공업수학

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#1 다음 수열의 수렴 여부를 보이시오. 수렴할 경우 극한값을 구하시오.

- $z_n = \frac{(1+i)^{2n}}{2^n}$ $\qquad \lim_{n\to\infty} \frac{(1+i)^{2n}}{2^n} = ?$
- $\lim_{n\to\infty} \frac{(1+i)^{2n}}{2^n} = \lim_{n\to\infty} \frac{\{(1+i)^2\}^n}{2^n} = \lim_{n\to\infty} \frac{(2i)^n}{2^n} = \lim_{n\to\infty} \left(\frac{2i}{2}\right)^n = \lim_{n\to\infty} (i)^n$
- $i^n = i, -1, -i, 1, i, \cdots$
- → $\lim_{n\to\infty} (i)^n$: Divergence (oscillate)

#2 다음 수열의 수렴 여부를 보이시오. 수렴할 경우 극한값을 구하시오.

- $z_n = (3+3i)^{-n}$ $\qquad\qquad \lim_{n\to\infty}(3+3i)^{-n} = ?$

- $|r| = \left|\frac{1}{(3+3i)}\right| = \frac{1}{3\sqrt{2}} < 1$ → Convergence

- $\lim_{n\to\infty}\frac{1}{(3+3i)^n} = 0$

#3 다음 급수의 수렴 발산을 구하시오.

- $\sum_{n=0}^{\infty} \frac{(20+30i)^n}{n!}$

Ratio test

Ratio = $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(20+30i)(20+30i)^n}{(n+1)n!}}{\frac{(20+30i)^n}{n!}}\right| = \left|\frac{20+30i}{n+1}\right| = \frac{\sqrt{1300}}{n+1}$

$\lim_{n\to\infty} \frac{\sqrt{1300}}{n+1} = 0 < 1$ → Convergence

#4 다음 급수의 수렴 발산을 구하시오.

- $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
- $\forall n > 1, \qquad \frac{1}{n} < \frac{1}{\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{n}$: Divergence

Then

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$: Divergence (P-series)

#5 다음 급수의 수렴 발산을 구하시오.

- $\Sigma_{n=1}^{\infty} \frac{i^n}{n}$ $\qquad\qquad$ $\Sigma_{n=1}^{\infty} \left|\frac{i^n}{n}\right| = \Sigma_{n=1}^{\infty} \frac{1}{n}$: Divergence

$$S_n = S_{4n-3} + S_{4n-2} + S_{4n-1} + S_{4n}$$

$$\Sigma_{n=1}^{\infty} \frac{i^n}{n} = \Sigma_{k=1}^{\infty} \left\{\frac{i}{4k-3} + \frac{-1}{4k-2} + \frac{-i}{4k-1} + \frac{1}{4k}\right\} = \Sigma_{k=1}^{\infty} \left\{\left(\frac{-1}{4k-1} + \frac{1}{4k-3}\right) i + \left(\frac{-1}{4k-2} + \frac{1}{4k}\right)\right\}$$

$$= \Sigma_{k=1}^{\infty} \left\{\frac{(-1)^k}{2k-1} i\right\} + \Sigma_{k=1}^{\infty} \left\{\frac{(-1)^k}{2k}\right\}$$

$\lim_{k\to\infty} \frac{(-1)^k}{2k-1} = \lim_{k\to\infty} \frac{(-1)^k}{2k} = 0$, alternating Series → Convergence

Then $\Sigma_{n=1}^{\infty} \frac{i^n}{n}$: Conditional Convergence

#5 다음 급수의 수렴 발산을 구하시오.

- $\sum_{n=1}^{\infty} \frac{i^n}{n}$ $\qquad$ $\sum_{n=1}^{\infty} \left|\frac{i^n}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n}$: Divergence

$$\sum_{n=1}^{\infty} \frac{i^n}{n} = \sum_{k=1}^{\infty} \left\{\frac{i}{4k-3} + \frac{-1}{4k-2} + \frac{-i}{4k-1} + \frac{1}{4k}\right\} = \sum_{k=1}^{\infty} \left\{\boxed{\frac{2i}{(4k-3)(4k-1)}} - \boxed{\frac{2}{4k(4k-2)}}\right\}$$

$$\boxed{\frac{2i}{(4k-3)(4k-1)}} = \frac{2i}{16k^2-16k+3} < \frac{2i}{16k^2-16k} = \frac{i}{8}\left(\frac{1}{k-1} - \frac{1}{k}\right)$$

$\sum_{k=2}^{\infty} \left(\frac{1}{k-1} - \frac{1}{k}\right) = 1$: Convergence → $\sum_{k=1}^{\infty} \frac{2i}{(4k-3)(4k-1)}$: Convergence

$$\boxed{\frac{2}{4k(4k-2)}} = \frac{2}{16k^2-8k} < \frac{2}{16k^2-16k} = \frac{1}{8}\left(\frac{1}{k-1} - \frac{1}{k}\right)$$

$\sum_{k=2}^{\infty} \left(\frac{1}{k-1} - \frac{1}{k}\right) = 1$: Convergence → $\sum_{k=1}^{\infty} \frac{2}{4k(4k-2)}$: Convergence

Then $\sum_{n=1}^{\infty} \frac{i^n}{n}$: Conditional Convergence