

9/17/24

MA-578

Homework 11) Hoff's book: problem 2.5)  
2.5)

Urn H is 40% green balls and 60% red balls

Urn T is 60% green and 40% red.

Someone will flip coin and then select a ball from H or T depending whether head or tails

Let  $X$  be 1 or 0 if the coin lands h or t andlet  $Y$  be 1 or 0 if the ball is green or red

a) Joint probability

Ball's

Urn's		Ball's	
		Green	Red
H	H	0.2	0.3
	T	0.3	0.2

$$P(H|G) = 0.5 \times 0.4 = 0.2$$

$$P(H|R) = 0.5 \times 0.6 = 0.3$$

$$P(T|G) = 0.5 \times 0.6 = 0.3$$

$$P(T|R) = 0.5 \times 0.4 = 0.2$$

b) Find  $E(Y)$ , when ball is green?

$$E(Y) = P(Y=0) \times 0 + P(Y=1) \times 1$$

$$= (0.2 + 0.3) \times 0 + (0.3 + 0.2) \times 1$$

$$E(Y) = 0.5, \text{ also can be said because it's 1 of 2 options}$$

c) Calc  $\text{Var}[Y|X=0]$ ,  $\text{Var}[Y|X=1]$ ,  $\text{Var}[Y]$ 

$$\text{Var}[Y|X=0] = P(Y=0|X=0) \times (0 - E[Y|X=0])^2 + P(Y=1|X=0) \times (1 - E[Y|X=0])^2$$

$$= (0.4) \times (0 - (0.6))^2 + 0.6 \times (1 - (0.6))^2$$

$$\text{Var}[Y|X=0] = 0.24$$

$$\text{Var}[Y|X=1] = P(Y=0|X=1) \times (0 - E[Y|X=1])^2 + P(Y=1|X=1) \times (1 - E[Y|X=1])^2$$

$$= (0.6) \times (0 - (0.4))^2 + (0.4) \times (1 - (0.4))^2$$

$$\text{Var}[Y|X=1] = 0.24$$

$$\text{Var}[Y] = P(Y=0) \times (0 - E[Y])^2 + P(Y=1) \times (1 - E[Y])^2$$

$$= (0.5) \times (0 - (0.5))^2 + (0.5) \times (1 - (0.5))^2$$

$$\text{Var}[Y] = 0.25$$

$\text{Var}[Y]$  is larger because there's greater uncertainty as no urn has been picked yet.