MA 578 — Bayesian Statistics

Homework 2
(Due: Tuesday, 10/01/24)

1. Hoff's book problem 3.3.

2. Hoff's book problem 3.4.

3. Jeffreys prior for *canonical* parameterization of binomial likelihood:

   (a) If $\theta \sim \text{Beta}(\alpha, \beta)$, show that $\lambda = \log(\theta/(1 - \theta)) = \text{logit}(\theta)$ has a "Beta-logit" distribution with density
   $$\mathbb{P}(\lambda) \propto \frac{e^{\alpha\lambda}}{(1 + e^\lambda)^{\alpha+\beta}}.$$

   (b) Suppose now that $X \mid \lambda \sim \text{Binom}(n, \text{logit}^{-1}(\lambda))$, that is, that
   $$\mathbb{P}(X \mid \lambda) \propto \exp\left\{\lambda X - n\log(1 + e^\lambda)\right\}.$$

   Show that Jeffreys prior for λ is Beta-logit$(\frac{1}{2}, \frac{1}{2})$ and so conclude that Jeffreys prior for $\theta = \text{logit}^{-1}(\lambda) \sim \text{Beta}(\frac{1}{2}, \frac{1}{2})$.