Heterogeneous Discounting and Frictional Unemployment*

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Abstract

This paper studies an environment where households are heterogeneous in their ability to smooth consumption, but where all workers face search frictions in the labour market. When households are heterogeneous in their income sources, and these different income sources respond differently to aggregate shocks, this implies heterogeneity in the value of employment across workers in the labour market. We show that when wages are determined with respect to joint surplus of a match, this has implications for the response of the economy to aggregate shocks. We preserve tractability by extending a standard TANK model as in Bilbiie (2008) to allow for frictional unemployment. In response to aggregate shocks there is heterogeneity in the stochastic discount factors of households such that workers from the two different types of household discount the lifetime value of an employment relationship at different rates. We show that under Nash bargaining this introduces an additional term equilibrium wage which reflects the fact that workers from different households value the continuation of the match differently, and that analytically the influence of this additional channel on the equilibrium wage depends critically on the response of profits relative to labour income. Under a standard calibration of the model, we show quantitatively that this channel amplifies the response of the economy to standard shocks on impact even in the absence of the income risk channel studied in Ravn and Sterk (2021), Broer, Druedahl, et al. (2021) and Gornemann et al. (2021), and that the influence of this channel is increasing in the sensitivity of firm profits to aggregate shocks.

Keywords: Search & matching; TANK models; Household heterogeneity; Stochastic discount factors

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1 Introduction

Over the past decade or so there has been a growing recognition the importance of household heterogeneity in shaping both the response of economies to shocks and the transmission of policy changes. Recently, there has been a particularly large literature that has grown around studying the relationship between household heterogeneity in asset holdings and aggregate demand. The seminal contribution is Kaplan et al. (2018), who show using a heterogeneous agent New Keynesian ('HANK') model that household heterogeneity is important for understanding the transmission mechanism of monetary policy to the real economy. Models with this flavour have since been developed to address a variety of important questions in macroeconomics: Gornemann et al. (2021) study the distributional consequences of monetary policy, McKay, Nakamura, et al. (2016) study the implications of household heterogeneity for the effectiveness of forward guidance, McKay and Reis (2016) apply a similar model to study the issue of automatic stabilisers, and Hagedorn et al. (2019) look at how household heterogeneity shapes the fiscal multiplier. All of the models in the papers mentioned above feature households are heterogeneous in their sources of income due to differences in their savings behaviour. In the extreme case, the wealthiest households in such models rely very little on income from the labour market to finance consumption, whilst households who are borrowing constrained rely *exclusively* on labour income, which is consumed in a hand-to-mouth fashion.

In principle, labour and financial income can respond very differently to the same aggregate shock. As households differ in their asset holdings, the consumption levels of different households can therefore respond very differently. We also know that when households are risk averse, in general this heterogeneity in consumption responses will also lead to heterogeneity in the rate at which households discount the future, usually given by the standard expression for the stochastic discount factor between two time periods t and $t+\tau$ for some household i:

$$\beta^i_{t,t+\tau} = \beta \frac{u'(c^i_{t+\tau})}{u'(c^i_t)}$$

Heterogeneity in discount factors is a feature of standard incomplete markets models in Huggett (1993) and Aiyagari (1994), which are incorporated into general equilibrium models with aggregate shocks and sticky prices in the HANK literature. In fact, stochastic discount factor heterogeneity across households reflects households optimally making different consumption-savings decisions due to the realization of idiosyncratic uncertainty. For instance, households receiving some positive income shock will decide to save relatively more, whilst households hit with a negative shock to income will prefer to dissave. These dynamics result in households constantly changing position within the aggregate wealth distribution for the economy. The largest difference is between those households who are borrowing constrained (and consequently have the highest MPC) versus households who are relatively wealthy. However this large dispersion in stochastic discount factors between these two groups plays no direct role by design in most standard models.[1] This is because the stochastic discount factor of households at the borrowing constraint does not directly affect the model's equilibrium.[2]

[1]Clearly this large dispersion in stochastic discount factors between the constrained and unconstrained households reflects the large difference in MPCs between the poor and rich households, which *does* play an important role for instance in how monetary policy is transmitted in a New Keynesian economy with heterogeneous households.

[2]This is because borrowing constrained households are by definition off their Euler equation. They would like to be able to borrow more, but they are prevented from doing so. As such their consumption path will deviate from its optimal level as defined by the Euler equation.

If the labour market is characterised by search frictions, the stochastic discount factors of constrained households may no longer be irrelevant for the model's equilibrium. If workers are heterogeneous in the rate they discount the future, this will induce heterogeneity in the worker's surplus from being in employment. In this paper we show that this has implications for the response of the labour market to shocks if wages are determined in such a way that depends on the joint match surplus (such as under Nash bargaining). We develop a tractable framework within which to study this channel and assess its implications for the response of the economy to standard macroeconomic shocks. The model extends the canonical TANK model in Bilbiie (2008) by incorporating frictional unemployment via search & matching frictions in the labour market in the spirit of Mortensen and Pissarides (1994). As is well known, TANK models employ the simple assumption that some fraction of households λ in every period are unable to participate in asset markets to smooth consumption (for some exogenous reason), whilst the remaining households behave as normal permanent income consumers. In this setting constrained households rely exclusively on labour income to finance consumption, whereas unconstrained households can smooth consumption and hold wealth in equilibrium (in the form of firm profits). In as far as labour income and firm profits respond differently to shocks, the consumption of the two households respond differently, inducing dispersion in their stochastic discount factors. The dispersion in how households discount future values implies that the value from being in employment for a worker will be different depending on the type of household the worker is from. We show that when wages depend on the joint match surplus, heterogeneity in stochastic discount factors across households introduces a new channel that affects the response of the economy to standard shocks via the labour market.

In this paper we study the specific case of the Nash-bargained wage, given that this is the conventional assumption as to how wages are determined in this setting. As is well known, the Nash bargained wage allocates a fraction of the joint match surplus to the worker and the remaining fraction to the firm. We show that heterogeneity in the rates at which firms and the average worker discount the future induces an additional term in the reservation wage of the firm under Nash bargaining. This additional term is a function of the discount factor dispersion between the two types of household in the economy, and consequently disappears when there is no household heterogeneity (i.e. when $\lambda = 0$). Firstly, we outline analytically that in a TANK environment the importance of this additional channel is ultimately driven by the responsiveness of firm profits *relative* to labour income, as well as other key parameters such as the fraction of constrained households λ, the degree of worker bargaining power, and parameters relating to household preferences. We find that this additional channel has consequences for the responsiveness of the economy to a shock on impact, but does not affect the persistence of the model's responses due to the forward-looking nature of the channel.

Under an intentionally standard calibration of the model, we find that in response to standard demand and supply shocks this channel emphasising the role of discount factor heterogeneity acts as an additional source of amplification. To our knowledge, this is the first such paper to outline and quantitatively evaluate this channel. We show that in response to a standard monetary policy (demand) shock where profits are very responsive, the amplification of the shock is fairly significant. In response to a standard supply shock however - where it is well known that standard matching models with Nash bargained wages are subject to the Shimer (2005) critique - firm profits are much less responsive and heterogeneity is quantitatively irrelevant. Versions of the model which address the Shimer critique by inducing greater responsiveness of firm profits to changes in productivity (e.g. small surplus calibration, rigid wages etc.) would allow discount factor dispersion to play a greater role in amplifying the response of the economy to supply shocks too. Finally, we illustrate that the degree of amplification becomes more powerful when the fraction of constrained households is larger, worker bargaining power is higher, and households are less willing to substitute consumption intertemporally.

Related literature: This paper is closely related to the vast literature studying heterogeneity in marginal propensities to consume (MPCs) across households, motivated by the empirical work by Campbell and Mankiw (1990) and Vissing-Jørgensen (2002) who document that a substantial fraction of US households (up to 40%) either save a very small fraction of their disposable income, or do not save at all. These households instead consume their current income in a "hand-to-mouth" fashion, in contrast to the theory of permanent income consumers in standard models. These findings precipitated a literature which has sought to embed embedding heterogeneity in MPCs across households into otherwise standard New Keynesian models. A simple approach of capturing household heterogeneity is to assume a two-agent structure, where some (usually a fixed) fraction of households consume their disposable income "hand-to-mouth" in every period whilst the remaining households behave as normal permanent income consumers. Two-agent models provide an environment to study the implications of heterogeneous MPCs whilst retaining tractability. This is the approach we take in this paper. Some notable examples are: Galí et al. (2007) study how the effects of government spending shocks are affected by household heterogeneity, whilst Bilbiie (2008) and Nisticò (2016) study the implications of limited asset market participation for monetary policy transmission and design. Broer, Harbo Hansen, et al. (2020) make a slightly different distinction between workers who earn labour income but are also able to save in bonds, and "capitalists" who only earn income from firm profits, and highlight the potential importance of wage rigidities in dampening the role heterogeneity plays in monetary transmission. Ascari et al. (2017) study the implications of household heterogeneity for the design of monetary policy in a more standard TANK model with wage rigidities, concluding that when wages are rigid the presence of constrained households does not matter for optimal monetary policy. Cantore and Freund (2021) use an estimated worker-capitalist model similar to Broer, Harbo Hansen, et al. (2020) to illustrate how government spending shocks can have significant distributional effects by increasing the labour share of income.

A more sophisticated (but computationally intensive) approach to modelling household heterogeneity has been popularised recently by heterogeneous agent New Keynesian ('HANK') models, following the seminal contribution by Kaplan et al. (2018).[3] These models feature much richer heterogeneity than in TANK models - households face uninsurable idiosyncratic income risk and are subject to borrowing constraints following the standard incomplete markets model of Huggett (1993) and Aiyagari (1994). When households are imperfectly insured against idiosyncratic income risk (due to incomplete markets), they face the risk of endogenously hitting the borrowing constraint if they experience negative income shocks persistently. Upon hitting this constraint, households are unable to smooth consumption and behave in a hand-to-mouth fashion. In contrast to TANK models, household are not a fixed 'type' over their lifetime and the fraction of constrained households will fluctuate in response to aggregate shocks. The key takeaway from the HANK model in Kaplan et al. (2018) is that the transmission of monetary

[3] Other notable contributions are McKay, Nakamura, et al. (2016), Acharya and Dogra (2020), Auclert (2019), Auclert et al. (2020), Nuno and Thomas (2020), Werning (2015), Bayer et al. (2020), and Luetticke (2021).

policy works at least as much through its effects on labour income, rather than almost entirely through the direct intertemporal substitution channel implied by the aggregate Euler equation in standard New Keynesian models.

Although HANK models are much richer, there is a close relationship between TANK and HANK models. Bilbiie (2020) transfers the insights of Kaplan et al. (2018) into a tractable framework based on the standard TANK model and shows that demand amplification increases when a larger fraction of households are constrained, as the average marginal propensity to consume is higher. Debortoli and Galí (2018) provide further evidence that TANK models can effectively approximate the quantitative responses of HANK models to aggregate shocks. They find that it is the size of the 'consumption gap' between constrained and unconstrained households at any point in time which is the most important channel determining the dynamic responses of HANK models, and this is captured in a much simpler way in TANK models.

This paper is also closely related to the large literature embedding the Mortensen and Pissarides (1994) search & matching model of the labour market into New Keynesian models.[4] This literature has traditionally studied the importance of the labour market frictions and institutions in determining the properties of aggregate fluctuations and inflation dynamics. The vast majority of these papers maintain the representative household assumption for tractability, although there are some important recent exceptions which highlight important interactions between labour market frictions and incomplete markets. Ravn and Sterk (2021) embed search & matching frictions into the two-agent setup of Broer, Harbo Hansen, et al. (2020), illustrating how under incomplete markets workers face endogenous unemployment risk which affects savings behaviour by introducing a wedge in the Euler equation of workers, and show how precautionary savings during recessions can amplify a downturnin the presence of nominal rigidities. A recent paper by Broer, Druedahl, et al. (2021) use a similar model augmented to allow for the slow adjustment of vacancies by removing the free entry condition and quantify the role unemployment risk plays in output fluctuations over the business cycle, finding that the interaction with endogenous separations and costly entry to be crucial. Gornemann et al. (2021) take a more quantitative approach by building a rich HANK model with a frictional labour market, allowing additionally for heterogeneity in both preferences and skills. They find that consumption becomes much more responsive to monetary policy shocks, that contractionary monetary policy leads to increases in inequality due to its effect on the labour market, and that households will tend to disagree about the appropriate stance of monetary policy based on their sources of income. In contrast these studies, in this paper we emphasise a very different channel for the interaction of household heterogeneity with labour market frictions Instead we maintain the standard approach of abstracting from unemployment risk as in Merz (1995), but show that even with full consumption risk-sharing within the household, heterogeneity in the worker's value of employment across households has implications for how the labour market responds to shocks.

Finally, this paper also contributes to a relatively recent literature documenting the link between discount rates and the labour market. This literature views job creation as another form of financial investment, and so should be subject to the same rate of discount as financial assets. The key contribution

[4]Notable contributions in this direction were made by Chéron and Langot (2000), Walsh (2005), Krause and Lubik (2007), Thomas (2008), Trigari (2009), Thomas and Zanetti (2009), Blanchard and Galí (2010), and Christiano et al. (2016).

here is Hall (2017), who illustrates that feeding an estimated process for discount factors on US stock market data into the standard search & matching model leads to empirically plausible fluctuations in unemployment. Kehoe et al. (2019) develop a standard search & matching model where household preferences are consistent with time-varying risk over the business cycle (e.g. recursive preferences) and find quantitatively that the model is able to replicate US unemployment fluctuations without generating further empirical puzzles. Finally, a recent contribution by Martellini et al. (2021) builds a rich search framework with heterogeneous match quality, and instead find that shocks to financial discount rates on their own do not generate sufficient volatility in labour market transition probabilities to induce large unemployment fluctuations. This paper suggests that heterogeneity in the rates of financial discounting, induced by heterogeneity in the ability of households to smooth consumption, potentially plays a role in amplifying shocks when the labour market is characterised by search frictions.

The rest of the paper is organised as follows. Section 2 details a simple TANK model extended to allow for frictional unemployment. Section 3 further outlines the role of heterogeneity in discount factors in the model. Section 4 details the quantitative analysis of the model where we study the implications of heterogeneity in discounting. Section 5 outlines some robustness analysis. Section 6 concludes.

2 Model

The model outlined below develops the standard TANK model presented in Bilbiie (2008) by including a frictional labour market in the spirit of Mortensen and Pissarides (1994). The resulting framework features both tractable household heterogeneity and frictional unemployment.

2.1 Households

The economy is populated by a continuum of infinitely-lived households. There are two types of household: "unconstrained" (U) households who save using all available assets in the economy, and "constrained" (C) households who do not save and consume their labour income in every period, i.e. hand-to-mouth. Households are indexed by $i \in \{U, C\}$. A fraction λ of households are constrained. Although there are employed and unemployed workers from both types of household, we assume there is perfect risk-sharing within each household as in Merz (1995) such that all members of the household have the same consumption level.[5]

A household of type i obtains utility from their consumption bundle, $u(C_t^i)$, and wish to maximise expected discount lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t^i)$$

where $\beta \in (0,1)$ is the common household discount factor. The stochastic discount factor of household i between any two time periods t and $t+\tau$ is defined as standard:

$$\beta_{t,t+\tau}^i = \beta \frac{u'(C_{t+\tau}^i)}{u'(C_t^i)} \tag{1}$$

[5]In contrast, Ravn and Sterk (2021) and Broer, Druedahl, et al. (2021) abstract from risk-sharing to study the interaction between labour market risk and precautionary savings. In this model, savings motives will purely be driven by intertemporal smoothing considerations of the unconstrained households as standard.

Goods: There are a continuum of differentiated goods sold by monopolistically competitive firms indexed by $k \in [0,1]$. $P_{k,t}$ is the price of good k at time t. Total consumption of household i is given by $C^i_t \equiv \left(\int_0^1 C^{i\,\frac{\nu-1}{\nu}}_{k,t} dk\right)^{\frac{\nu}{\nu-1}}$, where ν is the elasticity of substitution across goods. The overall demand for each good variety in the economy is standard:

$$C_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\nu} C_t$$

where $P_t \equiv \left(\int_0^1 P^{1-\nu}_{k,t} dk\right)^{\frac{1}{1-\nu}}$ is the relevant price index.

Budget constraints: Unconstrained households consume and save in every period. There are two types of financial asset in the economy: riskless one-period bonds B_t yielding the gross nominal return R_t, and shares in an equity fund which owns firm profits S_t which can be purchased at (real) price Q_t. The period budget constraint for the representative unconstrained household is therefore given by:

$$C^U_t + \frac{B_t}{P_t} + Q_t S_t \leq R_{t-1}\frac{B_{t-1}}{P_t} + \int_0^1 w_{k,t} N^U_{k,t} dk + u_t b + [Q_t + D_t] S_{t-1} + T^U_t$$

where T^U_t denotes the transfers received by the unconstrained household, $N_{k,t}$ is the employment rate of agents in the household at firm k who earn the real wage $w_{k,t} \equiv \frac{W_{k,t}}{P_t}$, u_t is the fraction of household members who are unemployed, and b denotes unemployment benefits received by each unemployed member of the household.[6]

Constrained households are excluded from asset markets. They consume their income in every period in a 'hand-to-mouth' fashion:

$$C^C_t = \int_0^1 w_{k,t} N^C_{k,t} dk + u_t b + T^C_t$$

where T^C_t are transfers received by constrained households. Hand-to-mouth behaviour implies perfect passthrough of fluctuations in labour income to consumption for these households.

Euler equations: Unconstrained households choose consumption and asset holdings to maximise utility subject to their period budget constraint . This gives rise to the standard Euler equations for bonds and shares:

$$\frac{1}{R_t} = \mathbb{E}_t\left[\beta^U_{t,t+1}\frac{1}{\pi_{t+1}}\right] \tag{2}$$

$$Q_t = \mathbb{E}_t\left[\beta^U_{t,t+1}(Q_{t+1} + D_{t+1})\right] \tag{3}$$

where $\pi_{t+1} \equiv P_{t+1}/P_t$ is the gross rate of price inflation in the economy. Note that the stochastic discount factor which matters for pricing assets belongs to unconstrained households due to the fact that constrained households are excluded from financial markets.

[6] Note that the real wage is assumed to be common across workers from different household types. This assumption will be discussed in detail below.

2.2 Labour Market

The labour market setup is very standard. Aggregate employment in the economy is given by N_t, which follows the law of motion:

$$N_t = (1-\rho)N_{t-1} + M_t \tag{4}$$

where $\rho \in (0,1)$ is an exogenous separation rate, and M_t are newly formed matches.[7] The labour force is assumed to be constant and normalized to 1 such that:

$$0 \leq N_t \leq 1$$

We define the stock of jobseekers at the beginning of the period as U_t, which consists of the unemployed from the previous period plus those workers separated from their jobs:

$$U_t = 1 - N_{t-1} + \rho N_{t-1} = 1 - (1-\rho)N_{t-1} \tag{5}$$

The unemployment rate in period t is therefore defined as the stock of unemployed net of those who successfully find jobs:

$$u_t = U_t - M_t = 1 - N_t \tag{6}$$

Matches are determined by a constant returns to scale aggregate matching function:

$$M_t = m(V_t, U_t)$$

where V_t are vacancies posted by firms at per period cost χ. Labour market tightness, the job filling rate and job finding rates are defined as standard:

$$\theta_t = \frac{V_t}{U_t}, \quad q(\theta_t) = \frac{M_t}{V_t}, \quad p(\theta_t) = \theta_t q(\theta_t) \tag{7}$$

2.3 Firms

There are a continuum of monopolistically competitive firms in the economy indexed by k who produce a differentiated good, which they then sell to consumers at price $P_{k,t}$. All firms employ a fraction of total workers $N_{k,t}$ who are each paid a real wage $w_{k,t}$. Firms hire workers by posting vacancies at per period cost χ and face quadratic price adjustment costs.[8] Overall, firm k will choose employment, vacancies and prices to maximise the present discounted value of their profits:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^U_{t,t+1} \left[\frac{P_{k,t}}{P_t} Y_{k,t} - \chi V_{k,t} - w_{k,t} N_{k,t} - \frac{\Psi}{2} \left(\frac{P_{k,t}}{P_{k,t-1}} - \pi \right)^2 Y_{k,t} \right]$$

where the parameter Ψ controls the severity of the adjustment costs and π is the steady state gross inflation rate. Profits are discounted using the discount factor of unconstrained households, as only these households trade shares in firm profits.

[7] This timing implies that new matches are able to produce in the same period they are matched. Timing conventions around when new matches begin to produce differ in the literature combining New Keynesian models with search & matching models of the labour market. This convention is adopted by Thomas and Zanetti (2009) and Blanchard and Galí (2010) .

[8] We assume Rotemberg (1982) price rigidities in order to stay closer to the TANK/HANK literature.

Production Technology: All producing firms have an identical linear production function in labour input:[9]

$$Y_{k,t} = A_t N_{k,t}$$

where A_t is productivity and $a_t \equiv \ln(A_t)$ follows an AR(1) process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{8}$$

where ε_t^a is an i.i.d. productivity shock with distribution $\mathcal{N}(0, \sigma_a^2)$.

Job creation & destruction: An exogenous fraction of matches $\rho \in (0,1)$ are destroyed at the end of every period. Firm k chooses employment and vacancy postings to maximise present discounted profits subject to a law of motion for firm employment

$$N_{k,t} = (1-\rho)N_{k,t-1} + q(\theta_t)V_{k,t}$$

the production function, and the standard demand curve for their output:

$$Y_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\nu} Y_t$$

As firms are identical, optimal choices are symmetric across firms which allows us to drop the k subscripts. Combining first-order conditions for employment and vacancies delivers a standard job creation condition:

$$\frac{\chi}{q(\theta_t)} = \varepsilon_t A_t - w_t + (1-\rho)\mathbb{E}_t\left[\beta^U_{t,t+1}\frac{\chi}{q(\theta_{t+1})}\right] \tag{9}$$

This condition equates the expected marginal cost of creating a job (the per period cost multiplied by its expected duration) to the expected marginal benefit of creating an additional job (the marginal product of an additional worker net of the real wage, plus the expected saving in period $t+1$). Note in this setup ε_t (the Lagrange multiplier on the production function) represents real marginal costs of production for the representative firm. Costly search for workers ($\chi > 0$) introduces a wedge into the expression for marginal costs relative to a standard New Keynesian model, capturing the additional cost of hiring a worker today due to the presence of costly vacancy posting, net of the future savings made from not having to hire in the next period (conditional on non-separation).

Pricing: Firms also make an optimal pricing decision, which delivers the usual first-order condition relating inflation and marginal costs:

$$1 - \Psi(\pi_t - \pi)\pi_t + \mathbb{E}_t[\beta^U_{t,t+1}\Psi(\pi_{t+1} - \pi)\pi_{t+1}\frac{Y_{t+1}}{Y_t}] = \nu(1-\varepsilon_t) \tag{10}$$

Note that if prices are fully flexible ($\Psi = 0$) then it is optimal for the firm to ensure that marginal costs are equal to the (constant) inverse price markup $\varepsilon_t \equiv \varepsilon = \frac{\nu-1}{\nu}$.

[9] We assume a linear production function for simplicity of exposition.

2.4 Wage Determination

Following convention, we assume that the real wage w_t in the economy is determined by Nash bargaining (Pissarides (2000)). This assumption implies crucially that wages depend on the worker's surplus from working, which in general depend on the type of household the worker is from.[10] This is becauase workers from different households will consume at different levels in equilibrium and discount future values at different rates in response to shocks (i.e $\beta^U_{t,t+1} \neq \beta^C_{t,t+1}$). To ensure there is a single equilibrium wage (for simplicity), we make the assumption that workers from both types of household are members of a union which bargains with the firm on their behalf. Therefore, we assume that the *average* discount factor is the relevant discount factor for wage bargaining:[11]

$$\beta_{t,t+1} = \lambda \beta^C_{t,t+1} + (1-\lambda)\beta^U_{t,t+1}$$

Value Functions: The marginal value to the firm of a filled vacancy $\mathbb{S}^f_t$ is given by:

$$\mathbb{S}^f_t = \varepsilon_t A_t - w_t + (1-\rho)\mathbb{E}_t[\beta^U_{t,t+1}\mathbb{S}^f_{t+1}]$$

where the assumption of free entry implies that the firm's surplus from a match is equal to the expected cost of posting a vacancy. The worker's surplus is given by:

$$\mathbb{S}^w_t = w_t - b + \mathbb{E}_t\left[\beta_{t,t+1}(1-\rho-p(\theta_{t+1}))\mathbb{S}^w_{t+1}\right]$$

where discounting occurs with respect to the union's discount factor, which captures all heterogeneity in discounting across the two households.

Deriving the Nash Wage: Combining the Nash sharing rule

$$\mathbb{S}^w_t = \eta(\mathbb{S}^w_t + \mathbb{S}^f_t)$$

with the expressions for firm and worker surpluses gives a standard expression for the bargained real wage as a weighted average of the worker's and the firm's reservation wage:

$$w_t = (1-\eta)\underline{w}_t + \eta\bar{w}_t \tag{11}$$

where $[\underline{w}_t, \bar{w}_t]$ are the worker's and firm's reservation wages respectively. The worker's reservation wage is simply equal to the level of unemployment benefit:

$$\underline{w}_t = b$$

whilst the firm's reservation wage is slightly more protracted:

$$\bar{w}_t = \varepsilon_t A_t + \mathbb{E}_t\left[\beta^U_{t,t+1}\chi\theta_{t+1}\right] + \lambda\mathbb{E}_t\left[\frac{\chi}{q(\theta_{t+1})}(1-\rho-p(\theta_{t+1}))(\beta^U_{t,t+1}-\beta^C_{t,t+1})\right]$$

[10] Nash bargaining is chosen for simplicity, but is just one possible way in which wages could be determined in relation to the worker's surplus. Under Nash bargaining workers obtain a fraction η of the total match surplus, which depends on the worker's surplus.

[11] Relaxing this assumption would imply that the bargained wage would depend on the type of household a firm matched with, so there would be two equilibrium wages to keep track of, corresponding to firms matched with workers from constrained and unconstrained households. As workers do not differ in terms of their productivity, the fraction of constrained households is fixed by assumption and search is random, the composition of the jobseekers pool would coincide with the composition of the population and therefore wouldn't be an additional source of amplification. Any additional amplification from relaxing this assumption would come only from any different behaviour of wages across the two types of workers.

Note crucially that the firm's reservation wage now depends on the degree of *dispersion* in the stochastic discount factors scross the two types of household, $\beta^U_{t,t+1} - \beta^C_{t,t+1}$. We defer detailed discussion of the implications of this for the model to Section 3. In the absence of household heterogeneity ($\lambda = 0$) this dependence disappears and we recover the standard expression for the Nash wage. A derivation of the expression for the Nash wage is provided in Appendix A.1.

2.5 Market Clearing

We can express aggregate output in the economy Y_t as:

$$Y_t = \int_0^1 Y_{k,t} dk = A_t \int_0^1 N_{k,t} dk = A_t N_t \tag{12}$$

In equilibrium the total supply of goods net of the costs of posting vacancies and changing prices must equal the total demand by households, given by $C_t \equiv \int_0^1 C_{k,t} dk = (1-\lambda)C^U_t + \lambda C^C_t$. This implies the following goods market clearing condition:

$$C_t = Y_t\left(1 - \frac{\Psi}{2}(\pi_t - \pi)^2\right) - \chi V_t \tag{13}$$

Given all unconstrained households make identical decisions and there is no government debt, bonds will be in zero net supply and the market for shares must clear:

$$B^U_t = 0$$

$$S^U_t = \frac{1}{1-\lambda}$$

2.6 Fiscal Policy

We assume that lump-sum taxes are levied on each type of household to finance their unemployment benefits implying the following consumption levels of each type of household:

$$C^U_t = w_t N_t + \frac{1}{1-\lambda} D_t \tag{14}$$

$$C^C_t = w_t N_t \tag{15}$$

These expressions illustrate that consumption differs across the two types of households due to the financial income unconstrained households obtain from holding shares in equilibrium.[12]

2.7 Monetary Policy

Finally, to close the model we need to specify a monetary policy rule. For simplicity, we assume the central bank follows a standard Taylor-type rule:

$$R_t = R_{ss} \pi_t^{\phi_\pi} \zeta_t \tag{16}$$

where ζ_t is a monetary policy shock, the log of which follows an AR(1) process:

$$\ln \zeta_t = \rho_\zeta \ln \zeta_{t-1} + \varepsilon^\zeta_t \tag{17}$$

[12]The importance of how firm profits are distributed across households in models of limited asset market participation models was first highlighted in Bilbiie (2008).

2.8 Equilibrium

The equilibrium of the model is described by the equations (1) - (17), in addition to specifying functional forms of the household's utility function and the aggregate matching function. The full list of equilibrium conditions used to solve the model under the functional form assumptions outined in Section 4 are provided in Appendix B.

3 Heterogeneous Discounting and the Labour Market

In a standard TANK model, heterogeneity in discounting is irrelevant by construction. As constrained households do not price assets, their stochastic discount factor plays no role in the model. In contrast, the key insight of the model presented in this paper is that when there are search frictions in the labour market heterogeneity in discounting *is* relevant. This is because when workers from different households discount future values differently, this induces heterogeneity in the present discounted values of employment across households in response to aggregate shocks. This has consequences for the labour market when wages in are determined with respect to the overall match surplus.

3.1 Role of Heterogeneous Discount Factors

What are the implications of heterogeneous discounting for the economy? We first address this question analytically by inspecting the role of heterogeneous discounting in the model outlined above. Recall from the previous section that discount factor heterogeneity was shown to affect the labour market via the Nash-bargained wage, more specifically the firm's reservation wage:

$$\bar{w}_t = \underbrace{\varepsilon_t A_t}_{\text{Prod. effect}} + \underbrace{\mathbb{E}_t\left[\beta^U_{t,t+1}\chi\theta_{t+1}\right]}_{\text{Cost effect}} + \underbrace{\lambda\mathbb{E}_t\left[\frac{\chi}{q(\theta_{t+1})}(1-\rho-p(\theta_{t+1}))(\beta^U_{t,t+1}-\beta^C_{t,t+1})\right]}_{\text{SDF hetero. effect}}$$

The firm's reservation wage has three components. The first two components are standard. There is an *additional* component of the firm's reservation wage that arises purely from household heterogeneity capturing dispersion in discount rates:

1. **Productivity effect:** The first term (in blue) reflects the fact that higher labour productivity increases the value of a worker to the firm. This is a standard component of the equilibrium wage in matching models where wages are determined by Nash bargaining (see Pissarides (2000)).

2. **(Hiring) Cost effect:** The second term (in red) captures the fact that higher (expected) future average hiring costs provide incentives for firms to hire today, so workers are rewarded in this cost saving for firms in the bargaining process via higher wages today. Again this term is completely standard.

3. **SDF heterogeneity effect:** The final term illustrates the effect of discount factor heterogeneity in this model. This additional term (in purple) appears because the two actors on either side of the wage bargaining discount the future value of the match differently. Firms are owned by subset of households (the unconstrained) who hold wealth (firm shares) in equilibrium, whereas workers come from both types of households in the economy. This means that the income of the average

firm owner is different to that of the average worker in the economy. Financial and labour income will in general respond differently to aggregate shocks, which induces *dispersion* in the discount factors across the different types of household in the economy and implies heterogeneity in the worker's surplus from employment across households.

In a representative household world ($\lambda = 0$) all households have identical income sources and there is no heterogeneity in discounting, so this term disappears as the two actors on each side of the wage bargain share a common stochastic discount factor.

Will the presence of discount factor heterogeneity amplifiy or dampen the response of the economy to aggregate shocks relative to a version of the model without household heterogeneity? This ultimately depends on how the dispersion in discount factors itself responds to aggregate shocks, or more specifically on the direction of the inequality:

$$\beta^U_{t,t+1} - \beta^C_{t,t+1} \lesseqgtr 0$$

Consider a shock that generates $\beta^U_{t,t+1} - \beta^C_{t,t+1} > 0$. This implies that firms (unconstrained households) place a relatively higher value on the total lifetime payoffs from a worker-firm match than workers from constrained households, who place relatively higher value on the immediate payoff from being in a match. This actually puts *upward* pressure on the reservation wage of the firm, as firms place a higher value on the total lifetime payoffs from the match than the average worker. This in turn puts upward pressure on the equilibrium wage, reducing hiring incentives for firms.

The quantitative importance of this channel on the firm's reservation wage depends on several other factors:

- The actual *size* of the discount factor dispersion induced by a shock, $\beta^U_{t,t+1} - \beta^C_{t,t+1} > 0$, which we discuss below.
- The response of labour market tightness θ_t, which governs how quickly workers and firms expect to find new matches.
- Two key parameters: the fraction of constrained households λ, and the degree of worker bargaining power η. A higher fraction of constrained households implies that the distance betwee the average worker's discount factor and that of firms will be relatively larger. The degree of worker bargaining power η is an important determinant of the significance of this channel, given that the channel operates exclusively through the firm's reservation wage.

3.2 Importance of Heterogeneous Discounting

What drives the size of dispersion in discount factors in the model? Fluctuations in $\beta^C_{t,t+1}$ are entirely determined by fluctuations in labour income, whereas $\beta^U_{t,t+1}$ captures changes to both labour and profit income in response to shocks. Hence discount factor dispersion will be predominantly driven by the response of profits *relative* to labour income.[13] If we assume a simple CRRA form for the utility function

[13] This reflects another dimension of the importance of firm profits in New Keynesian models with limited asset market participation, a point originally raised in Bilbiie (2008) relating to how firm profits are distributed. Here we take the simple case of no redistribution of firm profits.

such that $u'(c) = c^{-\sigma}$, it can be shown that to a first-order approximation discount factor dispersion can be expressed as:

$$\beta^U_{t,t+1} - \beta^C_{t,t+1} \approx \frac{\beta\sigma}{1+(1-\lambda)\gamma WN}\left[\mathbb{E}_t \Delta w_{t+1} N_{t+1} - \gamma \mathbb{E}_t \Delta D_{t+1}\right] \tag{18}$$

where $\gamma \equiv \frac{wN}{D}$ is the steady state ratio of labour income to firm profits.[14]

This equation confirms that discount factor dispersion is driven by two forces: (i) changes in labour income, and (ii) changes in firm profits. Which force dominates depends on the relative volatility of labour income and profits, as well as the size of labour income relative to profits in steady state (γ). The size of discount factor dispersion induced from movements in different income sources also depends crucially on λ, which controls the share of profits unconstrained households hold (and consume) in equilibrium. When λ is higher, firm profits make up a larger share of the income of unconstrained households, and so fluctuations in profits matter more for dispersion in discount factors across households. Finally, parameters determining the characteristics of household preferences (here captured by σ) are also important in determining the size of dispersion. When preferences are CRRA, a higher degree of risk aversion (lower degree of IES) implies a larger degree of dispersion in discount factors conditional on the responses of labour/profit income to a shock.

4 Quantitative Analysis

The analysis in the previous section establishes that discount factor heterogeneity has implications for the economy when the labour market is characterised by search frictions. In particular we showed that heterogeneity in discounting introduces an additional term into the expresion for the equilibrium wage which captures the fact that different actors in the bargaining process discount the future at different rates. This naturally leads us to the question of how important this channel is quantitatively for the behaviour of the model.

To address this issue we parameterize and solve the model and then study the response of the model to a monetary policy (demand) shock and a technology (supply) shock. We then investigate the quantitative importance of this channel by comparing the responses of the baseline model to a version of the model where household heterogeneity is shut down (i.e. $\lambda = 0$). The model is solved using standard perturbation techniques around a steady state with zero inflation.[15] A full description of the non-linear equilibrium conditions used to solve the model is given in Appendix B.

4.1 Functional forms

Preferences: We use a more general non-recursive household utility function popularised by Epstein and Zin (1989), in order to allow for a richer parameterization of household stochastic discount factors. The expected discounted lifetime utility of household i is given by J^i_t, which is defined as:

$$J^i_t = \left[(1-\beta){C^i_t}^{1-\frac{1}{\varphi}} + \beta\left(\mathbb{E}_t\left[{J^i_{t+1}}^{1-\sigma}\right]\right)^{\frac{1-\frac{1}{\varphi}}{1-\sigma}}\right]^{\frac{1}{1-\frac{1}{\varphi}}}$$

[14] See Appendix A.2 for a derivation.

[15] The model is initially solved using first-order perturbation, but later solved using higher order perturbation to allow the degree of risk aversion to influence the stochastic solution to the model when we deviate from using a power utility specification.

where $\sigma > 0$ measures the degree of risk aversion and $\varphi > 0$ captures the intertemporal elasticity of substitution (IES). Imposing the restriction $\sigma = \frac{1}{\varphi}$ gives the standard power utility (CRRA) specification. The corresponding expression for the stochastic discount factor of household i is given by:

$$\beta^i_{t,t+1} = \beta \left(\frac{C^i_{t+1}}{C^i_t} \right)^{-\frac{1}{\varphi}} \left(\frac{J^i_{t+1}}{\mathbb{E}_t \left[{J^i_{t+1}}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}} \right)^{\frac{1}{\varphi}-\sigma}$$

Matching: For the aggregate matching function, we assume a standard Cobb-Douglas form for simplicity:

$$M(V_t, U_t) = \bar{m} {V_t}^{1-\xi} U_t^{\xi}$$

where ξ is the elasticity of matches with respect to the stock of unemployed agents in the economy, and $\bar{m}$ is a match efficiency constant.

4.2 Parameterization

The baseline model is parameterized at a quarterly frequency and is intentionally kept very standard. For several values we calibrate them externally by setting them equal to standard values used in the literature on business cycles and search & matching models. Additionally we determine some parameters relating to the labour market by imposing external restrictions on steady state values such that the model matches key features of the US labour market. The parameterization is summarised in Table 1.

Externally calibrated parameters: We calibrate several model parameters using standard parameters in the business cycle literature. The household discount factor β is set to 0.9926 to target a 3% annual average real interest rate. The intertemporal elasticity of subsitution (IES) φ is set to 1.5, and we set $\sigma = 1/\varphi$ such that preferences are CRRA under our baseline calibration. The elasticity of substitution across varieties is set to $\nu = 6$, so that the steady state markup is 20%. The price adjustment costs parameter Ψ is chosen to match an average price duration of 1 year. We choose a value of the match elasticity $\xi = 0.5$ which is standard in the context of the survey by Petrongolo and Pissarides (2001), and then set $\eta = \xi$ such that the Hosios' condition is satisfied. We choose a Taylor rule coefficient that is standard, and normalise the steady state level of productivity A. Finally a key parameter in TANK models is the fraction of hand-to-mouth households in the economy, λ. We set this parameter based on Debortoli and Galí (2018) who target the fraction of constrained households in the steady state for a calibrated HANK model, which they find to be 21%.

External steady state restrictions: Several parameters relating to the labour market are set to match standard quarterly targets for the US. Specifically we set the values of $\{\rho, \chi, \bar{m}, b\}$ to match: (i) a 5.5% average unemployment rate, (ii) an average job filling rate equal to 0.7 as in Den Haan et al. (2000), (iii) an average quarterly job finding rate equal to 0.45 as in Shimer (2012), and (iv) a 40% replacement ratio as in Shimer (2005). Note that under this calibration of the replcement rate the standard Shimer (2005) critique will apply.

Exogenous processes: Finally, we assume that the persistence of the technology process $\rho_a = 0.9$, and the persistence of the monetary policy shock $\rho_\xi = 0.5$, which are standard values in the New Keynesian literature.

Table 1: Parameter Values

Parameter	Description	Value	Source
Externally calibrated parameters:			
β	Discount factor	0.9926	3% avg.real interest rate
φ	IES	1.5	Standard
σ	Risk aversion	$1/\varphi$	CRRA utility
ν	Substitution elasticity	6	Standard
ξ	Matching function elasticity	0.5	Petrongolo and Pissarides (2001)
η	Workers' bargaining power	0.5	Hosios' condition
Ψ	Quadratic price adjustment parameter	58.6969	avg. price duration of 1 year (Calvo)
ϕ_π	Taylor rule coefficient on inflation	2	Standard
A	Steady state productivity	1	Normalization
λ	Share of constrained households	0.21	Debortoli and Galí (2018)
External steady state restrictions:			
ρ	Separation rate	0.0476	$u = 0.055$
χ	Vacancy posting costs	0.6553	$q(\theta) = 0.7$
$\bar{m}$	Match efficiency constant	0.5612	$p(\theta) = 0.45$
b	Unemployment benefits	0.3129	40% replacement rate
Exogenous processes:			
ρ_a	Persistence of tech. shock	0.9	Standard
ρ_ξ	Persistence of mon. pol. shock	0.5	Standard

4.3 Monetary policy shock

We first study the response of the model to a 1% contractionary shock to the monetary policy rate in the model (the return on the risk-free bond). The responses of key variables in the model are presented in Figure 1. The contraction in the policy rate is associated with a fall in output and inflation as standard. This is driven by the fact that falling aggregate demand puts downward pressure on labour demand by firms, which leads to falling wages and vacancies resulting in lower labour market tightness and higher unemployment. The reduction in wages and vacancies also puts downward pressure on marginal costs which drives the fall in inflation as standard. This also results in an *increase* in firm profits. This countercyclical movement in firm profits is a well-known phenomenon in New Keynesian models.

Crucially, profits and labour income move in different directions in response to the shock. This induces dispersion in the discount factors across households as discussed in the previous section. This is illustrated in the top right panel in Figure 1, where profits increase by around 4.5% in response to the shock whilst aggregate labour income (equivalent to C_t^C in the model) falls by about 1% due to combination of falling employment and wages. This induces an increase in the gap between the discount factors of unconstrained and constrained households. As discussed above, an increase in discount factor dispersion $\beta_{t,t+1}^U - \beta_{t,t+1}^C > 0$ will amplify the effect of a contractionary shock due to its effect on the equilibrium wage, which firms take into account when they post vacancies. Hence the presence of heterogeneous discouting implies that contractionary monetary policy in this model has a greater impact on the labour market and therefore aggregate output.

To see the quantitative significance of this channel in the model in Figure 2 we plot the responses of key variables under the baseline calibration of $\lambda = 0.21$, and also the version of the model with no household heterogeneity ($\lambda = 0$) in response to the same shock. Overall we see that discount factor heterogeneity amplifies the effect of the shock quite substantially. Given the direction of discount factor dispersion induced by the shock, the additional component of the firm's reservation wage acts to further reduce firm's incentives to post jobs in response to the shock relative to a model without heterogeneity. The greater fall in vacancies induces an additional 0.2% increase in the unemployment response (around a 40% increase in amplitude), and leads to output falling by around an additional 0.01% (a 30% increase in amplitude). Note that because the discount factor heterogeneity channel is inherently forward-looking, its effect is entirely concentrated in the impact of the shock. Heterogeneity in discounting does not affect the persistence of the economy's response to the shock.

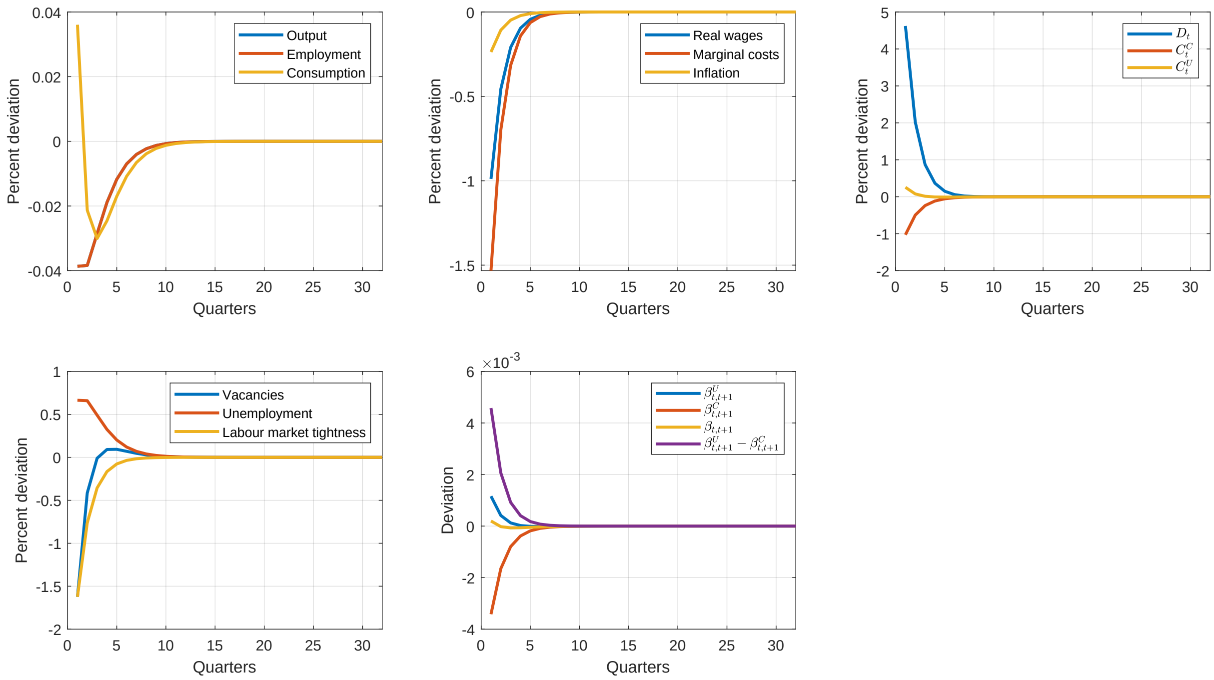

Figure 1: Impulse responses to a 1% monetary policy shock

4.4 Technology shock

Secondly we study the response of the model to a 1% positive (labour) productivity shock. This is the standard supply shock studied in the literature assessing the quantitative properties of matching models of the labour market. The responses of key variables in the model are presented in Figure 3. The increase in productivity is associated with a greater than one-for-one increase in output. However almost all of this is driven by the direct productivity effect on output (existing workers becoming more productive) rather than because firms hire substantially more workers from the unemployment pool such that aggregate employment grows. This is because the model suffers from the standard Shimer (2005) critique - under the assumptions of Nash bargained wages and large surplus calibration for the replacement rate, labour productivity shocks do not lead to much volatility in the labour market. Job creation

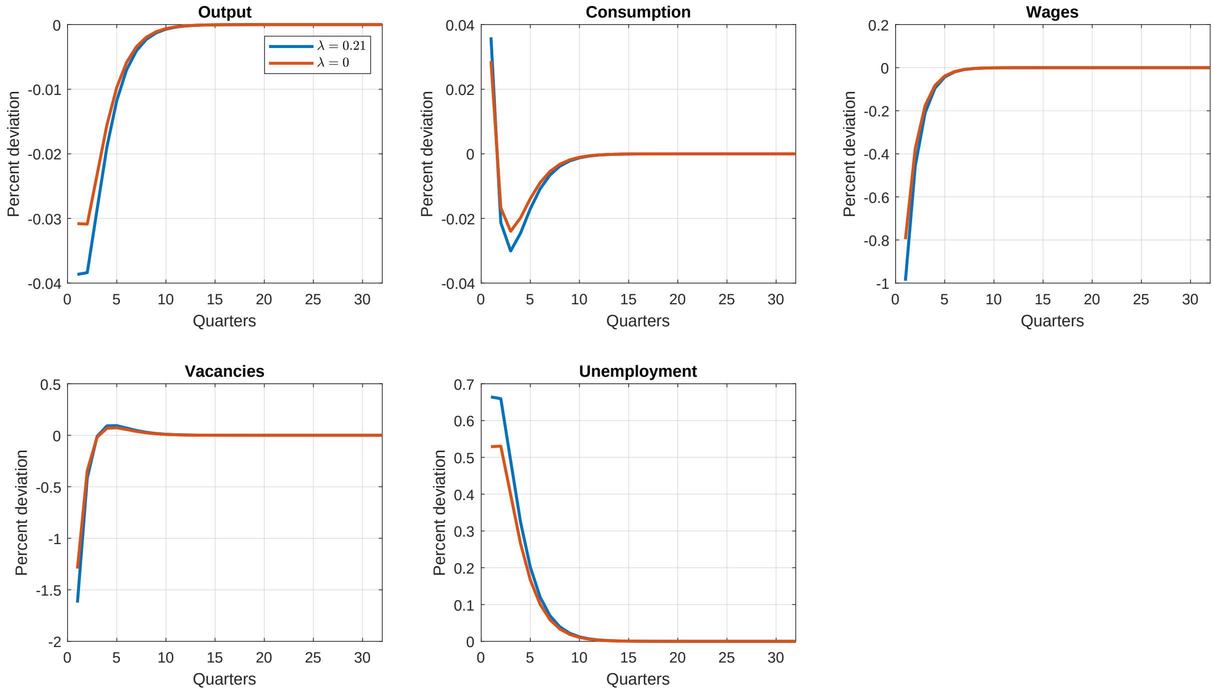

Figure 2: Monetary policy shock: Shutting down heterogeneity

still responds positively to the shock because the value of a worker to a firm increases, leading to falling unemployment and lower labour market tightness as standard. However the rise in real wages acts to reduce firms' incentives to hire more workers. Rising employment and wages implies rising labour costs for firms, however profits still respond positively overall because the increase in output (driven by higher productivity) dominates the rising labour costs (reflected in the muted response of marginal costs).

In resposne to a positive productivity shock, profits and labour income now move in the same direction (in contrast to a monetary policy shock). As the consumption level of constrained households increases by more (relative to steady state), the model predicts that the gap between the discount factors of constrained and unconstrained will move in the direction of constrained households. This will act in the opposite direction by stimulating job creation relative to a representative agent version of the model, once more amplifying the effect of the shock. Combined with the fact that profits are not very sensitive to supply shocks in matching models subject to the Shimer (2005) critique, we find that movements in discount factor dispersion are an order of magnitude smaller compared to the case of a monetary policy shock.

Given this significantly muted response of the dispersion in discount factors, this channel is quantiatively irrelevant in the case of a labour productivity shock. This is illustrated in Figure 4 where again we plot the responses of key variables under the baseline calibration of $\lambda = 0.21$, and also the version of the model with no household heterogeneity ($\lambda = 0$) in response to the same shock. Job creation responds only marginally more in the baseline model, and not enough to make a quantitative difference. Overall we see that the responses of the two models to a labour productivity shock are essentially identical.

In summary, we have shown that heterogeneity in the discounting of the value of match tends to amplifiy the response of the economy to standard demand and supply shocks. However, the quantitative significance of this channel is shown to depend on how firm profits respond to the aggregate shock

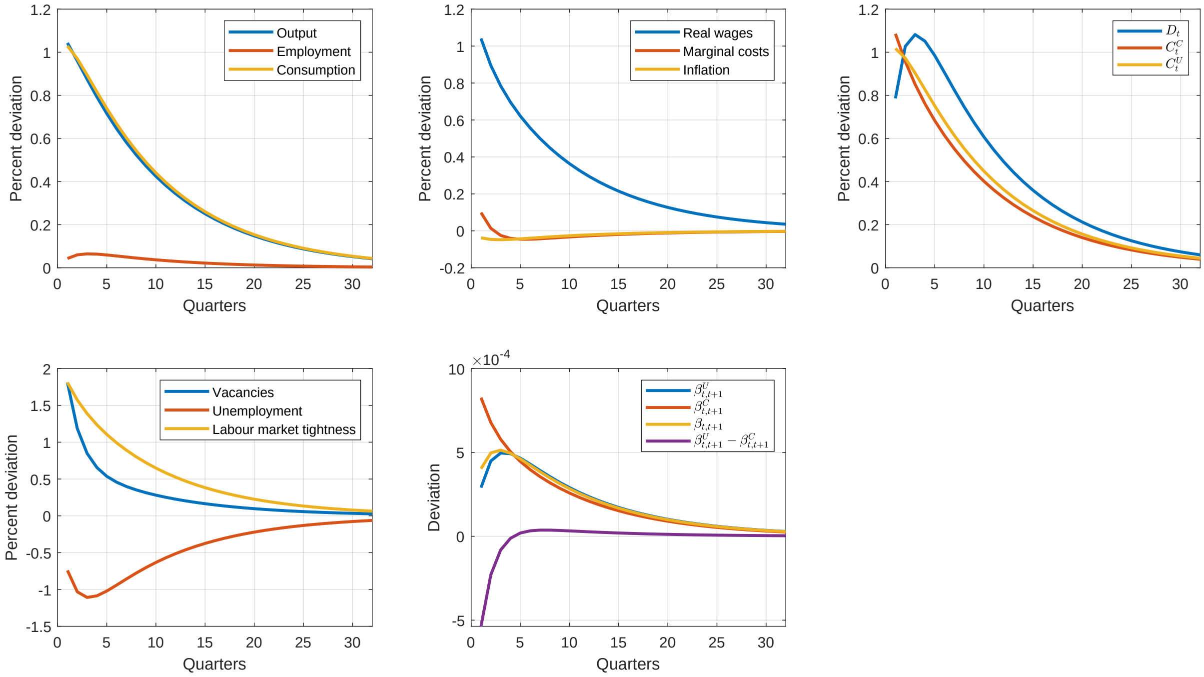

Figure 3: Impulse responses to a 1% technology shock

relative to aggregate labour income, as well as their relative volatilities. In the case of the monetary policy shock, labour income and profits respond in opposite directions and firm profits are roughly 4 times more volatile, so we find that heterogeneous discounting induces a fairly large amplification. On the other hand, in response to the productivity shock not only does labour income and profit income move in the same direction, they have roughly equal volatilities in response to the shock, partially a result of the usual unemployment volatility puzzle in matching models. In this case the amplification coming from heterogeneity in the value of the match to different households is quantitatively irrelevant.

5 Robustness

In Section 3 we illustrated that the strength of the discount heterogeneity channel on the labour market in response to shocks depends on several key parameters. We have already illustrated that the fraction of constrained households λ itself plays an important role in determining both the size of dispersion in discounts factors, and the importance of this channel for the equilibrium wage. Several other parameters were also shown to be potentially important for determining the strength of this channel, more specifically the parameters governing household preferences and the degree of worker bargaining power. In this section we investigate how re-calibrating these parameters changes the strength of the amplification from heterogeneous discounting.

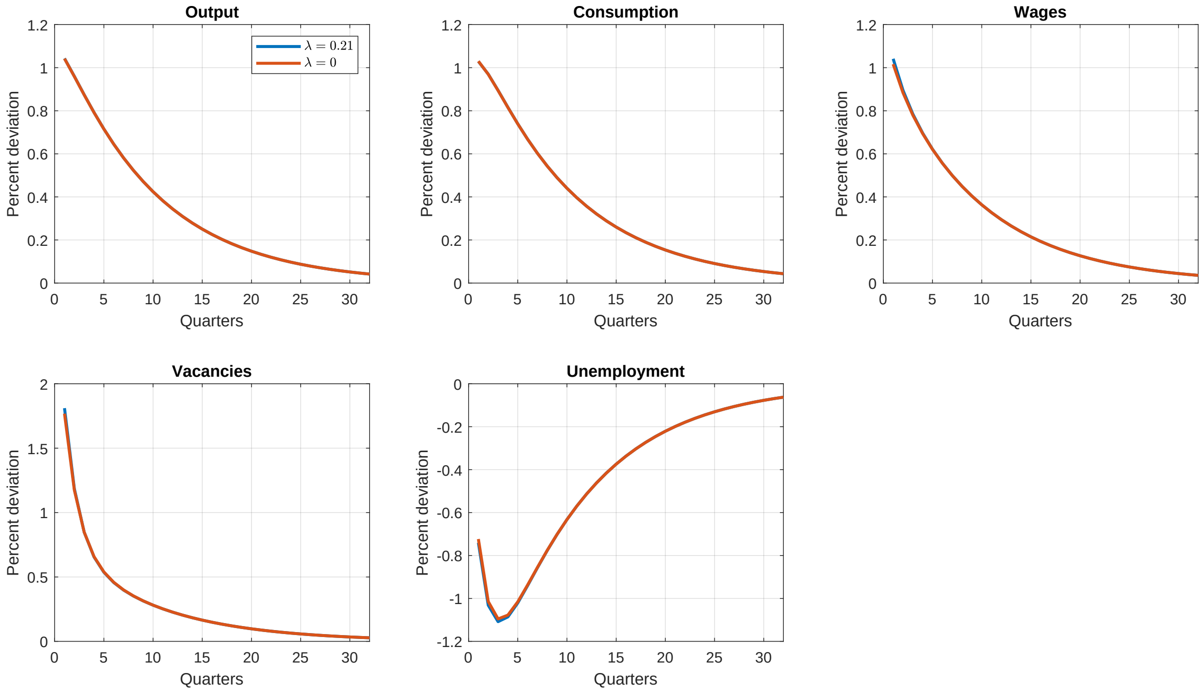

Figure 4: Technology shock: Shutting down heterogeneity

5.1 Household Preferences

In the first instance, in Section 3 we outlined how the utility function parameters which characterise the stochastic discount factor will play an important role in determining the size of discount factor dispersion. Under our baseline parameterization with Epstein-Zin preferences there are two parameters determining households preferences: the IES parameter φ and degree of risk aversion σ. We set the former to a standard value and determine the latter by imposing the CRRA restriction that $\sigma = 1/\varphi$.

To explore how sensitive the model's responses are to our chosen parameterization of household preferences, we recompute the responses of the model to a contractionary policy shock under two alternative calibrations of these parameters. Firstly, we maintain the CRRA restriction but increase the value of φ from 1.5 to 3. Under the CRRA restriction this both simultaneously reduces household risk aversion and increases the appetite of households to engage in intertemporal substitution. Secondly, we move away from the CRRA restriction that imposes a tight link between these two different concepts by fixing $\varphi = 1.5$ but increasing the degree of risk aversion σ to 5. This will allow us to understand whether risk aversion or intertemporal substitution appear to be more important for determining discount factor volatility in the model.

Responses of key model variables under the two alternative calibrations (as well as the baseline calibration) are plotted in Figure 5. Relative to the baseline calibration (blue line), we find that the first alternative calibration (red line) significantly reduces the degree to which discount factor dispersion amplifies the effect of the contractionary shock. In other words, simultaneously increasing the degree of substitution elasticity whilst reducing risk aversion dampens the importance of this channel, as illustrated in Section 3. Quantitatively the effect on the amplification on impact is quite strong - with higher IES the response of the model now looks quantitatively similar to the baseline parameterization when we shut down heterogeneity ($\lambda = 0$) in Figure 2.

The second alternative calibration allows us to understand whether changes in IES or risk aversion are more important in determining the size of discount factor dispersion in the model. When we fix the IES parameter φ but increase the degree of risk aversion σ to a higher but plausible value (yellow line), we also find that the amplification effect from heterogeneous discounting is dampened (though to a much lesser degree than when we increase household's willingness to do intertemporal substitution). In other words, increasing household risk aversion without also changing attitudes towards intertemporal substitution reducing the volatility of discount factor dispersion in the model. This suggests that household attitudes towards intertemporal substitution are more influential in determining volatility in stochastic discount factors (and therefore discount factor dispersion) than attitudes towards risk.

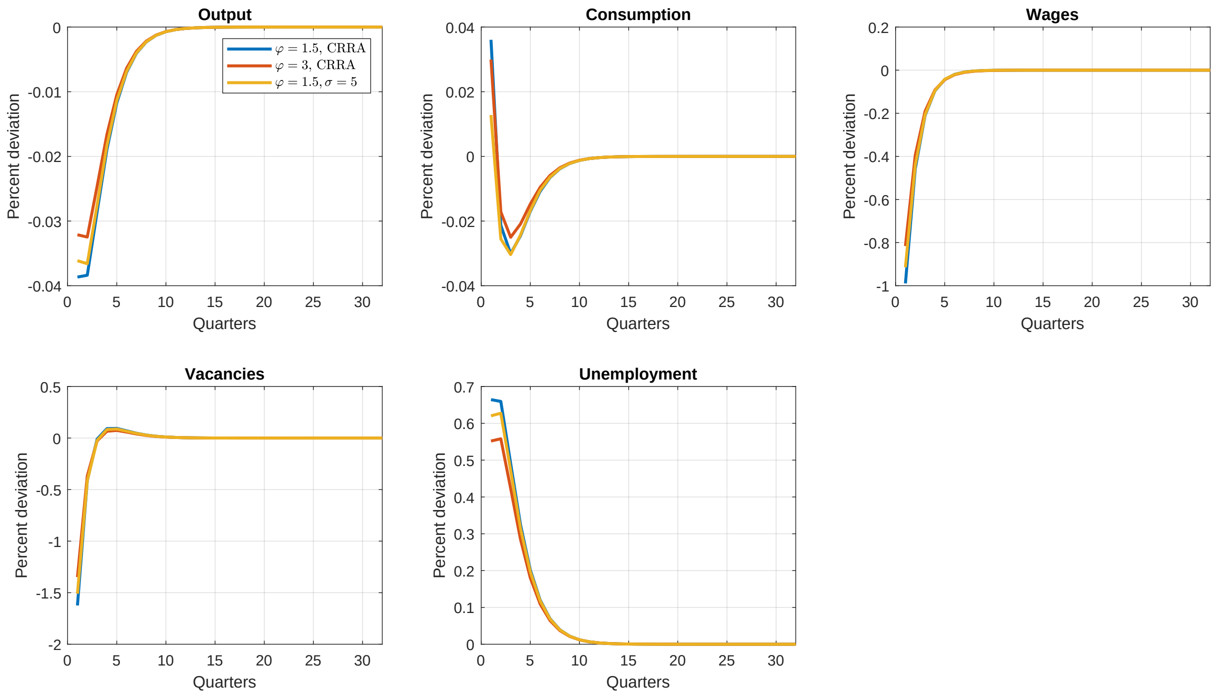

Figure 5: Exploring the implications of different household preferences, $\{\sigma, \varphi\}$

5.2 Bargaining Power

As discount factor heterogeneity operates through the firm's reservation wage in the baseline model, the degree of bargaining power η determines the importance of discount factor dispersion for the Nash bargain, and therefore its importance for the transmission of aggregate shocks. We recompute the model responses for an alternative calibration of the bargaining power parameter η, specifically setting $\eta = 0.4$. The results are plotted in Figure 6. Unsurprisingly we see that a lower degree of worker bargaining power reduces the amplification that comes from the discount factor dispersion channel. Again quantitatively this is similar to maintaining the baseline calibration, but shutting down this channel altogether by eliminating household heterogeneity.

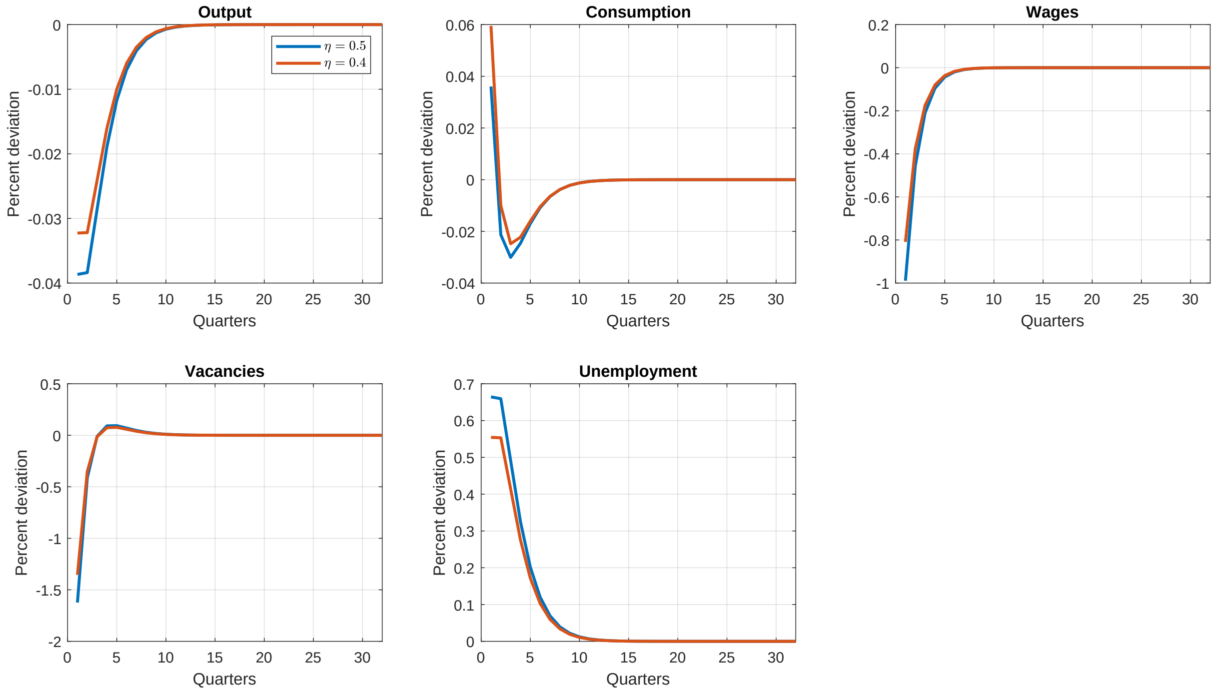

Figure 6: Exploring the implications of different degrees of worker bargaining power, η

6 Conclusion

This paper studies an environment where households are heterogeneous in their ability to smooth consumption, but where all workers face search frictions in the labour market. We show that in such an environment, household heterogeneity induces heterogeneity in the value of employment across workers in the labour market, which has implications for how the economy responds to macroeconomic shocks under the assumption that wages are determined with respect to joint surplus of a match. We preserve tractability by following the TANK literature in assuming that a fraction of households are unable to smooth consumption and instead consume hand-to-mouth, whereas the remainder of households behave as normal permanent income consumers. In equilibrium the two types of household differ in their sources of income: constrained households rely exclusively on labour income, whilst unconstrained households are able to smooth consumption and receive a share of firm profits in equilibrium. To the extent that these two sources of income respond to shocks differently, this induces heterogeneity in the stochastic discount factors of households such that workers from the two different types of household discount the lifetime value of an employment relationship at different rates. We show that under Nash bargaining this introduces an additional term equilibrium wage which reflects the fact that workers from different households value the continuation of the match differently, and that analytically the influence of this additional channel on the wage depends critically on the response of profits relative to labour income. Under a standard calibration of the model, we show that this channel amplifies the response of the economy to standard shocks even in the absence of the income risk channel studied in Ravn and Sterk (2021), Broer, Druedahl, et al. (2021) and Gornemann et al. (2021).

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A Model Derivations

A.1 Deriving the Nash wage

Recall that under Nash bargaining, the Nash wage satisfies the Nash sharing rule:

$$\mathbb{S}_t^w = \eta(\mathbb{S}_t^w + \mathbb{S}_t^f)$$

which states that workers receive a fraction η of the total match surplus i.e. the sum of the worker and firm surpluses. The worker and firm surpluses are defined as:

$$\mathbb{S}_t^w = w_t - b + \mathbb{E}_t\left[\beta_{t,t+1}(1-\rho-p(\theta_{t+1}))\mathbb{S}_{t+1}^w\right]$$

$$\mathbb{S}_t^f = \varepsilon_t A_t - w_t + (1-\rho)\mathbb{E}_t[\beta_{t,t+1}^U \mathbb{S}_{t+1}^f]$$

Substituting these expressions into the Nash sharing rule and rearranging, we can express the wage as:

$$w_t = (1-\eta)b + \eta\varepsilon_t A_t + \eta(1-\rho)\mathbb{E}_t\beta_{t,t+1}^U\mathbb{S}_{t+1}^f - (1-\eta)\mathbb{E}_t\beta_{t,t+1}(1-\rho-p(\theta_{t+1}))\mathbb{S}_{t+1}^w$$

Using the fact that $\mathbb{S}_{t+1}^w = \frac{\eta}{1-\eta}\mathbb{S}_{t+1}^f$, this expression becomes:

$$w_t = (1-\eta)b + \eta\left[\varepsilon_t A_t + (1-\rho)\mathbb{E}_t\mathbb{S}_{t+1}^f(\beta_{t,t+1}^U - \beta_{t,t+1}) + \mathbb{E}_t\beta_{t,t+1}p(\theta_{t+1})\mathbb{S}_{t+1}^f\right]$$

Using the free entry condition $\frac{\chi}{q(\theta_t)} = \mathbb{S}_t^f$ yields:

$$w_t = (1-\eta)b + \eta\left[\varepsilon_t A_t + (1-\rho)\mathbb{E}_t\frac{\chi}{q(\theta_{t+1})}(\beta_{t,t+1}^U - \beta_{t,t+1}) + \mathbb{E}_t\beta_{t,t+1}\chi\theta_{t+1}\right]$$

Finally, using $\beta_{t,t+1}^U - \beta_{t,t+1} = \lambda(\beta_{t,t+1}^U - \beta_{t,t+1}^C)$ and rearranging gives:

$$w_t = (1-\eta)b + \eta\left[\varepsilon_t A_t + \mathbb{E}_t\left[\beta_{t,t+1}^U\chi\theta_{t+1}\right] + \lambda\mathbb{E}_t\left[\frac{\chi}{q(\theta_{t+1})}(1-\rho-p(\theta_{t+1}))(\beta_{t,t+1}^U - \beta_{t,t+1}^C)\right]\right]$$

which can then be interpreted as a linear combination of the firm's reservation wage and the worker's reservation wage as standard.

A.2 Discount factor fluctuations

Assuming a CRRA form for the utility function, the stochastic discount factor of some household i can be expressed as:

$$\beta_{t,t+1}^i = \beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}$$

Taking a first-order Taylor expansion around the steady state gives:

$$\beta_{t,t+1}^i \approx \beta - \frac{\sigma\beta}{C^i}(C_{t+1}^i - C_t^i)$$

Hence we can approximate the dynamics of discount factor dispersion as:

$$\beta^U_{t,t+1} - \beta^C_{t,t+1} \approx -\frac{\sigma\beta}{C^U}(C^U_{t+1} - C^U_t) + \frac{\sigma\beta}{C^C}(C^C_{t+1} - C^C_t)$$

$$= \frac{\sigma\beta}{C^U C^C}\left[(C^U - C^C)\mathbb{E}_t \Delta W_{t+1} N_{t+1} - \frac{C^C}{1-\lambda}\mathbb{E}_t D_{t+1}\right]$$

where the second equality follows from substituting in the (linear) equilibrium expressions for each type of household's consumption. Substituting out for the steady state consumption levels and rearranging gives the expression in the text:

$$= \frac{\sigma\beta}{1 + \frac{(1-\lambda)(WN)^2}{D}}\left[\mathbb{E}_t \Delta W_{t+1} N_{t+1} - \frac{WN}{D}\mathbb{E}_t \Delta D_{t+1}\right]$$

$$= \frac{\sigma\beta}{1 + (1-\lambda)WN\gamma}\left[\mathbb{E}_t \Delta W_{t+1} N_{t+1} - \gamma\mathbb{E}_t \Delta D_{t+1}\right]$$

where the final equality comes from defining $\gamma \equiv \frac{WN}{D}$ as the ratio of steady state aggregate labour income to firm profits.

B List of Model Equilibrium Conditions

Table 2: Model Equations

$J_t^i = \left[(1-\beta){C_t^i}^{1-\frac{1}{\varphi}} + \beta\left(\mathbb{E}_t\left[{J_{t+1}^i}^{1-\sigma}\right]\right)^{\frac{1-\frac{1}{\varphi}}{1-\sigma}}\right]^{\frac{1}{1-\frac{1}{\varphi}}}$	(Household values)
$\beta_{t,t+1}^i = \beta\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\frac{1}{\varphi}}\left(\frac{J_{t+1}^i}{\mathbb{E}_t\left[{J_{t+1}^i}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}\right)^{\frac{1}{\varphi}-\sigma}$	(Stochastic discount factors)
$\frac{1}{R_t} = \mathbb{E}_t\left[\beta_{t,t+1}^U \frac{1}{\pi_{t+1}}\right]$	(Euler equation for bonds)
$Q_t = \mathbb{E}_t\left[\beta_{t,t+1}^U (Q_{t+1} + D_{t+1})\right]$	(Euler equation for firm shares)
$N_t = (1-\rho)N_{t-1} + p(\theta_t)U_t$	(Employment evolution)
$U_t = 1-(1-\rho)N_{t-1}$	(Definition of job seekers)
$u_t = 1 - N_t$	(Unemployment rate)
$\theta_t = \frac{V_t}{U_t}$	(Market tightness)
$q(\theta_t) = \bar{m}\theta_t^{-\xi}$	(Job filling rate)
$p(\theta_t) = \theta_t q(\theta_t)$	(Job finding rate)
$Y_t = A_t N_t$	(Aggregate production function)
$1 - \Psi(\pi_t - \pi)\pi_t + \mathbb{E}_t[\beta_{t,t+1}^U \Psi(\pi_{t+1} - \pi)\pi_{t+1}\frac{Y_{t+1}}{Y_t}] = \nu(1-\varepsilon_t)$	(Optimal pricing condition)
$\frac{\chi}{q(\theta_t)} = \varepsilon_t A_t - w_t + (1-\rho)\mathbb{E}_t\left[\beta_{t,t+1}^U \frac{\chi}{q(\theta_{t+1})}\right]$	(Job creation)
$w_t = (1-\eta)b + \eta\left[\varepsilon_t A_t + \mathbb{E}_t\left[\beta_{t,t+1}^U \chi\theta_{t+1}\right] + \lambda\mathbb{E}_t\left[\frac{\chi}{q(\theta_{t+1})}(1-\rho-p(\theta_{t+1}))(\beta_{t,t+1}^U - \beta_{t,t+1}^C)\right]\right]$	(Nash wage)
$C_t^U = w_t N_t + \frac{1}{1-\lambda}D_t$	(Consumption of unconstrained HHs)
$C_t^C = w_t N_t$	(Consumption of constrained HHs)
$D_t = y_t\left(1 - \frac{\psi}{2}(\pi_t - \pi)^2\right) - w_t N_t - \chi V_t$	(Firm profits)
$C_t = Y_t\left(1 - \frac{\psi}{2}(\pi_t - \pi)^2\right) - \chi V_t$	(Market clearing)
$R_t = R_{ss}\pi_t^{\phi_\pi}\zeta_t$	(Taylor rule)
$a_t = \rho_a a_{t-1} + \varepsilon_t^a$	(Aggregate productivity process)
$\ln\zeta_t = \rho_\zeta \ln\zeta_{t-1} + \varepsilon_t^\zeta$	(Monetary policy shock process)

Table 3: Steady State Equations

$J^i = \left[{C^i}^{1-\frac{1}{\varphi}} + \beta \left(\left[{J^i}^{1-\sigma} \right] \right)^{\frac{1-\frac{1}{\varphi}}{1-\sigma}} \right]^{\frac{1}{1-\frac{1}{\varphi}}}, \quad i = \{C, U\}$	(J^U, J^C)
$\frac{1}{R} = \beta \frac{1}{\pi}$	(R)
$N = (1-\rho)N + p(\theta)U$	(ρ)
$U = 1 - (1-\rho)N$	(U)
$u = 1 - N$	(N)
$\theta = \frac{V}{U}$	(V)
$q(\theta) = \bar{m}\theta^{-\xi}$	$(\bar{m})$
$p(\theta) = \theta q(\theta)$	(θ)
$Y = AN$	(Y)
$\varepsilon = \frac{\nu-1}{\nu}$	(ε)
$\frac{\chi}{q(\theta)} = \varepsilon A - w + (1-\rho)\beta \frac{\chi}{q(\theta)}$	(χ)
$w = (1-\eta)b + \eta(\varepsilon A + \beta\chi\theta)$	(w)
$C^U = wN + \frac{1}{1-\lambda}D$	(C^U)
$C^C = wN$	(C^C)
$D = Y - wN - \chi V$	(D)
$C = Y - \chi V$	(C)
$b/w = 0.4$	(b)