TechNote 13.2

Relative Efficiency of the Latin Square Design

We assume a repeated measures design with *a* subjects, each tested on a random sequence of *a* conditions. We also have a Latin square with the same dimensions. Assume that the population error variance for the repeated measures design is $\sigma^2_{e(RM)}$ and that for the Latin square design is $\sigma^2_{e(LS)}$. Because the Latin square design merely involves a rearrangement of the treatments for each subject, the expected total sums of squares are the same for the two designs, as are the expected sums of squares for subjects. Therefore,

$$\begin{aligned}(a-1)^2\sigma^2_{e(RM)} + (a-1)\left(\sigma^2_{e(RM)} + a\theta^2_A\right) &= (a-1)(a-2)\sigma^2_{e(LS)} \\ &+ (a-1)\left(\sigma^2_{e(LS)} + a\theta^2_A\right) + (a-1)\left(\sigma^2_{e(LS)} + a\theta^2_C\right)\end{aligned}$$

Canceling θ^2_A from both sides and combining terms where possible, we have

$$a(a-1)\sigma^2_{e(RM)} = (a-1)^2\sigma^2_{e(LS)} + (a-1)\left(\sigma^2_{e(LS)} + a\theta^2_C\right)$$

Dividing both sides by *a*(*a* - 1), and converting population variances to mean squares, we have the result in Equation 13.4.