9/17/24 MA-578

Homework 1

1) Hoff's book: problem 2.5)

2.5)

Urn H is 40% green balls and 60% red balls

urn T is 60% green and 40% red

Someone will flip coin and then select a ball from H or T depending wether head or Tails

~~Let~~ Let X be 1 or 0 if the coin lands h or t and

let Y be 1 or 0 if the ball is green or red

a) Joint probability

Ball's

Urn's		Green	Red
	H	0.2	0.3
	T	0.3	0.2

$P(H|G) = 0.5 * 0.4 = 0.2$

$P(H|r) = 0.5 * 0.6 = 0.3$

$P(T|G) = 0.5 * 0.6 = 0.3$

$P(T|R) = 0.5 * 0.4 = 0.2$

b) Find $E(Y)$, when ball is Green?

$E[Y] = P(Y=0) * 0 + P(Y=1) * 1$

$= (0.2 + 0.3) * 0 + (0.3 + 0.2) * 1$

$\boxed{E[Y] = 0.5}$, also can ~~said~~ bcz it's 1 of 2 options

c) Calc $Var[Y|x=0]$, $Var[Y|x=1]$, $Var[Y]$

$Var[Y|x=0] = P(Y=0|X=0) * (0 - E[Y|X=0])^2 + P(Y=1|X=0) * P(1 - E[Y|X=0])^2$

$= (0.4) * (0 - (0.6))^2 + 0.6 * (1 - (0.6))^2$

$\boxed{Var[Y|X=0] = 0.24}$

$Var[Y|X=1] = P(Y=0|X=1) * (0 - E[P|X=1])^2 + P(Y=1|X=1) * P(1 - E[Y|X=1])^2$

$= (0.6) * (0 - (0.4))^2 + (0.4) * (1 - (0.4))^2$

$\boxed{Var[Y|X=1] = 0.24}$

$VAR[Y] = P(Y=0) \cdot (0 - E[Y])^2 + P(Y=1) * (1 - E[Y])^2$

$= (0.5) * (0 - (0.5))^2 + (0.5) * (1 - (0.5))^2$

$\boxed{Var[Y] = 0.25}$

Var[Y] is larger bcz there's greater uncertainty as no urn has been picked yet.

2.5) d) Suppose you see that the ball is green. What is the probability that the coin turned up tails?

$$P(X=0 \mid Y=1) = \frac{P(Y=1 \mid X=0) * P(X=0)}{P(Y=1)}$$

$$= \frac{0.6 * \cancel{0.5}}{\cancel{0.5}}$$

$$\boxed{P(X=0 \mid Y=1) = 0.6}$$

2.6) Suppose events A and B are conditionally independent given C, which is written as $A \perp B \mid C$.

A^c = "not" A

We know... $P(A \cap B \mid C) = P(A \mid C) * P(B \mid C)$

and... $P(B \mid C) = P(A \cap B \mid C) + P(A^c \cap B \mid C)$

$\cancel{A^c \cap B \mid C}$

So... $P(A^c \cap B \mid C) = P(B \mid C) - P(A \cap B \mid C)$

$= P(B \mid C) - P(A \mid C) * P(B \mid C)$

$= P(B \mid C) - P(1 - P(A \mid C))$

$= P(A^c \mid C) * P(B \mid C) \Rightarrow A^c \perp B \mid C$

$\cancel{A \cap B^c \mid C}$

also... $P(A \cap B^c \mid C) = P(A \mid C) - P(A \cap B \mid C)$

$= P(A \mid C) - P(A \mid C) * P(B \mid C)$

$= P(A \mid C) - (1 - P(B \mid C))$

$= P(A \mid C) * P(B^c \mid C) \Rightarrow A \perp B^c \mid C$

$\cancel{A^c \cap B^c \mid C}$

and... $P(A^c \cap B^c \mid C) = P(A^c \mid C) - P(A^c \cap B \mid C)$

$= P(A^c \mid C) - P(A^c \mid C) * P(B \mid C)$

$= P(A^c \mid C) - P(1 - P(B \mid C))$

$= P(A^c \mid C) * P(B^c \mid C) \Rightarrow A^c \perp B^c \mid C$

3) Sampsons Paradox

a) Show that Treatment II is more helpful than treatment I, for each of 4 groups

i) Older Males

$T I \rightarrow \frac{120}{2(120)} = 0.5$

$T II \rightarrow \frac{20}{20+10} = 0.67$

→ Treatment II more helpful for older Males

ii) Young Males

$T I \rightarrow \frac{60}{60+20} = 0.75$

$T II \rightarrow \frac{40}{40+10} = 0.8$

→ Treatment II more helpful for young Males

iii) Older Females

$T I \rightarrow \frac{10}{10+50} = 0.17$

$T II \rightarrow \frac{20}{20+50} = 0.29$

→ Treatment II more helpful for older Females

iv) Young Females

$T I \rightarrow \frac{10}{10+10} = 0.5$

$T II \rightarrow \frac{160}{160+90} = 0.64$

→ Treatment II more helpful for young females

b)

		Helpful	Not
Older	I	130	170
	II	40	60
Young	I	70	30
	II	200	100

Older I $\frac{130}{300} = 0.43$ Treatment I better

II $\frac{40}{100} = 0.4$

Young I → 70/100 = 0.70; II → 200/300 = 0.67

Treatment I is better

3)

c) Marginal success rate of

Treatment I: $T_{Help} = 120 + 60 + 10 + 10 = 200$

$T_{Not} = 120 + 20 + 50 + 10 = 200$

$\boxed{r_{T_1} = 0.5} = (200/400)$

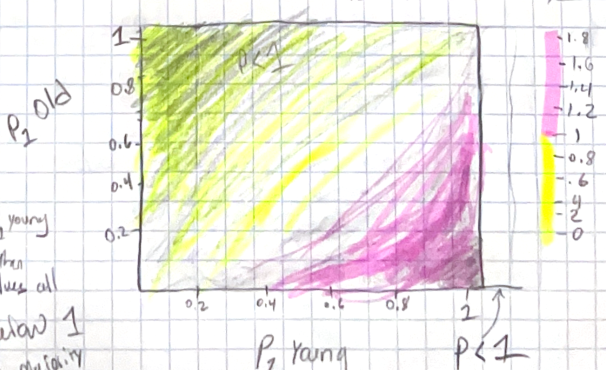
Treatment II: $T_{help} = 20 + 40 + 20 + 160 = 240$

$T_{not} = 10 + 10 + 50 + 90 = 160$

$\boxed{r_{T_2} = 0.6}$

These marginal success rates compares to conditions success rates of older males and young females. The older female success rates compare at a much lower success rate, while the young males compare at a much higher success rate. The Dispencies w/ the rates, could likely be because of Treatment II have overall better/higher success rate due to there be a greater number of older People as opposed to younger

d) Heatmap of Success Rate B/n Treatments I/II



where P_1 young is greater than P_1 older values all range below 1 which is majority of the map.