Q1: No, it is non linear.

Q2:

$$\begin{bmatrix} \phi_0(x_1) & \cdots & \phi_N(x_1) \\ \vdots & \ddots & \vdots \\ \phi_0(x_U) & \cdots & \phi_N(x_U) \end{bmatrix}$$

The dimensions = $U \times (N+1)$

(U: No. of samples)

Q3: $\tilde{y} = \phi(x)\, w$

$$\begin{bmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_U \end{bmatrix} = \begin{bmatrix} \phi_0(x_1) & \cdots & \phi_N(x_1) \\ \vdots & \ddots & \vdots \\ \phi_0(x_U) & \cdots & \phi_N(x_U) \end{bmatrix} \begin{bmatrix} w_0 \\ \vdots \\ w_{N+1} \end{bmatrix}$$

Q4: $$w^* = \underbrace{\left(\phi(x)^T \phi(x)\right)^{-1}}_{(N+1 \times N+1)} \underbrace{\phi(x)^T}_{(N+1 \times U)} \bar{y} \quad - (U \times 1)$$