Exercise Number: 2.7.4

Proposition. Let $\mathcal{F}_1$, $\mathcal{F}_2$, ... be a sequence of collections of subsets of Ω, such that $\mathcal{F}_n \subseteq \mathcal{F}_{n+1}$.

1. Suppose each F_i is an algebra. Then $\cup_{i=1}^{\infty} F_i$ is also an algebra.
2. Now suppose each F_i is a σ-algebra. Then $\cup_{i=1}^{\infty} F_i$ is not necessarily a σ-algebra.

Proof.

Recall the criteria required for a σ-algebra: finite additivity of sets, closed under complements, and includes the empty set.

1. First note that we have the following fact:
$$\cup_{i=1}^{n} \mathcal{F}_i = \mathcal{F}_n$$
since each successive algebra is just a refinement of the previous one.
2.

$\square$