

**Exercise Number: 1.3.2**

**Proposition.** Suppose  $\Omega = \{1, 2, 3\}$  and  $\mathcal{F}$  is the collection of all subsets of  $\Omega$ . The following are the necessary and sufficient conditions on the real numbers  $x, y$ , and  $z$  such that there exists a countably additive probability measure  $\mathbb{P}$  on  $\mathcal{F}$ , with  $x = \mathbb{P}\{1, 2\}$ ,  $y = \mathbb{P}\{2, 3\}$ ,  $z = \mathbb{P}\{1, 3\}$ :

1.  $x + y + z = 2$
2. Triangle inequality on  $x, y, z$  must hold true.
3.  $x, y, z$  must all be non-negative.

**Proof.** We first prove that individually all these conditions are necessary, and then that they are sufficient.

Necessary:

1. Since  $\mathbb{P}$  is countably additive, we have that  $\mathbb{P}\{1, 2\} = \mathbb{P}\{1\} + \mathbb{P}\{2\} = x$ , etc... for each  $x, y$ , and  $z$ .
2. This holds trivially
3. Obviously since  $P$  is a probability measure all of these values must be greater than 0.

Sufficient: