Mon. Not. R. astr. Soc. (1980) **190**, 413–420

The velocity of clusters of galaxies relative to the microwave background. The possibility of its measurement

R. A. Sunyaev and Ya. B. Zeldovich *Academy of Sciences, USSR, Space Research Institute, Profsoyuznaja 88, 117810 Moscow, USSR*

Received 1979 May 31

Summary. Observation of the microwave background intensity and polarization in the direction of clusters of galaxies permit us, in principle, to measure their peculiar velocities relative to the background radiation.

1 Introduction

It is well known according to the X-ray observations that rich clusters of galaxies contain a large amount of hot intergalactic gas (Forman *et al.* 1978; Cooke *et al.* 1978). Hence clusters of galaxies may be considered as fully ionized gas clouds with high temperature and finite optical depth with respect to Thomson scattering.

The scattering of 3 K microwave background radiation on these clouds of intergalactic gas opens the possibility of measuring the velocity of each cloud relative to the coordinate frame determined by the background radiation.

The radial motion changes the observed radiation temperature $\Delta T/T \sim -(v_r/c)\tau$ in the direction to the cloud. Here

$$\tau = \int_{-\infty}^{+\infty} \sigma_T N_e \, dr$$

is the optical depth of the cloud with respect to Thomson scattering, v_r is the radial component of the peculiar velocity of the cloud (positive v_r corresponds to a recession velocity exceeding that corresponding to Hubble's law).

In addition there is the possibility, at least in principle, of also finding the tangential component of the peculiar velocity of the cloud v_t. There are effects of the order of $0.1\beta_t^2\tau$ and $1/40\,\beta_t\tau^2$ (with $\beta_t = v_t/c$) in the microwave background polarization in the direction to the cloud. The linear polarization distribution depends on the direction of motion.

The effect proportional to $\tau^2\beta_t$ leads to different signs of polarization in the leading (with respect to the direction of motion) and trailing parts of the cloud.

Measurements of the large-scale anisotropy of the microwave background gave a motion velocity $V = 390\,\mathrm{km\,s^{-1}}$ for the solar system (Smoot, Gorenstein & Muller 1977). Using data on the rotation of the Galaxy Smoot *et al.* (1977) also estimated the velocity of the centre of the Galaxy relative to the microwave background to be $V = 600\,\mathrm{km\,s^{-1}}$. This is a rather small velocity. It is of great importance for cosmology to know what are typical peculiar velocities of other galaxies, clusters of galaxies and rich clusters of galaxies.

Earlier we analysed the distortion of the microwave background spectrum due to scattering on free hot electrons in such clusters of galaxies (Sunyaev & Zeldovich 1970, 1972). The scattering on hot electrons redistributes the photons over the spectrum. In the longwave (Rayleigh–Jeans, RJ) region of the spectrum, the radiation intensity and its brightness temperature decrease in the same ratio

$$\frac{\Delta T}{T}=\frac{\Delta J_\nu}{J_\nu}=-\frac{2kT_e}{m_e c^2}\tau, \tag{1}$$

where T_e is the electron temperature.

In the shortwave (Wien) region they increase. Below we write (see curves with $v_r = 0$ in Fig. 1) the dependence of the intensity distortions on the dimensionless frequency $x = h\nu/kT_r$ in the direction to the cluster (Zeldovich & Sunyaev 1969; see also Gould & Raphaeli 1978)

$$\left(\frac{\Delta J_\nu}{J_\nu}\right)_1 = 2\frac{kT_e}{m_e c^2}\tau\,\frac{x\exp(x)}{\exp(x)-1}\left\{\frac{x}{2\,th(x/2)}-2\right\} \tag{2}$$

or

$$\left(\frac{\Delta T}{T}\right)_1=\frac{2kT_e}{m_e c^2}\tau\left\{\frac{x}{2\,th(x/2)}-2\right\}.$$

The formulae are independent of the redshift. They are valid even for $z \gtrsim 1$.

The difference between equation (2) and the simple formula (1) increases with x. It is equal to 0.26 per cent with $x = 0.18$, $\lambda = 3$ cm; 3.7 per cent with $x = 0.67$, $\lambda = 8$ mm and 53 per cent with $x = 2.7$, $\lambda = 2$ mm.

The experimental study of this effect in the RJ spectral region was begun by Pariysky (1973) and Gull & Northover (1975). Now the effect has been definitely observed in the direction of the richest clusters of galaxies. For example Lake & Partridge (1977) (the observations were carried out on wavelength $\lambda = 9$ mm) and Birkinshaw *et al.* (1978) ($\lambda = 2.6$ cm) found a brightness diminution in the direction of the cluster of galaxies Abell 2218. According to Lake & Partridge (1977) (see also Shallwich & Wielebinsky (1979)) this diminution is equal to $\Delta T = (-2.65 \pm 0.23) \times 10^{-3}$ K. Let us assume that the gas temperature in this cloud of intergalactic gas is equal to $kT_e = 5$ keV or is of the same order as in the x-ray clusters of galaxies Coma, Virgo etc. (Mitchell & Culhane 1977), having similar velocity dispersion of the galaxies $\sqrt{\overline{V^2}} \sim 1500\ \mathrm{km\,s^{-1}}$ as in Abell 2218. Then from equation (1) it is easy to estimate the optical depth of the cloud:

$$\tau=\frac{1}{2}\frac{\Delta T}{T}\frac{m_e c^2}{kT_e}\approx\frac{1}{20}. \tag{3}$$

2 Radial component of the velocity

We turn now to the peculiar motion of the cluster. It was mentioned in our paper (Sunyaev & Zeldovich 1972) that the motion of the cloud as a whole relative to the background must lead (due to the Doppler effect) to an additional change of the radiation intensity $(\Delta J_\nu/J_\nu)_2$ and temperature $(\Delta T/T)_2$ in the direction to the cloud. Small effects superpose linearly,

$$\left(\frac{\Delta T}{T}\right)_{\text{total}} = \left(\frac{\Delta T}{T}\right)_1 + \left(\frac{\Delta T}{T}\right)_2.$$

If τ is small

$$\left(\frac{\Delta T}{T}\right)_2 = -\frac{v_r \tau}{c}; \quad \left(\frac{\Delta J_\nu}{J_\nu}\right)_2 = -\frac{x\exp(x)}{\exp(x)-1}\,\frac{v_r}{c}\tau. \tag{4}$$

(In the rest system of the cloud the background radiation has anisotropy of dipole type. After Thomson scattering radiation loses its dipole component, and in first approximation becomes isotropic. Doppler transformation to the observer's frame makes the scattered radiation field anisotropic, of dipole type again, increasing the effective temperature and total flux in the direction of cloud motion.)

The sign of the effect depends on the velocity direction. The amplitude of the temperature perturbation ΔT does not depend on the frequency; the perturbation of the intensity ΔJ_ν depends on x, but does not change its sign, contrary to the case of the thermal effect (2). Therefore, in principle, two measurements at different frequencies are enough to separate two effects of interest for us*

$$\frac{\Delta T}{T}(\nu) = \left[\frac{\Delta T}{T}(\nu)\right]_1 + \left(\frac{\Delta T}{T}\right)_2.$$

The index 1 corresponds to the thermal effect, and index 2 to the effect arising from radial peculiar velocity of the cloud. Fig. 1 shows that at wavelength $\lambda = 2$ mm the thermal effect decreases by a factor of 2 in comparison with the Rayleigh–Jeans region of the spectrum

$$\left(\frac{\Delta T}{T}\right)_1 \bigg|_{2\,\text{mm}} = -\frac{kT_e}{m_e c^2}\tau.$$

Observations at wavelength $\lambda = 2$ mm are possible from the Earth's surface (Fabbri *et al.* 1978). The thermal effect changes its sign in the Wien region of the spectrum, where $x = 3.83$ or $\lambda = 1.3$ mm. Unfortunately observations in the band $\lambda < 1.3$ mm are possible only from outside the Earth's atmosphere and this becomes a problem for the next decade. The curves with $v_r = \pm 3000\,\text{km s}^{-1}$ illustrate the change of the ΔT dependence on x or λ with simultaneous action of thermal effect and overall radial motion. They are normalized to equal effects in the Rayleigh–Jeans region.

In the Rayleigh–Jeans region the amplitudes of the effects are of the same order when

$$\left|\frac{v_r}{c}\right| = \frac{2kT_e}{m_e c^2} = \frac{1}{50} \quad \text{or} \quad v_r \approx 6000\,\text{km s}^{-1}.$$

The observations of Abell 2218 (Lake & Partridge 1977; Birkinshaw, Gull & Northover 1978; Shallwich & Wielebinski 1979) give an estimate of the upper limit to the radial peculiar velocity of the cluster

$$|v_r| < c\frac{\Delta T}{T}\frac{1}{\tau} = 6000\,\text{km s}^{-1}. \tag{5}$$

*We do not take into account here other effects, connected with non-thermal radio sources in the cluster, difference of the microwave background spectrum from the black body one, free–free emission of the cloud etc.

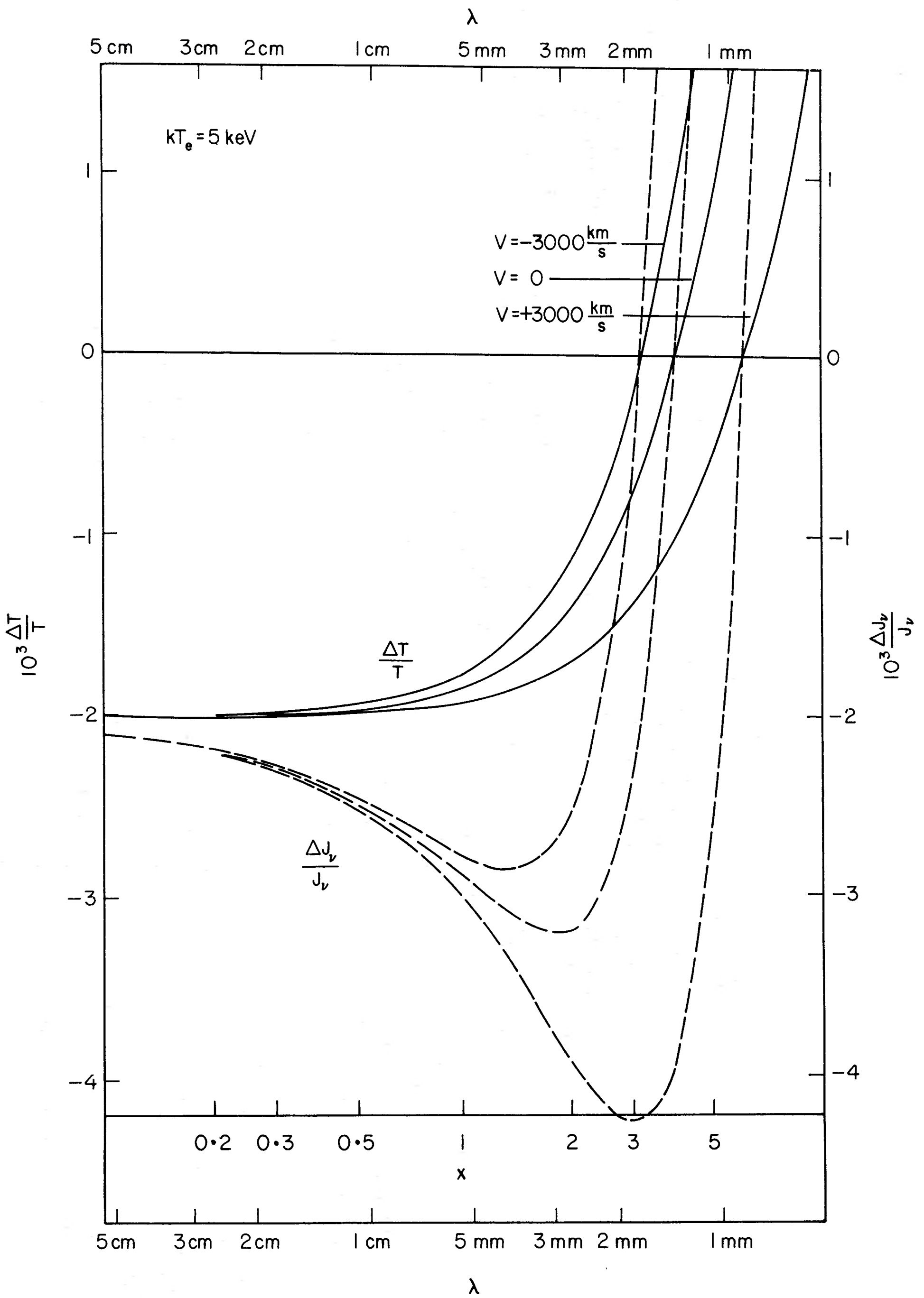

Figure 1. The change of microwave background radiation temperature in the direction to a cluster of galaxies containing hot intergalactic gas with $kT_e = 5.11$ keV: (a) the cloud moves towards the observer. The peculiar velocity is equal to $v_r = -3000\,\mathrm{km\,s^{-1}}$, $\tau = 0.2$. (b) The cluster is at rest relative to the microwave background; $\tau = 0.1$. (c) The peculiar recession velocity $v_r = 3000\,\mathrm{km\,s^{-1}}$, $\tau = 1/15$. All cases give equal effect in the Rayleigh–Jeans part of the spectrum.

Let us recall that the Hubble recession velocity for $z = 0.17$ is equal to $50\,000\,\mathrm{km\,s^{-1}}$. Precision of the distance and of the Hubble constant† estimates are much worse than 10 per cent. Therefore the usual methods of velocity estimating using the Doppler effect do not allow us to find a peculiar velocity of the order of $6000\,\mathrm{km\,s^{-1}}$ if the distance to the object

† Cavaliere, Danese & De Zotti (1977) and Silk & White (1978) proposed a new method of determining the Hubble constant based on the thermal effect.

corresponds to $z = 0.17$. Even the result (5) is of great interest in connection with the problems of large-scale structure of the Universe and of the spectrum of density perturbations in the Friedmannian model. Measurements of $\Delta T/T$ at $\lambda = 3$ cm and $\lambda = 2$ mm, with precision of the order of several per cent of the effect itself, could reveal velocities of the order of our Galaxy's motion velocity relative to the microwave background.

3 Tangential component of the peculiar velocity

The problem of the tangential velocity measurement is more difficult. It is possible to find v_t using the observation of the microwave background polarization of the scattered radiation. Existing equipment allows one to measure the microwave radiation polarization with precision of the order of 10^{-4} (Lubin & Smoot 1979). However, one can hope that the precision might be improved by two orders of magnitude. There are two different polarizational effects.

3.1 SCATTERING ON A SINGLE ELECTRON

For a single electron moving relative to the background the microwave radiation becomes anisotropic, transformed by the Doppler effect. In each direction the Planckian form of spectrum is conserved; however, the radiation temperature depends on the angle θ between the line of sight and the direction of motion according to the well-known formula (Landau & Lifshitz 1962)

$$T_0 = T_r \frac{\sqrt{1-\beta^2}}{1+\beta\mu_0} \tag{6}$$

where $\beta = v/c$; T_0, $\mu_0 = \cos\theta_0$ and the angle θ_0 are measured in the rest frame of the electron. The expansion of the formula for T_0 has, in the second order of β, a quadrupole component in the angular distribution

$$T_0 = T_r[1 - \beta\mu_0 + \beta^2(\mu_0^2 - 1/3) + \ldots]. \tag{7}$$

It is well known that the scattering of the unpolarized collimated light beam leads to linear polarization of the scattered light. We turn to the general case of a smooth anisotropic intensity distribution.

Polarization is a tensor quantity. Therefore it is proportional to the quadrupole term but does not depend on the dipole and higher terms in the intensity angular distribution.

After scattering on a free electron the linear polarization must arise in the direction perpendicular to the direction of its motion. The electric vector is perpendicular to the plane defined by the vectors of the motion velocity V and line of sight. If radiation angular distribution has a form

$$J = J_r[1 + a\mu_0 + \mathscr{C}(\mu_0^2 - 1/3) + \ldots),$$

the degree of linear polarization is equal to

$$p = \frac{J_\parallel - J_\perp}{J_\parallel + J_\perp} = 0.1\mathscr{C}\mu_0^2.$$

It does not depend on a and coefficients of higher harmonics. In Rayleigh–Jeans region $\mathscr{C} = \beta_t^2$. Therefore $p = 0.1\beta_t^2$.

In the case of small τ there is unscattered light in the direction to the cloud. The scattered light contributes only a small fraction τ to the total intensity. Therefore

$$p = 0.1\tau\beta_t^2.$$

The transformation to the observer's reference frame does not change this value. With $v_t = 6000\,\mathrm{km\,s^{-1}}$, $\beta_t = 0.02$ and $\tau = 1/20$ we have $p = 2 \times 10^{-6}$. This value must be compared with the observed $\Delta T/T \approx 10^{-3}$. It is difficult to find β_t using this method. With $x \gtrsim 1$ we must take into account the non-linear dependence of intensity at a given frequency on the temperature

$$J_\nu = B_\nu(T_0) + \left.\frac{dB_\nu}{dT}\right|_{T_0} (T - T_0) + \frac{1}{2}\left.\frac{d^2B_\nu}{dT^2}\right|_{T_0} (T - T_0)^2. \tag{8}$$

Using this expansion it is easy to find the coefficient $\mathscr{C}$ and the degree of polarization for any frequency.

3.2 EFFECT OF FINITE OPTICAL DEPTH

There is another effect connected with two consecutive scatterings, i.e. with finite optical depth of the cloud. Although τ is small, it is possible that effects of the order of τ^2 may become observable. These effects were absent when we considered scattering on a single electron.

In the rest frame of the cloud the unperturbed radiation field is in the first approximation on β,

$$T_0 = T_r(1 - \beta\mu_0), \quad J_0 = J_r(1 - k\beta\mu_0),$$

where coefficient k depends on x (see formula (8)).

Due to symmetry of the angular dependence of Thomson scattering relative to the change $\theta_0 \to \pi - \theta_0$, $\mu_0 \to -\mu_0$ the scattered radiation becomes isotropic, and the dipole term $k\beta\mu$ vanishes, in the scattered radiation field.

It is obvious that the isotropic radiation field does not change under the action of Thomson scattering (the scattered photons exactly compensate infalling photons).

Therefore we must take into account only the uncompensated decrease, due to scattering, of the dipole component (measured in a frame moving with the cloud) of the radiation.

For example, for a plasma sphere with constant electron density within its boundary, we obtain in the first approximation

$$J_0 = J_r(1 - k\beta\mu_0) \quad \text{for} \quad -1 < \mu_0 < 0,$$

and

$$J_0 = J_r[1 - k\beta\mu_0 \exp(-\tau\mu_0)] \quad \text{for} \quad 0 < \mu_0 < 1,$$

where

$$\tau = 2\sigma_T N_e R.$$

Such a distribution contains the second harmonics with coefficient $\sim k\beta\tau$. The scattered light becomes polarized, the polarization having opposite signs on different sides of the cloud.

Taking into account that only a small fraction τ of the detected radiation is scattered in the cloud we obtain finally the maximal degree of polarization

$$p_{\max} = \pm\frac{k}{40}\tau^2\beta_t.$$

In the Rayleigh–Jeans region $k = 1$ and with $\tau = 0.1$, $v_t = 3000\,\mathrm{km\,s^{-1}}$, $\beta_t = 0.01$ we find $p_{\max} = \pm 2.5 \times 10^{-6}$.

3.3 OTHER SOURCES OF POLARIZATION

The existence of non-thermal radio sources in clusters and radiation from intergalactic gas (of any nature) also contribute to the detected radiation flux and its polarization. Scattering

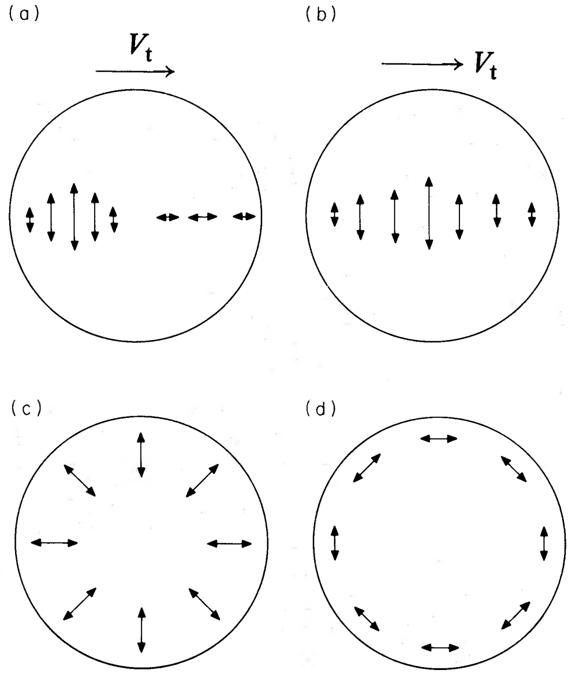

Figure 2. Predicted polarization of the microwave background in the direction to the cluster of galaxies. (a) Effect, connected with finite optical depth of the cloud and proportional to $\beta_t\tau^2$. (b) Effect, connected with scattering on a single electron and proportional to $\beta_t^2\tau$. (c) Polarization, connected with the thermal effect. (d) Polarization due to existence of the unpolarized radio radiation sources inside the cloud.

of such radiation in the cloud must lead to its polarization. The sign and degree of polarization must depend on the angle between the line of sight and the direction to the centre of the cloud. This effect is well known for optically thick objects (Chandrasekhar 1950; Sobolev 1967). Obviously it occurs also for $\tau < 1$. In this case it is proportional to τ and to the intensity of the source radiation. The coefficient is smaller than $1/40$.

The thermal effect decreases the intensity in the Rayleigh–Jeans region. Its influence can be compared with a 'a negative radiation source' inside the cloud. Therefore even in the case $V=0$ it can lead to the polarization of the microwave background in the direction to the cloud. In this case

$$p_{\max} = \gamma \frac{kT_e}{m_e c^2} \tau^2$$

and γ is smaller than $1/40$.

The polarization distribution over the cloud projection for different cases is given in Fig. 2.

4 Concluding remarks

Observations of the polarization distribution over the cloud image in several wavelengths must give information of extraordinary importance for cosmology. Possibly there are clusters of galaxies, superclusters and even intergalactic gas bridges between them, with $1/20 < \tau < 1$. Second-order effects become observable in this case. They do not depend on the electron temperature.

Even with $\tau = 1/20$ the cloud must scatter several per cent of the radiation of the galaxies and must appear as a diffuse radiation source in all spectral bands (in optical light, in particular). Narrow spectral lines must be absent in this scattered light due to the Doppler effect

$$\frac{\Delta\nu}{\nu} \sim \sqrt{\frac{2kT_e}{m_e c^2}} \sim \frac{1}{7}.$$

Let us remark also, that formulae (2) are derived using the Kompaneets (1956) differential equation, which was obtained assuming multiple scatterings. In reality, τ is small and we consider the contribution of the small fraction of single-scattered photons. However, the formulae (2) are exact for $x < \sqrt{m_e c^2/2kT_e} \approx 7$ (Sunyaev & Zeldovich 1980a, b). Important deviations from the formulae (2) occur only for $x > 7$ or $\lambda < 0.7$ mm, where the observations are very difficult.

References

Birkinshaw, M., Gull, S. F. & Northover, K. J. E., 1978. *Mon. Not. R. astr. Soc.*, **185**, 245.
Cavaliere, A., Danese, L. & De Zotti, G., 1977. *Astr. Astrophys.*, **217**, 6.
Chandrasekhar. S., 1950. *Radiative Transfer*, Oxford.
Cooke, B. A., Ricketts, M. J., Maccacaro, T., Pye, J. P., Elvis, M., Watson, M. G., Pounds, K. A., McHardy, I., Maccagni, D., Seward, F. D., Page, C. G. & Turner, M., 1978. *Mon. Not. R. astr. Soc.*, **182**, 489.
Fabbri, R., Melchiorri, F. & Natale, V., 1978. *Astrophys. Space Sci.*, **59**, 223.
Forman, F., Jones, C., Cominsky, L., Julian, P., Murray, S., Peters, G., Tananbaum, H. & Giacconi, R., 1978. *Astrophys. J. Suppl.*, **38**, 357.
Gould, R. J. & Raphaeli, Y., 1978. *Astrophys. J.*, **219**, 12.
Gull, S. F. & Northover, K. J. E., 1975. *Mon. Not. R. astr. Soc.*, **173**, 535.
Kompaneets, A. S., 1956. *Soviet Phys. JETP*, **31**, 876.
Lake, G. & Partridge, R. B., 1977. *Nature*, **270**, 502.
Landau, L. D. & Lifshiftz, E. M., 1962. *Theory of Fields*, Fizmatgiz, Moscow.
Lubin, P. M. & Smoot, G. F., 1979. *Phys. Rev. Lett.*, **42**, 129.
Mitchell, R. J. & Culhane, J. L., 1977. *Mon. Not. R. astr. Soc.*, **178**, 75.
Pariysky, Yu. N., 1973. *Soviet Astr. AJ*, **50**, 453.
Shallwich, D. & Wielebinski, R., 1979. *Astr. Astrophys.*, **71**, L15.
Silk, J. & White, S. D. M., 1978. *Astrophys. J.*, **226**, L103.
Smoot, G. F., Gorenstein, M. V. & Muller, R. A., 1977. *Phys. Rev. Lett.*, **39**, 14, 898.
Sobolev, V. V., 1967. *Course of Theoretical Astrophysics*, Nauka, Moscow.
Sunyaev, R. A. & Zeldovich, Ya. B., 1970. *Astrophys. Space Sci.*, **7**, 3.
Sunyaev, R. A. & Zeldovich, Ya. B., 1972. *Comments Astrophys. Space Phys.*, **4**, 173.
Sunyaev, R. A. & Zeldovich, Ya. B., 1980a. Astrophysics and space physics, *Soviet Sci. Rev.*, in press.
Sunyaev, R. A. & Zeldovich, Ya. B., 1980b. *A. Rev. Astr. Astrophys.*, in press.
Zeldovich, Ya. B. & Sunyaev, R. A., 1969. *Astrophys. Space Sci.*, **4**, 301.