Exercise Number: 4.5.3

Proposition. Let X and Y be two general random variables with finite means, and let Z = X + Y.

1.
$$Z^+ - Z^- = X^+ - X^- + Y^+ - Y^-$$
.

2.
$$\mathbb{E}(Z) = \mathbb{E}(X) + \mathbb{E}(Y)$$

3. The general definition of expectation is finitely linear, for general random variables with finite means.

Proof. Proven in the order presented.

1. Follows by definition of expectation.

2.