TechNote 13.1

Relative Efficiency of the Treatment *x* Blocks Design

The expected total sum of squares for the treatment *x* blocks design is

$$\begin{aligned} E(SS_{total(TxB)} &= (a-1)\times E(MS_{A(TxB)}) + (b-1)\times E(MS_B) \\ &+ (a-1)(b-1)\times E(MS_{AB}) + ab(n-1)\times E(MS_{S/AB}) \\ &= (a-1)(\sigma^2_{e(CR)} + bn\theta^2_A) + (b-1)(\sigma^2_{e(CR)} + bn\theta^2_B) \\ &+ (a-1)(b-1)(\sigma^2_{e(CR)} + n\theta^2_{AB}) + ab(n-1)\sigma^2_{e(CR)} \end{aligned}$$

Assuming the same total number of scores (N) for the completely randomized design, there are *bn* scores in each of the *a* treatment conditions, and the expected total sum of squares is

$$\begin{aligned} E(MS_{total(CR)} &= (a-1)\times E(MS_{A(CR)}) + a(bn-1)\times E(MS_{S/A}) \\ &= (a-1)(\sigma^2_{e(CR)} + bn\theta^2_A) + a(bn-1)\sigma^2_{e(CR)} \end{aligned}$$

Because $E(MS_{total(CR)}) = E(MS_{total(TxB)})$, setting the right hand sides of the two equations equal to each other, cancelling $(a-1)bn\theta^2_A$, and simplifying, yields

$$\begin{aligned} (abn-1)\sigma^2_{e(CR)} &= (abn-ab+a-1)\sigma^2_{e(TxB)} + (b-1)(\sigma^2_{e(TxB)} + an\theta^2_B) \\ &+ (a-1)(b-1)(\sigma^2_{e(TxB)} + n\theta^2_{AB}) \\ &= (abn-ab+a-1)\sigma^2_{e(TxB)} + E(SS_B) + E(SS_{AB}) \end{aligned}$$

Dividing by *abn*-1,

$$\sigma^2_{e(CR)} = \left(1 - \frac{a(b-1)}{abn-1}\right)\sigma^2_{e(TxB)} + \frac{E(SS_B) + E(SS_{AB})}{abn-1}$$

Replacing these population parameters by sample statistics, we have:

$$est(MS_{S/A}) = \left[1 - \frac{a(b-1)}{abn-1}\right] MS_{S/AB} + \frac{SS_B + SS_{AB}}{abn-1}$$

Dividing the right hand side of this equation by $MS_{S/AB}$ yields the expression for relative efficiency in Equation 13.1.