

**Exercise Number: 4.5.3**

**Proposition.** Let  $X$  be a random variable with finite mean, and let  $a \in \mathbb{R}$  be any real number. Then  $\mathbb{E}(\max(X, a)) \geq \max(\mathbb{E}(X), a)$ .

**Proof.** First consider the case  $\mathbb{E}(X) \geq a$ . Then evidently  $\max(\mathbb{E}(X), a) = \mathbb{E}(X)$ . Thus we want to show that

$$\mathbb{E}(\max(X, a)) \geq \mathbb{E}(X) \geq a.$$

Now we exploit the order preservation property of expectation (result of exercise 4.3.2). Define a random variable

$$Z(\omega) = \begin{cases} a, & \text{if } X(\omega) < a \\ X(\omega), & \text{otherwise.} \end{cases}$$

Then note that  $Z \geq X$  and therefore  $\mathbb{E}(Z) \geq \mathbb{E}(X)$ . But  $Z = \max(X, a)$ , completing the first portion of the proof.

Proving the second case,  $\mathbb{E}(X) < a$ , follows a similar but reversed line of argument.

□