TechNote 15.2

Deriving the *min F'* Statistic

When MS_{SAB} can't be calculated, we can conceive of the quasi-F as

$$\min F' = \frac{MS_A}{MS_{SA} + MS_{AB}}$$

Multiplying numerator and denominator by $MS_A / (MS_{SA} \cdot MS_{AB})$, the numerator of *min F'* becomes

$$\left(\frac{MS_A}{MS_{SA}}\right) x \left(\frac{MS_A}{MS_{AB}}\right) = F_1 \; x \; F_2$$

The denominator is

$$MS_{SA}\left(\frac{MS_A}{MS_{SA} \cdot MS_{AB}}\right) + MS_{AB}\left(\frac{MS_A}{MS_{SA} \cdot MS_{AB}}\right) = F_2 + F_1$$

Therefore, *min F'* $= F_1 F_2 / (F_2 + F_1)$. The error degrees of freedom for *min F'* are

$$df_{error} = \frac{(MS_{SA} + MS_{AB})^2}{MS_{AB}^2 / df_{AB} + MS_{SA}^2 / df_{SA}}$$

Multiplying the numerator by $MS_A^2 / (MS_{SA} \cdot MS_{AB})^2$, we have

$$\left[MS_{SA}\left(\frac{MS_A}{MS_{SA} \cdot MS_{AB}}\right) + MS_{AB}\left(\frac{MS_A}{MS_{SA} \cdot MS_{AB}}\right)\right]^2 = (F_2 + F_1)^2$$

and also multiplying the denominator by the same quantity, we have

$$\left(\frac{MS_{AB}^2}{df_{AB}}\right)\left(\frac{MS_A^2}{MS_{SA}^2 \cdot MS_{AB}^2}\right) + \left(\frac{MS_{SA}^2}{df_{SA}}\right)\left(\frac{MS_A^2}{MS_{SA}^2 \cdot MS_{AB}^2}\right)$$

$$= F_1^2 / df_{AB} + F_2^2 / df_{SA}$$

Therefore, $df_{error} = (F_1 + F_2)^2 / (F_1^2 / df_{AB} + F_2^2 / df_{SA})$.