2.2.5 Proposition. Define

$$\mathcal{B}_0 = \{\text{all finite unions of elements of J}\}$$

where J is defined as in the previous problem, being the collection of all intervals over $[0, 1]$. Then $\mathcal{B}_0$ is an algebra but not a sigma algebra.

Proof. Obviously $\emptyset$ and Ω are members. Now consider any two elements $A, B \in J$. Their intersection will be an intersection of intervals...since everything is finite it checks out. Also obviously closed under complements

The interest problem comes in proving that it is NOT a sigma algebra. For this we need to dig into our bag of tricks, so to speak. To prove this, we must demonstrate there exists a countable union of elements of $\mathcal{B}_0$ that is not a finite union of elements of $\mathcal{J}$: in other words, that it is not closed under the σ-algebra operations. Consider the set $\{\frac{1}{n} \mid n \in \mathbb{N}\}$. This set must be in the corresponding sigma algebra but it is clearly not a finite union of intervals of the $[0, 1]$.

$\square$