 1 & 2 & 2\end{bmatrix}\begin{bmatrix}3\\ 1\\ -1\end{bmatrix} = \begin{bmatrix}9 & -7 & 13\end{bmatrix}\begin{bmatrix}3\\ 1\\ -1\end{bmatrix} = [7] \qquad \big((1\times 3)(3\times 3)(3\times 1) = (1\times 1)\big)$$

- aBa^T

$$aBa^T = \begin{bmatrix}1 & -2 & 0\end{bmatrix}\begin{bmatrix}1 & -3 & 0\\ -3 & 1 & 0\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}1\\ -2\\ 0\end{bmatrix} = \begin{bmatrix}7 & -5 & 0\end{bmatrix}\begin{bmatrix}1\\ -2\\ 0\end{bmatrix} = [17] \qquad \big((1\times 3)(3\times 3)(3\times 1) = (1\times 1)\big)$$

- aCC^T

$$aCC^T = \begin{bmatrix}1 & -2 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 3 & 2\\ -2 & 0\end{bmatrix}\begin{bmatrix}0 & 3 & -2\\ 1 & 2 & 0\end{bmatrix} = \begin{bmatrix}-3 & -24 & 12\end{bmatrix} \qquad \big((1\times 3)(3\times 2)(2\times 3) = (1\times 3)\big)$$

- $C^T ba$

$$C^T ba = \begin{bmatrix}0 & 3 & -2\\ 1 & 2 & 0\end{bmatrix}\begin{bmatrix}3\\ 1\\ -1\end{bmatrix}\begin{bmatrix}1 & -2 & 0\end{bmatrix} = \begin{bmatrix}5\\ 5\end{bmatrix}\begin{bmatrix}1 & -2 & 0\end{bmatrix} = \begin{bmatrix}5 & -10 & 0\\ 5 & -10 & 0\end{bmatrix} \qquad \big((2\times 3)(3\times 1)(1\times 3) = (2\times 3)\big)$$

#3 $A + A^T$를 구하고 symmetric matrix가 됨을 확인하시오.

- $A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}, A^T = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix}, A + A^T = \begin{bmatrix} 8 & -4 & 4 \\ -4 & 2 & 8 \\ 4 & 8 & 4 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 8 & -4 & 4 \\ -4 & 2 & 8 \\ 4 & 8 & 4 \end{bmatrix} = (A + A^T)^T$ 이므로 symmetric matrix이다.

#4. $A - A^T$를 구하고 skew-symmetric matrix가 됨을 확인하시오.

- $A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}, A^T = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix}, A - A^T = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 4 \\ -2 & -4 & 0 \end{bmatrix}$

- $A - A^T = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 4 \\ -2 & -4 & 0 \end{bmatrix} = -(A - A^T)^T$ 이므로 skew-symmetric matrix이다.

#5. $(AB)^T = B^T A^T$가 성립하는 것을 위의 A, B로 구하시오.

- $(AB)^T = \left(\begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 10 & -14 & 6 \\ -5 & 7 & 12 \\ -5 & -1 & 4 \end{bmatrix}^T = \begin{bmatrix} 10 & -5 & -5 \\ -14 & 7 & -1 \\ 6 & 12 & 4 \end{bmatrix}$
- $B^T A^T = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 10 & -5 & -5 \\ -14 & 7 & -1 \\ 6 & 12 & 4 \end{bmatrix}$
- $\therefore (AB)^T = B^T A^T$

#6. MATLAB - #1

```
>> A = [0 2 4; 6 5 5; 1 0 -3], B = [0 5 2; 5 3 4; -2 4 -2],C =[5 2; -2 4; 1 0], D = [-4 1; 5 0; 2 -1], E = [0 2; 3 4;3 -1]

A =

     0     2     4
     6     5     5
     1     0    -3


B =

     0     5     2
     5     3     4
    -2     4    -2


C =

     5     2
    -2     4
     1     0


D =

    -4     1
     5     0
     2    -1


E =

     0     2
     3     4
     3    -1
```

```
>>  u = [1.5;0;-3.0], v = [-1;3;2], w=[-5;-30;10]

u =

    1.5000
         0
   -3.0000


v =

    -1
     3
     2


w =

    -5
   -30
    10
```

#6. MATLAB - #1

#1-1

```
>> A+2*B

ans =

     0    12     8
    16    11    13
    -3     8    -7
```

#1-2

```
>> C+2*A
Arrays have incompatible sizes for this operation.
```

#1-3

```
>> (D+2*E)+C

ans =

     1     7
     9    12
     9    -3
```

#1-4

```
>> u+(3*v-w)

ans =

    3.5000
   39.0000
   -7.0000
```

#6. MATLAB - #2~5

```
>> A=[4 -2 3; -2 1 6; 1 2 2], B = [1 -3 0; -3 1 0; 0 0 2], C = [0 1; 3 2; -2 0]

A =

     4    -2     3
    -2     1     6
     1     2     2


B =

     1    -3     0
    -3     1     0
     0     0     2


C =

     0     1
     3     2
    -2     0
```

```
>> a = [1 -2 0], b = [3;1;-1]

a =

     1    -2     0


b =

     3
     1
    -1
```

#6. MATLAB - #2

$b^T Ab$

```
>> transpose(b)*A*b

ans =

     7
```

aCC^T

```
>> a*C*transpose(C)

ans =

    -3   -24    12
```

aBa^T

```
>> a*B*transpose(a)

ans =

    17
```

$C^T ba$

```
>> transpose(C)*b*a

ans =

     5   -10     0
     5   -10     0
```

#6. MATLAB - #3

```
>> A+transpose(A)

ans =

     8    -4     4
    -4     2     8
     4     8     4

>> transpose(A+transpose(A))

ans =

     8    -4     4
    -4     2     8
     4     8     4
```

#6. MATLAB - #4

```
>> A-transpose(A)

ans =

     0     0     2
     0     0     4
    -2    -4     0

>> -1*transpose(A-transpose(A))

ans =

     0     0     2
     0     0     4
    -2    -4     0
```

#6. MATLAB - #5

```
>> transpose(A*B)

ans =

    10    -5    -5
   -14     7    -1
     6    12     4

>> transpose(B)*transpose(A)

ans =

    10    -5    -5
   -14     7    -1
     6    12     4
```