공업수학

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#1

1. 다음 선형 연립 방정식의 해를 구하시오. (Gauss Elimination 사용)

1−1.

$$\boxed{\begin{aligned} 4y + 3z &= 8 \\ 2x - z &= 2 \\ 3x + 2y &= 5 \end{aligned}}$$

1−2.

$$\begin{aligned} 10x + 4y - 2z &= -4 \\ -3w - 17x + y + 2z &= 2 \\ w + x + y &= 6 \\ 8w - 34x + 16y - 10z &= 4 \end{aligned}$$

#1-1

$$\begin{aligned} 4y + 3z &= 8 \\ 2x \quad - z &= 2 \\ 3x + 2y \quad &= 5 \end{aligned}$$

→

$$\left[\begin{array}{ccc|c} 0 & 4 & 3 & 8 \\ 2 & 0 & -1 & 2 \\ 3 & 2 & 0 & 5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & 0 & -1 & 2 \\ 0 & 4 & 3 & 8 \\ 3 & 2 & 0 & 5 \end{array}\right] \xrightarrow[\frac{1}{4}R_2 \to R_2]{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{3}{4} & 2 \\ 3 & 2 & 0 & 5 \end{array}\right] \xrightarrow{-3R_1 + R_3 \to R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{3}{4} & 2 \\ 0 & 2 & \frac{3}{2} & 2 \end{array}\right] \xrightarrow{-2R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{3}{4} & 2 \\ \underline{0} & \underline{0} & \underline{0} & 2 \end{array}\right]$$

$0x + 0y + 0z = 2$ → 불능, 해가 존재하지 않음

#1-2

$$\begin{aligned} 10x + 4y - 2z &= -4 \\ -3w - 17x + y + 2z &= 2 \\ w + x + y &= 6 \\ 8w - 34x + 16y - 10z &= 4 \end{aligned}$$

$$\rightarrow \left[\begin{array}{cccc|c} 0 & 10 & 4 & -2 & -4 \\ -3 & -17 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & 6 \\ 8 & -34 & 16 & -10 & 4 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ -3 & -17 & 1 & 2 & 2 \\ 0 & 10 & 4 & -2 & -4 \\ 8 & -34 & 16 & -10 & 4 \end{array}\right]$$

$$\xrightarrow[-8R_1 + R_4 \to R_4]{3R_1 + R_2 \to R_2} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ 0 & -14 & 4 & 2 & 20 \\ 0 & 10 & 4 & -2 & -4 \\ 0 & -42 & 8 & -10 & -44 \end{array}\right] \xrightarrow{-3R_2 + R_4 \to R_4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ 0 & -14 & 4 & 2 & 20 \\ 0 & 10 & 4 & -2 & -4 \\ 0 & 0 & -4 & -16 & -104 \end{array}\right] \xrightarrow[-\frac{1}{4}R_4 \to R_4]{-\frac{1}{14}R_2 \to R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ 0 & 1 & -\frac{2}{7} & -\frac{1}{7} & -\frac{10}{7} \\ 0 & 10 & 4 & -2 & -4 \\ 0 & 0 & 1 & 4 & 26 \end{array}\right] \xrightarrow{-10R_2 + R_3 \to R_3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ 0 & 1 & -\frac{2}{7} & -\frac{1}{7} & -\frac{10}{7} \\ 0 & 0 & \frac{48}{7} & -\frac{4}{7} & \frac{72}{7} \\ 0 & 0 & 1 & 4 & 26 \end{array}\right] \xrightarrow{\frac{7}{48}R_3 \to R_3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ 0 & 1 & -\frac{2}{7} & -\frac{1}{7} & -\frac{10}{7} \\ 0 & 0 & 1 & -\frac{1}{12} & \frac{3}{2} \\ 0 & 0 & 1 & 4 & 26 \end{array}\right]$$

$$\xrightarrow{-R_3 + R_4 \to R_4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ 0 & 1 & -\frac{2}{7} & -\frac{1}{7} & -\frac{10}{7} \\ 0 & 0 & 1 & -\frac{1}{12} & \frac{3}{2} \\ 0 & 0 & 0 & \frac{49}{12} & \frac{49}{2} \end{array}\right] \xrightarrow{\frac{12}{49}R_4 \to R_4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 6 \\ 0 & 1 & -\frac{2}{7} & -\frac{1}{7} & -\frac{10}{7} \\ 0 & 0 & 1 & -\frac{1}{12} & \frac{3}{2} \\ 0 & 0 & 0 & 1 & 6 \end{array}\right] \text{ REF}$$

#1-2

- REF :

$$\begin{bmatrix} 1 & 1 & 1 & 0 & | & 6 \\ 0 & 1 & -\frac{2}{7} & -\frac{1}{7} & | & -\frac{10}{7} \\ 0 & 0 & 1 & -\frac{1}{12} & | & \frac{3}{2} \\ 0 & 0 & 0 & 1 & | & 6 \end{bmatrix}$$

R_4 : $1z = 6$ $\quad\therefore z = 6$

R_3 : $y - \frac{1}{12}z = \frac{3}{2} \quad \rightarrow y - \frac{1}{2} = \frac{3}{2}$ $\quad\therefore y = 2$

R_2 : $x - \frac{2}{7}y - \frac{1}{7}z = -\frac{10}{7} \quad \rightarrow \quad x - \frac{4}{7} - \frac{6}{7} = -\frac{10}{7}$ $\quad\therefore x = 0$

R_1 : $w + x + y = 6 \quad \rightarrow \quad w + 0 + 2 = 6$ $\quad\therefore w = 4$

#2

- $E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$

2. E_1, E_2, E_3 가 위와 같다. 임의의 4x4 행렬 A를 만들었을 때, $B = E_3E_2E_1A$와 $C = E_1E_2E_3A$는 동일한가?

#2

- $E_3E_2E_1 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&8\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\-5&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&0&1&0\\-5&1&0&0\\0&0&0&8\end{bmatrix}$

- $E_1E_2E_3 = \begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\-5&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&8\end{bmatrix} = \begin{bmatrix}1&0&0&0\\-5&0&1&0\\0&1&0&0\\0&0&0&8\end{bmatrix}$

- $B = \begin{bmatrix}1&0&0&0\\0&0&1&0\\-5&1&0&0\\0&0&0&8\end{bmatrix}A\,, C = \begin{bmatrix}1&0&0&0\\-5&0&1&0\\0&1&0&0\\0&0&0&8\end{bmatrix}A$

#2

- $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} A$, $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} A$
- Let $A = [a_1\ \ a_2\ \ a_3\ \ a_4]^T, a_i\ (i = 1, \cdots, 4)\ is\ row\ vector, a_i \in R^4$

If $a_1 = \mathbf{0}$ → $B = C$

If $a_1 \neq \mathbf{0}$ → $B \neq C$

#3

- $E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$

3. E_1, E_2, E_3 는 각각 elementary row operations 중에서 어떤 연산을 수행하는가? (예를 들어 E_1A 는 A 행렬을 어떻게 변경하는가?)

#3-1

- E_1

$$E_1A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

E_1은 I_4에서 2행과 3행을 교환한 형태이다 $(R_2 \leftrightarrow R_3)$.

어떤 행렬에 기본 행(열)연산을 적용하는 것은 그 행렬의 앞(뒤)에 단위행렬에서 동일한 기본 행(열)연산이 수행된 행렬을 곱하는 것과 동일하다.

따라서 E_1A의 결과는 A의 2행과 3행을 교환한$(R_2 \leftrightarrow R_3)$ 행렬이다.

#3-2

- E_2

$$E_2A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} - 5a_{11} & a_{32} - 5a_{12} & a_{33} - 5a_{13} & a_{34} - 5a_{14} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

E_2 는 I_4에서 $-5R_1 + R_3 \rightarrow R_3$ 의 기본 행 연산을 한 것이다.

#3-1과 마찬가지로 어떤 행렬(A)의 앞에 단위행렬에서 $-5R_1 + R_3 \rightarrow R_3$ 의 기본 행 연산이 수행된 행렬을 곱하는 것은 행렬 A에 기본 행 연산 $-5R_1 + R_3 \rightarrow R_3$ 를 취하는 것과 같다.

따라서 E_2A 는 A에 기본 행 연산 $-5R_1 + R_3 \rightarrow R_3$을 취한 행렬이다.

#3-3

- E_3

$$E_3A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 8a_{41} & 8a_{42} & 8a_{43} & 8a_{44} \end{bmatrix}$$

E_3는 I_4에 $8R_4 \to R_4$의 기본 행 연산을 실행한 것이다.

따라서 E_3A는 A에 기본 행 연산 $8R_4 \to R_4$을 실행한 행렬이다.

#4-1 다음 행렬의 rank를 구하시오.

- $$\begin{bmatrix} 2 & 4 & 8 & 16 \\ 16 & 8 & 4 & 2 \\ 4 & 8 & 16 & 2 \\ 2 & 16 & 8 & 4 \end{bmatrix} = 2\begin{bmatrix} 1 & 2 & 4 & 8 \\ 8 & 4 & 2 & 1 \\ 2 & 4 & 8 & 1 \\ 1 & 8 & 4 & 2 \end{bmatrix} \xrightarrow[-R_1+R_4 \to R_4]{\substack{-8R_1+R_2 \to R_2 \\ -2R_1+R_3 \to R_3}} \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & -12 & -30 & -63 \\ 0 & 0 & 0 & -15 \\ 0 & 6 & 0 & -6 \end{bmatrix} \xrightarrow{\substack{-\frac{1}{12}R_2 \to R_2 \\ -\frac{1}{15}R_3 \to R_3 \\ \frac{1}{6}R_4 \to R_4}}$$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & \frac{5}{2} & \frac{21}{4} \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \xrightarrow{\substack{-R_4+R_2 \to R_2 \\ R_3+R_4 \to R_4 \\ \frac{2}{5}R_2 \to R_2}} \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & 1 & \frac{21}{10} \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{21}{10} \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_4} \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{21}{10} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

REF : $\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{21}{10} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ → leading entry : 4개 → rank = 4

#4-2 다음 행렬의 rank를 구하시오

- $$\begin{bmatrix} 5 & -2 & 1 & 0 \\ -2 & 0 & -4 & 1 \\ 1 & -4 & -11 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_4]{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & -4 & -11 & 2 \\ 0 & 1 & 2 & 0 \\ 5 & -2 & 1 & 0 \\ -2 & 0 & -4 & 1 \end{bmatrix} \xrightarrow[2R_1 + R_4 \to R_4]{-5R_1 + R_3 \to R_3} \begin{bmatrix} 1 & -4 & -11 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 18 & 56 & -10 \\ 0 & -8 & -26 & 5 \end{bmatrix}$$

$$\xrightarrow[8R_2 + R_4 \to R_4]{-18R_2 + R_3 \to R_3} \begin{bmatrix} 1 & -4 & -11 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 20 & -10 \\ 0 & 0 & -10 & 5 \end{bmatrix} \xrightarrow[\frac{1}{10}R_4 \to R_4]{\frac{1}{20}R_3 \to R_3} \begin{bmatrix} 1 & -4 & -11 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix} \xrightarrow{R_3 + R_4 \to R_4} \begin{bmatrix} 1 & -4 & -11 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

REF : $\begin{bmatrix} 1 & -4 & -11 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$ → leading entry : 3개 → rank = 3

#5. 다음 벡터들은 linearly independent 인가? – sol1.

[4 -1 3], [0 8 1], [1 3 -5], [2 6 1]

$c_1\begin{bmatrix}4\\-1\\3\end{bmatrix}+c_2\begin{bmatrix}0\\8\\1\end{bmatrix}+c_3\begin{bmatrix}1\\3\\-5\end{bmatrix}+c_4\begin{bmatrix}2\\6\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ 를 만족하는 nontrivial solution $c_i(i=1,\cdots,4)$ 가 존재하면 일차 결합이다.

$$\begin{bmatrix}4 & 0 & 1 & 2 & | & 0\\ -1 & 8 & 3 & 6 & | & 0\\ 3 & 1 & -5 & 1 & | & 0\end{bmatrix} \rightarrow \text{REF}: \begin{bmatrix}1 & -8 & -3 & 6 & | & 0\\ 0 & 1 & \frac{13}{32} & \frac{13}{16} & | & 0\\ 0 & 0 & 1 & \frac{181}{197} & | & 0\end{bmatrix}$$

이므로 한 개의 free variable이 존재한다. 따라서 nontrivial solution (c_1, c_2, c_3, c_4)가 존재하고, 위의 벡터는 일차 종속이다.

#5. 다음 벡터들은 linearly independent 인가? – sol2.

[4 -1 3], [0 8 1], [1 3 -5], [2 6 1]

$c_1\begin{bmatrix}4\\-1\\3\end{bmatrix}+c_2\begin{bmatrix}0\\8\\1\end{bmatrix}+c_3\begin{bmatrix}1\\3\\-5\end{bmatrix}+c_4\begin{bmatrix}2\\6\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ 를 만족하는 nontrivial solution $c_i(i=1,\cdots,4)$ 가 존재하면 일차 결합이다.

행렬 $\begin{bmatrix}4 & 0 & 1 & 2 & | & 0\\-1 & 8 & 3 & 6 & | & 0\\3 & 1 & -5 & 1 & | & 0\end{bmatrix}$을 직접 풀어서 계산할 수도 있지만 행렬의 행의 수가 3이기 때문에 $rank \leq 3$ 이다. 따라서 (c_1,c_2,c_3,c_4)에 대하여 적어도 하나 이상의 free variable이 존재하고 nontrivial solution (c_1,c_2,c_3,c_4)가 존재하므로 주어진 벡터는 일차 결합으로 표현된다.

#6 MATLAB - #2

```
>> E_1 = [1 0 0 0; 0 0 1 0; 0 1 0 0; 0 0 0 1]
E_2 = [1 0 0 0; 0 1 0 0; -5 0 1 0; 0 0 0 1]
E_3 = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 8]

E_1 =

     1     0     0     0
     0     0     1     0
     0     1     0     0
     0     0     0     1


E_2 =

     1     0     0     0
     0     1     0     0
    -5     0     1     0
     0     0     0     1


E_3 =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     8
```

```
>> E_3*E_2*E_1

ans =

     1     0     0     0
     0     0     1     0
    -5     1     0     0
     0     0     0     8

>> E_1*E_2*E_3

ans =

     1     0     0     0
    -5     0     1     0
     0     1     0     0
     0     0     0     8
```

1. $a_1 \neq \mathbf{0} \rightarrow B \neq C$

```
>> A1 = [0 1 2 4; 5 6 7 8; 4 5 6 7; 1 1 2 2];
>> B = E_3*E_2*E_1*A1

B =

     0     1     2     4
     4     5     6     7
     5     1    -3   -12
     8     8    16    16

>> C = E_1*E_2*E_3*A1

C =

     0     1     2     4
     4     0    -4   -13
     5     6     7     8
     8     8    16    16
```

2. $a_1 = \mathbf{0} \rightarrow B = C$

```
>> A2 = [0 0 0 0; 1 4 5 2; 4 4 2 3; 1 2 1 2];
>> B = E_3*E_2*E_1*A2

B =

     0     0     0     0
     4     4     2     3
     1     4     5     2
     8    16     8    16

>> C = E_1*E_2*E_3*A2

C =

     0     0     0     0
     4     4     2     3
     1     4     5     2
     8    16     8    16
```

#6 MATLAB #3

$R_2 \leftrightarrow R_3$

```
E_1 =

     1     0     0     0
     0     0     1     0
     0     1     0     0
     0     0     0     1
```

$-5R_1 + R_3 \rightarrow R_3$

```
E_2 =

     1     0     0     0
     0     1     0     0
    -5     0     1     0
     0     0     0     1
```

$8R_3 \rightarrow R_3$

```
E_3 =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     8
```

```
>> A = [1 1 1 1; 2 2 2 2; 3 3 3 3; 4 4 4 4]

A =

     1     1     1     1
     2     2     2     2
     3     3     3     3
     4     4     4     4
```

$E_1A\ : R_2 \leftrightarrow R_3$

```
>> E_1*A

ans =

     1     1     1     1
     3     3     3     3
     2     2     2     2
     4     4     4     4
```

$E_2A\ :\ -5R_1 + R_3 \rightarrow R_3$

```
>> E_2*A

ans =

     1     1     1     1
     2     2     2     2
    -2    -2    -2    -2
     4     4     4     4
```

$E_3A\ :\ 8R_3 \rightarrow R_3$

```
>> E_3*A

ans =

     1     1     1     1
     2     2     2     2
     3     3     3     3
    32    32    32    32
```

#6 MATLAB #4

```
>> A = 2*[1 2 4 8; 8 4 2 1; 2 4 8 1; 1 8 4 2]

A =

     2     4     8    16
    16     8     4     2
     4     8    16     2
     2    16     8     4

>> rank(A)

ans =

     4
```

```
>> B = [5 -2 1 0; -2 0 -4 1; 1 -4 -11 2;0 1 2 0]

B =

     5    -2     1     0
    -2     0    -4     1
     1    -4   -11     2
     0     1     2     0

>> rank(B)

ans =

     3
```

#6 MATLAB #5

```
A =

     4     0     1     2
    -1     8     3     6
     3     1    -5     1

>> rank(A)

ans =

     3
```

```
>> A = [4 0 1 2; -1 8 3 6; 3 1 -5 1]

A =

     4     0     1     2
    -1     8     3     6
     3     1    -5     1

>> c = null(A) %Ax=0

c =

   -0.3356
   -0.5453
   -0.1602
    0.7512
```

$Ax = 0$을 만족하는 $\mathbf{0}$이 아닌 x존재