Expectation Maximization - Missing Data

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Stat 624 Project 2

December 11, 2017

Overview

1. Introduction
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3. Simulation Study
4. Application
5. Conclusion

Introduction/Motivation

Problem

Parameter estimation is important, and there are various methods used to solve these kinds of problems.

Dilemma

Many approaches however, are not robust when they encounter non-ideal circumstances (e.g. missing data).

Solution

It is possible though, to work around this dilemma through various data imputation methods, such as the Expectation-Maximization Algorithm

Questions of Interest

Question 1

Can we find estimates of the means and covariances between variables?

Question 2

Can we come up with a method to determine if the data are missing at random or if there is a pattern to the missingness?

Question 3

Can we determine which variables are most and least correlated?

Methodology

In this study we will look at 3 different methods of data imputation for data that follow a multivariate normal distribution.
Methods:

- Throw-away Method
- Expectation Maximization
- Conditional Sampling

MVN PDF:

$$f(x|\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} exp\{-\frac{1}{2}(x-\mu)^\top \Sigma^{-1}(x-\mu)\}$$

Methodology

Throw-away Method:

- This method isn't actually an imputation method. It simply entails the removal of all non-complete observations, and then using sample mean and sample covariance as parameter estimates.

Mean Estimate (MLE)

$$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N}(x_n)$$

Covariance Estimate (Unbiased)

$$\hat{\Sigma} = \frac{1}{N-1}\sum_{n=1}^{N}(x_n - \bar{x})(x_n - \bar{x})^\top$$

Methodology

Expectation-Maximization Algorithm:

- Instead of throwing away the incomplete data, we can iteratively update estimates for the mean. The algorithm will continue to iterate through 'expectation' and 'maximization' steps until it converges. The updates are made using the condition multivariate normal distribution.

$$(\mathbf{x}_1|\mathbf{x}_2 = a) \sim N(\bar{\mu}, \bar{\Sigma})$$

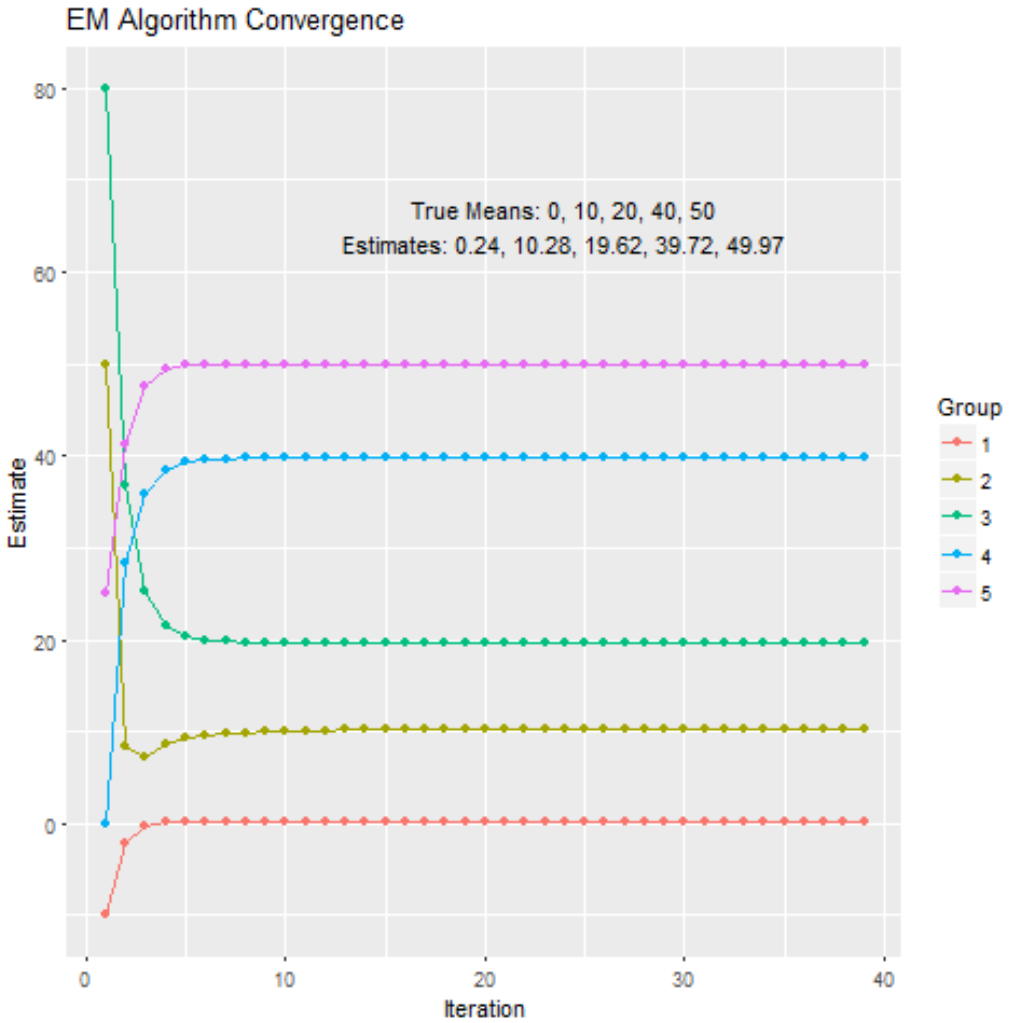
$$\bar{\mu} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(a - \mu_2)$$

$$\bar{\Sigma} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

Note: subscript 1 refers to missing values, and subscript 2 refers to non-missing values

Methodology

Convergence Plot:



Methodology

"Conditional Sampling":

- Similar to the EM Algorithm, but instead of imputing the mean for the missing values, we draw random values from the distribution of the estimates of μ and Σ for each iteration. The draws of values are then used to estimate the mean and covariance. (This method will not converge)

$$(\mathbf{x}_1|\mathbf{x}_2 = a) \sim N(\bar{\mu}, \bar{\Sigma})$$

$$\bar{\mu} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(a - \mu_2)$$

$$\bar{\Sigma} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$$

Note: subscript 1 refers to missing values, and subscript 2 refers to non-missing values

MC Simulated Data for non-random missing data:

	Estimate	Bias	MSE
Throw Away	0.0010	0.0010	0.0621
	10.0021	0.0021	0.0612
	20.0023	0.0023	0.0615
	40.0026	0.0026	0.0615
	50.0034	0.0034	0.0620
EM Algorithm	0.0008	0.0008	0.0124
	10.0049	0.0049	0.0135
	20.0048	0.0048	0.0139
	40.0022	0.0022	0.0125
	50.0027	0.0027	0.0144
Conditional Sampling	-0.0062	-0.0062	0.0122
	9.9961	-0.0039	0.0126
	19.9973	-0.0027	0.0142
	39.9920	-0.0080	0.0118
	49.9990	-0.0010	0.0157

Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

MC Simulated Means for randomly missing data:

Method	Estimate	Bias	MSE
Throw Away	0.0010	0.0010	0.0621
	10.0021	0.0021	0.0612
	20.0023	0.0023	0.0615
	40.0026	0.0026	0.0615
	50.0034	0.0034	0.0620
EM Algorithm	0.0008	0.0008	0.0124
	10.0049	0.0049	0.0135
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Application

We'll test these methods using data of characteristics of hepatitis patients:

	Age	Bilirubin	AlkPhosphate	Sgot	AlbuMin	ProTime
1	30	1.00	85	18	4.00	
2	50	0.90	135	42	3.50	
3	78	0.70	96	32	4.00	
4	31	0.70	46	52	4.00	80
5	34	1.00		200	4.00	
6	34	0.90	95	28	4.00	75

Application

	Throw Away	EM Algorithm	Conditional Sampling
Age	41.06	41.20	41.20
Bilirubin	1.25	1.43	1.43
AlkPhosphate	102.51	106.30	106.31
Sgot	86.39	85.89	85.92
AlbuMin	3.83	3.81	3.81
ProTime	62.16	61.81	61.74

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)

Title of the publication

Journal Name 12(3), 45 – 678.

The End