Exercise Number: 4.3.4

Proposition. Let X and Y be two general independent random variables with finite means, and let $Z = XY$. Then the following holds true:

(a) $X^{\#_x}$ and $Y^{\#_y}$ are independent, where $\#_x, \#_y \in \{+, -\}$

(b) $Z^+ = X^+Y^+ + X^-Y^-$ and $Z^- = X^-Y^+ + X^+Y^-$

(c) $\mathbb{E}(Z) = \mathbb{E}(X)\mathbb{E}(Y)$.

Proof. Proven in the order presented.

1. Consider any two borel sets $B_x, B_y \subseteq \mathbb{R}$. We are given that

$$\mathbb{P}[X^{-1}(B_x) \cap Y^{-1}(B_y)] = \mathbb{P}[X^{-1}(B_x)]\mathbb{P}[Y^{-1}(B_y)].$$

   Now consider

$$X^{-1}(B_x) = (X^+ - X^-)^{-1}(B_x).$$

   The claim

2. Fairly straight forward: details fairly trivial here. Z^+ is going to on the omega where both X and Y are positive or negative: sinceit is defined as a product. When one omega maps both to negative, X^+Y^+ is 0 and vice versa, allowing the desired result. Same logic applies to Z^-.

3. Just apply the results

□

DISCUSSION: The point of this proposition (and the previous one) is that this new general definition of expectation which involves its decomposition into negative/positive components really does not change the properties of expectation that had been previously examined ...in a "finite" setting.