

Exercise Number: 2.7.9

Proposition. Let \mathcal{F} be a σ -algebra, and write $|\mathcal{F}|$ to indicate the total number of sets in this collection. If $|\mathcal{F}|$ is finite, then $|\mathcal{F}| = 2^m$ for some $m \in \mathbb{N}$.

Proof.

Discussion. This result is interesting, counter-intuitive, and made have consequences for theory involving refinements of σ -algebras later on.