Exercise Number: 4.5.12

Proposition. Let X and Y be independent general non-negative random variables, and let $X_n = \Psi_n(x) = \min(n, 2^{-n}\lfloor 2^n x \rfloor)$.

a. The sequence of functions $\Phi_n(x) = \min(n, 3^{-n}\lfloor 3^n x \rfloor)$ is an example of a sequence other than $\Psi_n(x)$ such that for all x, $0 \leq \Phi_n(x) \leq x$ and $\{\Phi_n(x)\} \nearrow x$.

b. If $Y_n = \Phi_n(Y)$, then X_n and Y_n must be independent

c.

d.

Proof.

□