Evidence for cosmic neutrino background form CMB circular polarization

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The primordial anisotropies of the cosmic microwave background are linearly polarized via Compton-scattering. On the other hand, a primordial degree of circular polarization of the Cosmic Microwave Background is not observationally excluded. In this work, we discuss the generation of the circular polarization of CMB via their scattering on the cosmic neutrino background since the epoch of recombination. We show that photon-neutrino interaction can transform plane polarization into circular polarization through processes $\gamma + \nu \to \gamma + \nu$ and the Stokes-V parameter of CMB has linear dependence on the wavelength and the cosmic neutrino background perturbations of pressure and shear stress and also the maximum value of C^V is estimated in range of a few Nano-Kelvin square.

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I. INTRODUCTION

Modern cosmological observations of the Cosmic Microwave Background (CMB) radiation contain valuable information about our universe. The CMB photons have decoupled from matter about 3×10^5 years after the Big-Bang (BB), so we are unable to probe the universe closer than 300 000 years to the BB by using CMB. Cosmological information encoded in the CMB radiation concerns not only temperature fluctuations and the spectrum of anisotropy pattern, but also the intensity and spectrum of linear and circular polarizations. From a result of the anisotropic Compton scattering around the epoch of recombination, it is generally expected that some relevant linear polarizations (about 10%) of CMB radiation should be present [1–3], and polarization fluctuations are smaller than the temperature fluctuations [5]. Currently, there are several ongoing experiments [6–11, 40] attempting to measure CMB polarizations. Theoretical studies of CMB polarizations were carried out in Refs. [1–4], and numerical calculations [13, 14] have confirmed that about 10% of the CMB radiation is linearly polarized, via the Compton and Thompson scattering of unpolarized photons at the last scattering surface (the redshift $z \sim 10^3$). Polarized light is conventionally described in terms of the Stokes parameters, a linearly polarized radiation is described by non-zero values for the Stokes parameters Q and/or U and the possibility of the generation of circular polarization can be determined by the Stokes parameter V [15]. On the basis of the mechanism discussed in [1], the linear polarization of the CMB in the presence of a large-scale magnetic field B can be converted to the circular polarization under the formalism of the generalized Faraday rotation (FR) [19, 20] known as the Faraday conversion (FC). The evolution of the Stokes parameter V given by this mechanism is obtained as

$$\dot{V} = 2U \frac{d}{dt}(\Delta\phi_{FC}), \tag{1}$$

where $\Delta\phi_{FC} \propto B^2$ is the Faraday conversion phase shift [20]. There are several papers which have attempted to discuss the probability of the generation of circular polarization of CMB photons. Giovannini has shown that if the CMB photons are scattered via electrons in the presence of a magnetic field, a non-vanishing V mode can be produced [22, 23]. Furthermore, Cooray, Melchiorri and Silk have discussed that the CMB radiation observed today is not exactly the same as the field last scattered [20], Bavarsad *et al* have shown that CMB polarization acquires a small degree of circular polarization when a background magnetic field is considered or the quantum electrodynamic sector of standard model is extended by Lorentz non-invariant operators as well as non-commutativity [21], Motie and Xue have discussed that the circular polarizations of radiation fields can be generated from the effective Euler-Heisenberg Lagrangian [24] and the transform plane polarization into circular polarization via photon-photon interactions mediated by the neutral hydrogen background, $\gamma + \gamma + atom \to \gamma + \gamma + atom$, through completely forward processes, has been discussed by Sawyer [25]. We would like to point out that photon - neutrino scattering can generate circular polarization. The reason for that is: in context of standard model we have the purely left-handed interaction for neutrinos which caused linearly polarized photons achieve circular polarizations by interacting with left-handed neutrinos, in contrast they do not acquire circular polarizations by interacting with electrons in the forward scattering terms of [1]. We can consider any linear polarization as two equal component, lift and right handed circular polarization. Due to lift handed interaction of neutrino, only one part of this linear polarization (lift handed component) is affected by neutrino. Finally after neutrino - photon scattering, the total number of lift and right handed circular polarization become different and we have a net circular polarization for photons. In this study, we are going to check and calculate the generation of circular polarization for CMB due to their scattering with the cosmic neutrinos background (CνB).

On the other hand, a similar probe like the CMB is the cosmic neutrinos background (CνB) which can give us very helpful information about the early universe. Due to their weak interaction they decouple earlier about 1 second after the BB from matter at a temperature of $T_\nu \approx 1$MeV (10^{10} Kelvin). The Cosmic Neutrino Background CνB of today (with temperature $T_{0\nu} \approx 1.95$ K) therefore contains information of the universe already 1 second after the BB. The CMB measurements have already constrained the neutrino and antineutrino masses sum within LambdaCDM framework to about $\sum_{i=e,\mu,\tau}(m_{\nu_i} + m_{\bar{\nu}_i}) \sim 0.6$ eV, implying a constraint of ~ 0.1 eV on the individual neutrino and antineutrino mass. We consider this limit for mass of each type of neutrino and antineutrino for the rest. But we should remind that the detection of this CνB seems to be hardly possible due to the weak interaction of neutrinos with matter and due to their low energy. Nevertheless, several methods have been discussed in the literature to search for these relic neutrinos [26–35]. Here we discuss the possibility to find any effects of CνB on the circular polarization of the CMB photons via photon-neutrino scattering. As is well known, photon-neutrino cross-section in the context of standard model is very small because neutrinos are neutral particles with very small electromagnetic dipole moment $\mu_\nu \propto m_\nu$ and also the leading order of photon-neutrino interaction (one-loop) contain the weak interaction. But this is not really bad news for our idea because we are going to consider the last scattering surface for photon-neutrinos around the epoch of recombination. This means if coherent photon-neutrino forward scattering after recombination age can provide any sources for the circular polarization of CMB, $\Delta\phi_{FC}$ grows due to large distance (larger than Mpc) or equivalency large time scale of evolution (see Eq. 1). In principle, under effects of background fields, particle scattering and temperature fluctuations, linear polarizations of CMB radiation field propagating from the last scattering surface can rotate each other and convert to circular polarizations. In this study we will study the distribution of neutrino-photon scattering for the generation of the circular polarization of the CMB. First we give a brief introduction on Stokes parameters and derive the time evolution of these parameters in terms of the photon-particle scattering. Then by considering the weak and electrodynamic interactions, we will find the time evolution of Stokes parameters in terms of photon-neutrino interactions. Finally we try to estimate the maximum value of the V-mode polarization by using the relevant values of energy and number density of cosmic neutrinos around recombination epoch.

II. STOKES PARAMETERS

As usual, we characterized the polarization of CMB by means of the Stokes parameters of radiation: I, Q, U and V. Assume a quasi-monochromatic electromagnetic wave propagating in the $\hat{z}$-direction which is described by:

$$E_x = a_x(t)\cos[\omega_0 t - \theta_x(t)], \quad E_y = a_y(t)\cos[\omega_0 t - \theta_y(t)], \tag{2}$$

where amplitudes $a_{x,y}$ and phase angles $\theta_{x,y}$ are slowly varying functions with respect to the period $\mathcal{T}_0 = 2\pi/\omega_0$. Stokes parameters, which describe polarization states of a nearly monochromatic electromagnetic wave, are defined as the following time averages [15]:

$$\begin{aligned} I &= \langle a_x^2\rangle + \langle a_y^2\rangle, \\ Q &= \langle a_x^2\rangle - \langle a_y^2\rangle, \\ U &= \langle 2a_x a_y \cos(\theta_x - \theta_y)\rangle, \\ V &= \langle 2a_x a_y \sin(\theta_x - \theta_y)\rangle, \end{aligned} \tag{3}$$

where the parameter I is total intensity, Q and U intensities of linear polarizations of electromagnetic waves, whereas the V parameter indicates the difference between left- and right- circular polarizations intensities. Linear polarization can also be characterized through a vector of modulus $P_L \equiv \sqrt{Q^2 + U^2}$. The time evolution of these Stokes parameters is given through Boltzmann equation. Boltzmann equation is a systematic mechanism in order to describe the evolution of the distribution function under gravity and collisions. Ones can consider each polarization state of the CMB radiation as the phase space distribution function ξ. The classical Boltzmann equation generally is written as

$$\frac{d}{dt}\xi = \mathcal{C}(\xi) \tag{4}$$

where the left hand side is known as the Liouville term deals with the effects of gravitational perturbations about the homogeneous cosmology. The right hand side of the Boltzmann equation contains all possible collision terms. By considering the contribution of the neutrino-photon scattering on the right hand side of above equation, we calculate the time evolution of the each polarization state of the CMB photons. For the rest of calculations, Stokes parameters are given in a quantum-mechanical description. An arbitrary polarized state of a photon ($|k^0|^2 = |\mathbf{k}|^2$), propagating in the $\hat{z}$-direction, is given by

$$|\epsilon\rangle = a_1 \exp(i\theta_1)|\epsilon_1\rangle + a_2 \exp(i\theta_2)|\epsilon_2\rangle, \tag{5}$$

where linear bases $|\epsilon_1\rangle$ and $|\epsilon_2\rangle$ indicate the polarization states in the x- and y-directions. Quantum-mechanical operators in this linear bases, corresponding to Stokes parameter, are given by

$$\begin{aligned}
\hat{I} &= |\epsilon_1\rangle\langle\epsilon_1| + |\epsilon_2\rangle\langle\epsilon_2|, \\
\hat{Q} &= |\epsilon_1\rangle\langle\epsilon_1| - |\epsilon_2\rangle\langle\epsilon_2|, \\
\hat{U} &= |\epsilon_1\rangle\langle\epsilon_2| + |\epsilon_2\rangle\langle\epsilon_1|, \\
\hat{V} &= i|\epsilon_2\rangle\langle\epsilon_1| - i|\epsilon_1\rangle\langle\epsilon_2|.
\end{aligned} \tag{6}$$

An ensemble of photons in a general mixed state is described by a normalized density matrix $\rho_{ij} \equiv (\,|\epsilon_i\rangle\langle\epsilon_j|/\mathrm{tr}\rho)$, and the dimensionless expectation values for Stokes parameters are given by

$$I \equiv \langle\hat{I}\rangle = \mathrm{tr}\rho\hat{I} = 1, \tag{7}$$

$$Q \equiv \langle\hat{Q}\rangle = \mathrm{tr}\rho\hat{Q} = \rho_{11} - \rho_{22}, \tag{8}$$

$$U \equiv \langle\hat{U}\rangle = \mathrm{tr}\rho\hat{U} = \rho_{12} + \rho_{21}, \tag{9}$$

$$V \equiv \langle\hat{V}\rangle = \mathrm{tr}\rho\hat{V} = i\rho_{21} - i\rho_{21}, \tag{10}$$

where "tr" indicates the trace in the space of polarization states. These above equations determine the relationship between four Stokes parameters and the 2×2 density matrix ρ of photon polarization states. In this section, we use notations which used in [24].

III. THE GENERATION OF POLARIZED CMB VIA PHOTON-NEUTRINOS SCATTERING.

The density operators describing a system of photons is given by

$$\hat{\rho} = \frac{1}{\mathrm{tr}(\hat{\rho})} \int \frac{d^3k}{(2\pi)^3} \rho_{ij}(k) a_i^\dagger(k) a_j(k), \tag{11}$$

where $\rho_{ij}(k)$ is the general density-matrix (7-10) in the space of polarization states with a fixed energy-momentum "k". The number operator $D^0_{ij}(k) \equiv a_i^\dagger(k) a_j(k)$. Then the expectation value of this number operator is defined by

$$\langle\, D^0_{ij}(k)\,\rangle \equiv \mathrm{tr}[\hat{\rho} D^0_{ij}(k)] = (2\pi)^3 \delta^3(0)(2k^0)\rho_{ij}(k). \tag{12}$$

And on the other hand, the time evolution of the operator $D^0_{ij}(k)$, considered in the Heisenberg picture, is

$$\frac{d}{dt} D^0_{ij}(k) = i[H, D^0_{ij}(k)], \tag{13}$$

where H is the full Hamiltonian. Taking the expectation value of both sides of above equation gives the Boltzmann equation (4) for the system's density matrix (as well as polarization states) which is a generalization of the usual classical Boltzmann equation for particle occupation numbers. By substituting Eq. (12) in Eq. (13), the time evolution of $\rho_{ij}(k)$ as well as Stokes parameters is given [1],

$$(2\pi)^3 \delta^3(0)(2k^0)\frac{d}{dt}\rho_{ij}(k) = i\langle \left[H^0_I(t); D^0_{ij}(k)\right]\rangle - \frac{1}{2}\int dt \langle \left[H^0_I(t); \left[H^0_I(0); D^0_{ij}(k)\right]\right]\rangle, \tag{14}$$

where $H^0_I(t)$ is the first order of the interacting Hamiltonian. The first term on the right-handed side is a forward scattering term, and the second one is a higher order collision term. In order to find effects of photon- neutrinos scattering on the polarization of the CMB, we start with following lagrangian

$$\pounds_I = \pounds_{QED} + \pounds_{e\nu}, \tag{15}$$

where the first term $\pounds_{QED}$ is the quantum electrodynamic lagrangian (QED), and the second term $\pounds_{e\nu}$ is the lagrangian of weak interaction containing electron-neutrino vertex. In context of standard model, there is no direct vertex for photon-neutrino however the first order of the interaction between photon-neutrino appears during one-loop interaction where photons and neutrinos both interact with electrons and weak gauge bosons (see Fig. 1).

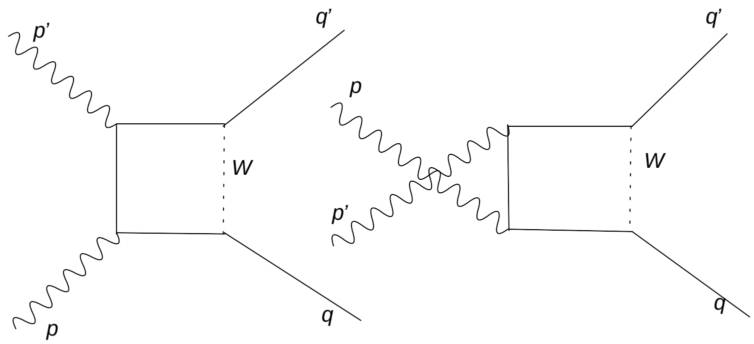

FIG. 1. The typical diagrams of photon-neutrino scattering is given in this plot.

We express the electromagnetic free gauge field A_μ in terms of plane wave solutions in the Coulomb gauge [36],

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2k^0} \left[a_i(k)\epsilon_{i\mu}(k)e^{-ik\cdot x} + a_i^\dagger(k)\epsilon_{i\mu}^*(k)e^{ik\cdot x}\right], \tag{16}$$

where $\epsilon_{i\mu}(k)$ are the polarization four-vectors and the index $i = 1, 2$, representing two transverse polarizations of a free photon with four-momentum k and $k^0 = |\mathbf{k}|$. $a_i(k)$ $[a_i^\dagger(k)]$ are the creation [annihilation] operators, which satisfy the canonical commutation relation as following

$$\left[a_i(k), a_j^\dagger(k')\right] = (2\pi)^3 2k^0 \delta_{ij}\delta^{(3)}(\mathbf{k} - \mathbf{k}'). \tag{17}$$

Also the free fermion field ψ is given:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_\mathbf{p}}} \sum_r \left[b_r(p)U_r(p)e^{-ip\cdot x} + d_r^\dagger(p)\mathcal{V}_r(p)e^{ip\cdot x}\right], \tag{18}$$

where U_r and $\mathcal{V}_r$ are Dirac spinors, b_r (d_r) and $b_r^\dagger$ $(d_r^\dagger)$ are creation and inhalation operators for fermions (anti-fermions), which satisfy following relations,

$$\left\{b_s(p), b_r^\dagger(p')\right\} = \left\{d_s(p), d_r^\dagger(p')\right\} = (2\pi)^3 \delta_{sr}\delta^{(3)}(\mathbf{p} - \mathbf{p}'). \tag{19}$$

Now by using Lagrangian (15) and Fig.1, first order photon-neutrino Hamiltonian interaction is given by

$$\begin{aligned} H_I^0 = &\int d\mathbf{q}d\mathbf{q}'d\mathbf{p}d\mathbf{p}'(2\pi)^3\delta^3(\mathbf{q}' + \mathbf{p}' - \mathbf{p} - \mathbf{q}) \\ &\times \exp[it(q'^0 + p'^0 - q^0 - p^0)] \left(b_{r'}^\dagger a_{s'}^\dagger(\mathcal{M}_1 + \mathcal{M}_2)a_s b_r\right) \end{aligned} \tag{20}$$

where

$$\begin{aligned} \mathcal{M}_1 + \mathcal{M}_2 = &-\frac{1}{8}e^2 g_w^2 \int \frac{d^4k}{(2\pi)^4} D_{\alpha\beta}(q-k)\bar{U}_{r'}(q')\gamma^\alpha(1-\gamma_5)S_F(k+p-p') \\ &\times \left[\not{\epsilon}_{s'} S_F(k+p)\not{\epsilon}_s + \not{\epsilon}_s S_F(k-p')\not{\epsilon}_{s'}\right] S_F(k)\gamma^\beta(1-\gamma_5)U_r(q), \end{aligned} \tag{21}$$

here $D_{\alpha\beta}$ and S_F are boson and fermion propagators, g_W is the weak coupling constant and our notation $d\mathbf{q} = d^3q/[(2\pi)^3 2q^0]$, the same for $d\mathbf{p}, d\mathbf{p}'$ and $d\mathbf{q}'$. By using above result H_I^0 and Eq.(14), we are ready to find the commutator in the photon-neutrino forward scattering term

$$[H_I^0, D_{ij}^0(\mathbf{k})] = \int d\mathbf{q}d\mathbf{q}'d\mathbf{p}d\mathbf{p}'(2\pi)^3\delta^3(\mathbf{q}' + \mathbf{p}' - \mathbf{p} - \mathbf{q})(\mathcal{M}_1 + \mathcal{M}_2)$$
$$\times (2\pi)^3[b_{r'}^\dagger b_r a_{s'}^\dagger a_s 2p^0 \delta_{is}\delta^3(\mathbf{k} - \mathbf{p}) - b_{r'}^\dagger b_r a_i^\dagger a_s 2p'^0 \delta_{js'}\delta^3(\mathbf{k} - \mathbf{p}')]. \tag{22}$$

On using the above expectation values and below operator expectation values [1],

$$\langle\, a_1 a2 ... b_1 b_2 ... \,\rangle = \langle\, a_1 a_2 ... \,\rangle \langle\, b_1 b_2 ... \,\rangle \tag{23}$$
$$\langle\, a_{s'}^\dagger(p') a_s(p) \,\rangle = 2p^0 (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}')\rho_{ss'}(\mathbf{x}, \mathbf{p}), \tag{24}$$
$$\langle\, b_{r'}^\dagger(q') b_r(q) \,\rangle = (2\pi)^3 \delta^3(\mathbf{q} - \mathbf{q}')\delta_{ss'}\frac{1}{2} n_\nu(\mathbf{x}, \mathbf{q}), \tag{25}$$

it follows that

$$i\langle[H_I^0, D_{ij}^0(\mathbf{k})]\rangle = -\frac{i}{16} e^2 g_w^2 \int d\mathbf{q}(\rho_{s'j}(\mathbf{k})\delta_{is} - \rho_{is}(\mathbf{k})\delta_{js'}) n_\nu(\mathbf{x}, \mathbf{q})$$
$$\times \int \frac{d^4 l}{(2\pi)^4} D_{\alpha\beta}(q - l)\bar{U}_r(q')\gamma^\alpha(1 - \gamma_5) S_F(l)$$
$$\times [\,\not{\epsilon}_{s'}, S_F(l + k)\not{\epsilon}_s + \not{\epsilon}_s\, S_F(l - k)\not{\epsilon}_{s'}\,]\, S_F(l)\gamma^\beta(1 - \gamma_5) U_r(q), \tag{26}$$

where integrating on l comes from the loop interaction of photon-neutrino and $n_\nu(\mathbf{x}, \mathbf{q})$ represents the number density of neutrinos of momentum $\mathbf{q}$ per unit volume (CνB distribution function). By helping Dimensional regularization and Feynman parameters, we go forward to obtain the leading order term of the right side of the above equation, then

$$i\langle[H_I^0, D_{ij}^0(\mathbf{k})]\rangle = -\frac{1}{16}\frac{1}{4\pi^2} e^2 g_w^2 \int d\mathbf{q}(\rho_{s'j}(\mathbf{k})\delta_{is} - \rho_{is}(\mathbf{k})\delta_{js'}) n_\nu(x, q)$$
$$\times \int_0^1 dy \int_0^{1-y} dz \frac{(1 - y - z)}{z M_W^2} \bar{U}_r(q)(1 + \gamma_5)(2z\not{q}\epsilon_{s'}.\epsilon_s$$
$$+\ 2z(\not{\epsilon}_{s'}\ \mathbf{q}.\epsilon_s\ + \not{\epsilon}_s\, \mathbf{q}.\epsilon_{s'}\,) + (3y - 1)\not{k}\,(\not{\epsilon}_s\not{\epsilon}_{s'}\ - \not{\epsilon}_{s'}\not{\epsilon}_s\,)\,) U_r(q). \tag{27}$$

Here we use the gamma-matrix identity $\not{A}\not{B} = 2A.B - \not{B}\not{A}$, the polarization vector properties $k.\epsilon_i = 0$ and $\epsilon_i.\epsilon_j = -\delta_{ij}$. Now every thing is ready to see the time evolution of stocks parameters as well as each polarization state of CMB photons. We are interested in V parameter which gives the contribution of the circular polarization, by considering Eqs.(14,27), dV/dt is given as following

$$\frac{dV(\mathbf{x}, \mathbf{k})}{dt} \approx \frac{1}{6}\frac{1}{(4\pi)^2}\frac{e^2 g_w^2}{M_W^2 k^0}\int d\mathbf{q}\ n_\nu(x, q)\bar{U}_r(q)(1 + \gamma_5)$$
$$\times [(\not{\epsilon}_1\, q.\epsilon_1 - \not{\epsilon}_2\, q.\epsilon_2) Q(\mathbf{k}) - (\not{\epsilon}_1\, q.\epsilon_2 + \not{\epsilon}_2\, q.\epsilon_1) U(\mathbf{k})]\, U_r(q), \tag{28}$$

we should remind that $U(k)$ is one of stocks parameters which represents linear polarization while U_r is Dirac spinor. We neglect the terms with $1/M_w^4$ order and smaller than one. In order to go further, let's introduce Dirac spinors and our frame work in more details. γ^μ, γ^5 and $U_r(q)$ are given

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \quad U_r(q) = \begin{pmatrix} \sqrt{p.\sigma}\xi^r \\ \sqrt{p.\bar{\sigma}}\xi^r \end{pmatrix}, \tag{29}$$

where $\sigma^\mu = (1, \vec{\sigma})$, $\bar{\sigma}^\mu = (1, -\vec{\sigma})$ and ξ is two-component spinor normalized to unity and also belove equations are useful

$$\bar{U}_r(q)\gamma^\mu U_s(q) = 2q^\mu \delta^{rs}, \quad \frac{1}{2}\sum_r \bar{U}_r(q)\gamma^\mu(1-\gamma^5)U_r(q) = 2q^\mu. \tag{30}$$

Then by making average on the spin of neutrinos, $\frac{1}{2}\sum_r$, and substituting above equations in Eq.(28), we arrive

$$\begin{aligned} \frac{dV(\mathbf{x},\mathbf{k})}{dt} &\approx \frac{\sqrt{2}}{3\pi k^0}\alpha\, G^F \int d\mathbf{q}\; n_\nu(x,q) \\ &\times [\,(q.\epsilon_1\; q.\epsilon_1 - q.\epsilon_2\; q.\epsilon_2)Q(\mathbf{k}) - (q.\epsilon_1\; q.\epsilon_2 + q.\epsilon_2\; q.\epsilon_1)U(\mathbf{k})\,]\,, \end{aligned} \tag{31}$$

where

$$G^F = \frac{\sqrt{2}}{8}\frac{g_W^2}{M_W^2} \approx 1.16\times 10^{-5}(GeV)^{-2}, \quad \alpha = \frac{e^2}{4\pi} = 1/137. \tag{32}$$

This equation contains an integration on neutrinos momentum which should determine. In next section, by introducing CνB distribution function and perturbations, we try to estimate the integration in (31).

IV. CνB DISTRIBUTION FUNCTION AND PERTURBATIONS

It is convenient to write the phase space distribution of neutrino $f_\nu(\vec{\mathbf{x}},\vec{\mathbf{q}},\tau)$ as a zeroth-order distribution $f_{\nu 0}(\vec{\mathbf{x}},\vec{\mathbf{q}},\tau)$ plus a perturbed piece $\Psi(\vec{\mathbf{x}},\vec{\mathbf{q}},\tau)$ as following [16–18]

$$f_\nu(\vec{\mathbf{x}},\vec{\mathbf{q}},\tau) = f_{\nu 0}[1+\Psi(\vec{\mathbf{x}},\vec{\mathbf{q}},\tau)], \tag{33}$$

where $\vec{\mathbf{q}} = q\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}$ indicates the direction of neutrino velocity. This phase space distribution evolves according to the Boltzmann equation. In terms of our variables $(\vec{\mathbf{x}}, q, \hat{\mathbf{n}}, \tau)$, we have

$$\frac{\partial f_\nu}{\partial\tau} + i\frac{q}{\varepsilon_\nu}(\vec{\mathbf{K}}\cdot\hat{\mathbf{n}})\Psi + \frac{d\ln f_{\nu 0}}{d\ln q}[\dot{\phi} - i\frac{\varepsilon_\nu}{q}(\vec{\mathbf{K}}\cdot\hat{\mathbf{n}})\psi] = 0 \tag{34}$$

where $\mathbf{K}$ (wave number) is the Fourier conjugate of $\mathbf{x}$, the collision terms on the RHS of Eq.(34) is neglected due to weak interactions of neutrino and two scalar potentials ϕ and ψ characterize metric perturbations, they appear in the line element as

$$ds^2 = a^2(\tau)\{-(1+2\psi)d\tau^2 + (1+2\phi)dx_i\, dx^j\}. \tag{35}$$

Notice, the terms in the Boltzmann equation depend on the direction of the momentum $\hat{n}$ only through its angle with $\vec{\mathbf{K}}$ ($\mu' = \hat{\mathbf{K}} \cdot \hat{n}$). The conformal Newtonian gauge (also known as the longitudinal gauge) advocated by Mukhanov et al.[37] is a particularly simple gauge to use for the scalar mode of metric perturbations. It should be emphasized that the conformal Newtonian gauge is a restricted gauge since the metric is applicable only for the scalar mode of the metric perturbations; the vector and the tensor degrees of freedom are eliminated from the beginning. By considering the collision-less Boltzmann equation (34) and expanding the angular dependence of the perturbation Ψ in a series of Legendre polynomials $P_l(\mu')$ as following

$$\Psi(\vec{\mathbf{K}}, q, \mu', \tau) = \sum_{l=0} (-i)^l (2l+1)\Psi_l(\vec{\mathbf{K}}, \tau) P_l(\mu'), \tag{36}$$

Instead of assuming a form for f_ν, it will be helpful taking the perturbations of energy density $\delta\rho_\nu$, pressure δP_ν, energy flux $\delta T^0_{\nu i}$, and shear stress as following [17]

$$\begin{aligned} \delta\rho_\nu &= 4\pi a^{-4} \int q^2\, dq\, \varepsilon_\nu\, f_{\nu 0} \Psi_0, \qquad \delta P_\nu = \frac{4\pi}{3}\, a^{-4} \int q^2\, dq\, \frac{q^2}{\varepsilon_\nu} f_{\nu 0}\Psi_0 \\ (\bar{\rho}_\nu + \bar{P}_\nu)\theta_\nu &= 4\pi a^{-4} \int q^2\, dq\; q\, f_{\nu 0}\Psi_1, \quad (\bar{\rho}_\nu + \bar{P}_\nu)\sigma_\nu = \frac{8\pi}{3}\, a^{-4} \int q^2\, dq\, \frac{q^2}{\varepsilon_\nu}\, f_{\nu 0}\Psi_2, \end{aligned} \tag{37}$$

where $\varepsilon_\nu = (q^2 + a^2\, m_\nu^2)^{1/2}$ ($m_\nu \simeq 0.1$eV) and a is scalar factor. Next we can rewrite Eq.(31) in terms of Ψ_l and μ (indicates the angle between CMB photons direction and $\vec{\mathbf{K}}$) as following

$$\begin{aligned} &\frac{1}{(2\pi)^3} \left(\left(\hat{\mathbf{K}} \cdot (\epsilon_1 - \epsilon_2)\right)^2 Q - 2\hat{\mathbf{K}} \cdot \epsilon_1\; \hat{\mathbf{K}} \cdot \epsilon_2\, U \right) \int q^2\, dq\, d\Omega\, \mu'^2\, \frac{q^2}{\varepsilon_\nu}\, f_{\nu 0} \Psi(\vec{\mathbf{K}}, q, \mu', \tau), \\ &= \frac{1}{(2\pi)^3} \left(\left(\hat{\mathbf{K}} \cdot (\epsilon_1 - \epsilon_2)\right)^2 Q - 2\hat{\mathbf{K}} \cdot \epsilon_1\; \hat{\mathbf{K}} \cdot \epsilon_2\, U \right) \frac{4\pi}{3} \int q^2\, dq\, \frac{q^2}{\varepsilon_\nu}\, f_{\nu 0}[\Psi_0 - 2\Psi_2], \\ &\simeq \left(\left(\hat{\mathbf{K}} \cdot (\epsilon_1 - \epsilon_2)\right)^2 Q - 2\hat{\mathbf{K}} \cdot \epsilon_1\; \hat{\mathbf{K}} \cdot \epsilon_2\, U \right) \tilde{\rho}_\nu \left[\frac{\delta P_\nu}{\tilde{\rho}_\nu} - 2\sigma_\nu \right] \end{aligned} \tag{38}$$

where

$$\begin{aligned} \dot{\Psi}_0 &= -\frac{qK}{\varepsilon_\nu}\Psi_1 - \dot{\phi}\frac{d\ln f_{\nu 0}}{d\ln q} \\ \dot{\Psi}_1 &= \frac{qK}{3\varepsilon_\nu}(\Psi_0 - 2\Psi_2) + \frac{\varepsilon_\nu\, K}{3q}\psi\frac{d\ln f_{\nu 0}}{d\ln q} \\ \dot{\Psi}_l &= \frac{qK}{(2l+1)\varepsilon_\nu}(l\Psi_{l-1} - (l+1)\Psi_{l+1}), \quad l \geq 2, \end{aligned} \tag{39}$$

and

$$\tilde{\rho}_\nu = \frac{1}{2\pi^2}\, a^{-4} \int q^2\, dq\, \varepsilon_\nu\, f_{\nu 0}, \qquad \tilde{P}_\nu = \frac{1}{6\pi^2}\, a^{-4} \int q^2\, dq\, \frac{q^2}{\varepsilon_\nu}\, f_{\nu 0}, \tag{40}$$

The evolution equations derived for neutrino perturbations can be solved numerically once the initial perturbations are specified. By starting the integration at early times when a given K-mode is still outside the horizon $K\tau << 1$ and implementing very basic iso-curvature and adiabatic initial conditions given by [17], we can obtain an estimation for the value of the perturbations. The value of $\eta_\nu = \frac{\delta P_\nu}{\tilde{\rho}_\nu} - 2\sigma_\nu$ depends on time or red-shift and the wave number K, but for simplicity we consider the time average value of this quantity at $K = K_* = 0.05/Mpc$ which is given by

$$\bar{\eta}_\nu =< [\frac{\delta P_\nu}{\tilde{\rho}_\nu} - 2\sigma_\nu] >= \frac{1}{\tau_0 - \tau_{lss}} \int_{\tau_0}^{\tau_{lss}} d\tau\, [\frac{\delta P_\nu}{\tilde{\rho}_\nu} - 2\sigma_\nu(\tau)](\tau, K_*) \simeq 0.3\frac{\Delta T}{T}\Big|_\nu \leq 10^{-5}, \tag{41}$$

where τ_{lss} indicates the time at last scattering surface and τ_0 is present time. Next by using Eqs.(38), (41), the time evolution of V mode is given

$$\begin{aligned} \frac{dV(\mathbf{x}, \mathbf{k})}{dt} \approx & \frac{\sqrt{2}}{3\pi}\, \frac{1}{k^0} \alpha\, G^F\, \tilde{\rho}_\nu(\mathbf{x}) \bar{\eta}_\nu \\ & \times \left(\left(\hat{\mathbf{K}} \cdot (\epsilon_1 - \epsilon_2) \right)^2 Q - 2\hat{\mathbf{K}} \cdot \epsilon_1\; \hat{\mathbf{K}} \cdot \epsilon_2\, U \right). \end{aligned} \tag{42}$$

We go forward by considering $k^0 \approx T_\gamma$ and also to avoid from the angular distribution of each mode

$$V(\mathbf{x}, k) = \int \frac{d\Omega}{4\pi} V(\mathbf{x}, \mathbf{k}), \quad k = |\mathbf{k}| = k^0, \tag{43}$$

where $d\Omega$ is the differential solid angle. We consider $Q(\mathbf{x}, k)$, $U(\mathbf{x}, k)$ and $I(\mathbf{x}, k)$ modes in the same way as well as $V(\mathbf{x}, k)$. Then

$$\frac{dV}{dt}(\mathbf{x}, k) \approx \frac{\sqrt{2}}{3\pi}\, \frac{1}{k^0} \alpha\, G^F\, \tilde{\rho}_\nu(\mathbf{x}) \bar{\eta}_\nu\, (C_U + C_Q) \tag{44}$$

where

$$C_U = -2 \int \frac{d\Omega}{4\pi} (\hat{\mathbf{K}}.\hat{\epsilon}_1\; \hat{\mathbf{K}}.\hat{\epsilon}_2) \tag{45}$$

$$C_Q = \int \frac{d\Omega}{4\pi}\, (\hat{\mathbf{K}}.\hat{\epsilon}_1\; \hat{\mathbf{K}}.\hat{\epsilon}_1 - \hat{\mathbf{K}}.\hat{\epsilon}_2\; \hat{\mathbf{K}}.\hat{\epsilon}_2) Q(\mathbf{x}, \mathbf{k}). \tag{46}$$

To estimate the V mode, we integrate over time $\int dt = \int dz\, a/H(z)$, where the redshift $z \in [0, 10^3]$, the Hubble function $H(z) = H_0[\Omega_M(z+1)^3 + \Omega_\Lambda)]^{1/2}$ for $\Omega_M \simeq 0.3$, $\Omega_\Lambda \simeq 0.7$ and $H_0 = 75$ km/s/Mpc, and the temperature $T_{\gamma,\nu} = T_{0,\gamma,\nu}(1+z)$ [$T_{0,\gamma} \approx 2.725K^\circ = 2.349 \times 10^{-4}$eV =

$(0.511\mathrm{cm})^{-1}]$ in the standard cosmology [16]. And also $\tilde{\rho}_\nu = \tilde{\rho}^0_\nu F(z)$ where $\tilde{\rho}^0_\nu \simeq m_\nu n^0_\nu(\mathbf{x}) \sim m_\nu 112/cm^3$ is today's CνB density and

$$F(z) = \frac{4\pi}{\tilde{\rho}^0_\nu}(1+z)^4 \int q^2\, dq\, \left(q^2 + (1+z)^{-2}\, m_\nu^2\right)^{1/2} f_{\nu 0}. \tag{47}$$

Finally V is given by

$$V \approx \frac{\sqrt{2}}{3\pi}\frac{1}{k^0}\alpha\, G^F\, \tilde{\rho}^0_\nu\, \bar{\eta}_\nu \int_0^{1000} \frac{dz}{(1+z)^2}\frac{F(z)}{H(z)}(C_U + C_Q), \tag{48}$$

where we can substitute

$$H_0^{-1} \approx 6\times 10^{41}(GeV)^{-1}, \quad T_{0,\nu} \approx 1.67\times 10^{-13} GeV, \quad T_{0,\gamma} \approx 2.34\times 10^{-13} GeV, \tag{49}$$

into equation (48) and integrate on redshift z. By assuming the independence $(C_U + C_Q)$ from redshift, we arrive

$$V(\mathbf{x}, k) \approx \frac{1}{2}\left(\frac{T_{0,\gamma}}{k}\right)\left(\frac{m_\nu}{0.1eV}\right)\left(\frac{n^0_\nu(\mathbf{x})}{112/cm^3}\right)\bar{\eta}_\nu\,(C_U + C_Q), \tag{50}$$

where $k \simeq T_{0,\gamma}$ is the values of average energy of CMB at present universe. Now we need to have some estimations of C_U and C_Q as well as U and Q polarization modes. First approximation for these quantities is given by knowing that $V/I < \delta T/T$ [38], this implies that $\bar{\eta}_\nu\,(C_U + C_Q)/I$ should be smaller than 10^{-7}. Let's investigate C_U and C_Q more precisely. A Fourier transform over the spatial dependence $\mathbf{x}$ of the equation (50) gives

$$V(\mathbf{K}, k) \approx 10^2\left(\frac{T_{0,\gamma}}{k}\right)\left(\frac{m_\nu}{0.1eV}\right)\left(\frac{n^0_\nu}{112/cm^3}\right)\bar{\eta}_\nu\,\left(C_U(\mathbf{K},k) + C_Q(\mathbf{K},k)\right). \tag{51}$$

For the rest of paper by neglecting the effects of tensor perturbations and assuming θ' as the angel between the $\mathbf{K}$ mode perturbation and the photon direction $\mathbf{k}$ [see Fig.(2)], then we have

$$C_U(\mathbf{K}, k) = -\int \frac{d\Omega'}{4\pi}(\sin^2\theta'\, \sin 2\phi')U(\mathbf{K},\mathbf{k}) \tag{52}$$

$$C_Q(\mathbf{K}, k) = \int \frac{d\Omega'}{4\pi}(\sin^2\theta'\, \cos 2\phi')Q(\mathbf{K},\mathbf{k}), \tag{53}$$

where $k = |\mathbf{k}|$. Then ones can expand the incident intensities U and Q in spherical harmonics (around $\mathbf{K}$ direction) as following

$$U(\mathbf{K},\mathbf{k}) = \sum_{lm} \mathfrak{u}_{lm}(k) Y_{l,m}(\theta',\, \phi'), \quad Q(\mathbf{K},\mathbf{k}) = \sum_{lm} \mathfrak{q}_{lm}(k) Y_{l,m}(\theta',\, \phi'). \tag{54}$$

By using equations (50)-(54), the photon-neutrino scattering generates circular polarization CMB from initially linear polarized CMB if this linear intensity (C_U and C_Q) at a given point as a

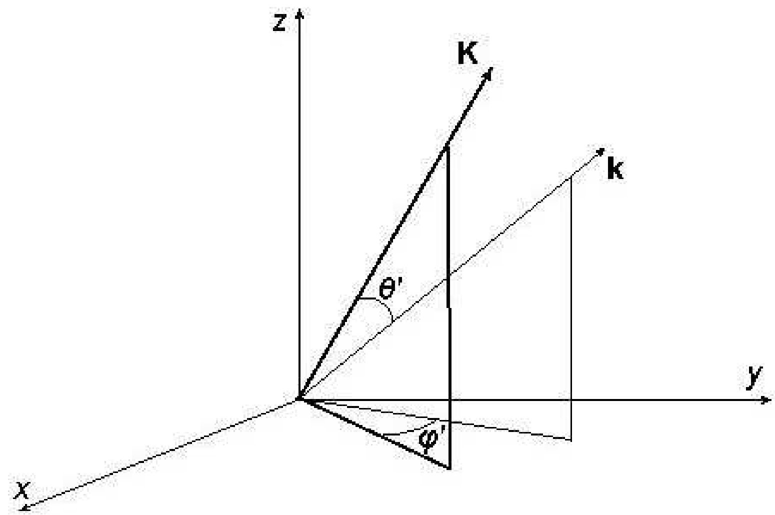

FIG. 2. Angles and directions to determine the angular dependence of equation (51) is given in above plot.

function of direction has no-zero component Y_{22}.

$$C_U(\mathbf{K},k) = -\frac{2}{\pi}(\sqrt{\frac{2\pi}{15}})\mathfrak{u}_{22}(k) \tag{55}$$

$$C_Q(\mathbf{K},k) = \frac{2}{\pi}(\sqrt{\frac{2\pi}{15}})\mathfrak{q}_{22}(k). \tag{56}$$

As a result of this calculation, the exactly value of V-parameter in each $\mathbf{K}$-mode depends to the quadruple components of the incident intensity distribution and the CνB perturbations δP_ν and σ_ν.

V. THE ANISOTROPY OF THE PHOTON DISTRIBUTION.

The pervious section has devoted for the right side of the Boltzman equation, collision and scattering terms. In this section we discuses the lift side term which describes the propagation of photons in the background space-time. As discussed in [1–3, 16], the first order deviation from flat space-time in the metric perturbation leads to an incommodiously for the photon CMB distribution function which is necessary to generate unpolarized CMB due to Thomson scattering. To go further, we expand the photon distribution function $f_\gamma(\mathbf{k},\mathbf{x})$ about its zero-order Bose-Einstein value as following

$$\begin{aligned} f_\gamma(\mathbf{k},\mathbf{x},t) &= [\exp\{\frac{k}{T_\gamma(t)(1+\Theta(\mathbf{k},\mathbf{x},t))}\}-1]^{-1} \\ &\simeq f_\gamma^0 + k\frac{\partial f_\gamma^0}{\partial k}\Theta(\mathbf{k},\mathbf{x},t), \end{aligned} \tag{57}$$

where $f^0_\gamma = [\exp\{\frac{k}{T_\gamma}\} - 1]^{-1}$ and $\Theta = \delta T/T$. Here the zero-order temperature $T_\gamma(t)$ is a function of time only, not space. The perturbation to the distribution function is characterized by $\Theta(\mathbf{k}, \mathbf{x}, t)$. In the smooth zero-order universe, photons are distributed homogeneously, that is, T_γ is independent of $\vec{x}$ and isotropically, so T_γ is independent of the direction of propagation $\mathbf{k}$. We decomposed the perturbation Θ into a sum over Legendre polynomials,

$$\Theta(\mathbf{x}, \mathbf{k}, t) = \sum_l \Theta_l(k)\mathcal{P}_l(\mu), \tag{58}$$

where μ is the dot product of the wave vector $\mathbf{k}$ and the direction of propagation [16] where in conjugate coordinate

$$\Theta(\mathbf{K}, \mathbf{k}, t) = \sum_l \Theta_l(k)\mathcal{P}_l(\mathbf{K}.\mathbf{k}). \tag{59}$$

By using Eqs. (57,61), we can expand the intensity of the CMB radiation as

$$I(\mathbf{K}, \mathbf{k}, t) \simeq I_0(k, t) + 4k\frac{\partial I_0}{\partial k}\Delta I(\mathbf{K}, \mathbf{k}, t) + ... \simeq= I_0(k, t)(1 + 4\Theta(\mathbf{K}, \mathbf{k}, t)), \tag{60}$$

where $\Delta I(\mathbf{K}, \mathbf{k}, t)$ depends on $\Theta(\mathbf{K}, \mathbf{k}, t)$, so this quantity can represent into a sum over Legendre polynomials like Eq.(61),

$$\Delta I(\mathbf{K}, \mathbf{k}, t) = \sum_l \Delta I_l(k)\mathcal{P}_l(\mathbf{K}.\mathbf{k}). \tag{61}$$

On the other hand, at first glance, Compton scattering is a perfect mechanism for producing polarized radiation. But to produce polarized radiation, the incoming radiation must have a nonzero quadruple component [1, 16].

$$\begin{aligned} &\frac{d}{dt}\Delta Q(\mathbf{K}, \mathbf{k}, t) \approx \sigma_T \bar{n}_e \Delta I_2(\mathbf{K}, k, t) \sin^2\theta' \cos 2\phi', \\ &\frac{d}{dt}\Delta U(\mathbf{K}, \mathbf{k}, t) \approx -\sigma_T \bar{n}_e \Delta I_2(\mathbf{K}, k, t) \sin^2\theta' \sin 2\phi' \end{aligned} \tag{62}$$

where θ' and ϕ' are determined in Fig.(2) and

$$\Delta Q(\mathbf{K}, \mathbf{k}, t) = \left(4k\frac{\partial I_0}{\partial k}\right)^{-1} Q(\mathbf{K}, \mathbf{k}, t), \quad \Delta U(\mathbf{K}, \mathbf{k}, t) = \left(4k\frac{\partial I_0}{\partial k}\right)^{-1} U(\mathbf{K}, \mathbf{k}, t), \tag{63}$$

here $\Delta Q(\Delta U)$ are dimensionless quantities which should be smaller than $\delta T/T$, $\bar{n}_e$ is the electron number density and $I_0(k, t)$ is unpolarized intensity of the CMB which depends on T^4_γ. By substituting equation (62) into (48) and choosing the spherical coordinates for $\hat{\mathbf{k}}$ with axis in the $\mathbf{K}$ direction (as shown in Fig.(2)), we obtain the reasonable estimation for V-mode in terms of Θ_2 at present time t_0

$$\begin{aligned} \Delta V(\mathbf{K}, k, t_0) \approx 3 \times 10^{-3} \left(\frac{T_{0,\gamma}}{k}\right)\left(\frac{m_\nu}{0.1eV}\right)\left(\frac{n^0_\nu}{112/cm^3}\right) \bar{\eta}_\nu\, \sigma_T H_0^{-1} \bar{n}_e\, \Delta I_2(\mathbf{K}, k, t_0) \\ \times \int_0^{1000} \frac{dz}{(1+z)^2}\frac{F(z)}{H(z)/H_0}\int_z^{1000}\frac{dz'}{H(z)/H_0}, \end{aligned} \tag{64}$$

where ΔV is defined as dimensionless quantity as well as $\Delta Q(\Delta U)$. Then by considering relevant values for quantities which appear in above equation and making integrations, we obtain

$$\Delta V(\mathbf{K},k,t_0) \approx 30\left(\frac{T_{0,\gamma}}{k}\right)\left(\frac{m_\nu}{0.1eV}\right)\left(\frac{n_\nu^0}{112/cm^3}\right)\left(\frac{\bar{n}_e}{0.1cm^{-3}}\right)\bar{\eta}_\nu\,\Delta I_2(\mathbf{K},k,t_0). \tag{65}$$

The above equation is given for each mode of $\mathbf{K}$, but we interest to measure the value of V-parameter in real space coordinate $\mathbf{x}$ (we have to make inverse Furrier transform of above equation) and then two point function $<\Delta V(\mathbf{x},k,t_0)\Delta V(\mathbf{x},k,t_0)>$ where

$$\Delta V(\mathbf{x},k,t_0) = \int \frac{d^3K}{(2\pi)^3}\,e^{i\mathbf{K}.\mathbf{x}}\Delta V(\mathbf{K},k,t_0) \tag{66}$$

Before doing this transformation, we introduced $\xi(\mathbf{K})$, which is a random variable used to characterize the initial amplitude of each mode of $\mathbf{K}$. It has the following statistical property

$$\left\langle \xi^*(\mathbf{K}')\xi(\mathbf{K})\right\rangle = (2\pi)^3\delta(\mathbf{K}'-\mathbf{K})\,p_s(K) \tag{67}$$

$p_s(K)$ is so called primordial scaler power spectrum (index "s" shows scaler perturbations). As discussed in [39], this quantity can be described as

$$p_s(K) = p_s(K_*)\left(\frac{K}{K_*}\right)^{n_s-1} \tag{68}$$

where $p_s(K_*)$ and n_* are determined at pivot scale $K_* \simeq 0.05/Mpc$. In general n_s and $p_s(K_*)$ depend on the pivot scale. As shown in above equation, the simplest case, neglecting a possible tensor component, the initial conditions are characterized by only two parameters n_s and $p_s(K_*)$. For simplicity, we consider scale-invariant (Harrison- Zel'dovich spectrum) case with $n_s \simeq 1$ and $p_s(K_*) \propto A_s\frac{2\pi^2}{K_*^3}$. However, in principle, other initial condition are also possible. The total value of two point correlation function of ΔV-mode can be written as

$$\begin{aligned} C^V &= <\Delta V(\mathbf{x},k,t_0)\Delta V(\mathbf{x},k,t_0)> \simeq \int \frac{d^3K}{(2\pi)^3}p_s(K)|\Delta V(\mathbf{K},k,t_0)|^2 \\ &\simeq 10^3\bar{\eta}_\nu^2\left(\frac{n_\nu^0(\mathbf{x})}{112/cm^3}\right)^2\left(\frac{m_\nu/0.1eV}{k/T_{0,\gamma}}\right)^2\left(\frac{\bar{n}_e}{0.1cm^{-3}}\right)^2 C_2^T, \end{aligned} \tag{69}$$

where

$$C_2^T = \int \frac{dK}{4\pi^2}\,K^2\,p_s(K)\,|\Delta I_2(\mathbf{K},k,t_0)|^2, \tag{70}$$

Here we obtain the two point correlation function of ΔV-mode as function two point correlation function of the angular power spectrum of temperature fluctuations [for more details about C_l^T, see [1, 2, 39]]. Finally from equation (41) and (69), the value of C^V approximately is given

$$C^V \simeq 10^{-6}\left(\frac{N_\nu}{3}\right)^2\left(\frac{\bar{\eta}_\nu}{10^{-5}}\right)^2\left(\frac{n_\nu^0(\mathbf{x})}{112/cm^3}\right)^2\left(\frac{m_\nu/0.1eV}{k/T_{0,\gamma}}\right)^2\left(\frac{\bar{n}_e}{0.1cm^{-3}}\right)^2 C_2^T, \tag{71}$$

where N_ν is the number of neutrino flavors. Here we estimate the maximum value of C^V in terms of two point correlation function of the angular power spectrum of temperature fluctuations C_2^T where the value of C_2^T is about a thousand micro Kelvin square [see for example [2, 39]]. As a result, the maximum value of C^V is about 10^{-6} of the quadruple component of the the temperature power spectrum C_2^T or larger then Nano-Kelvin square. Of course we can generalize the above calculation for other components. We should emphasize that we consider the contribution of anti-neutrino photon scattering too. The calculation shows that the contribution of anti-neutrino photon scattering is the same as neutrino photon scattering with the same sing [see appendix VIII], so in above calculation $n_\nu^0 \equiv n_\nu^0 + n_{\bar{\nu}}^0 \sim 112/cm^3$.

VI. AFTER THE LAST SCATTERING:

By considering photon-neutrino scattering at last scattering surface, we discuss and calculate the generation of the CMB's circular polarization. But we must remind during the propagation from the last scattering surface to us, CMB photons encounter large-scale structures and undergo significant changes due to effects related to structure formation [12]. The polarization modifications may occur during propagating in this large structure formation. On the other hand the presence of thermal electrons and large-scale diffuse synchrotron emission towards galaxy clusters suggest the presence of the large scale magnetic fields [44]. The presence of this large scale magnetic filed causes some modifications on the circular polarization of CMB due to the compton scattering which is discussed in [20–23].The generation of circular polarization in the process of transfer of CMB within a large scale magnetic field and structure, due to the presence of the electrons, is discussed in [20]. They show the V-mode is about 10^{-9} for $\lambda = 1cm$, $z = 1000$ and for length scale about 1 Mpc and number density of electrons about 0.1 per cm^3,

$$V(k) \propto 10^{-9}\frac{\bar{n}_e}{0.1cm^{-3}}(\frac{B}{10\mu G})^2(\frac{\lambda}{1cm})^3\frac{L}{1Mpc}, \tag{72}$$

where is smaller than the maximum value of V-mode due to photon-neutrino scattering discussed in past section. As shown in Eq. (65), the value of V-mode due to neutrino-photon has the linear dependence on the wavelength $\lambda = 1/k^0$ unlike the cubic dependence of the result of [20].

The electron-photon scattering generates the linear polarization of CMB from unpolarized CMB [1] but this process doesn't give any contribution for the CMB's circular polarization in the absence of magnetic field. The probability of the generation of circular polarization via compton scattering in presence of magnetic field is discussed in [21]. The maximum value of the V-mode is given by

$$V(k) \propto 10^5 \frac{\bar{n}_e}{0.1cm^{-3}}(\frac{B}{10\mu G})(\frac{\lambda}{1cm})^3 \frac{L}{1Mpc}(\frac{T_e}{m_e})^3, \tag{73}$$

where the maximum of $\frac{T_e}{m_e}$ is about 10^{-6}. By considering $\frac{T_e}{m_e}$, the maximum value of V-model is about 10^{-13} where is smaller than result is given in Eq. (65). Also the above equation has the linear dependence on magnetic field (unlike the result of [20]) and the cubic dependence on the wavelength (like the result of [20]) while in case of photon-neutrino scattering (65), the linear dependence on the wavelength appears. Of course we can determine the exact value for V- mode in Eqs. (65), (72) and (73) and we can be sure that the contribution of photon-neutrino scattering V is the dominant contribution because there are unknown parameters in each equation such as the value of the large scale magnetic field, $\delta T/T(\vec{x},\vec{k})$ as well as $I(\vec{x},\vec{k})$. But by considering relevant values for the large scale magnetic field and anisotropic, we can find that the contribution of photon-neutrino scattering to generate CMB's circular polarization is dominant.

Another effect of the large-scale structure and magnetic field which needs to be discussed may appear as the circular polarization convert to linear one due to photon-neutrino and Compton scattering, that means

$$\frac{dQ(U)}{dt}(\vec{k}) \propto V(\vec{k}) \tag{74}$$

First, we investigate the conversion of circular polarization to linear one via Compton scattering. The Compton scattering in the absent of magnetic field doesn't give any term like (74), see section IV in [1]. But the Compton scattering in the presence of magnetic field gives [21]

$$Q(U)(\vec{k}) \propto 10^5 \frac{\bar{n}_e}{0.1cm^{-3}}(\frac{B}{10\mu G})(\frac{\lambda}{1cm})^3 \frac{L}{1Mpc}(\frac{T_e}{m_e})^3 V(\vec{k}) \tag{75}$$

If we substitute the value of the $V(\vec{k})$ from equation (48) and the relevant value of parameters which appear in above equation, The maximum value of $Q(U)(\vec{k})$ becomes very small. So we can neglect the conversion of the circular to linear polarization via Compton scattering. To investigate the conversion of the circular to linear polarization via photon-neutrino scattering, we use Eqs. (14) and (27) which give

$$\begin{aligned} \frac{dQ(\mathbf{k})}{dt} \approx \frac{1}{6}\frac{1}{(4\pi)^2}\frac{e^2 g_w^2}{M_W^2 k^0} \int d\mathbf{q}\; n_\nu(x,q)[\bar{U}_r(q)(1+\gamma_5) \\ \times (\not{\epsilon}_1 q.\epsilon_1 - \not{\epsilon}_2 q.\epsilon_2)U_r(q)]V(\mathbf{k}). \end{aligned} \tag{76}$$

By substituting $V(\mathbf{k})$ from equation (48), the value of $Q(\mathbf{k})$ is proportional to $(G^F)^2$ which become very small. It is so negligible

VII. CONCLUSION.

In this letter, by approximately solving the first order of Quantum Boltzmann Equation for the density matrix of a photon ensemble, and time-evolution of Stokes parameters, we show that the linear polarizations of the CMB can convert to circular polarizations by scattering the CMB photon on cosmic neutrinos background CνB. The maximum value of the V-Stokes parameter in $\mathbf{K}$ direction is given by (65) at frequencies of a few GHz with the linear dependence on the wavelength and the CνB perturbations of pressure δP_ν and shear stress σ_ν. To have a measurable quantity for circular polarization, we calculate $C^V =< VV >$. By considering the average value of $\bar{\eta}_\nu$ about the fluctuation temperature $\bar{\eta}_\nu \le \delta T/T\big|_\nu$, the maximum value of C^V is about 10^{-6} of C_2^T angular power spectra or lager than Nano-kelvin square. We should mention, we only try to estimate analytically the value of circular polarization due to CMB and CνB scattering. But to have the exactly value, we should numerically solve Boltzmann equation for CMB and CνB in during a model for expansion universe which leave it here.

It is expected that the polarization data, which will become available with the Plank 2014 data, provide valuable information on the nature of the CMB anomalies (with resolution in range of Nano-kelvin) [40] and there are also other high resolution polarization experiments such as ACTPol [41], PIXIE [42], SPIDER [43]. Of course we can not exactly compare the effects of photon-neutrino scattering on the circular polarization of the CMB with other interactions without any knowledge about the pattern and distribution of the initial linear polarization but we can discuss about its maximum. Our value for circular polarization (65) is lager than the one which is given by [20] and comparable with the bound reported in [5]. In the work reported in [20], they show that Faraday conversion process during the propagation of polarized CMB photons through regions of the large-scale structure containing magnetized relativistic plasma, such as galaxy clusters, will lead to a circularly polarized contribution of order 10^{-9} at frequencies of $10GHz$ with the cubic dependence on the wavelength (our result has the linear dependence on the wavelength) and square dependence on the large scale magnetic field. Refs. [22, 23] have argued that the presence of a large-scale magnetic field prior to equality can affect the photon-electron and the photon-ion scattering, this leads to the radiation becomes circularly polarized and the induced VV angular power spectra have been computed. Their results are comparable with the result of [20]. In [21], the effect of the large-scale of magnetic field on the compton scattering has been discussed which leads to generate circular polarization for CMB. The band on V-mode reported in [21] is very smaller than our result and has the cubic dependence on the wavelength (our result has the linear dependence on the wavelength), the linear dependence on the large scale magnetic field and the cubic dependence on T_e/m_e. The band on V-mode reported in [20] can be larger than our result if we have the large-scale of magnetic field in order of $B > 10mG$. Also in [21], they show that CMB polarization acquires a small degree of circular polarization when the quantum electrodynamic sector of standard model is extended by Lorentz non-invariant operators as well as non-commutativity theory. These results contain Lorentz non-invariant and non-commutativity parameters which we don't know the exactly values of them. In Ref. [24], it has been shown that circular polarizations of radiation fields can be generated from the effective Euler-Heisenberg Lagrangian in order of 10^{-10}K which is very small. The transformation plane of the polarization into circular polarization via photon-photon interactions mediated by the neutral hydrogen background, $\gamma+\gamma+atom \to \gamma+\gamma+atom$, through completely forward processes, has been discussed in [25]. The ratio of circular to plane polarization intensities V/Q is predicted to be at the level of several times 10^{-5} for some regions of angular size less than 1/300 and with large plane polarizations. So the value of the circular polarization (from CMB and CνB forward scattering) seems to be large enough to detect. And the other hand, as we already mentioned that the detection of CνB is hardly possible due to the weak interaction of neutrinos with matter and due to their low energy, however the measuring of CMB's circular polarization may give us the good experimental testifier for cosmic neutrino back ground CνB.

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VIII. APPENDIX: ANTI-NEUTRINO-PHOTON SCATTERING

The first order of photon - anti-neutrino Hamiltonian interaction is given

$$\begin{aligned} H_I^0 &= \int d\mathbf{q}d\mathbf{q}'d\mathbf{p}d\mathbf{p}'(2\pi)^3\delta^3(\mathbf{q}'+\mathbf{p}'-\mathbf{p}-\mathbf{q}) \\ &\times \exp[it(q'^0+p'^0-q^0-p^0)]\left(d_r a^\dagger_{s'}(\mathcal{M}_1'+\mathcal{M}_2')a_s\, d^\dagger_{r'}\right) \end{aligned} \tag{77}$$

where the amplitude of the diagram shown in Fig.(3) and its crossing is given by

$$\begin{aligned} \mathcal{M}_1'+\mathcal{M}_2' &= \frac{1}{8}e^2g_w^2\int\frac{d^4l}{(2\pi)^4}D_{\alpha\beta}(l-q)\bar{\mathcal{V}}_r(q)\gamma^\alpha(1-\gamma_5)S_F(p'-p-l) \\ &\times\left[\not\epsilon_s\, S_F(-l-p)\not\epsilon_{s'}\ +\not\epsilon_{s'}\, S_F(p'-l)\not\epsilon_{s'}\right] S_F(-l)\gamma^\beta(1-\gamma_5)\mathcal{V}_{r'}(q'), \end{aligned} \tag{78}$$

Next we substitute above equation into (22) then we make average of this exsection value of anti-neutrino-photon forward scattering (similar to equation (26) for photon neutrino scattering) and then we have

$$\begin{aligned} i\langle[H_I^0,D_{ij}^0(\mathbf{k})]\rangle &= +\frac{i}{16}e^2g_w^2\int d\mathbf{q}(\rho_{s'j}(\mathbf{k})\delta_{is}-\rho_{is}(\mathbf{k})\delta_{js'})n_{\bar\nu}(x,q) \\ &\times\int\frac{d^4l}{(2\pi)^4}D_{\alpha\beta}(-q+l)\bar{\mathcal{V}}_r(q)\gamma^\alpha(1-\gamma_5)S_F(-l) \\ &\times[\not\epsilon_s\, S_F(-l-k)\not\epsilon_{s'}\ +\not\epsilon_{s'}\, S_F(-l+k)\not\epsilon_s\ ]\, S_F(-l)\gamma^\beta(1-\gamma_5)\mathcal{V}_r(q). \end{aligned} \tag{79}$$

By helping Dimensional regularization and Feynman parameters, we will go forward to obtain the leading order term of the right side of the above equation, then

$$\begin{aligned} i\langle[H_I^0,D_{ij}^0(\mathbf{k})]\rangle &= -\frac{1}{16}\frac{1}{4\pi^2}e^2g_w^2\int d\mathbf{q}(\rho_{s'j}(\mathbf{k})\delta_{is}-\rho_{is}(\mathbf{k})\delta_{js'})n_{\bar\nu}(x,q) \\ &\times\int_0^1dy\int_0^{1-y}dz\frac{(1-y-z)}{zM_W^2}\bar{\mathcal{V}}_r(q)(1+\gamma_5)(2z\not q\epsilon_{s'}.\epsilon_s \\ &+\ 2z(\not\epsilon_{s'}\,\mathbf{q}.\epsilon_s\ +\not\epsilon_s\,\mathbf{q}.\epsilon_{s'}\ )-(3y-1)\not k\,(\not\epsilon_s\not\epsilon_{s'}\ -\not\epsilon_{s'}\not\epsilon_s\ )\,)\mathcal{V}_r(q). \end{aligned} \tag{80}$$

By use the gamma-matrix identity, the polarization vector properties $k.\epsilon_i=0$ and $\epsilon_i.\epsilon_j=-\delta_{ij}$. Finally dV/dt is given as following

$$\begin{aligned} \frac{dV(\mathbf{x},\mathbf{k})}{dt} &\approx +\frac{1}{6}\frac{1}{(4\pi)^2}\frac{e^2g_w^2}{M_W^2k^0}\int d\mathbf{q}\ n_{\bar\nu}(x,q)\bar{\mathcal{V}}_r(q)(1+\gamma_5) \\ &\times[(\not\epsilon_1\ q.\epsilon_1\ -\not\epsilon_2\,q.\epsilon_2)Q(\mathbf{x},\mathbf{k})-(\not\epsilon_1\,q.\epsilon_2+\not\epsilon_2\,q.\epsilon_1)U(\mathbf{x},\mathbf{k})]\,\mathcal{V}_r(q), \end{aligned} \tag{81}$$

And also belove equations are useful:

$$\bar{\mathcal{V}}_r(q)\gamma^\mu\mathcal{V}_s(q)=2q^\mu\delta^{rs},\quad \frac{1}{2}\sum_r\bar{\mathcal{V}}_r(q)\gamma^\mu(1\pm\gamma^5)\mathcal{V}_r(q)=2q^\mu. \tag{82}$$

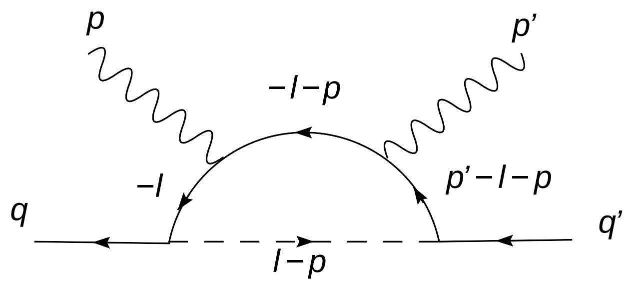

FIG. 3. The typical diagram of photon-antineutrino scattering is given in this plot.

By substituting above equation into (81) and assuming $\nu_{\bar{\nu}} = n_\nu$, we arrive to

$$\frac{dV(\mathbf{x},\mathbf{k})}{dt}\Big|_{\bar{\nu}} = \frac{dV(\mathbf{x},\mathbf{k})}{dt}\Big|_{\nu} \tag{83}$$

This means that the anti-neutrino photon scattering affects the circular polarization of CMB as same as the neutrino photon scattering with the same value and sing.