Microlensing parameters in `MulensModel`

Radek Poleski
last update: Jul 2025

Microlensing parameters of the `ModelParameters` class in the `MulensModel` package are presented on the next page:

Parameter	Name in `MulensModel`	Unit	Description
t_0	`t_0`		The time of the closest approach between the source and the lens.
u_0	`u_0`		The impact parameter between the source and the lens center of mass.
t_E	`t_E`	d	The Einstein crossing time.
t_eff	`t_eff`	d	The effective timescale, $t_\mathrm{eff} \equiv u_0 t_\mathrm{E}$.
ρ	`rho`		The radius of the source as a fraction of the Einstein ring.
$t_\star$	`t_star`	d	The source self-crossing time, $t_\star \equiv \rho t_\mathrm{E}$.
$\pi_{\mathrm{E},N}$	`pi_E_N`		The North component of the microlensing parallax vector.
$\pi_{\mathrm{E},E}$	`pi_E_E`		The East component of the microlensing parallax vector.
$t_{0,\mathrm{par}}$	`t_0_par`		The reference time for parameters in parallax models.[a]
K	`convergence_K`		External mass sheet convergence.
G	`shear_G`		External mass sheet shear; complex valued to represent both the magnitude and angle relative to the binary axis.
s	`s`		The projected separation between the lens primary and its companion as a fraction of the Einstein ring radius.
q	`q`		The mass ratio between the lens companion and the lens primary $q \equiv m_2/m_1$.
α	`alpha`	deg.	The angle between the source trajectory and the binary axis.
ds/dt	`ds_dt`	yr^{-1}	The rate of change of the separation.
$d\alpha/dt$	`dalpha_dt`	deg. yr^{-1}	The rate of change of α.
s_z	`s_z`		Position along the line of sight as a fraction of the Einstein ring radius. Positive axis points to the observer.[b]
ds_z/dt	`ds_z_dt`	yr^{-1}	The rate of change of s_z.[b]
$t_{0,\mathrm{kep}}$	`t_0_kep`		The reference time for lens orbital motion calculations.[a]
$x_{\mathrm{caustic,in}}$	`x_caustic_in`		Curvelinear coordinate of caustic entrance for a binary lens model.[c]
$x_{\mathrm{caustic,out}}$	`x_caustic_out`		Curvelinear coordinate of caustic exit for a binary lens model.[c]
$t_{\mathrm{caustic,in}}$	`t_caustic_in`		Epoch of caustic exit for a binary lens model.[c]
$t_{\mathrm{caustic,out}}$	* (t_caustic_out		Epoch of caustic exit for a binary lens model.[c]
ξ_P	`xi_period`	d	The orbital period of xallarap.
ξ_a	`xi_semimajor_axis`		The semimajor axis of a xallarap orbit as a fraction of the Einstein ring.
ξ_i	`xi_inclination`	deg	The inclination of a xallarap orbit.[d]
ξ_Ω	`xi_Omega_node`	deg	The longitude of the ascending node of a xallarap orbit.[d]

ξ_u	`xi_argument_of_latitude_reference`	deg	The argument of latitude at the reference epoch ($t_{0,\chi}$). The argument of latitude is a sum of true anomaly (ν, changes with time) and the argument of periapsis (ω, orbit parameter, i.e., does not change with time): $u = \nu + \omega$.[d]
ξ_e	`xi_eccentricity`		The eccentricity of a xallarap orbit.
ξ_ω	`xi_omega_periapsis`	deg	The argument of periapsis of a xallarap orbit.[d]
$\xi_{\rm A}$	`xi_A`		Thiele-Innes parameter $\mathrm{A} = a * (\cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i)$[e]
$\xi_{\rm B}$	`xi_B`		Thiele-Innes parameter $\mathrm{B} = a * (\sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i)$[e]
$\xi_{\rm F}$	`xi_F`		Thiele-Innes parameter $\mathrm{F} = -a * (\cos\Omega\sin\omega + \cos\Omega\cos\omega\cos i)$[e]
$\xi_{\rm G}$	`xi_G`		Thiele-Innes parameter $\mathrm{G} = -a * (\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i)$[e]
$q_{\rm source}$	`q_source`		Mass ratio of source components: $m_{s,2}/m_{s,1}$. It is valid only for xallarap models.
$t_{0,\xi}$	`t_0_xi`		The reference epoch for parameters in xallarap models.[a]

Table 1: Notes:

[a] – $t_{0,\rm par}$, $t_{0,\rm kep}$, and $t_{0,\chi}$ are reference parameters, hence, do not change these during fitting.

[b] – To include keplerian motion of the lenses on a circular orbit either s_z or ds_z/dt has to be defined.

[c] – The four parameters of binary lens in Cassan (2008) parameterization ($x_{\rm caustic,in}$, $x_{\rm caustic,out}$, $t_{\rm caustic,in}$, $t_{\rm caustic,out}$) are used instead of (t_0, u_0, $t_{\rm E}$, α).

[d] – The orbital angles are illustrated in Figure 1.

[e] – Thiele-Innes parameters are an alternative way of defining the orbit. They should not be used together with standard orbital parameters (ξ_a, ξ_i, ξ_Ω, ξ_ω). For details see *An Introduction to Close Binary Stars, R. W. Hilditch* (2001).

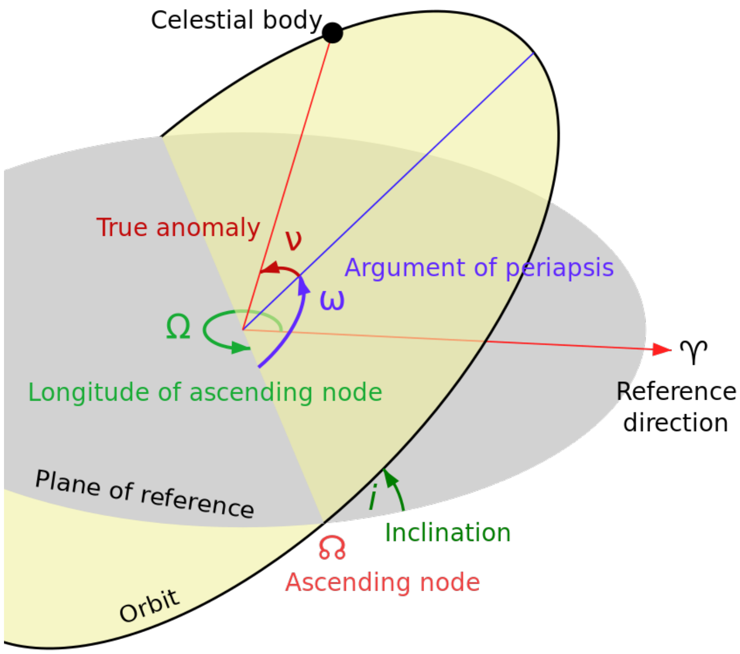

Figure 1: Definition of orbital angles. There are Wikipedia articles that give more details: orbital elements and argument of latitude. For xallarap, the reference direction is the relative lens-source proper motion direction and the reference plane is the plane of the sky.

Some of the parameters can be defined separately for each of the sources in binary source models. In that case, add _1 or _2 to parameter name. These are:

- `t_0_1`, `t_0_2`,
- `u_0_1`, `u_0_2`,
- `rho_1`, `rho_2`,
- `t_star_1`, `t_star_2`.

Also note that there are properties of the microlensing events that are not considered parameters in the `ModelParameters` class, but are implemented in other parts of the `MulensModel`. The most important are:

- source and blending fluxes – `Event` and `FitData`; also see use case 38,
- sky coordinates – `Model.coords`,
- limb-darkening coefficients – `Model.set_limb_coeff_gamma` and `Model.set_limb_coeff_u`,
- flux ratio for binary source models – `Model.set_source_flux_ratio` and `Model.set_source_flux_ratio_for_band`,
- methods used to calculate magnification – `Model.set_magnification_methods`,
- coordinates of space telescopes – `Model.get_satellite_coords`.