A Search for Tau Neutrino Appearance with IceCube-DeepCore

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8.3.1 Choice of Binning

In order to understand the potential for IceCube's measurement of ν_τ appearance, a choice of binning must be decided upon. The analysis discussed here uses two variables to describe the oscillations: the reconstructed energy and zenith angle. These dimensions form an integral part of the standard oscillation analysis and are often used in measurements of atmospheric mixing parameters .

list of atmo disappearance measurements that use zenith and energy binning

The choice of binning for zenith angles is selected to be similar, but somewhat finer than previous work . For this work, we use the fully sky, including upgoing events ($\cos(\phi) = -1$) corresponding to events passing through the full diameter of the Earth where we expect the strongest oscillation effects to very downgoing events ($\cos(\phi) = 1$) where events are originating in showers above the Antarctic. The energy binning is selected to match previous work from DeepCore and consists of 8 bins logarithmically spaced from 5.6 GeV to 56 GeV .

dragon, leesard 3 year papers

dragon, leesard

In addition, recent work with DeepCore has shown that a third dimension separating the sample into cascade-like and track-like events may provide better sensitivity than using solely track-like events. Two variables are available in the GRECO sample. The first, the reconstructed length of a muon track, provides a simple separation between events with a clear muon track from those without one. This, in general, leads to reasonable separation between the ν_μ events undergoing disappearance and ν_τ events undergoing appearance. This may be seen in , where the cumulative distribution of the various simulation components are shown as a function of the reconstructed track length. The separation between the ν_μ and ν_τ charged-current samples occurs between 30-50 meters. By separating the sample into cascade-like events (eg. L < 50 m) and track-like events (L ≥ 50 m), the disappearance and appearance may be partially disentangled.

cumulative plot of track length

The second potential separating variable is the likelihood ratio between a cascade and PegLeg's mixed cascade+track reconstructions. A higher likelihood (lower log-likelihood) in the casade fit implies that the event is more likely intrinsically cascade-like while the reverse is true for intrinsically track-like events. The cumulative plot of the likelihood ratio is shown in . There exists a broad choice of values with similar separation properties from approximuately $-4 < \Delta\text{LLH} < -2$. Once again, separating events into two samples using the likelihood ratio may improve the ability of the analysis to disentangle the disappearance and appearance effects.

cumulative plot of deltallh

Both variables clearly show some separating power and likely have similar behavior: an event with a longer reconstructed muon track should be expected to prefer the PegLeg reconstruction over a cascade reconstruction. In order to choose between the parameters, the efficacy of separating each of the simulation samples from the ν_τ charged current signal roc curves for track length

was evaluated. The results are shown in , which give the fraction of each type rejected and the number of ν_τ events accepted into the "cascade-like" sample for various choices of the separating parameters. Values further from a diagonal indicate better separation between the ν_τ and other event types. Here we see that the track length performs uniformly better than the likelihood ratio in separating the disappearing ν_μ charged-current and appearing ν_τ charged current events. Furthermore, the reconstructed track length performs significantly better in separating the neutrino components from the atmospheric muon background. The reconstructed track length is therefore selected as the separating variable for this analysis.

this sentence needs to be reworded. its too verbose

8.3.2 The MC Fit Templates

A choice of 50 meters of reconstructed track length is selected for this analysis. Because the PegLeg reconstruction assumes the muon track to be minimally ionizing, the division of track-like and cascade-like has an effect on the minimum energy of each sample. In particular, no track-like events ($L \geq 50$ m) may have less than 10 GeV in total reconstructed energy. Both track- and cascade-like events may reconstruct with higher energies than 10 GeV.

mc templates!

nufit 2.2

The binned expectations used in the fit are shown in assuming the oscillation parameters given by . The lack of expected events in the track-like histogram is clearly visible. As expected, atmospheric muon background events tend to reconstruct as downgoing events, primarily visible in the track-like channel. The signal ν_τ events occur in the very upgoing cascade channel and make up, at most, approximately 10% of the events in those bins.

8.4 Parametrizing the Tau Neutrino Appearance

In order to properly measure the appearance of ν_τ events, a choise of "appearance parameter" must be selected. Here, we discuss the choice of parameter used in this analysis.

8.4.1 CC vs CC+NC

As described in 1.2, neutrinos may interact in two distinct ways to produce light in the IceCube detector. These two methods, the charged-current and neutral-current interactions, provide separate windows into neutrino interactions. Tau neutrino events may interact in either of these channels depending on the neutrino energy.

PDG

With a mass of 1776.82 ± 0.16 MeV and a lifetime of 290.3 ± 0.5 femtoseconds , τ leptons produced during neutrino oscillations in DeepCore tend to travel very short differences before decaying. The charged-current interactions of the ν_τ result in a variety of signatures due to the unique decay behavior of the τ lepton.

$$\tau^- \rightarrow \begin{cases} \mu^- \bar{\nu}_\mu \nu_\tau & 17.41 \pm 0.04\% \\ e^- \bar{\nu}_e \nu_\tau & 17.83 \pm 0.04\% \\ \text{Hadrons} & \text{Otherwise} \end{cases} \tag{8.1}$$

In either the muonic or the electronic decay modes, a fraction of the energy is lost to outgoing neutrinos, resulting in a smaller observed charge than would be associated with a corresponding interaction of another neutrino type. Furthermore, the muonic decay mode may lead to a visible muon track for the ν_τ interaction. These muon tracks associated with the appearance of ν_τ would appear at lower energies than the tracks corresponding to the ν_μ disappearance, allowing both effects to be observed simultaneously.

Unlike the varied results of the charged current interactions, neutral current interactions of neutrinos are assumed to have identical coupling and behavior, regardless of flavor and, therefore, undergo no observable change due to oscillations. Because of this, studies of the standard unitary PMNS matrix tend to treat neutral current events as effectively non-oscillating . In contrast, searches for new physics and sterile neutrinos result can result in a change in the apparent number of neutral current interactions in the detector.

superk paper, opera paper sources for un-oscillating NC

For this analysis, both approaches have been adopted. A fit using charged-current events as the signal is used to provide limits on the modifications to a 3x3 mixing matrix without the introduction of neutral-current altering behavior . A second fit, including both neutral current and charged current ν_τ events, provides more insight into possible extra flavors of neutrinos.

non-sterile explanations of non-unitarity? maybe the neutrino decay paper?

8.4.2 The ν_τ Normalization

Because effectively all ν_τ events observable in DeepCore are the result of neutrino oscillations, the total number of observed ν_τ interactions is a direct measure of the appearance itself. The number of ν_τ events interacting in DeepCore is, however, affected by many of the previously-discussed systematics. In particular, the number of events is strongly related to the assumed atmospheric oscillation parameters.

In order to provide a quantitative measure of the appearance, the overall normalization of signal events is used as a final physics parameter. The normalization is a fit parameter, defined to be a total modification of the number of candidate ν_τ events after all other systematic parameters are applied.

think up a better phrasing to introduce the tau normalization

$$f'_{ijk} = \sum_{m \neq \nu_\tau} f^{m}_{ijk}\left(\theta_{23}, \Delta m^2, ...\right) + N_{\nu_\tau} f^{\nu_\tau}_{ijk}\left(\theta_{23}, \Delta m^2, ...\right) \tag{8.2}$$

In this case, we end up with two general cases for the result. In the expected case, $N_{\nu_\tau} = 1.0$, we find that the number of candidate events is consistent with our modeling of the ν_τ and unitary PMNS mixing. If the value is significantly different from 1.0, we may have hints of either mismodeled cross-sections or of novel physics. Due to the large existing uncertainties in the PMNS matrix described in 8.1, either situation is likely to yield valuable information.

Crazy shit that I will probably take out. but maybe find the neutrino decay paper again?

8.4.3 Limits on the ν_τ Normalization

This analysis is not the first to search for appearance in this way. Two other experiments, OPERA and Super-Kamiokande, have reported previous measurements parametrized in the same way.

The OPERA Limit

The Oscillation Project with Emulsion-tRacking Apparatus, better known by the acronym OPERA, is an experiment designed to search for ν_τ appearance. Unlike IceCube's use of atmospheric neutrinos, OPERA uses muon neutrinos produced in the CERN Neutrinos to Gran Sasso (CNGS) beamline. OPERA uses an bricks of photographic films in order to accurately track and reconstruct neutrino interactions in the fiducial volume. This technique allows analyzers to clearly identify not only the initial neutrino interactionv vertex, but also the decay products along the path of the charged lepton produced in charged current interactions. In OPERA, the muon and tau lepton produce significantly different signals due to the short lifetime and unique decay properties of the tau lepton. The impressive ability to

Interaction Mode	Non-tau-like	tau-like	All
CC nue	3071.0	1399.2	4470.2
CC numu	4231.9	783.4	5015.3
CC nutau	49.1	136.1	185.2
NC	291.8	548.3	840.1

Table 8.1 – The rates expected for each of the neutrino types in the Super-Kamiokande search for ν_τ appearance. Reproduced from.

identify the particle dynamics is balanced by the small fiducial volume of the experiment, yielding only 5408 useful events for analysis from five years of data-taking.

In 2015, OPERA released their final result in the search for ν_τ appearance. Five candidate events, shown in , were identified in the data sample with a signal expectation of 2.64 $\pm$ 0.53 and a background expectation of 0.25 $\pm$ 0.05. The data unambiguously rules out the no-appearance hypothesis, with a rejection at 5.1σ .

opera tau neutrino event views

opera paper: https://arxiv.org/abs/1507.01417

In terms of the normalization described above, OPERA reported a final value of $1.8^{+1.8}_{-1.1}$ at the 90% level. This value is consistant with the standard unitary oscillation scheme, but with large errors.

The Super-Kamiokande Limit

Super-Kamiokande, described in 2.2, also has reported results in searches for ν_τ appearance. The Super-Kamiokande collaboration developed a new event selection in the search for ν_τ events, including the implementation of a neural net to identify τ-like and non-τ-like events. The neural net itself includes information about the energy of the event and is trained against a background sample of simulated events. Events are analyzed in terms of the zenith angle and the neural net output variable.

superk paper on appearance https://arxiv.org/pdf/1711.09436.pdf

These two categories of events are fit with an unbinned likelihood including 28 systematic effects included in the analysis.

here

https://arxiv.org/pdf/1711.09436.pdf again

The Super-Kamiokande measurement yields an expectation of 185.2 ν_τ events in 5326 days or appoximately 12.7 events per year . After fitting, the final rejection of the no-appearance hypothesis is found to be 4.6σ. Like OPERA, Super-Kamiokande finds more ν_τ candidate events than expected, with a best-fit normalization of 1.47 ± 0.32.

wtf is going on here? table 1 in the paper gives an expectation of 185.2 events, but the stuff at the top right of page 11 says the expectation is 224...?! and NEITHER of these give the 1.47 that they quote. wtf

figure 14 of https://arxiv.org/pdf/1711.09436.pdf

8.5 Systematics Considerations

8.5.1 Oscillation Parameters

8.5.2 Flux Uncertainties

Atmospheric Muon Flux

Neutrino Flux

8.5.3 Propagation Uncertainties

8.5.4 Cross-section Uncertainties

Axial Masses

DIS Cross-sections

8.5.5 Detector Systematics

While the previous systematics have been concerned with global physics parameters, the remainder are dedicated to understanding the uncertainties associated with the IceCube detector itself, such as the properties of the PMTs and the ice. These parameters, collectively referred to as the **detector systematics**, do not have known analytic forms and may affect the rate of events, the reconstruction properties of a given event, or both. The effect of these uncertainties must be evaluated using additional Monte Carlo simulations.

The GRECO event selection uses a number of simulation sets, shown in 8.2 for signal and 8.3 for background, to characterize the effects of these detector systematics. Each set contains at least one simulation parameter changed from the baseline set and are run through the full GRECO processing.

Set Number	Coincident Fraction	DOM Eff	Hole Ice	Forward Coeff	Absorption	Scattering	Livetime
Baseline	0%	100%	25	0	100%	100%	30 years
640C	100%	100%	25	0	100%	100%	30 years
641	0%	88%	25	0	100%	100%	30 years
643		94%					
644		97%					10 years
645		103%					5 years
646		106%					10 years
648		112%					
660	0%	100%	15	0	100%	100%	10 years
661			20				
662			30				
663			35				
670	0%	100%	25	2.0	100%	100%	10 years
671				-5.0			
672				-3.0			
673				1.0			
674				-1.0			
681	0%	100%	25	0.0	92.9%	92.9%	30 years
682					110%	100%	
683					100%	110%	

Table 8.2 – Systematics sets used for the characterization of the signal neutrino events. While all listed sets have up to 30 years of effective livetime available, not all events are processed in each set.

Set Number	Oversizing	DOM Eff	Hole Ice	Forward Coeff	Absorption	Scattering	Livetime	Comments
Baseline	1.0	99%	25	0	100%	100%	5 years	1 year standard + 4 years KDE Prescale
A	1.0	69.3%	30	0	100%	100%	1 year	1 year standard
B		79.2%						
C		79.2%	25				4 years	4 years KDE Prescale
D		89.1%						
E		105%					1 year	1 year KDE Prescale
F	1.0	99%	15	0	100%	100%	5 years	1 year standard + 4 years KDE Prescale
G			30					
H	1.0	99%	30	-2	100%	100%	5 years	1 year standard + 4 years KDE Prescale
I				-4				
J	3.0	99%	25	0	100%	100%	1 year	1 year KDE Prescale
K					110%			
L					80%			
M					100%	80%		
N						110%		
O						120%		
P					92.9%	92.9%		
Q					114.2%	114.2%		

Table 8.3 – Systematics sets used for the characterization of the atmospheric muon background.

Coincident Fraction

The GENIE simulation sets are produced with exactly one neutrino interaction per event. In the actual detector, a fraction of triggered events will consist of a temporally coincident muon and neutrino pair which may be from the same air shower or from indepdent showers. In order to account for this possibility, a sample of such events are simulated assuming independent showers. In this case, every produced event contains at least one atmospheric muon in addition to exactly one neutrino interaction. By interpolating between this "100% coincident" sample and the standard "0% coincdent" sets, the effect of these events may be included in the final analysis.

The GRECO event selection actively selects against atmospheric muon-like events. The lowest-order effect of this choice is that increasing the coincident event fraction leads to a correspondingly lower total event rate, as shown in **??**. In order to distinguish the effect of the coincident events from a global normalization factor, the coincident event fraction is treated in a manner such that the total rate of events remains unchanged. The effect of this systematic in the final analysis is therefore shown in instead.

coin fraction figure

In most analyses in IceCube, a coincident event fraction of approximately 10% is assumed. This is derived from a combination of the atmospheric neutrino and muon fluxes assuming independent poissonian rates. At final level, the true fraction of coincident events is unknown, but previous oscillation analyses have found no clear issues using the standard simulation sets assuming no coincident events. A generous prior is therefore assumed to be a one-sided Gaussian distribution centered at 0% with a 10% width.

DOM Efficiency

As with all PMTs, the light detection probability of the IceCube DOMs is not perfect. Indeed, the total efficiency of detecting incident photons as measured by Hammamatsu, shown in **??**, is about 30% for the R7081-02 PMT used in standard IceCube DOMs. Before and during deployment, the net quantum efficiency of some DOMs were tested . The efficiency of the DOMs was again measured in-situo in order to better account for local effects like cable shadowing and the glass-ice interface. Dedicated measurement spost-deployment have used minimum ionizing muons in data and simulation and derived a modification of the assumed efficiency, hereafter referred to as the **DOM efficiency**, of 99%$\pm$10%.

Hammamatsu quantum efficiency? http://www.hamamatsu.com/resources/pdf/etd/LARGE_AREA_PMT_TPMH1286E.pdf

How many were tested in a lab before deployment?

Where does the domeff prior come from?

The DOM efficiency scales the probability of observing photons incident at the face of the DOM. A higher DOM efficiency leads to more information about individual particle interactions, leading to better reconstructions. The improved reconstructions lead to higher neutrino event rates at final level as well as more well-defined oscillation features in the reconstructed space. In addition, higher DOM efficiency increases the number of hits observed along atmospheric muon tracks, yielding improved veto efficiency. The net effect of changing the DOM efficiency by 10% is shown in .

domeff

Bulk Absorption and Scattering

The ice model used in IceCube is fit in-situo using various data from the deployment and detector operation in a process similar to the one described in 5.3. The model, here referred to as the **bulk ice model**, consists of scattering and absorption coefficients fit as a function of depth within the detector as well as information about anisotropy in the scattering properties of the ice . Uncertainties for these scattering and absorption coefficients, shown in , provide a significant source of uncertainty for physics measurements

ice model

bulk ice uncertainties vs depth

in IceCube. To handle these effects, global scale factors are used to modify all scattering or absorption coefficients in the bulk ice model simultaneously. Using the most recent published uncertainties on our ice model, a total uncertainty of approximately 10% is assumed for these global scale factors. Three variations are typically used, corresponding to sets with 10% larger absorption coefficients, 10% larger scattering coefficients, and a 7.1% reduction to both sets of coefficients.

The scattering and absorption exhibit different behaviors at final level in the GRECO sample. In general, the absorption behaves in a similar manner to the DOM efficiency, as both parameters modify the number of observed photons at the face of the PMT. In the signal samples, the effects of absorption uncertainties is relatively small. The most notable feature is an overall rate decrease (increase) for larger (smaller) absorption coefficients. As in the DOM efficiency, the depth of the oscillation minimum is also affected by the absorption coefficients due to a change in the reconstruction resolution.

The absorption, shown in , affects the atmospheric muons much more strongly than the neutrinos. Once again, this is due to the event selection: with weaker absorption (ie, smaller coefficients), more photons from the muon track may be detected. The observation of additional photons from the muon track improves the veto efficiency, leading to a significant decrease in the number of muons at final level.

absorption

The scattering, in contrast, has very little effect on the muon distribution, as seen in . No changes in rate or in reconstruction resolution are observed in the muon distributions.

scattering

In the neutrinos, the effects of the scattering are more important. In particular, stronger scattering (larger coefficients) lead to a reconstruction bias, with more events reconstructing as downgoing. This is a known effect of the reconstruction, where we use a version of the ice model which interprets off-time hits as being due to backscattered photons in a downgoing event.

Hole Ice and Foward Scattering

While the bulk ice refers to the scattering and absorption properties of the entire interaction volume, additional care must be taken for the ice close to the face of a PMT. During deployment, contaminants, including air, were introduced into the drill holes. These contaminants have been seen to form a dense column with unique scattering properties near the center of the drill holes. This bubble column, known as the **hole ice**, has properties separate from the rest of the ice model.

The uncertainties associated with the hole ice are significant and tend to elicit more attention than bulk ice uncertainties in searches for oscillations with DeepCore. The simulation of the hole ice model used here, discussed briefly in 4.2.4, requires two free parameters which will be referred to as the **lateral** and **forward** scattering parameters here for clarity. The lateral scattering modifies the efficiency of accepting photons incident from the horizon at each DOM while the forward scattering modifies only the acceptance of the very-forward region. The models of the angular acceptance are shown in .

hole ice and hifwd

The effects of the two hole ice parameters show very similar behavior to that of the scattering uncertainty in the bulk ice, as all three coefficients are modeling the scattering properties of different locations in the ice.

Parametrizing with Hyperplanes

For each of the simulation sets and each particle type, histograms are produced using the reconstructed energy, zenith, and track length. These systematic histograms give information about the expected change of the final histogram as a function of the changing systematics parameters, but the information is encoded in discretized points with statistical fluctuations due to the finite simulation statistics. In order to produce continuous systematics for analysis, the discrete detector systematics must be parametrized.

how do i flesh this out?

For this work, a hyperplane is fit to the detector systematics sets for each particle type and for each bin in the analysis histogram.

For neutrinos, a simple linear model is assumed for each detector systematic, with one free coefficient associated with each systematic as well as one free constant term independent of the systematics. The form of the hyperplane for each neutrino type in the bin ijk is given by 8.3.

$$f'_{ijk} = \left(\sum_{m}^{detsys} \left(a_m^{ijk} (x_m - x_m^0) \right) + b^{ijk} \right) f_{ijk} \tag{8.3}$$

muongun rates vs domeff and absorption to justify exponentials

For atmospheric muons, the form is slightly modified due to the strong effects observed from both the DOM efficiency and the absorption. In these two cases, an exponential model is selected to better describe the observed effects in simulation.

$$f_{ijk}\prime = \left(\sum_{m \neq DE,Abs}^{detsys} \left(a_m^{ijk} (x_m - x_m^0) \right) + \sum_{m}^{DE,Abs} \left(a_m^{ijk} e^{b^{ijk}(x_m - x_m^0)} \right) + c^{ijk} \right) f_{ijk} \tag{8.4}$$

Need to include some discussion of th goodness of fit for these sets. Maybe a plot of the chi2 values for all of the sets?

chi2 values for hyperplane parametrizations

8.6 The Method of Maximum Likelihood

8.6.1 The χ^2 Fit

The simplest implementation of a fitting algorithm begins with an assumption of the true and observed distributions. Namely, that the observed number of events in each bin of the histogram is drawn from a distribution approximately Gaussian with a mean μ equal to the expectation from simulations and a variance σ^2 calculated from the Poisson uncertainty on the expectation.

$$P(x|\mu) = N e^{\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} \tag{8.5}$$

where N is a normalization constant and, in the case of Poissonian statistics of simple histograms, the variance is given by the event weights in the specified bin.

$$\sigma^2 = \mu = \sum w \tag{8.6}$$

This needs work. can't even be called a derivation. its just crap.

From this point, taking the logrithm yields the standard χ^2 definition for the likelihood after dropping the constant terms.

$$\chi^2 = \frac{(x-\mu)^2}{\mu} \tag{8.7}$$

8.6.2 Finite Statistics

The χ^2 distribution above implicitly assumes that the dominant source of uncertainty at the best-fit point comes from the statistical fluctuations of the data around the true distribution represented by the Monte Carlo simulation. While this is true in te ideal case, in practice the statistical properties of the simulation sets themselves cannot be ignored. In general, every attempt should be made to ensure the statistical fluctuations of the simulation sets are negligible compared to those of the data. This typically leads to requests for at least an order of magnitude larger simulation statistics than expected from the data itself. In the situation where this is infeasible, modifications to the likelihood space itself may be used to account for the additional uncertainties. For this analysis the statistical uncertainties of the underlying simulation sets are added to the weighted uncertainties in quadrature.

make a plot showing chi2 value as a function of mc stats scale factor to justify the 10x rule

$$\chi_1^2 = \frac{1}{2} \frac{\left(x - \sum w\right)^2}{\left(\sum w\right)^2 + \sum w^2} \tag{8.8}$$

Due to the large uncertainties associated with the atmospheric muon sample, further considerations are necessary. In particular, the large uncertainties associated with atmospheric muon simulation statistics may be used by the fitter in order to reduce the χ^2_{FS} value. This situation proceeds with the minimization process as normal until a runaway effect is observed by increasing the statistical uncertainties at the expense of data/simulation agreement. In this case, the numerator becomes simply

$$\lim_{N_\mu \to \infty} \left(x - \sum w\right)^2 = \left(\sum w\right)^2 \tag{8.9}$$

The resulting limit in each bin as the event weights become large is therefore

$$\lim_{N_\mu \to \infty} \chi_1^2 = \frac{\left(\sum w\right)^2}{\left(\sum w\right)^2 + \sum w^2} \tag{8.10}$$

$$\lim_{N_\mu \to \infty} \chi_1^2 = 0 \tag{8.11}$$

While this is a particular concern for all simulation types, the dominant contribution to the $\sum w^2$ term is the atmospheric muons. In addition, the atmospheric muons have the strongest impacts from non-normalization systematic uncertaintines, particularly the DOM efficiency and the absorption. Modifying either of these parameters or the normalization systematics in the fit may lead to this runaway behavior.

In order to prevent this situation, a further modification of the χ^2 is necessary. For this analysis, the total scale of the statistical uncertainty is assumed to be set by the seed values of the fit.

$$N_{w^2} = \frac{\sum w^2_{Seed}}{\sum w^2} \tag{8.12}$$

With this modification, the χ^2 is now defined to be

this is only true for muons! how do i explain that?

$$\chi^2_{FS} = \frac{1}{2} \frac{\left(x - \sum w\right)^2}{\left(\sum w\right)^2 + N_{w^2} \sum w^2} \tag{8.13}$$

Taking the limit of this χ^2 in a single bin as the event weights become large now gives

$$\lim_{N_\mu \to \infty} \chi^2_{FS} = \frac{1}{2} \frac{\left(x - \sum w\right)^2}{\left(\sum w\right)^2 + N_{w^2} \sum w^2} \tag{8.14}$$

$$\lim_{N_\mu \to \infty} \chi^2_{FS} = 1 \tag{8.15}$$

This is in agreement with the limit obtained from the standard χ^2.

8.6.3 Fit Priors

In many cases, the systematics listed in 8.4.3 have known constraints from external measurements. This information can be useful and should be included in the analysis to bias the minimization toward the most likely systematics values. These constraints are included in the form of **priors** in the analysis. Priors are additional terms included multiplicatively (additively) in the likelihood (log-likelihood) calculation. These often take the form of a Gaussian distribution with mean μ and variance σ^2 given by external measurements. For this measurement, most priors are handled assuming a standard Gaussian form. For a systematic i with value x_i, these additional terms take the form

$$\chi^2_i = \frac{\left(x_i - x_0\right)^2}{\sigma^2} \tag{8.16}$$

These additional terms are added to 8.6.2 in order to calculate the final χ^2_{FS} used in the minimization for this analysis.

$$\chi^2_{Total} = \sum_{bins} \chi^2_{FS} + \sum_{systematics} \chi^2_i \tag{8.17}$$

All of these references for the priors...

A list of priors is shown in 8.4.Note that the coincident event fraction is effectively a one-sided Gaussian due to physical constraints on the value.

	Systematic	Unit	Type	Baseline/Seed Value	Prior	Allowed Range	Reference
Physics Parameter	N_{ν_τ}	-	Analytic	1.0	-	0.0 - 2.0	-
Oscillations	Δm^2_{3j}	$10^{-3}\ eV^2$	Analytic	2.526	-	2.0 - 3.0	nufit
	$\mathrm{Sin}^2\theta_{23}$	-	Analytic	0.440 (NO), 0.66 (IO)	-	0.0 - 1.0	nufit
Total Rates	N_nu, N_mu	Years	Analytic	2.25	-	0.0 - 10.0	-
Cross-section	Axial Mass (QE)	σ	Analytic	0.0	0.0 ± 1.0	-5.0 - 5.0	GENIE
	Axial Mass (RES)	σ	Analytic	0.0	0.0 ± 1.0	-5.0 - 5.0	GENIE
	$N_{\nu^{NC}}$	-	Analytic	1.0	1.0 ± 0.2	0.0 - 2.0	NC prior
Flux	γ_ν	-	Analytic	0.0	0.0 ± 0.10	-0.50 - 0.50	Honda
	γ_μ	σ	Analytic	0.0	0.0 ± 1.0	-5.0 - 5.0	ste and davide's paper
	Up/Horizontal Ratio	σ	Analytic	0.0	0.0 ± 1.0	-5.0 - 5.0	Barr
	$\nu/\bar{\nu}$ Ratio	σ	Analytic	0.0	0.0 ± 1.0	-5.0 - 5.0	Barr
	Φ_{ν_e}	-	Analytic	1.0	1.0 ± 0.05	0.8 - 1.2	Barr
	Coincident Fraction	-	Hyperplane	0.0	$0.0 + 0.10$	0.0 - 1.0	coin fraction...?
Detector	DOM Efficiency	-	Hyperplane	1.0	1.0 ± 0.1	0.7 - 1.3	DOM eff
	Hole Ice (Lateral)	-	Hyperplane	0.25	0.25 ± 0.10	0.0 - 0.5	hole ice
	Hole Ice (Forward)	-	Hyperplane	0.0	-	-5.0 - 5.0	hole ice
	Absorption	-	Hyperplane	1.0	1.0 ± 0.1	0.5 - 1.5	bulk ice
	Scattering	-	Hyperplane	1.0	1.0 ± 0.1	0.5 - 1.5	bulk ice

Table 8.4 – Priors and allowed ranges for each systematic included in this analysis.

8.7 Expected Sensitivity to Appearance

8.7.1 Fitting Code

Maybe all of this oscfit stuff should just be moved to just before the systematics section.

should i even talk about oscfit itself? it seems a bit awkward

msu and desy disappearance

Flowchart of oscfit fitting. at least broadly

prob3++

Checks in this analysis are first performed using solely simulation files. In order to understand the expected sensitivity of this analysis, a fitting package previously used to fit the ν_μ disappearance . The code, known as **OscFit**, works in multiple stages. After separating the simulation into separate channels consisting of ν_e^{CC}, ν_μ^{CC}, ν_τ^{CC}, ν^{NC}, μ_{atm}, and accidental triggers, the analytic systematics are applied. These systematics solely rely on information about the particle interaction in order to calculate correction factors to the event weights and are not sensitive to the order of application. The oscillation calculations are performed at this stage and are based on the Prob3++ code to calclulate the full three-flavor unitary oscillations including matter effects within the Earth.

When including the neutral current interactions from ν_τ events in the signal definition, the neutral current events are reweighted for oscillations at this stage. The OscFit code assumes the neutral current interaction rate is unaffected by oscillations and the ν_τ^{NC} events are not directly included in favor of the significantly higher simulation statistics from the other sets. Because no charged leptons are produced in the neutral current interactions, no differences in event topology are expected based on flavor of neutrino interaction. For the purposes of this analysis, the neutral current interactions from ν_e and ν_μ events are instead used to model the effect of the ν_τ^{NC} events. The Prob3++ code calculates oscillation probabilities for these events given the expected contribution to the neutral current event rate from ν_τ^{NC} events.

$$R_{\nu_\tau^{NC}} = R_{\nu_e^{NC}} P_{\nu_e \to \nu_\tau}\left(\theta_{23}, \Delta m^2_{3i}\right) + R_{\nu_\mu^{NC}} P_{\nu_\mu \to \nu_\tau}\left(\theta_{23}, \Delta m^2_{3i}\right) \tag{8.18}$$

The modification to the total neutral current rate given the ν_τ normalization, N_{ν_τ}, is then given by

$$R\prime_{\nu^{NC}} = R_{\nu^{NC}} + R_{\nu_\tau^{NC}}\left(N_{\nu_\tau} - 1\right) \tag{8.19}$$

The modified weights are then used to histogram the simulated event samplesinto one histogram per simulation channel. After histogramming, the detector systematics are applied to the each of the binned templates bin-by-bin using hyperplanes calculated for each bin as described in 8.5.5. The hyperplanes themselves are created using the same process, but are created only once using the baseline systematics values and oscillation parameters taken from . More in-depth tests have shown little change when accounting for changes in the hyperplane coefficients as a function of different oscillation parameters.

nufit 2.2

Once all systematics have been applied, the normalization terms representing the overall scale factors for the neutrino rate, N_ν, the muon rate, N_μ, and the accidental rate, N_{noise}, are mutliplied to the respective histograms. The final histograms are summed together to form the final simulation expectation to be compared to the data using the χ^2_{FS} described in 8.6.2.

The value of the χ^2_{FS} is minimized as a function of the various systematics using the iMinuit2 package. The minimization continues until the requested tolerance, 10^{-16}, is reached by the minimizer, after which the best fit histogram and systematics values are returned to the user.

iminuit

8.7.2 Sensitivity of the Analysis

To evaluate the expected sensitivity of this analysis, the OscFit code is used to find the best-fit value of the χ^2_{FS}. Two methods are used to evaluate both the average expected sensitivity and range of variation of the sensitivity due to both the data and simulation statistics.

The first method, known as the **Asimov** expectation , begins by creating the expected histogram using baseline values of the systematics and oscillations. The produced histogram, representing an exact PDF of the expected events, is then used as an estimate of the data. The χ^2_{FS} is then minimized while the value of N_{ν_τ} is fixed at regularly spaced points in the interval [0,2] in order to produce a contour. A final minimization is performed allowing the minimizer to identify the global best-fit value of N_{ν_τ}.

asimov expectation?

The final expected sensitivity in the Asimov approach is given by calculating the difference between the values of the χ^2_{FS} at each point and the global best fit.

$$\Delta\chi^2\left(N_{\nu_\tau}\right) = \chi^2_{FS}\left(N_{\nu_\tau}\right) - \chi^2_{FS}\left(N^{Global}_{\nu_\tau}\right) \tag{8.20}$$

The value of $\Delta\chi^2_{FS}$ as a function of N_{ν_τ} is shown in . These values may be converted into expected significance levels using Wilk's theorem assuming one degree of freedom.

asimov sensitivity

wilks theorem

The second method, producing what is known as a **Brazilian flag** plot due to the color scheme, provides an estimate of the expected range of sensitivities in this analysis. The production of a Brazilian flag begins with the production of a pseudo-data histogram from the Asimov histogram. Because the simulation sets used here have significant uncertainties due to limited simulation statistics, the first step is to vary the event rate in each bin within the statistical uncertainties of the Monte Carlo. To do so, histograms of the uncertainty in each bin are produced using the baseline systematics values

$$\sigma^{MC}_{ijk} = \sqrt{\sum_{m}^{Evts} w^2_{ijkm}} \tag{8.21}$$

This uncertainty is assumed to be approximately Gaussian. The event rate in each bin of each simulation template is then varied using a Gaussian distribution using the expected rate as the mean and σ^{MC} as the uncertainty. The new templates are them summed together and each bin is fluctuated around the new expectation assuming Poisson statistics, creating a representation of one possible realization of the data in the analysis. The OscFit minimization then proceeds as described in the Asimov case using each of 500 realizations of pseudo-data, with the calculation of the $\Delta\chi^2$ as described in 8.7.2. The Brazilian flag shows the 1σ and 2σ range of $\Delta\chi^2$ values around the median at each value of N_{ν_τ}. This provides a graphical representation, shown in , of the expected range of variation of the sensitivity given solely statistical uncertainties.

brazilian flag

8.7.3 Impact of Systematics

There are various ways to measure the impact of the included systematics in this analysis. Described here are methods to evaluate, in order of increasing importance, the total systematics impact, the impact of each systematic individually, the correlation between systematics, and the effect of non-baseline values. Each of these test different aspects of the sensitivity and all are included for completeness.

Total Systematics Impact

The total impact of the systematics on the sensitivity may be measured by comparing the total Asimov sensitivity to an Asimov sensitivity calculated using no systematics. This is shown in . It is clear from the comparison that the analysis is very sensitive to the included systematics set.

comparison of stat-only fit to full systematics fit

N+1 Tests: Sensitivity of Analysis to Systematic

A different test is also possible: Instead of calculating likelihoods with no systematics included, a single systematic may be used at a time. This test, called an N+1 test for the addition of one systematic at a time, yields useful information on a sample's sensitivity to single systematics. A small change in sensitivity between the no-systematics case above and an N+1 Asimov sensitivity may have two possible explanations. The first, that the current analysis is unaffected by changes in

N-1 Tests: Redundancy Between Systematics

"Hidden Potential" Tests: Non-Baseline Values

8.7.4 Expected Sensitivity over Time

8.7.5 Feldman-Cousins vs WIlk's Theorem

8.8 Fitting Data

8.8.1 Burn Sample Fits: Testing the Fitting Code

8.8.2 Blind Fits: Checking the Goodness-of-Fit

8.9 Results from the Analysis

8.9.1 Individual Years

8.9.2 Combining Years of Data

8.9.3 Systematics Pulls in the Final Result

8.9.4 Implications and Future Work

Notes