공업수학

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#1 z가 다음과 같을 때 e^z를 $u + iv$형태로 쓰시오.

- $z = 2 + \pi i$
- $e^z = e^{2+\pi i} = e^2 e^{\pi i} = e^2(\cos\pi + i\sin\pi) = \underbrace{e^2\cos\pi}_{u} + i\underbrace{e^2\sin\pi}_{v}$

#2 다음 방정식의 모든 해를 구하시오.

- $e^z = -2$

$e^z = e^{x+yi} = e^x(\cos y + i \sin y) = -2$

$|e^z| = 2 = e^x$ → $x = \ln 2$

$\cos y = -1, \sin y = 0$ → $y = \pi + 2n\pi$

$\therefore z = \ln 2 + ((2n+1)\pi)i$

#3 다음 값을 $u + iv$ 형태로 구하시오.

- $\sin 2\pi i = \frac{1}{2i}\left(e^{i(2\pi i)} - e^{-i(2\pi i)}\right) = \frac{1}{2i}\left(e^{-2\pi} - e^{2\pi}\right) = i\left(\frac{e^{2\pi} - e^{-2\pi}}{2}\right)$

- $u = 0, v = \left(\frac{e^{2\pi} - e^{-2\pi}}{2}\right)$

- $\sin 2\pi i = \sin 0 \cosh 2\pi + i \cos 0 \sinh 2\pi$

$$= i \sinh 2\pi = i\left(\frac{e^{2\pi} - e^{-2\pi}}{2}\right)$$

```
>> sin(2*pi*i)

ans =

   0.0000e+00 + 2.6774e+02i

>> sinh(2*pi)

ans =

  267.7449
```

#4 다음 값을 $u + iv$ 형태로 구하시오.

- $\sinh(3+4i) = \frac{e^{3+4i} - e^{-(3+4i)}}{2} = \frac{e^{3}e^{4i} - e^{-3}e^{-4i}}{2}$

$= \frac{1}{2}\{e^{3}(\cos 4 + i\sin 4) - e^{-3}(\cos 4 - i\sin 4)\}$

$= \frac{1}{2}\{(e^{3}\cos 4 - e^{-3}\cos 4) + i\,(e^{3}\sin 4 + e^{-3}\sin 4)\}$

$u = \frac{e^{3}\cos 4 - e^{-3}\cos 4}{2}, \quad v = \frac{e^{3}\sin 4 + e^{-3}\sin 4}{2}$

```
>> sinh(3+4*i)

ans =

  -6.5481 - 7.6192i

>> sinh(3)*cos(4)

ans =

   -6.5481

>> cosh(3)*sin(4)

ans =

   -7.6192
```

#5 다음 방정식의 모든 해를 구하시오.

- $\sin z = 10$

$\sin z = \sin x \cosh y + i \cos x \sinh y = 10$

$cosx \cdot \sinh y = 0$ → $\cos x = 0$ or $\sinh y = 0$

$i)$ $\sinh y = 0$ → $y = 0, \cosh y = 1$

$-1 \leq \sin x \leq 1 , -1 \leq \sin x \cosh y \leq 1 \neq 10$

$ii) \cos x = 0$ → $\sin x = \begin{cases} -1, & x = -\frac{\pi}{2} + 2n\pi \\ 1, & x = \frac{\pi}{2} + 2n\pi \end{cases}$

$\cosh y \geq 1, \sin x \geq 0 \rightarrow \boldsymbol{x = \frac{\pi}{2} + 2n\pi},$ $\sin x = 1$

$\cosh y = \frac{e^y + e^{-y}}{2} = 10 \rightarrow e^y - 20 + e^{-y} = 0 \rightarrow e^{2y} - 20e^y + 1 = 0$

$\boldsymbol{y = ln\left(10 \pm \sqrt{100 - 1}\right) = ln\left(10 \pm 3\sqrt{11}\right)}$

$\therefore \boldsymbol{z = x + yi = \left(\frac{\pi}{2} + 2n\pi\right) + ln\left(10 \pm 3\sqrt{11}\right)i}$

#5 - MATLAB

```
>> n = pi/2 : 2*pi : 50

n =

    1.5708    7.8540   14.1372   20.4204   26.7035   32.9867   39.2699   45.5531

>> sin(n+log(10 + sqrt(99))*i)

ans =

  1 ~ 7번 열

  10.0000 + 0.0000i  10.0000 + 0.0000i  10.0000 + 0.0000i  10.0000 - 0.0000i  10.0000 - 0.0000i  10.0000 - 0.0000i  10.0000 - 0.0000i

  8번 열

  10.0000 - 0.0000i

>> sin(n+log(10 - sqrt(99))*i)

ans =

  1 ~ 7번 열

  10.0000 - 0.0000i  10.0000 - 0.0000i  10.0000 - 0.0000i  10.0000 + 0.0000i  10.0000 + 0.0000i  10.0000 + 0.0000i  10.0000 + 0.0000i

  8번 열

  10.0000 + 0.0000i
```

#6 다음 값을 $u + iv$형태로 구하시오.

- $Ln(3-3i) = \ln|z| + iArg(z)$

$|z| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$Arg(z) = \arctan\left(-\frac{3}{3}\right) = -\frac{\pi}{4}$

$\therefore\ Ln(3-3i) = \ln\left(3\sqrt{2}\right) - \frac{\pi}{4}i$

```
>> log(3-3*i)

ans =

   1.4452 - 0.7854i

>> log(3*sqrt(2))

ans =

    1.4452

>> -1*pi/4

ans =

   -0.7854
```

#7 다음 값을 $u + iv$형태로 구하시오.

- $\ln(4+3i) = \ln|z| + \arg(z)$

$|z| = \sqrt{4^2 + 3^2} = 5$

$\arg(z) = \arctan\left(\frac{3}{4}\right) \pm 2n\pi \;\; \approx 0.6435 \pm 2n\pi$

$\therefore \ln(4+3i) = \ln(5) + (0.6435 \pm 2n\pi)i$

```
>> atan(3/4)

ans =

    0.6435

>> log(4+3*i)

ans =

   1.6094 + 0.6435i

>> log(5)

ans =

    1.6094
```

#8 다음 방정식의 해를 구하시오.

- $\ln(z) = -\frac{\pi i}{2}$

$$z = e^{-\frac{\pi i}{2}} = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right) = -i$$

```
>> syms z
>> solve(log(z)+pi*i/2)

ans =

-1i
```

#9 다음 값을 구하시오.

- $(1-i)^{1+i} = k$

$\ln(1-i)^{1+i} = (1+i)\ln(1-i) = \ln k$

$\ln(1-i) = \ln\left(\sqrt{2}\right) + i\left(-\frac{\pi}{4} \pm 2n\pi\right)$

$\ln k = (1+i)\left(\ln\sqrt{2} + i\left(-\frac{\pi}{4} \pm 2n\pi\right)\right)$

$\therefore\ k = e^{(1+i)\left(\ln\sqrt{2}+i\left(-\frac{\pi}{4}\pm 2n\pi\right)\right)}$

```
>> (1-i)^(1+i)

ans =

   2.8079 - 1.3179i

>> exp((1+i)*(log(sqrt(2))+i*(-pi/4)))

ans =

   2.8079 - 1.3179i
```