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Jaemin Shin

We want to solve an ill-posed problem arising in some inverse problems using DNNs and compare the proposed method with known methods. We start with a simple ill-posed problem.

1. Consider a simple linear equation
$$Ax = b. \tag{1}$$
Here, A is an $n \times n$ non-singular matrix with a sufficiently small condition number.

   Train a model to solve (1) for a fixed A. You may generate data set first. Randomly take $b \in \{(b_1, b_2, \cdots, b_n) : -M \leq b_i \leq M\}$ and then solve the equation (1) for x, i.e. $x := A^{-1}b$. The data set for training should be $\{x^j, b^j\}_{j=1}^J$. This problem is similar to #5 assigned on March 19. Give your result, including errors for a test set.

2. Now add some noise to $\{b\}$ and solve (1) using the developed model in #1. Is your solution operator (model) well-posed?

3. Simulate #1 and #2 again for a different matrix A which is ill-conditioned. How are the results different?

4. Consider Tikhonov regularization technique for (1);
$$\min_x \{\|Ax - b\|^2 + \alpha \|x|^2\}. \tag{2}$$
Develop a DNN model for (2).