PHYSICAL REVIEW D **94**, 065033 (2016)

Generation of circular polarization of gamma ray bursts

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(Received 22 May 2016; published 22 September 2016)

The generation of the circular polarization of gamma ray burst (GRB) photons is discussed in this paper via their interactions with astroparticles in the presence or absence of background fields such as magnetic fields and noncommutative space-time geometry. Solving the quantum Boltzmann equation for GRB photons as a photon ensemble, we discuss the generation of circular polarization (as Faraday conversion phase shift $\Delta\phi_{\mathrm{FC}}$) of GRBs in the following cases: (i) intermediate interactions, i.e., the Compton scattering of GRBs in the galaxy cluster magnetic field and in the presence of noncommutative space-time geometry, as well as the scattering of GRBs in the cosmic neutrino background (CNB) and cosmic microwave background (CMB); (ii) interactions with particles and fields in shockwaves, i.e., the Compton scattering of GRBs with accelerated charged particles in the presence of magnetic fields. We found that (i) after shockwave crossing, the greatest contribution of $\Delta\phi_{\mathrm{FC}}$ for energetic GRBs (of the order of GeV and larger) comes from GRB-CMB interactions, but for low-energy GRBs the contributions of the Compton scattering of GRBs in the galaxy cluster magnetic field dominate; (ii) in shockwave crossing, the magnetic field has significant effects on converting a GRB's linear polarization to a circular one, and this effect can be used to better understand the magnetic profile in shockwaves. The main aim of this work is to study and measure the circular polarization of GRBs for a better understanding of the physics and mechanism of the generation of GRBs and their interactions before reaching us.

DOI: 10.1103/PhysRevD.94.065033

I. INTRODUCTION

Gamma ray burst is short-lived, transient (milliseconds to hundreds of seconds) γ-ray radiation that is the most energetic explosion in the Universe, taking place at cosmological distances. The early phase of GRB emission is called prompt emission, which is followed by an afterglow, long-lasting emission in the x-ray, optical, and radio wavelengths [1,2].

A certain degree of linear polarization has been measured in several GRB afterglows (see [3] for review) and also circular polarization has been recently measured in GRB121024A of about 0.6% [4]. For synchrotron emission, the polarization level depends on (i) the local magnetic field orientation, (ii) the geometry of the emitting region with respect to the line of sight, and (iii) the electron pitch-angle distribution. The magnetic and geometric properties of the jet could be investigated by studying the afterglow polarization [5]. In this article, we present an estimation of circular polarization for GRBs by considering different configurations (i.e., magnetic fields or geometries) and interactions. For each different scenario, we study the conditions for reaching the maximal and minimal polarizations and we estimate their values. We discuss the implications of our results to the microphysics of GRB afterglows in view of recent polarization measurements. Low degrees of linear polarization are predicted in theoretical models [6–8].

In Ref. [9], the linear polarization of the prompt emission of GRB 140206A was investigated and the linear polarization level of the second peak of this GRB was constrained to be larger than 28% at 90% confidence level. Degrees of linear polarization of $P = 28^{+4}_{-4}\%$ in the immediate afterglow of the Swift γ-ray burst GRB120308A have been reported [10]. Four minutes after its discovery, the polarization level decreased to $P = 16^{+5}_{-4}\%$ over the subsequent 10 minutes. The first claim of detection of circular polarization in GRB afterglow radiation was recently reported in the optical afterglow of GRB 121024A, which was detected by the Swift satellite in 2012 [11]. The linear polarization of this burst was measured at the level of ~4% and the circular polarization was detected at the level of ~0.6% [4]. This shows that the circular polarization was intrinsic to the afterglow of GRB 121024A.

The circular polarization of GRB can be generated due to several interactions, such as Compton scatterings in noncommutative space-time [12], photon propagation in the presence of magnetic fields [13,14], photon scattering with neutrinos [15], photon-photon scattering, and so on [16]. Generally, photon interaction with a charged particle causes the outgoing photon to be linearly polarized, whereas there is no physical mechanism to generate a circular polarization

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by the mentioned interaction. However, a degree of circular polarization can be generated by Compton scattering in the presence of a large-scale background magnetic field. By the definition known as Faraday conversion [13,17], the linear polarization of the CMB can be converted to circular polarization under the mentioned mechanisms. The converted Stokes-V contribution is given as

$$\dot{V} = 2U\frac{d\Delta\phi_{\rm FC}}{dt}, \tag{1}$$

where $\Delta\phi_{\rm FC}$ is called the Faraday conversion phase shift. In the following, we estimate this phase shift for GRB due to several interactions.

The paper is organized as follows. In Sec. II, the density operator and Stokes parameters are briefly reviewed. In Sec. III, we calculate the evolution of Stokes parameters via photon-photon scattering using the Euler-Heisenberg effective Lagrangian. In Sec. IV, we investigate the Faraday conversion phase shift of GRB photons by considering CMB-GRB and CNB-GRB interactions, as well as Compton scattering in an electromagnetic background and in noncommutative space-time. In Sec. V, we estimate the Faraday conversion phase shift of GRBs due to their interactions in internal and external shock waves in both fireball and fireshell scenarios of GRBs. Finally, the results summary and conclusion are given in the last section.

II. STOKES PARAMETERS AND BOLTZMANN EQUATION

The density operator of an ensemble of photons in terms of the Stokes parameter is defined as [18]

$$\hat{\rho} = \frac{1}{{\rm tr}(\hat{\rho})}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\rho_{ij}(\mathbf{k})D_{ij}(\mathbf{k}),$$
$$\rho = \frac{1}{2}\begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}, \tag{2}$$

where I is the total intensity, Q and U describe linear polarization, and V indicates circular polarization:

$$I = \rho_{11}+\rho_{22}, \tag{3}$$

$$Q = \rho_{11}-\rho_{22}, \tag{4}$$

$$U = \rho_{12}+\rho_{21}, \tag{5}$$

$$V = i(\rho_{12}-\rho_{21}). \tag{6}$$

$\rho_{ij}(\mathbf{k})$ is the density matrix, which is related to the photon number operator $D^0_{ij}(\mathbf{k}) \equiv a^\dagger_i(\mathbf{k})a_j(\mathbf{k})$. The expectation value of the number operator is defined as

$$\langle D^0_{ij}(\mathbf{k})\rangle \equiv {\rm tr}[\hat{\rho}D^0_{ij}(\mathbf{k})] = (2\pi)^3\delta^3(0)(2k^0)\rho_{ij}(\mathbf{k}). \tag{7}$$

The time evolution of the operator $D^0_{ij}(\mathbf{k})$, considered in the Heisenberg picture, is

$$\frac{d}{dt}D^0_{ij}(\mathbf{k}) = i[H, D^0_{ij}(\mathbf{k})], \tag{8}$$

where H is the full Hamiltonian. The evolution equation, i.e., quantum Boltzmann equation, for the density matrix is given by

$$(2\pi)^3\delta^3(0)(2k^0)\frac{d}{dt}\rho_{ij}(\mathbf{k})$$
$$= i\langle[H^0_I(t), D^0_{ij}(\mathbf{k})]\rangle - \frac{1}{2}\int dt\langle[H^0_I(t),[H^0_I(0), D^0_{ij}(\mathbf{k})]]\rangle, \tag{9}$$

where $H^0_I(t)$ is the first order of the interaction Hamiltonian. The first term on the right-hand side is a forward scattering term, and the second one is the higher order collision term.

III. GRB POLARIZATION DUE TO THE EULER-HEISENBERG EFFECTIVE LAGRANGIAN

The photon-photon scattering in the vacuum does not occur in classical electrodynamics, owing to the fact that Maxwell's equations are linear. The Euler-Heisenberg Lagrangian describes the nonlinear dynamics of electromagnetic fields in the vacuum. In this Lagrangian four photons interact through one vertex, which in the original theory is mediated by an electron loop. The Euler-Heisenberg effective Lagrangian is given by [19,20]

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90m_e^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right], \tag{10}$$

where the first term $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is the classical Maxwell Lagrangian. We express the electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the field strength $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, and the gauge field A_μ in terms of plane wave solutions

$$A_\mu(x) = \int\frac{d^3\mathbf{k}}{(2\pi)^3 2k^0}[a_i(k)\epsilon_{i\mu}(k)e^{-ik\cdot x} + a^\dagger_i(k)\epsilon^*_{i\mu}(k)e^{ik\cdot x}], \tag{11}$$

where $\epsilon_{i\mu}(k) = (0, \vec{\epsilon}_i(\mathbf{k}))$ are the polarization four-vectors chosen to be real and the index $i = 1,\ 2$ represents two transverse polarizations of a free photon with four-momentum k and $k^0 = |\mathbf{k}|$. The $a_i(k)$ and $a^\dagger_i(k)$ are creation and annihilation operators, which satisfy the canonical commutation relation

$$[a_i(k), a_j^\dagger(k')] = (2\pi)^3 2k^0 \delta_{ij} \delta^{(3)}(\mathbf{k} - \mathbf{k}'). \tag{12}$$

The Euler-Heisenberg effective Lagrangian (10) gives the interaction Hamiltonian $H_I^0(t)$ in Eq. (9):

$$H_I^0 = H_I^{\rm EH} = -\frac{\alpha^2}{90 m_e^4} \int d\mathbf{p_1} d\mathbf{p_2} d\mathbf{p_3} d\mathbf{p_4} (2\pi)^3 \delta^3(\mathbf{p_1} + \mathbf{p_2} - \mathbf{p_3} - \mathbf{p_4}) [a_{s'}^\dagger(p_4) a_{r'}^\dagger(p_3) \mathcal{M} a_s(p_2) a_r(p_1)]. \tag{13}$$

In the quantum Boltzmann equation (9), $H_I^{\rm EH}$ is of the order of α^2; therefore we consider the forward scattering term only and neglect the higher order collision term. First we use the Wick theorem to arrange all creation operators to the left and all annihilation operators to the right. In this way the expectation value of the interaction Hamiltonian and number operator commutator in the forward scattering term is

$$\langle [H_I^0, D_{ij}^0(\mathbf{k})] \rangle = -\frac{\alpha^2}{90 m_e^4} \int d\mathbf{p_1} d\mathbf{p_2} d\mathbf{p_3} d\mathbf{p_4} (2\pi)^3 \delta^3(\mathbf{p_1} + \mathbf{p_2} - \mathbf{p_3} - \mathbf{p_4}) \mathcal{M} \langle [a_{s'}^\dagger(p_4) a_{r'}^\dagger(p_3) a_s(p_2) a_r(p_1), a_i^\dagger(k) a_j(k)] \rangle, \tag{14}$$

where $d\mathbf{p}_i \equiv \frac{d^3\mathbf{p}_i}{(2\pi)^3 2p_i^0}$, $i = 1, 2, 3, 4$ and

$$\begin{aligned}
\mathcal{M} &= 2\left(g^{\mu\mu'} g^{\nu\nu'} g^{\alpha\alpha'} g^{\beta\beta'} + \frac{7}{4} \epsilon^{\mu\nu\mu'\nu'} \epsilon^{\alpha\beta\alpha'\beta'}\right) \mathcal{I}, \\
\mathcal{I} &= (p_{1\mu} \epsilon_{r\nu}(p_1) - p_{1\nu} \epsilon_{r\mu}(p_1))(p_{2\mu'} \epsilon_{s\nu'}(p_2) - p_{2\nu'} \epsilon_{s\mu'}(p_2))(p_{3\alpha} \epsilon_{r'\beta}(p_3) - p_{3\beta} \epsilon_{r'\alpha}(p_3))(p_{4\alpha'} \epsilon_{s'\beta'}(p_4) - p_{4\beta'} \epsilon_{s'\alpha'}(p_4)) \\
&\quad + (p_{1\mu} \epsilon_{r\nu}(p_1) - p_{1\nu} \epsilon_{r\mu}(p_1))(p_{3\mu'} \epsilon_{r'\nu'}(p_3) - p_{3\nu'} \epsilon_{r'\mu'}(p_3))(p_{2\alpha} \epsilon_{s\beta}(p_2) - p_{2\beta} \epsilon_{s\alpha}(p_2))(p_{4\alpha'} \epsilon_{s'\beta'}(p_4) - p_{4\beta'} \epsilon_{s'\alpha'}(p_4)) \\
&\quad + (p_{1\mu} \epsilon_{r\nu}(p_1) - p_{1\nu} \epsilon_{r\mu}(p_1))(p_{4\mu'} \epsilon_{s'\nu'}(p_4) - p_{4\nu'} \epsilon_{s'\mu'}(p_4))(p_{2\alpha} \epsilon_{s\beta}(p_2) - p_{2\beta} \epsilon_{s\alpha}(p_2))(p_{3\alpha'} \epsilon_{r'\beta'}(p_3) - p_{3\beta'} \epsilon_{r'\alpha'}(p_3)).
\end{aligned} \tag{15}$$

Here we use the commutation relation [18]

$$\langle a_m^\dagger(p') a_n(p) \rangle = 2p^0 (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}') \rho_{mn}(\mathbf{p}), \tag{16}$$

operator expectation value

$$\begin{aligned}
\langle a_{s_1'}^\dagger(p_1') a_{s_1}(p_1) a_{s_2'}^\dagger(p_2') a_{s_2}(p_2) \rangle &= 4 p_1^0 p_2^0 (2\pi)^6 \delta^3(\mathbf{p}_1 - \mathbf{p}_1') \delta^3(\mathbf{p}_2 - \mathbf{p}_2') \rho_{s_1' s_1}(\mathbf{p}_1) \rho_{s_2' s_2}(\mathbf{p}_2) \\
&\quad + 4 p_1^0 p_2^0 (2\pi)^6 \delta^3(\mathbf{p}_1 - \mathbf{p}_2') \delta^3(\mathbf{p}_2 - \mathbf{p}_1') \rho_{s_1' s_2}(\mathbf{p}_2) [\delta_{s_2' s_1} + \rho_{s_2' s_1}(\mathbf{p}_1)],
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
&\langle [a_{s'}^\dagger(p_4) a_{r'}^\dagger(p_3) a_s(p_2) a_r(p_1), a_i^\dagger(k) a_j(k)] \rangle \\
&\quad = 2p_1^0 2p_2^0 2k^0 (2\pi)^9 \{ \delta^3(\mathbf{p_1} - \mathbf{k}) \delta_{ri} [\delta^3(\mathbf{p_2} - \mathbf{p_4}) \delta^3(\mathbf{p_3} - \mathbf{k}) \rho_{s's}(\mathbf{p_2}) \rho_{r'j}(\mathbf{k}) + \delta^3(\mathbf{p_2} - \mathbf{p_3}) \delta^3(\mathbf{p_4} - \mathbf{k}) \rho_{r's}(\mathbf{p_2}) \rho_{s'j}(\mathbf{k})] \\
&\qquad + \delta^3(\mathbf{p_2} - \mathbf{k}) \delta_{si} [\delta^3(\mathbf{p_1} - \mathbf{p_4}) \delta^3(\mathbf{p_3} - \mathbf{k}) \rho_{s'r}(\mathbf{p_1}) \rho_{r'j}(\mathbf{k}) + \delta^3(\mathbf{p_3} - \mathbf{p_1}) \delta^3(\mathbf{p_4} - \mathbf{k}) \rho_{s'j}(\mathbf{k}) \rho_{r'r}(\mathbf{p_1})] \\
&\qquad - \delta^3(\mathbf{p_3} - \mathbf{k}) \delta_{r'j} [\delta^3(\mathbf{p_1} - \mathbf{p_4}) \delta^3(\mathbf{p_2} - \mathbf{k}) \rho_{is}(\mathbf{p_2}) \rho_{s'r}(\mathbf{p_1}) + \delta^3(\mathbf{p_4} - \mathbf{p_2}) \delta^3(\mathbf{p_1} - \mathbf{k}) \rho_{ir}(\mathbf{p_1}) \rho_{s's}(\mathbf{p_2})] \\
&\qquad - \delta^3(\mathbf{p_4} - \mathbf{k}) \delta_{s'j} [\delta^3(\mathbf{p_1} - \mathbf{p_3}) \delta^3(\mathbf{p_2} - \mathbf{k}) \rho_{is}(\mathbf{p_2}) \rho_{r'r}(\mathbf{p_1}) + \delta^3(\mathbf{p_3} - \mathbf{p_2}) \delta^3(\mathbf{p_1} - \mathbf{k}) \rho_{ir}(\mathbf{p_1}) \rho_{r's}(\mathbf{p_2})] \}.
\end{aligned} \tag{18}$$

The time-evolution equation for the density matrix is approximately obtained as

$$\begin{aligned}(2\pi)^3\delta^3(0)2k^0\frac{d}{dt}\rho_{ij}(\mathbf{k}) &\approx i\langle[H_I^{\mathrm{EH}}, D_{ij}^0(\mathbf{k})]\rangle\\ &= -i\frac{\alpha^2}{45m_e^4}(2\pi)^3\delta^3(0)\int\frac{d^3\mathbf{p}}{(2\pi)^3 2p^0}4(\rho_{ss'}(\mathbf{p})+\rho_{s's}(\mathbf{p}))[\delta_{ri}\rho_{r'j}(\mathbf{k})-\delta_{r'j}\rho_{ir}(\mathbf{k})]\\ &\quad\times(k_\mu\epsilon_{r\nu}(k)-k_\nu\epsilon_{r\mu}(k))(k_{\alpha'}\epsilon_{r'\beta'}(k)-k_{\beta'}\epsilon_{r'\alpha'}(k))(p_{\mu'}\epsilon_{s\nu'}(p)-p_{\nu'}\epsilon_{s\mu'}(p))(p_\alpha\epsilon_{s'\beta}(p)-p_\beta\epsilon_{s'\alpha}(k))\\ &\quad\times\left(g^{\mu\mu'}g^{\nu\nu'}g^{\alpha\alpha'}g^{\beta\beta'}+\frac{7}{4}\epsilon^{\mu\nu\mu'\nu'}\epsilon^{\alpha\beta\alpha'\beta'}\right).\end{aligned} \tag{19}$$

The time evolution of Stokes parameter I (3) is given as

$$\frac{d}{dt}I(\mathbf{k}) = 0. \tag{20}$$

This implies that the total intensity of the ensemble of photons does not depend on photon-photon forward scattering, which is excepted because the forward scattering cannot change the momenta of interacting photons. However, the time evolution of Stokes parameters Q, U, and V are calculated as

$$\frac{d}{dt}Q(\mathbf{k}) = \frac{1}{2k^0}\frac{\alpha^2}{45m_e^4}\int\frac{d^3p}{(2\pi)^3 2p^0}8BV(\mathbf{k}), \tag{21}$$

$$\frac{d}{dt}U(\mathbf{k}) = -\frac{1}{2k^0}\frac{\alpha^2}{45m_e^4}\int\frac{d^3p}{(2\pi)^3 2p^0}4AV(\mathbf{k}), \tag{22}$$

and

$$\frac{d}{dt}V(\mathbf{k}) = \frac{1}{2k^0}\frac{\alpha^2}{45m_e^4}\int\frac{d^3p}{(2\pi)^3 2p^0}4(AU(\mathbf{k})-2BQ(\mathbf{k})), \tag{23}$$

where

$$\begin{aligned}A &= \left(g^{\mu\mu'}g^{\nu\nu'}g^{\alpha\alpha'}g^{\beta\beta'}+\frac{7}{4}\epsilon^{\mu\nu\mu'\nu'}\epsilon^{\alpha\beta\alpha'\beta'}\right)[(k_\mu\epsilon_{1\nu}(k)-k_\nu\epsilon_{1\mu}(k))(k_\alpha\epsilon_{1\beta}(k)-k_\beta\epsilon_{1\alpha}(k))\\ &\quad-(k_\mu\epsilon_{2\nu}(k)-k_\nu\epsilon_{2\mu}(k))(k_\alpha\epsilon_{2\beta}(k)-k_\beta\epsilon_{2\alpha}(k))]\{(I(\mathbf{p})+Q(\mathbf{p}))[(p_{\mu'}\epsilon_{1\nu'}(p)-p_{\nu'}\epsilon_{1\mu'}(p))(p_{\alpha'}\epsilon_{1\beta'}(p)-p_{\beta'}\epsilon_{1\alpha'}(p))]\\ &\quad(I(\mathbf{p})-Q(\mathbf{p}))[(p_{\mu'}\epsilon_{2\nu'}(p)-p_{\nu'}\epsilon_{2\mu'}(p))(p_{\alpha'}\epsilon_{2\beta'}(p)-p_{\beta'}\epsilon_{2\alpha'}(p))]\\ &\quad+2U(\mathbf{p})[(p_{\mu'}\epsilon_{1\nu'}(p)-p_{\nu'}\epsilon_{1\mu'}(p))(p_{\alpha'}\epsilon_{2\beta'}(p)-p_{\beta'}\epsilon_{2\alpha'}(k))]\},\\ &= 4\{7\epsilon^{\mu\nu\mu'\nu'}\epsilon^{\alpha\beta\alpha'\beta'}k_\mu k_\alpha p_{\mu'}p_{\alpha'}[\epsilon_{1\nu}(k)\epsilon_{1\beta}(k)-\epsilon_{2\nu}(k)\epsilon_{2\beta}(k)][(I+Q)(\mathbf{p})\epsilon_{1\nu'}(p)\epsilon_{1\beta'}(p)+(I-Q)(\mathbf{p})\epsilon_{2\nu'}(p)\epsilon_{2\beta'}(p)\\ &\quad+2U(\mathbf{p})\epsilon_{1\nu'}(p)\epsilon_{2\beta'}(p)]+(I+Q)(\mathbf{p})\{[(k.p)\epsilon_1(k).\epsilon_1(p)-k.\epsilon_1(p)p.\epsilon_1(k)]^2-[(k.p)\epsilon_2(k).\epsilon_1(p)-k.\epsilon_1(p)p.\epsilon_2(k)]^2\}\\ &\quad+(I-Q)(\mathbf{p})\{[(k.p)\epsilon_1(k).\epsilon_2(p)-k.\epsilon_2(p)p.\epsilon_1(k)]^2-[(k.p)\epsilon_2(k).\epsilon_2(p)-k.\epsilon_2(p)p.\epsilon_2(k)]^2\}\\ &\quad+2U(\mathbf{p})\{[(k.p)\epsilon_1(k).\epsilon_1(p)-k.\epsilon_1(p)p.\epsilon_1(k)][(k.p)\epsilon_1(k).\epsilon_2(p)-k.\epsilon_2(p)p.\epsilon_1(k)]\\ &\quad-[(k.p)\epsilon_2(k).\epsilon_1(p)-k.\epsilon_1(p)p.\epsilon_2(k)][(k.p)\epsilon_2(k).\epsilon_2(p)-k.\epsilon_2(p)p.\epsilon_2(k)]\}\},\end{aligned} \tag{24}$$

and

$$\begin{aligned}B &= \left(g^{\mu\mu'}g^{\nu\nu'}g^{\alpha\alpha'}g^{\beta\beta'}+\frac{7}{4}\epsilon^{\mu\nu\mu'\nu'}\epsilon^{\alpha\beta\alpha'\beta'}\right)(k_\mu\epsilon_{1\nu}(k)-k_\nu\epsilon_{1\mu}(k))(k_\alpha\epsilon_{2\beta}(k)-k_\beta\epsilon_{2\alpha}(k))\\ &\quad\times\{(I(\mathbf{p})+Q(\mathbf{p}))(p_{\mu'}\epsilon_{1\nu'}(p)-p_{\nu'}\epsilon_{1\mu'}(p))(p_{\alpha'}\epsilon_{1\beta'}(p)-p_{\beta'}\epsilon_{1\alpha'}(p))\\ &\quad+(I(\mathbf{p})-Q(\mathbf{p}))(p_{\mu'}\epsilon_{2\nu'}(p)-p_{\nu'}\epsilon_{2\mu'}(p))(p_{\alpha'}\epsilon_{2\beta'}(p)-p_{\beta'}\epsilon_{2\alpha'}(p))\\ &\quad+U(\mathbf{p})[(p_{\mu'}\epsilon_{1\nu'}(p)-p_{\nu'}\epsilon_{1\mu'}(p))(p_{\alpha'}\epsilon_{2\beta'}(p)-p_{\beta'}\epsilon_{2\alpha'}(p))+(p_{\mu'}\epsilon_{2\nu'}(p)-p_{\nu'}\epsilon_{2\mu'}(p))(p_{\alpha'}\epsilon_{1\beta'}(p)-p_{\beta'}\epsilon_{1\alpha'}(p))]\},\end{aligned}$$

$$
\begin{aligned}
&= 4\{7\epsilon^{\mu\nu\mu'\nu'}\epsilon^{\alpha\beta\alpha'\beta'}k_\mu k_\alpha p_{\alpha'}p_{\mu'}\epsilon_{1\nu}(k)\epsilon_{2\beta}(k)[(I+Q)(\mathbf{p})\epsilon_{1\nu'}(p)\epsilon_{1\beta'}(p)+(I-Q)(\mathbf{p})\epsilon_{2\nu'}(p)\epsilon_{2\beta'}(p)\\
&\quad + U(\mathbf{p})(\epsilon_{2\nu'}(p)\epsilon_{1\beta'}(p)+\epsilon_{1\nu'}(p)\epsilon_{2\beta'}(p))] + (I+Q)(\mathbf{p})[(k.p)\epsilon_2(k).\epsilon_1(p) - k.\epsilon_1(p)p.\epsilon_2(k)][(k.p)\epsilon_1(k).\epsilon_1(p)\\
&\quad - k.\epsilon_1(p)p.\epsilon_1(k)] + (I-Q)(\mathbf{p})[(k.p)\epsilon_2(k).\epsilon_2(p) - k.\epsilon_2(p)p.\epsilon_2(k)][(k.p)\epsilon_1(k).\epsilon_2(p) - k.\epsilon_2(p)p.\epsilon_1(k)]\\
&\quad + U(\mathbf{p})\{[(k.p)\epsilon_1(k).\epsilon_1(p) - k.\epsilon_1(p)p.\epsilon_1(k)][(k.p)\epsilon_2(k).\epsilon_2(p) - k.\epsilon_2(p)p.\epsilon_2(k)]\\
&\quad + [(k.p)\epsilon_1(k).\epsilon_2(p) - k.\epsilon_2(p)p.\epsilon_1(k)][(k.p)\epsilon_2(k).\epsilon_1(p) - k.\epsilon_1(p)p.\epsilon_2(k)]\}\}. \qquad (25)
\end{aligned}
$$

It is shown that the time evolutions of Q, U, and V gain their sources from the combinations of Stokes parameters, which indicate a rotation or conversion between linear and circular polarizations due to the effective Euler-Heisenberg Lagrangian.

IV. GRB FARADAY CONVERSION DUE TO INTERMEDIATE INTERACTIONS

Precise measurement of GRB polarization is one of the major goals for future GRB observations that can provide valuable information about their interactions, especially new physics, before reaching us. The study of polarizations can also provide important information on the cluster magnetic field strength and structure. The linear polarization of photons can be converted to circular polarization in the presence of a magnetic field or by scattering off cosmic particles. The Stokes parameter V in this mechanism evolves in time,

$$\dot{V} = 2U\frac{d\Delta\phi_{\rm FC}}{dt}, \qquad (26)$$

where $\Delta\phi_{\rm FC}$ is the Faraday conversion phase shift [13]. The integral over time can be transformed into the integral over redshift as follows:

$$\int_t^0 dt' \to \frac{1}{H_0}\int_0^z \frac{dz'}{(1+z')\hat{H}(z')}, \qquad (27)$$

where H_0 is the Hubble parameter and the function $\hat{H}(z)$ is given by

$$\hat{H}(z) = [\Omega_r(1+z)^4 + \Omega_M(1+z)^3 + \Omega_\Lambda]^{1/2}, \qquad (28)$$

and $\Omega_r \le 10^{-4}$, $\Omega_M = 0.3$, and $\Omega_\Lambda = 0.7$ are the present densities of radiation, matter, and dark energy, respectively. Energy, the wavelength of radiation, and the number density of particles depend on the redshift as cosmic expansion results of the Universe:

$$E = E_0(1+z), \qquad \lambda = \lambda_0(1+z)^{-1}, \qquad n = n_0(1+z)^3, \qquad (29)$$

where E_0, λ_0, n_0 are measured at the present time.

A. CMB-GRB forward scattering

In order to calculate the time evolution of the Stokes parameter V, we consider GRB-photon wave number $\mathbf{k}$ in the $\hat{z}$-direction, its polarization vectors $\vec{\epsilon}_1(\mathbf{k})$ in the $\hat{x}$-direction, and $\vec{\epsilon}_2(\mathbf{k})$ in the $\hat{y}$-direction. In this coordinate, CMB-photon wave number $\mathbf{p}$ and its polarization vectors $\vec{\epsilon}_1(\mathbf{p})$ and $\vec{\epsilon}_2(\mathbf{p})$ are represented by

$$
\begin{aligned}
\hat{\mathbf{p}} &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\\
\vec{\epsilon}_1(\mathbf{p}) &= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)\\
\vec{\epsilon}_2(\mathbf{p}) &= (-\sin\phi, \cos\phi, 0). \qquad (30)
\end{aligned}
$$

Linear polarization is a second-rank symmetric and traceless tensor, which can be decomposed on a sphere into spin-2 spherical harmonics. These are the analog of the spherical harmonics used in the temperature maps and obey the same completeness and orthogonality relations. By applying Eq. (30) in Eq. (24) and expanding the Stokes parameters of the CMB photons (target) as a function of spherical harmonics, we obtain

$$
\begin{aligned}
U(\mathbf{p}) &= \sum_{lm} U_{l,m}(p)Y_{l,m}(\theta,\phi),\\
Q(\mathbf{p}) &= \sum_{lm} Q_{l,m}(p)Y_{l,m}(\theta,\phi),\\
I(\mathbf{p}) &= \sum_{lm} I_{l,m}(p)Y_{l,m}(\theta,\phi). \qquad (31)
\end{aligned}
$$

Then the time evolution of the V-mode polarization of GRB photons is given by

$$\frac{d}{dt}V(\mathbf{k}) = \frac{1}{30\pi}\bar{I}(\bar{p})\sigma_T\frac{k}{m_e}\frac{U(\mathbf{k})}{m_e}G, \qquad (32)$$

where

$$
\begin{aligned}
G = &-12\frac{1}{\bar{I}}\int\frac{d^3p}{(2\pi)^3}p\sum_{lm}Y_{l,m}(\theta,\phi)(1-\cos(\theta))^2\\
&\times[Q_{lm}(\mathrm{p})\cos(2\phi) + U_{lm}(\mathrm{p})\sin(2\phi)], \qquad (33)
\end{aligned}
$$

where

$$\int dp \frac{p^3}{2\pi^2} I(p) = \bar{I}(\bar{p}) \simeq \bar{p} n_\gamma, \qquad (34)$$

and $\bar{p} = |\mathrm{p}|$ is the average value of the momentum of the target (CMB photons). As a result, the Faraday conversion phase shift of GRBs with redshift z and energy at the present time k^0 due to CMB-GRB forward scattering is given by

$$\Delta\phi_{\mathrm{FC}}|_{\mathrm{CG}} \simeq 10^{-7}\ \mathrm{rad} \frac{k_0}{\mathrm{GeV}} \frac{\bar{p}_0}{2.3 \times 10^{-4}\ \mathrm{eV}} \frac{n_{0\gamma}}{411\ \mathrm{cm}^{-3}} \times \int_0^z \frac{dz'(1+z')^4}{\hat{H}(z')} \frac{G}{10^{-5}}, \qquad (35)$$

where $\bar{p}_0$ and $n_{0\gamma}$ are the average energy and number density of CMB at the present time. We just assume $G \sim \frac{\delta T}{T} \simeq 10^{-5}$, which are in that order the CMB's anisotropies [see Table I, third column (E–H), to find the values of the Faraday conversion phase shift for the electromagnetic spectrum regarding $z = 1$].

B. GRBs and cosmic neutrino background forward scattering

Cosmic neutrino background (CNB) decoupling occurred only one second after the big bang. Therefore, similar to CMB, it contains very helpful information about the early Universe. Here we use the time evolution of GRB polarization due to photon-neutrino interactions [15,21]:

$$\frac{dV}{dt} = C_Q Q + C_U U, \qquad (36)$$

where

$$C_Q = -\frac{\sqrt{2}\alpha G^F n_\nu}{3\pi k^0} \langle v_{\nu\alpha} q_\beta \rangle \epsilon_2^\alpha \epsilon_1^\beta,$$
$$C_U = -\frac{\sqrt{2}}{6\pi k^0} \alpha G^F n_\nu (\langle v_{\nu\alpha} q_\beta \rangle \epsilon_1^\alpha \epsilon_1^\beta - \langle v_{\nu\alpha} q_\beta \rangle \epsilon_2^\alpha \epsilon_2^\beta), \qquad (37)$$

where $n_{\nu 0}$ is the number density of the cosmic neutrino background at the present time. The energy of the cosmic neutrino background is at the same order of its temperature $T_{0,\nu} \simeq 1.95$ K. $\vec{v}_\nu = v_\nu \hat{k}$ is the bulk velocity of the cosmic neutrino background and its average value is assumed to be $\bar{v}_\nu = \delta T/T \sim 10^{-5}$ [15]. Finally we obtain the Faraday conversion phase shift due to GRBs and cosmic neutrino background forward scattering:

$$\Delta\phi_{\mathrm{FC}}|_{\nu\mathrm{G}} \simeq 10^{-23}\ \mathrm{rad} \frac{q_0}{1.6 \times 10^{-4}\ \mathrm{eV}} \left(\frac{k_0}{\mathrm{GeV}}\right)^{-1} \times \frac{n_{\nu 0}}{312\ \mathrm{cm}^{-3}} \frac{\bar{v}_\nu}{10^{-5}} \int_0^z \frac{dz'(1+z')^2}{\hat{H}(z')} \times (\langle \hat{v}_\alpha \hat{q}_\beta \rangle \epsilon_1^\alpha \epsilon_1^\beta - \langle \hat{v}_\alpha \hat{q}_\beta \rangle \epsilon_2^\alpha \epsilon_2^\beta). \qquad (38)$$

This result shows that $\Delta\phi_{\mathrm{FC}}$ due to GRB-$C\nu$B interaction for high-energy GRBs is negligible, compared with the contribution from GRBs and CMB interaction (see Table I, fourth column from the right, to find the value of the Faraday conversion phase shift for the electromagnetic spectrum regarding $z = 1$). This is expected from the energy dependence of weak interactions of neutrinos and photons.

C. Compton scattering in the presence of magnetic fields

When linearly polarized light propagates through relativistic magnetized plasma, it undergoes Faraday rotation and Faraday conversion, which describe the interconversion of linearly and circularly polarized light. The conversion measures an angle related to Faraday conversion in a magnetized relativistic plasma as [13]

$$\Delta\phi_{\mathrm{FC}}|_{B13} = \frac{e^4 \lambda^3}{\pi^2 m_e^3} \left(\frac{\beta - 1}{\beta - 2}\right) \int dl n_e(l) \gamma_{\min} |\mathbf{B}|^2 (1 - \mu^2), \qquad (39)$$

where n_e (n_{e0}) is the number density of the electron (at the present time) and β defines the power-law distribution of the particles, in terms of the Lorentz factor, such that $N(\gamma) = N_0 \gamma^{-\beta}$ and $\gamma_{\min} < \gamma < \gamma_{\max}$, and μ is the cosine of the angle between the line-of-sight direction and the

TABLE I. GRB Faraday conversion phase shift due to intermediate interactions for the electromagnetic spectrum regarding $z = 1$.

GRB types	λ cm	$\Delta\phi_{\mathrm{FC}}\|_{\mathrm{CG}}$	$\Delta\phi_{\mathrm{FC}}\|_{\nu\mathrm{G}}$	$\Delta\phi_{\mathrm{FC}}\|_{B13}$	$\Delta\phi_{\mathrm{FC}}\|_{B14}$	$\Delta\phi_{\mathrm{FC}}\|_{NC(1\ \mathrm{TeV})}$
Prompt	10^{-13}	$\sim 10^{-6}$	$\sim 10^{-23}$	$\sim 10^{-47}$	$\sim 10^{-40}$	$\sim 10^{-19}$
γ-ray	10^{-10}	$\sim 10^{-9}$	$\sim 10^{-20}$	$\sim 10^{-38}$	$\sim 10^{-31}$	$\sim 10^{-16}$
x-ray	10^{-8}	$\sim 10^{-11}$	$\sim 10^{-18}$	$\sim 10^{-32}$	$\sim 10^{-25}$	$\sim 10^{-14}$
UV	10^{-6}	$\sim 10^{-13}$	$\sim 10^{-16}$	$\sim 10^{-26}$	$\sim 10^{-19}$	$\sim 10^{-12}$
Visible	10^{-4}	$\sim 10^{-15}$	$\sim 10^{-14}$	$\sim 10^{-20}$	$\sim 10^{-13}$	$\sim 10^{-10}$
Infrared	10^{-3}	$\sim 10^{-16}$	$\sim 10^{-13}$	$\sim 10^{-17}$	$\sim 10^{-10}$	$\sim 10^{-9}$
Microwave	1	$\sim 10^{-19}$	$\sim 10^{-10}$	$\sim 10^{-8}$	$\sim 10^{-1}$	$\sim 10^{-6}$
Radio	10^5	$\sim 10^{-24}$	$\sim 10^{-5}$	$\sim 10^{7}$	$\sim 10^{14}$	$\sim 10^{-1}$

magnetic field B in galaxy clusters. Suppose that reasonable parameters for galaxy clusters are $B = 10\ \mu$G; a path length of 1 Mpc, which is a typical size for a massive cluster; $\gamma_{\rm min} = 100$ for relativistic particles; an observed frequency of 10 GHz; and $\Delta\phi_{\rm FC}$ is estimated to be about a few $\times 10^{-3}$ [13]. In the case of GRBs interacting with nonrelativistic particles and cosmic charged particles in the presence of an intergalactic magnetic field, we ignore $\gamma_{\rm min}$ and $\frac{\beta-1}{\beta-2}$, so the Faraday conversion phase shift is estimated as

$$\begin{aligned}\Delta\phi_{\rm FC}|_{B13} &= \frac{e^4\lambda^3}{\pi^2 m_e^3}\int dl n_e(l)|\mathbf{B}|^2(1-\mu^2),\\ &\approx 10^{-8}\ \text{rad}\left(\frac{\lambda_0}{1\ \text{cm}}\right)^3\frac{n_{e0}}{10^{-7}\ \text{cm}^{-3}}\\ &\quad\times\left(\frac{|\mathbf{B}|}{10\ \mu\text{G}}\right)^2(1-\mu^2)\int_0^z\frac{dz'}{(1+z')\hat{H}(z')}.\end{aligned}\tag{40}$$

Using Eq. (40), $\Delta\phi_{\rm FC}|_{B13}$ is about 10^{-8} radians for $\lambda_0 \sim 1$ cm and its values for other electromagnetic wavelengths regarding $z = 1$ are given in the fifth column of Table I.

As mentioned in above paragraphs, the linear polarization of GRBs is converted to circular polarization due to Compton scattering in the presence of an intergalactic magnetic field, while $\Delta\phi_{\rm FC}|_{B13}$ depends on $|\mathbf{B}|^2$. In [14], the effect of magnetic fields on electron wave functions, in addition to the electron propagators in the case of a very weak magnetic field compared to the critical value $B_c = \frac{m_e^2}{e} = 4.414\times 10^{13}$ G, was considered, and then the Faraday conversion phase shift was calculated. In that study the circular polarization was linearly proportional to the background magnetic field and came from the first term on the right-hand side of Eq. (9) (the forward scattering term). The time evolution of the Stokes parameter V up to the order of e^4 is given as [14]

$$\begin{aligned}\dot{V}^{(1)} &= i\frac{\pi e^4}{4m^2k}\int d\mathbf{q}d\mathbf{p}\delta(k-p)\left(\frac{1}{q.k}-\frac{1}{q.p}\right)\left(\frac{1}{(q.k)^2}-\frac{1}{(q.p)^2}\right)(\tilde{q}.\epsilon_1(k)q.\epsilon_2(k)-\tilde{q}.\epsilon_2(k)q.\epsilon_1(k))\\ &\quad\times n_e(\mathbf{q})[((q.\epsilon_1(p))^2+(q.\epsilon_2(p))^2)(I^{(1)}(\mathbf{k})-I^{(1)}(\mathbf{p}))-(q.\epsilon_1(p)q.\epsilon_1(p)-q.\epsilon_2(p)q.\epsilon_2(p))Q^{(1)}(\mathbf{p})\\ &\quad-2q.\epsilon_1(p)q.\epsilon_2(p)U^{(1)}(\mathbf{p})]+\mathcal{O}(k,p),\end{aligned}\tag{41}$$

where $\tilde{q}_\mu = -eB_{\mu\nu}q^\nu$, $d\mathbf{q} \equiv \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{m_f}{q^0}$ and $d\mathbf{p} \equiv \frac{d^3\mathbf{p}}{(2\pi)^3 2p^0}$.

$$\begin{aligned}\dot{V}^{(1)} &= \frac{e^4 m\lambda^3}{8\pi}\int\frac{d\Omega}{4\pi}\bar{n}_e(\mathbf{v}_e.(\hat{\mathbf{k}}-\hat{\mathbf{p}}))^2\left(\frac{eB}{m^2}\right)\\ &\quad\times F\left(v,\hat{\mathbf{p}},\epsilon_1,\epsilon_2,I,U,Q,\frac{B_{\mu\nu}}{B}\right),\end{aligned}\tag{42}$$

and F can be easily defined by comparing (41) and (42) [14]. Here again we can estimate the Faraday conversion phase shift as

$$\begin{aligned}\Delta\phi_{\rm FC}|_{B13} &\simeq 10^{-1}\ \text{rad}\left(\frac{B}{10\ \mu\text{G}}\right)\left(\frac{\lambda_0}{1\ \text{cm}}\right)^3\left(\frac{\bar{n}_{e0}}{10^{-7}\ \text{cm}^{-3}}\right)\\ &\quad\times\left(\frac{v_e}{10^{-5}}\right)^2\int_0^z\frac{dz'}{(1+z')\hat{H}(z')}\frac{d\Omega}{4\pi}(\hat{v}.(\hat{\mathbf{k}}-\hat{\mathbf{p}}))^2\\ &\quad\times F\left(\hat{v}_e,\hat{\mathbf{p}},\epsilon_1,\epsilon_2,I,U,Q,\frac{B_{\mu\nu}}{B}\right),\end{aligned}\tag{43}$$

where v_e is the electron bulk velocity, which is about $v_e \sim \delta T/T \simeq 10^{-5}$. The values of $\Delta\phi_{\rm FC}$ for other electromagnetic wavelengths regarding $z = 1$, using Eq. (43), are given in the sixth column of Table I.

D. Compton scattering in noncommutative space-time

The circular polarization for GRBs can also be generated due to photon-charged particles (electrons and protons) forward scattering in noncommutative space-time [12,14].

Noncommutative quantum field theory is a generalization of ordinary quantum field theory, to describe the physics at the Planck scale or quantum gravity scale. In noncommutative field theory, coordinates turn to operators which do not commute. The noncommutative relation of space-time is described as [22]

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu},\tag{44}$$

where $\theta^{\mu\nu} \propto 1/\Lambda_{\rm NC}^2$ is a real antisymmetric tensor, and $\Lambda_{\rm NC}$ is the scale at which the NC effects become relevant.

The time evolution of the Stokes parameter V in noncommutative space-time is calculated as [12]

$$\dot{V}(\mathbf{k}) = i\sum_{f=e,p}\frac{3}{4}\frac{m_f}{k^0}\frac{\sigma^T}{\alpha}\frac{m_e^2}{\Lambda^2}\bar{n}_f v_f Q_f^2(AQ+BU),\tag{45}$$

where

$$A = -\hat{\theta}^{0i}(\epsilon_{1i}\hat{v}_f\cdot\epsilon_2 + \epsilon_{2i}\hat{v}_f\cdot\epsilon_1)$$
$$B = \hat{\theta}^{0i}(\epsilon_{1i}\hat{v}_f\cdot\varepsilon_1 - \epsilon_{2i}\hat{v}_f\cdot\epsilon_2), \tag{46}$$

where m_f and v_f are the mass and velocity of fermions, σ^T is the Thomson cross section, and $\alpha = e^2/4\pi$. In usual space-time, the time evaluation of photon Stokes parameters depends on the cross section of the usual electron-photon Compton scattering σ_T. Since the usual Compton cross section of photon-fermions at low energy depends on the inverse square mass of fermions, we just consider the electron, but in the case of NC forward scattering of photon-fermions, the time evaluation of Stokes parameters has a linear dependence on the mass of fermions (45). Therefore the contribution of photon-proton forward scattering in NC space-time is larger than the photon-electron one by a factor m_p/m_e, while the average number of electrons approximately equals the average number of protons $\bar{n}_p = \bar{n}_e$ due to electric neutrality in cosmology. So the Faraday conversion phase shift is obtained as

$$\Delta\phi_{\rm FC}|_{\rm NC} \simeq 10^{-19}\,{\rm rad}\left(\frac{k_0}{\rm GeV}\right)^{-1}\frac{n_{p0}}{10^{-7}\,{\rm cm}^{-3}}\left(\frac{\Lambda}{1\,{\rm TeV}}\right)^{-2}\frac{v_p}{10^{-5}} \times\int_0^z\frac{dz'(1+z')}{\hat{H}(z')}\hat{\theta}^{0i}(\epsilon_{1i}\hat{v}_p\cdot\varepsilon_1 - \epsilon_{2i}\hat{v}_p\cdot\epsilon_2). \tag{47}$$

Therefore the Faraday conversion phase shift for $\Lambda_{\rm NC} = 1$ TeV will be of the order 10^{-19} radians, which is comparable to the Faraday conversion phase shift in the case of GRB-CNB interaction. The Faraday conversion phase shift due to forward Compton scattering in noncommutative space-time for $\Lambda_{\rm NC} = 1$ TeV is shown in the last column of Table I.

V. FARADAY CONVERSION OF GRBS DUE TO THEIR INTERACTIONS IN INTERNAL AND EXTERNAL SHOCKS

A. Fireball model

The most common types of gamma-ray bursts are considered to be from a dying massive star that collapses and forms a black hole, by driving a particle jet into space. Light across the spectrum arises from hot gas near the black hole, collisions within the jet, and the jet's interaction with its surroundings. In most accepted fireball models, internal shocks take place around 10^{13}–10^{15} cm in the presence of a magnetic filed about $B_{\rm fb}^{\rm in} = 10^6$ G. These shocks accelerate the electrons to ultrarelativistic energies (the typical Lorentz factor of an electron is 1000); the needed magnetic field is carried from the inner engine or is generated and amplified by the shocks. The electrons emit the observed prompt γ-rays (with energy about a few GeV) via synchrotron radiation. External shocks take place around 10^{16}–10^{18} cm from the center in the presence of an estimated magnetic field up to $B_{\rm fb}^{\rm ex} \sim 1$ G. At this stage, a counterpart at longer wavelengths (x-ray, UV, optical, infrared, and radio) is generated, known as the afterglow, that generally remains detectable for days or longer after the first detection of high-energy GRBs [23].

B. Fireshell model

In the fireshell model, it is assumed that an optically thick e^-e^+-baryon plasma is created in the process of black hole formation and self-accelerated as a spherically symmetric fireshell with a Lorentz factor in the range $200 < \Gamma < 3000$. After the e^-e^+ plasma self-acceleration phase, the transparency condition is obtained and the proper-GRB (P-GRB) is emitted. As a consequence, the huge value of the magnetic field is not needed, so the average value of the magnetic field in this model is about $\sim\mathcal{O}(1)$ G. Then an optically thin fireshell of baryonic matter remains, which expands with an ultrarelativistic velocity and the afterglow emission begins due to losing its kinetic energy via collision with the circumburst medium (CBM) [24].

The most important difference between fireball and fireshell models comes from the mechanism of P-GRB generation as well as the value of the magnetic field in internal shock. The value of the magnetic field in the fireball model with internal shock $B_{\rm fb}^{\rm in}$ is about six orders of magnitude larger than the one for the fireshell model's $B_{\rm fs}$. This event could make a big difference between the generated circular polarization of P-GRBs for each model.

Another point which should be mentioned, x-ray, optical, and radio GRBs generated in the afterglow area only experience the conditions in external shock, where the value of magnetic field and number density of accelerated charged particles are almost equal for both fireball and fireshell models. As a result, apart from the details of afterglow mechanisms for fireball and fireshell models, it is expected that the values of generated circular polarization for x-ray, optical, and radio GRBs in both models belong in the same order of magnitude.

C. Faraday conversion for prompt emission

The prompt emission of γ-rays with energy of GeV propagates crossing from both external and internal shocks; their linear polarization can convert to a circular one due to their intermediate interactions. The Faraday conversion phase shift due to CMB-GRB forward scattering $\Delta\phi_{\rm FC}|_{\rm CG}$ in internal and external shockwaves for both the fireball and fireshell models is almost the same; it can be estimated as

$$\Delta\phi_{\rm FC}|_{\rm CG} \simeq 10^{-17}\,{\rm rad}\frac{k_0}{\rm GeV}\frac{\bar{p}_0}{2.3\times10^{-4}\,{\rm eV}}\frac{n_{0\gamma}}{411\,{\rm cm}^{-3}}(1+z)^2 \times\int\frac{dl}{10^{18}}\frac{G}{10^{-5}}. \tag{48}$$

The Faraday conversion phase shift due to CνB-GRB forward scattering $\Delta\phi_{\rm FC}|_{\nu\rm G}$ in internal and external shockwaves for both models is the same too, and it is given as

$$\Delta\phi_{\rm FC}|_{\nu\rm G} \simeq 10^{-33}\ \text{rad}\frac{q_0}{1.6\times10^{-4}\ \text{eV}}\left(\frac{k_0}{\rm GeV}\right)^{-1}\frac{n_{\nu0}}{312\ \text{cm}^{-3}} \times\frac{\bar{v}_\nu}{10^{-5}}\int\frac{dl}{10^{18}}(\langle\hat{v}_\alpha\hat{q}_\beta\rangle\epsilon_1^\alpha\epsilon_1^\beta-\langle\hat{v}_\alpha\hat{q}_\beta\rangle\epsilon_2^\alpha\epsilon_2^\beta). \tag{49}$$

Note that the results in (48) and (49) can be applied also for all x-ray, optical, and radio GRBs; it just needs to choose a suitable energy for GRBs. The Faraday conversion phase shift due to Compton scattering in a magnetic field, Eqs. (39) and (42), was suggested in [13] and [14], respectively, which based on [13] in the fireball model $\Delta\phi_{\rm FC}|_{\rm fb}^{B13}$ is

$$\begin{aligned}\Delta\phi_{\rm FC}|_{\rm fb}^{B13} &= \frac{e^4\lambda^3}{\pi^2 m_e^3}\left(\frac{\beta-1}{\beta-2}\right)\int dl n_e(l)\gamma_{\min}|\mathbf{B}|^2(1-\mu^2),\\ &\approx 10^{-28}\,\text{rad}\left(\frac{\beta-1}{\beta-2}\right)_{\beta=2.5}\left(\frac{\gamma_{\min}}{300}\right)\left(\frac{\bar{n}_e}{0.1\,\text{cm}^{-3}}\right)\\ &\times\left(\frac{\lambda}{10^{-13}\,\text{cm}}\right)^3(1+z)^{-3}\left(\int\frac{dl}{10^{16}\,\text{cm}}\left(\frac{|\mathbf{B}_{\rm fb}^{\rm in}|}{10^6\,\text{G}}\right)^2\right.\\ &\left.\times(1-\mu^2)+10^{-10}\int\frac{dl}{10^{18}\,\text{cm}}\left(\frac{|\mathbf{B}_{\rm fb}^{\rm ex}|}{1\,\text{G}}\right)^2(1-\mu^2)\right).\end{aligned} \tag{50}$$

In the fireshell model, the magnitude of the magnetic field in internal and external shockwaves is about 1 G, and the Faraday conversion phase shift $\Delta\phi_{\rm FC}|_{\rm fs}^{B13}$ is given by

$$\begin{aligned}\Delta\phi_{\rm FC}|_{\rm fs}^{B13} &\simeq 10^{-38}\,\text{rad}\left(\frac{\beta-1}{\beta-2}\right)_{\beta=2.5}\left(\frac{\gamma_{\min}}{300}\right)\left(\frac{\bar{n}_e}{0.1\,\text{cm}^{-3}}\right)\\ &\times\left(\frac{\lambda}{10^{-13}\,\text{cm}}\right)^3(1+z)^{-3}\left(\int\frac{dl}{10^{18}\,\text{cm}}\left(\frac{|\mathbf{B}_{\rm fs}|}{1\,\text{G}}\right)^2\right.\\ &\left.\times(1-\mu^2)+10^{-2}\int\frac{dl}{10^{16}\text{cm}}\left(\frac{|\mathbf{B}_{\rm fs}|}{1\,\text{G}}\right)^2(1-\mu^2)\right).\end{aligned}$$

The above equations show that the mechanism suggested in [13] does not have a significant effect on the generation of circular polarization for γ-ray GRBs. Let us check the mechanism reported in [14] in the fireball model,

$$\begin{aligned}\Delta\phi_{\rm FC}|_{\rm fb}^{B14} &\simeq 10^{-24}\,\text{rad}\left(\frac{\beta-1}{\beta-2}\right)_{\beta=2.5}\left(\frac{\gamma_{\min}}{300}\right)\left(\frac{\bar{n}_e}{0.1\,\text{cm}^{-3}}\right)\\ &\times\left(\frac{v_e}{0.1}\right)^2\left(\frac{\lambda}{10^{-13}\,\text{cm}}\right)^3(1+z)^{-3}\\ &\times\left(\int\frac{dl}{10^{16}\,\text{cm}}\frac{|\mathbf{B}_{\rm fb}^{\rm in}|}{10^6\,\text{G}}+10^{-4}\int\frac{dl}{10^{18}\,\text{cm}}\frac{|\mathbf{B}_{\rm fb}^{\rm ex}|}{1\,\text{G}}\right),\end{aligned} \tag{51}$$

and in the fireshell model

$$\begin{aligned}\Delta\phi_{\rm FC}|_{\rm fs}^{B14} &\simeq 10^{-28}\,\text{rad}\left(\frac{\beta-1}{\beta-2}\right)_{\beta=2.5}\left(\frac{\gamma_{\min}}{300}\right)\left(\frac{\bar{n}_e}{0.1\,\text{cm}^{-3}}\right)\\ &\times\left(\frac{v_e}{0.1}\right)^2\left(\frac{\lambda}{10^{-13}\,\text{cm}}\right)^3(1+z)^{-3}\\ &\times\left(\int\frac{dl}{10^{18}\,\text{cm}}\frac{|\mathbf{B}_{\rm fs}|}{1\,\text{G}}+10^{-2}\int\frac{dl}{10^{16}\,\text{cm}}\frac{|\mathbf{B}_{\rm fs}|}{1\,\text{G}}\right).\end{aligned} \tag{52}$$

The evolution of Stokes parameter V due to Compton scattering on noncommutative space-time $\dot{V}(\mathbf{k})|_{\rm NC}$ for relativistic fermions is calculated as

$$\dot{V}(\mathbf{k})|_{\rm NC} = i\frac{3}{4}\frac{\sigma^T}{\alpha k^0}\frac{m_e^2}{\Lambda_T^2}\frac{\bar{\epsilon}_f}{g_f}(AQ+BU), \tag{53}$$

where A and B are defined in (46). g_f is the fermion spin state (degrees of freedom) and $\bar{\epsilon}_f$ is the averaged energy density of fermions, which is related to the Lorentz factor as $\bar{\epsilon}_f = \bar{n}_f m_f \gamma^2$ [25,26]. Therefore, the Faraday conversion phase shift for Compton scattering of GRB protons on noncommutative space-time $\Delta\phi_{\rm FC}|_{\rm NC}$ in internal and external shockwaves for both the fireball and fireshell models is estimated as

$$\begin{aligned}\Delta\phi_{\rm FC}|_{\rm NC} &\simeq 10^{-14}\,\text{rad}\left(\frac{k_0}{\rm GeV}\right)^{-1}\frac{\bar{n}_{p0}}{10^{-1}\,\text{cm}^{-3}}\left(\frac{\Lambda}{1\,\text{TeV}}\right)^{-2}\\ &\times\left(\frac{\gamma}{300}\right)^2\frac{1}{(1+z)}\int\frac{dl}{10^{18}}\hat{\theta}^{0i}(\epsilon_{1i}\hat{v}_p\cdot\varepsilon_1-\epsilon_{2i}\hat{v}_p\cdot\epsilon_2).\end{aligned} \tag{54}$$

The Faraday conversion phase shifts for prompt emission due to the interactions in the fireball and fireshell models are estimated in Table II.

TABLE II. Prompt GRB Faraday conversion phase shift due to the interactions in shockwaves.

Prompt emission	λ cm	$\Delta\phi_{\rm FC}\vert_{\rm CG}$	$\Delta\phi_{\rm FC}\vert_{\nu\rm G}$	$\Delta\phi_{\rm FC}\vert_{B13}$	$\Delta\phi_{\rm FC}\vert_{B14}$	$\Delta\phi_{\rm FC}\vert_{NC(1\text{ TeV})}$
Fireball	10^{-13}	$\sim10^{-17}$	$\sim10^{-33}$	$\sim10^{-28}$	$\sim10^{-24}$	$\sim10^{-14}$
Fireshell	10^{-13}	$\sim10^{-17}$	$\sim10^{-33}$	$\sim10^{-38}$	$\sim10^{-28}$	$\sim10^{-14}$

TABLE III. GRB Faraday conversion phase shift due to the interactions in the external shockwaves in the fireball and fireshell models.

GRB types	λ cm	$\Delta\phi_{FC}\vert_{CG}$	$\Delta\phi_{FC}\vert_{\nu G}$	$\Delta\phi_{FC}\vert_{B13}$	$\Delta\phi_{FC}\vert_{B14}$	$\Delta\phi_{FC}\vert_{NC(1\ \mathrm{TeV})}$
γ-ray	10^{-10}	$\sim 10^{-20}$	$\sim 10^{-30}$	$\sim 10^{-29}$	$\sim 10^{-19}$	$\sim 10^{-11}$
x-ray	10^{-8}	$\sim 10^{-22}$	$\sim 10^{-28}$	$\sim 10^{-23}$	$\sim 10^{-13}$	$\sim 10^{-9}$
UV	10^{-6}	$\sim 10^{-24}$	$\sim 10^{-26}$	$\sim 10^{-17}$	$\sim 10^{-7}$	$\sim 10^{-7}$
Visible	10^{-4}	$\sim 10^{-26}$	$\sim 10^{-24}$	$\sim 10^{-11}$	$\sim 10^{-1}$	$\sim 10^{-5}$
Infrared	10^{-3}	$\sim 10^{-27}$	$\sim 10^{-23}$	$\sim 10^{-8}$	$\sim 10^{2}$	$\sim 10^{-4}$
Microwave	1	$\sim 10^{-30}$	$\sim 10^{-20}$	~ 10	$\sim 10^{11}$	$\sim 10^{-1}$
Radio	10^{5}	$\sim 10^{-35}$	$\sim 10^{-15}$	$\sim 10^{16}$	$\sim 10^{26}$	$\sim 10^{4}$

D. Faraday conversion for x-ray, optical, and radio GRBs

At distances of about 10^{16}–10^{18} cm from the center, the GRB afterglow is formed. The magnetic field in this region for both the fireball and fireshell models is about $B_{\mathrm{af}} \sim \mathrm{G}$, so the Faraday conversion phase shift due to Compton scattering in the magnetic field given in [13] for afterglow GRBs $\Delta\phi_{FC}|^{B13}_{\mathrm{afterglow}}$ can be estimated as follows:

$$\begin{aligned}\Delta\phi_{FC}|^{B13}_{\mathrm{afterglow}} &= \frac{e^4\lambda^3}{\pi^2 m_e^3}\left(\frac{\beta-1}{\beta-2}\right)\int dl n_e(l)\gamma_{\min}|\mathbf{B}|^2(1-\mu^2),\\ &\approx 10^{-29}\ \mathrm{rad}\left(\frac{\beta-1}{\beta-2}\right)_{\beta=2.5}\left(\frac{\gamma_{\min}}{300}\right)\left(\frac{\bar{n}_e}{0.1\ \mathrm{cm}^{-3}}\right)\\ &\quad\times\left(\frac{\lambda}{10^{-10}\ \mathrm{cm}}\right)^3(1+z)^{-3}\\ &\quad\times\int\frac{dl}{10^{18}\ \mathrm{cm}}\left(\frac{|\mathbf{B}_{\mathrm{af}}|}{1\ \mathrm{G}}\right)^2(1-\mu^2),\end{aligned}\tag{55}$$

and for the mechanism reported in [14],

$$\begin{aligned}\Delta\phi_{FC}|^{B14}_{\mathrm{afterglow}} &\simeq 10^{-19}\ \mathrm{rad}\left(\frac{\beta-1}{\beta-2}\right)_{\beta=2.5}\left(\frac{\gamma_{\min}}{300}\right)\\ &\quad\times\left(\frac{\bar{n}_e}{0.1\ \mathrm{cm}^{-3}}\right)\left(\frac{v_e}{0.1}\right)^2\left(\frac{\lambda}{10^{-10}\ \mathrm{cm}}\right)^3\\ &\quad\times(1+z)^{-3}\int\frac{dl}{10^{18}\ \mathrm{cm}}\frac{|\mathbf{B}_{\mathrm{af}}|}{1\ \mathrm{G}}.\end{aligned}\tag{56}$$

Since GRB-CMB, GRB-$C\nu$B, and Compton scattering in noncommutative space-time do not depend on an electromagnetic field, the Faraday conversion phase shift for these interactions has the same values as estimated in the previous section. The Faraday conversion phase shifts for γ-ray emission in the range of MeV energy and the afterglow spectrum in the fireball and fireshell models are estimated in Table III.

VI. CONCLUSION

Gamma-ray bursts are transients of γ-ray radiation and are the most energetic explosions in the Universe. The burst can last from milliseconds to hundreds of seconds. Many observational events and theoretical works have led us to understand the nature of GRBs and there are several possible models of GRBs. The linear polarization of photons can be converted to circular polarization by scattering from cosmic particles or being in a background field. The polarized radiation incoming from galaxy clusters experiences a rotation of the plane of polarization as it passes through the magnetized medium. When GRBs travel through a region containing a magnetic field, linear polarization can generate circular polarization via a process called Faraday conversion. In this study we discussed other interactions that can generate circular polarization for GRBs in additional to traveling GRBs through a region of magnetic field.

In this paper we calculated the Stokes parameters in photon-photon scattering through the Euler-Heisenberg effective Lagrangian and estimated the Faraday conversion phase shift in photon-photon scattering, photon-neutrino scattering, Compton scattering in the presence of the background magnetic field, and noncommutative quantum electrodynamics. These interactions are considered in two parts: intermediate interactions (from leaving shockwave to detectors) and interactions that take place in shockwaves.

The results for GRB interactions in the intermediate part are summarized in Table I. From these results, it is concluded that photon-photon scattering through the Euler-Heisenberg effective Lagrangian is the prevailing interaction for producing circular polarization of high energy GRBs; also, the Faraday conversion phase shift is large for Compton scattering in a magnetic field and in noncommutative space-time for high-wavelength GRBs. As can be seen in Table II, the magnetic field is strong for internal shock in the fireball model, so Compton scattering in magnetic field plays an important role in producing circular polarization of high-energy GRBs. The Faraday conversion phase shift in external shock for the afterglow through intermediate interactions is expected to be almost the same in both the fireball and fireshell models (see Table III). Therefore it seems that studying and measuring the circular polarization of GRBs are very helpful for a better understanding of physics, the methods of generating GRBs, and their interactions before reaching us.

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