공업수학

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1, 2 .$z_1 = -1 + 3i, z_2 = 2 - i$ 일 때 다음을 구하시오.

- 1. $\frac{4(z_1+z_2)}{z_1-z_2}$

$z_1 + z_2 = 1 + 2i,\ 4(z_1 + z_2) = 4 + 8i$

$z_1 - z_2 = -3 + 4i$

$\frac{4(z_1+z_2)}{z_1-z_2} = \frac{4+8i}{-3+4i} = \frac{(4+8i)(-3-4i)}{(-3+4i)(-3-4i)} = \frac{20-40i}{25} = \frac{4}{5} - \frac{8}{5}i$

- 2. $(z_1^2 - z_2^2) = (z_1 + z_2)(z_1 - z_2) = (1 + 2i)(-3 + 4i) = -11 - 2i$

3. 다음을 polar form으로 표현하시오.

- 3. $z = -3$

$$z = x + yi = -3 + 0i \quad \rightarrow \quad (x = -3, y = 0)$$

$$x = r\cos\theta\,, \quad y = r\sin\theta\,, \quad r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2} = 3$$

$$-3 = 3\cos\theta\,, \quad 0 = 3\sin\theta \rightarrow \theta = \pi$$

$$\therefore z = -3 = 3(\cos\pi + i\sin\pi)$$

4. 다음을 polar form으로 표현하시오.

- 4. $z = 1 + \frac{\pi}{2} i$

$$z = x + yi = 1 + \frac{\pi}{2} i \quad \rightarrow \left(x = 1, y = \frac{\pi}{2}\right)$$

$$x = r\cos\theta\,, \qquad y = r\sin\theta\,, \qquad r = \sqrt{x^2 + y^2} = \sqrt{\frac{4+\pi^2}{4}}$$

$$1 = \sqrt{\frac{4+\pi^2}{4}}\cos\theta \quad , \frac{\pi}{2} = \sqrt{\frac{4+\pi^2}{4}}\sin\theta \rightarrow \theta = \arccos\left(\sqrt{\frac{4}{4+\pi^2}}\right) = \arcsin\left(\sqrt{\frac{\pi^2}{4+\pi^2}}\right) \approx 57.5184$$

$$\therefore z = 1 + \frac{\pi}{2} i = \sqrt{\frac{4+\pi^2}{4}}(\cos\theta + i\sin\theta) = \sqrt{\frac{4+\pi^2}{4}}\left(\sqrt{\frac{4}{4+\pi^2}} + i\sqrt{\frac{\pi^2}{4+\pi^2}}\right) = \sqrt{\frac{4+\pi^2}{4}}(\cos 57.5184 + i\sin 57.5184)$$

5. $w^4 = i$를 만족하는 모든 w를 구하시오.

- Let $z = \ i \ = r(\cos\theta + i\sin\theta)$ → $r = 1, \theta = \frac{\pi}{2} + 2k\pi \quad , k \in \{0, 1, 2, 3\}$

Then $z = \left(\cos\left(\frac{\pi}{2} + 2k\pi\right) + i\sin\left(\frac{\pi}{2} + 2k\pi\right)\right)$

- Let $w = R(cos\phi + i\sin\phi)$

Then $w^4 = R^4(\cos 4\phi + i\sin 4\phi) = (\cos 4\phi + i\sin 4\phi) \quad , (\because R^4 = r = 1)$

$w^4 = z \quad \rightarrow 4\phi = \frac{\pi}{2} + 2k\pi \quad \rightarrow \phi = \frac{\pi}{8} + \frac{k\pi}{2}$

→$\phi = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

$\therefore w = \left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\ \right), \left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right), \left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right), \left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$

6. $z^2 + z + 1 - i = 0$을 푸시오.

- $z^2 + z + 1 - i = 0$

→$z^2 + 1 = -(z - i)$

→$(z + i)(z - i) = -(z - i)$

$i)$ $(z - i) = 0$ → $z = i$

$ii)$ $(z - i) \neq 0$ → $(z + i) = -1, z = -i - 1$

$\therefore z = i, -i - 1$

7. MATLAB

#1

```
>> z1 = -1+3i; z2 = 2-i;
>> 4*(z1+z2)/(z1-z2)

ans =

   0.8000 - 1.6000i
```

#2

```
>> z1 = -1+3i; z2 = 2-i;
>> z1^2-z2^2

ans =

 -11.0000 - 2.0000i
```

#3

```
>> z = -3;
>> [theta, r] = cart2pol(real(z), imag(z))

theta =

    3.1416


r =

     3
```

$$\theta = \pi, r = 3$$
$$\rightarrow z = 3(\cos\pi + i\sin\pi)$$

7. MATLAB

#4

```
>> z = 1 + (pi/2)*i;
>> [theta, r] = cart2pol(real(z), imag(z))

theta =

    1.0039


r =

    1.8621

>> a = sqrt(4/(4+pi^2));
>> acos(a)

ans =

    1.0039

>> 1/a

ans =

    1.8621
```

$$a = \sqrt{\frac{4}{4+\pi^2}}$$

$$\theta = \arccos\left(\sqrt{\frac{4}{4+\pi^2}}\right)$$

$$r = \sqrt{\frac{4+\pi^2}{4}} = \frac{1}{a}$$

#5

```
>> syms w
>> solve(w^4-i,w)

ans =

- (2^(1/2) + 2)^(1/2)/2 - ((2 - 2^(1/2))^(1/2)*1i)/2
  (2^(1/2) + 2)^(1/2)/2 + ((2 - 2^(1/2))^(1/2)*1i)/2
  (2 - 2^(1/2))^(1/2)/2 - ((2^(1/2) + 2)^(1/2)*1i)/2
  ((2^(1/2) + 2)^(1/2)*1i)/2 - (2 - 2^(1/2))^(1/2)/2
```

#6

```
>> syms z
>> solve(z^2+z+1-i, z)

ans =

- 1 - 1i
      1i
```

```
>> - (2^(1/2) + 2)^(1/2)/2 - ((2 - 2^(1/2))^(1/2)*1i)/2
  (2^(1/2) + 2)^(1/2)/2 + ((2 - 2^(1/2))^(1/2)*1i)/2
  (2 - 2^(1/2))^(1/2)/2 - ((2^(1/2) + 2)^(1/2)*1i)/2
  ((2^(1/2) + 2)^(1/2)*1i)/2 - (2 - 2^(1/2))^(1/2)/2

ans =

  -0.9239 - 0.3827i


ans =

   0.9239 + 0.3827i


ans =

   0.3827 - 0.9239i


ans =

  -0.3827 + 0.9239i
```

```
>> cos(9*pi/8)+i*sin(9*pi/8)

ans =

  -0.9239 - 0.3827i
```

```
>> cos(pi/8)+i*sin(pi/8)

ans =

   0.9239 + 0.3827i
```

```
>> cos(13*pi/8)+i*sin(13*pi/8)

ans =

   0.3827 - 0.9239i
```

```
>> cos(5*pi/8)+i*sin(5*pi/8)

ans =

  -0.3827 + 0.9239i
```