بسم الله الرحمن الرحیم

University of Tehran
College of Engineering

Digital Image Processing

Instructor: Dr. Hamid Soltanian-Zadeh

Homework Assignment 2:

Intensity Transformations and Spatial Filtering

Due date: 1403/12/17

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Contents

1 Instructions

Please answer the following questions based on Chapter 3 of the textbook by Gonzalez and Woods. Submit your solutions by the due date. Read the following carefully and follow these instructions when submitting your answers:

Requirement	Description	Consideration
Standard Due Date	1403/12/17 - 23:59	
Elearn HW Upload	**Only**	Only use ELearn to submit your homeworks.
Email Address	adanayidet@gmail.com	Feel free to say hello!
Submit format	Read Note 4	Extremely important
Late Submit	Penalty	$5 - 10\%$ penalty per day

Note 1: *This request is not intended to impose any hardship on you, but to help with better management of the class. Your cooperation is greatly appreciated in making the process more efficient. Thank you!*

Note 2: *We will also have a Telegram group for Q&A. The link (clickable) is provided here:* [open the link]

Note 3: *Use camera scanners if you wish to submit your handwritten answers. While readability is always important, it is especially critical in the DIP course, which focuses on quality enhancement.*

Note 4: *You must send a zip or rar file named* `DIP-HWx-std.no` *where x is the number of the homework (e.g.* `DIP-HW2-810199034`*). Inside the zipped file will be a folder for each question containing its code and output images. There should also be a single pdf file in the root folder as your main report.*

Academic Integrity: *Plagiarism, cheating, or using unauthorized external resources (including AI tools, solution manuals, or copying from peers) is strictly prohibited. All assignments will be checked for originality, and any violations will result in academic penalties. Please submit only your own work.* ***It's easy to detect whether your answers are generated by AI or are truly your own work, so be sure to submit original solutions.***

Homework Integrity Please note that Questions 2–7 pertain to a single problem. However, Q8 and Q9 are separate and correspond to two different problems.

2 Problem Definition

Assume there is a factory which creates hemisphere light bulbs with a specific intensity pattern. There are two important Quality Control (QC) factors:

1. The output shape must be a hemisphere with a given radius R.
2. The intensity pattern must follow the next instructions:

Assuming the center of the sphere as the origin of the cylindrical coordinate system, the intensity of each point on the hemisphere should be equal to:

$$I(\rho, \phi, z) = I_{\mathsf{max}} z$$

where $z = R\cos(\theta)$ in spherical coordinates.

2-a. What is the mean intensity of the surface?

Hint: You will need to calculate an integral over the surface of the hemisphere. Consider that you are integrating over the surface area, not the volume.

2-b. Interpret the result from Part 1.

Discuss the physical meaning of the result in terms of how intensity varies across the hemisphere. What does the result say about the brightness distribution? We want to eliminate all geometrical via choosing a proper value for I_{max} to eliminate all geometrical effects. An engineer has suggested to use $I_{max} = \frac{\alpha}{R}$. Is this wise? Discuss.

From now on, we accept your modification in 2-b for I_{max}.

2-c. Generate a 512x512px image of ideal top-view projection of the object. The image is fit to the object. Register your mathematical calculations, code and final image.

Hint: It is better to define a variable or constant in your code for width and height of the final image an avoid hard-coded numbers.

Hint2: As you may have noticed, α is an important factor here. Consider it as the maximum possible value based on your intensity depth.

Hint3: You may face complex numbers due to floating-point processing errors. Take real of them carefully.

2-d. Calculate the mean intensity of the top-view image. How is it relevant to that of 2-a?

Hint: Only consider the circle image and do not involve the full frame.

2-e. This time consider your circle is fit within a square frame (the most narrow possible square) with zero intensity. Calculate the formula again. Discuss.

2-f. Now calculate the mean intensity of all pixels in your generated image and compare with the previous answer. Discuss.

3 Pixel Intensity Calibration Issues

Now, place the hemisphere on a flat black surface, and position a camera directly above the hemisphere, aligned with its center. The monitor shows a circle. We fix the distance between the top point of the hemisphere and the camera lens as $2R$.

Given the imaging setup and assuming an orthogonal projection (neglecting perspective and other geometric disturbances), consider that we use a 2D coordinate system on the image plane centered at the middle. The image is cropped properly so that the screen square exactly fits the borders of the hemisphere's projection as a circle.

3-a - 3-d: For each scenario in the *'images/q3'* folder, identify the issue displayed on the factory's monitor. Use the methods covered in the textbook's Section 3.2 to fix the problem. For each item, calculate the mean intensity of the processed image and compare it with that of *2-f*. Discuss your findings.

4 Watermark Logo

The factory decides to add a watermark logo on the front surface of the production. However, it is barely visible on the monitor. Help them enhance the logo's visibility for better representation in their marketing sessions!

4-a: The input is given in the *'images/q4'* folder. Investigate the photo and design your algorithm based on the materials in Section 3-3 of the textbook.

Hint: The base approach is Histogram Equalization, but a more advanced technique is required beyond simple histogram equalization.

5 Linear Spatial Smoothing

As you've noticed, all of the captured photos contain strong noise, which is undesirable. We can use smoothing techniques to restore the image.

5-a: Calculate the mean intensity of *'images/q5/a.png'* and compare it to the ideal mean intensity of the photo given in 'images/q5/b.png'. Discuss.

5-b: Apply box filters with sizes 9, 25, 49, and 81. Register the output. Compare the results to the ideal and noisy images, analyze mean intensities, and discuss.

5-c: Visually inspecting the processed images provides valuable insights, but this approach is *subjective*. To analyze the results in a *global sense*, we need an *objective* metric. One such metric is MSE. Since you have the ideal image, you can use this metric. Conduct a quick research on MSE (do not report) and compute the MSE for each kernel used in 5-b. Discuss.

5-d: The MSE analysis suggests using larger kernels. Start with a kernel size of 3×3 and incrementally increase it up to 127×127 (using only odd numbers). Calculate the MSE for each kernel size and plot it against increasing k, where the kernel size is $k \times k$. Identify the optimal size, register the plot, and save the

best output. Discuss.

5-e: Repeat 5-d using low-pass Gaussian kernels. Compare the results with those from 5-d and discuss. Assume $\sigma = 5$. Compare the MSE of the best results from 5-d and 5-e.

5-f: Gaussian filters have two parameters: σ and k. In the previous question, only k was explored. Now, investigate the effect of both σ and k. The resulting plot will be a 2D image. Assign the x-axis to k and the y-axis to σ, using a descriptive intensity map. Identify the optimal point and compare it with the best box filter result. Register the 2D-MSE plot and the best output plot. Assume $\sigma \in [3, 7]$ and use equally spaced values in this interval, with the same number of points as k.

6 Separable Kernels

6-a: Repeat 5-f, but this time, use two equal 1D Gaussian filters instead of a 2D Gaussian filter. Measure the runtime of both implementations and compare the results.

7 Modality

Modality and Multi-Modal Signal Processing Quality control (QC) often relies on camera-captured images, utilizing *visual* information to assess a product's conformity. However, determining QC results solely from visual data can be challenging, especially when factors such as lighting conditions, reflections, or occlusions affect the captured image. In cases where both intensity and shape must be verified, relying on a single modality may introduce uncertainty.

To enhance QC reliability, integrating an additional modality—such as ultrasonic scanning—can provide a more robust evaluation. Unlike cameras, ultrasonic sensors measure vertical distance through Time-of-Flight calculations, offering an independent structural assessment of the product. By analyzing the voltage signal variations (v_{min} to v_{max}), a precise height profile can be reconstructed. This multi-modal approach allows for cross-validation of shape and structural integrity, reducing dependency on visual cues alone.

In practice, processing ultrasonic data requires appropriate filtering techniques to mitigate noise. Since an ideal ground-truth image is not readily available, generating an estimated reference based on known physics is essential. This enables objective evaluation using metrics like MSE, ensuring that QC decisions are based on reliable and consistent data. Through this multi-modal fusion of image and ultrasonic data, the factory can achieve higher accuracy in defect detection and product validation.

As you have noticed, a camera-captured image can be used for QC by analyzing *visual* information. However, can it be reliably useful for determining the QC result?

7-a: Assume we are assured that the desired intensity pattern of a product is correct. How can we check (using the camera photo) if the shape is also correct? Discuss.

7-b: Assume we are not sure whether the intensity is correct. Can we verify the shape correctness using the camera's output? Discuss and compare.

To enhance QC, the company utilizes an ultrasonic scanner. The sensor measures vertical distance and moves over the object on a fixed surface. It is a linear sensor that measures Time-of-Flight distance and converts it into a voltage signal v. When positioned over the object at $(x, y) = (0, 0)$, its value is v_{min}, and when over the surface, it is v_{max}.

7-c: One output sample is provided in *images/q7/sensor.png*. We aim to reduce noise using the method in question 5. However, the original *target* image is unavailable for MSE calculation. Fortunately, we understand the physics of the problem. Calculate and generate a 512x512 pixel representation of the ideal target image.

Hint: A normalization coefficient (scaling factor) is needed. Try to extract it from the given sample image. A simple min or max approach is not reliable in the presence of noise!

7-d: With the ideal target image obtained, perform the search as in 5-f. Register the 2D MSE plot and the best result. Interpret.

Hint: Reuse the algorithm from 5-f where applicable to avoid rewriting everything.

8 Into the Maps!

You are given a grayscale satellite image located at `images/q8/image.png`. This exercise focuses on concepts from Chapter 3 of the textbook, particularly high-pass filtering, sharpening, and high-boost filtering.

Please answer following questions and report them as 8-a to 8-e.

a Apply high-pass filtering using 3×3 kernels for the following directions:

- Horizontal edges
- Vertical edges
- Diagonal (both main and anti-diagonal) edges

Show the resulting images and compute the mean intensity of each result. Interpret how the mean intensity relates to the structures in the image. (Hint: Higher mean values may indicate a predominance of edges aligned with the given filter direction.)

b Apply the following edge-detection filters:

- Sobel (horizontal and vertical)
- Prewitt (horizontal and vertical)
- Roberts Cross

Compare their results with those obtained in part a. Discuss differences in edge detection performance.

c Perform image sharpening using the Laplacian filter. Compare it with the result of the high-pass filter in part a.

d Implement high-boost filtering with a boost factor of $A = 1.5$. Compare this with the previous results and interpret the effect of increasing A.

e Examine other kernel sizes and then discuss how the choice of kernel size impacts the results of high-pass and high-boost filtering.

Hint: Be sure to properly normalize and visualize your results.

9 Directional Edge Analysis

You are given three grayscale images located in:

`images/q9/`

Apply directional filtering and analyze the results.

Please answer following questions and report them as 9-a to 9-e.

a Apply high-pass filtering using 3×3 kernels for the following directions on each image:

- Horizontal edges
- Vertical edges
- Diagonal edges (both main and anti-diagonal)

b Compute the mean intensity of each filtered image.

c Compare the mean intensities across different directions for each image. Interpret the results in terms of dominant structures in the images. (Hint: A higher mean intensity may indicate a prevalence of edges in that direction.)

d Compare the results across the three images. Discuss how structural patterns differ in each image based on the filtering results.

Hint: Consider how the edge structures in natural, urban, or textured images affect the mean intensity in each direction.

"Solving the textbook's chapter questions is a great way to get additional practice and is strongly suggested. However, these problems are not graded in this homework!"