공업수학

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#1 Maclaurin Series를 구하시오.

- $\sin 2z^2 = ?$

$$\sin z = \Sigma_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!}$$

$$z \leftarrow 2z^2$$

$$\sin 2z^2 = \Sigma_{n=0}^{\infty} \frac{(-1)^n \left(2z^2\right)^{2n+1}}{(2n+1)!}$$

#2 Maclaurin Series를 구하시오.

- $\frac{1}{(1-z)^3} = ?$

$$\frac{d^2}{dz^2}(1-z)^{-1} = \frac{d}{dz}(1-z)^{-2} = 2(1-z)^{-3} \rightarrow \frac{d^2}{dz^2}\frac{1}{2}(1-z)^{-1} = (1-z)^{-3}$$

$$\frac{1}{2}\frac{d^2}{dz^2}\frac{1}{1-z} = \frac{1}{2}\frac{d^2}{dz^2}\ \Sigma_{n=0}^{\infty} z^n = \frac{1}{2}\frac{d}{dz}\Sigma_{n=0}^{\infty} n z^{n-1} = \frac{1}{2}\Sigma_{n=0}^{\infty} n(n-1)z^{n-2}$$

$$\therefore \frac{1}{(1-z)^3} = \frac{1}{2}\ \Sigma_{n=0}^{\infty} n(n-1)z^{n-2}$$

#3 다음 주어진 포인트를 center로 하는 Tayler Series를 구하시오.

- $e^z, \qquad z_0 = \pi i$

$$f(z_0) = f'(z_0) = f^{(2)}(z_0) = \cdots = e^{\pi i} = -1$$

$$\begin{aligned}
e^z &= f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \cdots + \frac{f^{(k)}(z_0)}{k!}(z - z_0)^k + \cdots \\
&= f(\pi i) + \frac{f'(\pi i)}{1!}(z - \pi i) + \cdots + \frac{f^{(k)}(\pi i)}{k!}(z - \pi i)^k + \cdots \\
&= -1\left\{1 + \frac{z-\pi i}{1!} + \frac{(z-\pi i)^2}{2!} + \cdots + \frac{(z-\pi i)^k}{k!} + \cdots\right\} \\
&= -\Sigma_{n=0}^{\infty}\left\{\frac{(z-\pi i)^n}{n!}\right\}
\end{aligned}$$

#4 Matlab으로 Maclaurin Series를 이용하여 e의 값을 구해보시오.

- $e^x = \Sigma_{n=0}^{\infty} \frac{x^n}{n!}$ → $e = \Sigma_{n=0}^{\infty} \frac{1}{n!}$

```
>> for n = 0 : 100
sum = sum + 1/factorial(n)
end
```

```
>> vpa(sum, 30)

ans =

2.71828182845904553488480814849
```

#5 다음 f(x)의 Fourier Series를 찾으시오. (period = 2π)

- $f(x) = x^2, (-\pi < x < \pi)$

$f(x) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

$x^2: even \ \rightarrow b_n = 0$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2\, dx = \frac{1}{\pi}\int_0^{\pi} x^2\, dx = \frac{1}{\pi}\cdot\frac{\pi^3}{3} = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx\, dx = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\, dx \qquad (\because x^2 \cos nx : even)$$

#5 다음 f(x)의 Fourier Series를 찾으시오. (period = 2π)

$$a_n = \frac{2}{\pi}\int_0^\pi x^2 \cos nx\, dx$$

$$a_n = \frac{2}{\pi}\cdot\frac{2\pi}{n^2}\cos n\pi = \frac{4}{n^2}\cos n\pi$$

$$\int_0^\pi x^2 \cos nx\ dx = \left[x^2\cdot\frac{1}{n}\sin nx\right]_0^\pi - \int 2x\cdot\frac{1}{n}\sin nx\, dx$$

$$= \cancel{\left[x^2\cdot\frac{1}{n}\sin nx\right]_0^\pi} - \left\{\left[2x\cdot\left(-\frac{1}{n^2}\right)\cos nx\right]_0^\pi - \int 2\cdot\left(-\frac{1}{n^2}\right)\cos nx\right\}$$

$$= \left[\frac{2x}{n^2}\cos nx\right]_0^\pi - \frac{2}{n^2}\int \cos nx\ dx$$

$$= \left[\frac{2x}{n^2}\cos nx\right]_0^\pi - \cancel{\frac{2}{n^2}\left[\frac{1}{n}\sin nx\right]_0^\pi} = \frac{2\pi}{n^2}\cos n\pi$$

$$\rightarrow f(x) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos nx) = \frac{\pi^2}{3} + \Sigma_{n=1}^{\infty}\frac{4}{n^2}\cos n\pi \cos nx$$

#6 MATLAB

$$f(x) = \frac{\pi^2}{3} + \Sigma_{n=1}^{\infty} \frac{4}{n^2} \cos n\pi \cos nx$$

```
>> sum = pi^2/3;
for n = 1:50
sum = sum + 4*cos(n*pi)*cos(n*x)/n^2;
end
>> ezplot(sum)
```