Exercise Number: 2.6.1.

The setup is as follows. We are attempting to translate coin tossing into a measure-theoretic framework. In the case of finite number of coin tosses, say n, this translates simply into binary sequences of n tuples, with tails being 0 and heads being 1, i.e.:

$$\Omega_n = \{(r_1, r_2, ..., r_n) \mid r_i \in \{0, 1\}\}.$$

Since these are finite sets, we can simply define $\mathcal{F} = \mathcal{P}(\Omega))$, i.e. the power set. Finally the probability measure for any $A \in \mathcal{F}$ follows as $\mathbb{P}(A) = |A|/2^n$.

However...what happens in the limit as n grows to infinity? Now defining $\mathcal{F}$ in a suitable manner is no longer trivial, and so what to do about $\mathbb{P}$ is as unknown as its domain. The book proposes to utilize the extension theorem with a semi-algebra defined as follows:

$$\mathcal{J} = \{A_{a_1 a_2 \cdots a_n} ; n \in \mathbb{N} \text{ and } a_1, a_2, \cdots, a_n \in \{0, 1\}\} \cup \{\emptyset, \Omega\}$$

where $A_{a_1 a_2 \cdots a_n} = \{(r_1, r_2, ...,) \mid r_i = a_i \;\; 1 \leq i \leq n\}$.

Proposition.

Proof.