

Proposition. Suppose $\Omega = 1, 2$ with $\mathbb{P}(\emptyset) = 0$ and $\mathbb{P}(\{1, 2\}) = 1$. Suppose $\mathbb{P} = \frac{1}{4}$. Then \mathbb{P} is countably additive if and only if $\mathbb{P}(\{2\}) = \frac{3}{4}$.

Proof. First assume \mathbb{P} is countably additive. Then

$$\mathbb{P}(\{1, 2\}) = \mathbb{P}(\{1\}) + \mathbb{P}(\{2\}) = 1/4 + \mathbb{P}(\{2\}) = 1 \Rightarrow \mathbb{P}(\{2\}) = 3/4.$$

Next assume that