1) $$\frac{\partial L}{\partial b^{(2)}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial b^{(2)}}$$

$$\frac{\partial L}{\partial y} = \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{\partial y_2} \end{bmatrix}$$

$$\frac{\partial y}{\partial b^{(2)}} = \begin{bmatrix} \frac{\partial y_1}{\partial b_1^{(2)}} & \frac{\partial y_1}{\partial b_2^{(2)}} \\ \frac{\partial y_2}{\partial b_1^{(2)}} & \frac{\partial y_2}{\partial b_2^{(2)}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial y_1}{\partial b_1^{(2)}} & 0 \\ 0 & \frac{\partial y_2}{\partial b_2^{(2)}} \end{bmatrix}$$

$$\frac{\partial L}{\partial b^{(2)}} = \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{\partial y_2} \end{bmatrix} \begin{bmatrix} \frac{\partial y_1}{\partial b_1^{(2)}} & 0 \\ 0 & \frac{\partial y_2}{\partial b_2^{(2)}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial L}{\partial y_1} \cdot \frac{\partial y_1}{\partial b_1^{(2)}} & \frac{\partial L}{\partial y_2} \cdot \frac{\partial y_2}{\partial b_2^{(2)}} \end{bmatrix}$$

$$\frac{\partial L}{\partial b^{(1)}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial h} \cdot \frac{\partial h}{\partial u} \cdot \frac{\partial u}{\partial b^{(1)}}$$

$$\frac{\partial L}{\partial y} = \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{\partial y_2} \end{bmatrix}$$

$$\frac{\partial y}{\partial h} = \begin{bmatrix} \partial y_1/\partial h_1 & \partial y_1/\partial h_2 \\ \partial y_2/\partial h_1 & \partial y_2/\partial h_2 \end{bmatrix}$$

$$\frac{\partial h}{\partial u} = \begin{bmatrix} \frac{\partial h_1}{\partial u_1} & 0 \\ 0 & \frac{\partial h_2}{\partial u_2} \end{bmatrix}$$

$$\frac{\partial v}{\partial b^{(1)}} = \begin{bmatrix} \frac{\partial v_1}{\partial b_1^{(1)}} & 0 \\ 0 & \frac{\partial v_2}{\partial b_2^{(1)}} \end{bmatrix}$$

$$\frac{\partial L}{\partial b^{(1)}} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial h} \cdot \frac{\partial h}{\partial v} \cdot \frac{\partial v}{\partial b^{(1)}}$$

$$= \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{y_2} \end{bmatrix} \begin{bmatrix} \partial y_1/\partial h_1 & \partial y_1/\partial h_2 \\ \partial y_2/\partial h_1 & \partial y_2/\partial h_2 \end{bmatrix} \begin{bmatrix} \frac{\partial h_1}{\partial v_1} & 0 \\ 0 & \frac{\partial h_2}{\partial v_2} \end{bmatrix} \begin{bmatrix} \frac{\partial v_1}{\partial b_1^{(1)}} & 0 \\ 0 & \frac{\partial v_2}{\partial b_2^{(1)}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{y_2} \end{bmatrix} \begin{bmatrix} \partial y_1/\partial h_1 & \partial y_1/\partial h_2 \\ \partial y_2/\partial h_1 & \partial y_2/\partial h_2 \end{bmatrix} \begin{bmatrix} \frac{\partial h_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial b_1^{(1)}} & 0 \\ 0 & \frac{\partial h_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial b_2^{(1)}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial L}{\partial y_1} & \frac{\partial L}{y_2} \end{bmatrix} \begin{bmatrix} \frac{\partial y_1}{\partial h_1} \cdot \frac{\partial h_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial b_1^{(1)}} & \frac{\partial y_1}{\partial h_2} \frac{h_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial b_2^{(1)}} \\ \frac{\partial y_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial b_1^{(1)}} & \frac{\partial y_2}{\partial h_2} \frac{\partial h_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial b_2^{(1)}} \end{bmatrix}$$

$$= \left[\frac{\partial L}{\partial y_1} \cdot \frac{\partial y_1}{\partial h_1} \cdot \frac{\partial h_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial b_1^{(1)}} + \frac{\partial L}{y_2} \frac{\partial y_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial b_1^{(1)}} \quad \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial h_2} \cdot \frac{h_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial b_2^{(1)}} + \frac{\partial L}{y_2} \cdot \frac{\partial y_2}{\partial h_2} \cdot \frac{h_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial b_2^{(1)}} \right]$$

2) $$\frac{\partial L}{y} = \frac{\partial}{\partial y} \left[\|t - \hat{y}\|^2 \right]$$

$$= \frac{\partial}{\partial y} \begin{bmatrix} (t_1 - y_1) & (t_2 - y_2) \end{bmatrix} \begin{bmatrix} t_1 - y_1 \\ t_2 - y_2 \end{bmatrix}$$

$$= \frac{\partial}{\partial y} (t_1 - y_1)^2 + (t_2 - y_2)^2$$

$$\frac{\partial L}{\partial y_1} = -2(t_1 - y_1), \quad \frac{\partial L}{\partial} = -2(t_2 - y_2)$$

$$\frac{\partial L}{\partial y_1} = -2(t_1 - y_1), \quad \frac{\partial L}{\partial y_2} = -2(t_2 - y_2)$$

$$y_1 = w_{11}^{(2)} h_1 + w_{12}^{(2)} h_2 + b_1^{(2)}$$

$$y_2 = w_{21}^{(2)} h_1 + w_{22}^{(2)} h_2 + b_2^{(2)}$$

$$\frac{\partial y_1}{\partial b_1^{(2)}} = 1 \qquad \frac{\partial y_2}{\partial b_2^{(2)}} = 1$$

$$\frac{\partial L}{\partial b^{(2)}} = \begin{bmatrix} -2(t_1 - y_1) & -2(t_2 - y_2) \end{bmatrix}$$

$$\frac{\partial y_1}{\partial h_1} = w_{11} \qquad \frac{\partial y_1}{\partial h_2} = w_{12}$$

$$\frac{\partial y_2}{\partial h_1} = w_{21} \qquad \frac{\partial y_2}{\partial h_2} = w_{22}$$

$$\frac{\partial h_1}{\partial v_1} = 1 \quad \text{if } v_1 > 0$$

$$\frac{\partial h_2}{\partial v_2} = 1 \quad \text{if } v_2 > 0$$

$$\frac{\partial v_1}{\partial b_1^{(1)}} = 1 \qquad \frac{\partial v_2}{\partial b_2^{(1)}} = 1$$

$$\frac{\partial L}{\partial B^{(1)}} = \begin{bmatrix} -2(t_1 - y_1)(w_{11}) - 2(t_2 - y_2)(w_{21}) & -2(t_1 - y_1)(w_{12}) - 2(t_2 - y_2)(w_{22}) \end{bmatrix}$$