THE CIRCULAR POLARIZATION OF SOURCES OF SYNCHROTRON RADIATION*

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Abstract. The degree of circular polarization p_c is calculated for two models of a source of synchrotron radiation:

(1) A source with an inhomogeneous magnetic field and isotropic angular distribution of the electrons with respect to the magnetic field;

(2) A source with a homogeneous magnetic field and anisotropic angular distribution of the electrons in which the anisotropy of angular distribution substantially increases with the electron energy.

The first model can be used to describe extended radio-sources; and the second, to describe compact radio-sources.

For those sources, whose observed polarization properties correspond to the first model, we obtain an integral equation which connects the observed distribution of the sources with the extent of their linear and circular polarization (p_l and p_c) and the unknown distribution of the sources over the strength B and the degree of homogeneity $\varepsilon = (B_0/B)^2$ of the magnetic field; B_0 is a homogenous field, $B_0 \ll B$. A solution of the integral equation obtained is found for a particular case. This solution makes it possible to determine the distribution of different types of sources over ε if the distribution of these sources in the extent of linear polarization is known. The formulae obtained make it possible to indicate which sources with a known degree of linear polarization should be expected to exhibit highest circular polarization.

In the discussion of the first model the question is raised as to the information one can get about the magnetic field by using observations of both linear and circular polarization for a separate source, and for a number of sources.

It is shown that the determination of the most probable values of B and ε in a separate source based on the known values of p_l and p_c for the source, is possible only if one knows the distribution over B and ε of the sources of the type to which the source in question belongs. The observational data now available make it possible to find the distribution of the sources only over ε. Since the distribution over B and ε is at present unknown, even a very strong upper limit for p_c in the case of a separate source does not enable us to give an exact upper limit for the strength of the magnetic field in this source.

In the first model the upper limit for the magnetic field can be obtained only if the upper limit of p_c is known for a certain number of sources N, with $N \gg 1$. This limit allows for much stronger fields than are usually admitted. This last fact should be taken into consideration when one deals with the results of observations of circular polarization in sources with strong magnetic fields.

The first model presents some difficulties when we compare it with observations of some compact sources. The second model can explain why one observes in these sources a violation of the law $p_c \sim \nu^{-1/2}$ and a change of sign in p_c when the frequency of the observations ν changes.

1. Introduction

1.1. As a rule, non-thermal radiation from various cosmic sources is linearly polarized. In the radio frequency range the degree of linear polarization p_l is usually given by $p_l = 3 \pm 10\%$. The degree of circular polarization p_c, in objects of relatively low

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brightness temperature $T < 10^{10}$–10^{12} K (in this paper we consider only such objects), is significantly less than p_l. Komesaroff *et al.* (1968), Biraud and Veron (1968), Seielstad (1969), and Berge and Seielstad (1969), obtained upper estimates for a great number of radio-sources, and it was shown that in various sources $p_c < 0.1 \pm 3\%$. Biraud (1969), Seaquist (1970), Gilbert and Conway (1970), and Conway *et al.* (1971) produced quite reliable measurements of the degree of circular polarization in certain, mainly extragalactic, sources. According to the latest results Gilbert and Conway (1970) the quantity p_c attained a value of $p_c = 0.18\%$*

The small value of p_c in comparison to p_l qualitatively agrees with the synchrotron model of radioemission and, at the same time, provides the basis for a theoretical treatment and interpretation of the observations (1–8) within the scope of this model.

1.2. In this work we shall consider a model of the source of synchrotron emission as an inhomogeneous magnetic field with an isotropic angular distribution of relativistic electrons (model I), and a model of the source with a homogeneous magnetic field and anisotropic angular distribution of the relativistic electrons relative to the magnetic field, where the anisotropy of the angular distribution significantly increases with growth of the electron energies (model II). In both cases the source seems to be transparent – i.e. – re-absorption of the radiation or rotation of the plane of polarization do not occur.

Model I may be used to describe the extended sources (for example radiogalaxies) and the 'quiet' compact sources – i.e., those which do not show large variations in the optical and radio ranges (for example, quasi-stellar galaxies). Actually there is reason to believe that in similar sources the relativistic electrons, after acceleration, (or during the process of acceleration) manage to become isotropic, and that the magnetic field in a relatively large radiating volume may acquire an inhomogeneous character. On the other hand, in the variable compact sources (for example in quasars or the nuclei of *N*-galaxies) the observed variable radio emission may be generated by relativistic electrons (for which isotropy has not been established) in the relatively small radiating volume where the magnetic field, has a quasi-homogeneous character. Model II is intended to describe such sources.

The assumption of transparency of the sources is, in our opinion, reasonable first approximation for the constructing of preliminary theoretical models. In every case this assumption may be easily checked, since in the transparent source the emission is characterized by a power-law spectrum, and the degree of linear polarization does not depend on the frequency.

* Note that the presence of circular polarization of the radiation implies, quite definitely and independently of the nature of this radiation, the existence of a certain axial vector, essentially connected with the process of generation or transport of radiation in the source. On the basis of the synchrotron theory of radio emission such a vector would represent the magnetic field. In our view, the necessity to relate the radiation source with an axial vector, makes difficult the interpretation of the radio emission from extragalactic sources on the basis of the Rees model (1971) – i.e., by regarding it as a Compton emission of relativistic electrons in a low frequency field of magnetic dipole radiation from an aggregate of pulsars situated at the source centre. According to the Rees model it is difficult to associate an axial vector to the extragalactic source as a whole, and not to its separate parts.

1.3. Later on we shall calculate the degree of circular polarization of a source in the presence of a homogeneous magnetic field and isotropic angular distribution of electron (Section 2), and shall consider such information as may be obtained from the combined observations of linear and circular polarization from agglomerations of sources (Section 3), or such information as may be obtained from separate sources (Section 4).

In Section 5 the degree of circular polarization is calculated, and qualitative particulars of the homogeneous magnetic field source with anisotropic angular distribution of electrons, in which the anisotropy of the angular distribution significantly grows with increase of electron energies, are briefly discussed.

In Section 6 a comparison of theory and observations is made, and the results obtained in Sections 2–5 are discussed. It is shown how interpretations of the observations can be given by making use of formulae from Sections 2–5. In particular, it will be shown that if we know the distribution of sources in terms of their linear polarization, we can find the distribution of the sources in terms of the degree of uniformity of the magnetic field and determine in which sources we can expect a maximum degree of circular polarization.

2. Circular Polarization of the Radiation from Sources with a Non-Homogeneous Magnetic Field

2.1. Let us suppose that the spectrum of relativistic electrons in the source is given by the power law relation $N(E) \sim E^{-\gamma}$ and that the synchrotron radiation, after its generation, emerges from the source without absorption or change of polarization. In this case, when the magnetic field in the source may be considered to be uniform, the degree of linear polarization is given (see, for example, Ginzburg and Syrovatsky, 1955) by

$$p_l = \frac{\gamma + 1}{\gamma + \frac{7}{3}}.$$

As was shown by Sciama and Rees (1967), the order of magnitude of the degree of circular polarization is given by

$$p_c \simeq -\operatorname{ctg}\varphi \left(\frac{3e}{2\pi mc} \frac{B \sin\varphi}{\nu} \right)^{1/2}. \tag{2.1}$$

A more exact expression defining the degree of circular polarization was obtained by Legge and Westfold (1968) and Sazonov (1969) as

$$p_c = -\frac{4(\gamma+1)(\gamma+2)}{3\gamma(\gamma+\frac{7}{3})} \frac{\Gamma\left(\frac{3\gamma+4}{12}\right)\Gamma\left(\frac{3\gamma+8}{12}\right)}{\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+7}{12}\right)} \times$$
$$\times \left(\operatorname{ctg}\varphi + \frac{1}{\gamma+2} \frac{1}{Y} \frac{\mathrm{d}Y}{\mathrm{d}\varphi} \right) \left(\frac{B \sin\varphi}{B_\nu} \right)^{1/2}. \tag{2.2}$$

where φ is the angle between the direction of the magnetic field and the line of sight; $Y(\theta)$ is a function describing the distribution of the relativistic electrons in terms of the angle θ between their momenta and the magnetic field $\mathbf{B}$ for an isotropic angular distribution $Y=1$. Rotation of the electric vector in the wave occurs in the clockwise sense if $p_c<0$ and conversely.

The quantity B_ν is given by

$$B_\nu \equiv \frac{2\pi}{3} \frac{mc}{e} \nu \simeq \frac{\nu}{10^6},$$

where ν is the observation frequency, all numerical quantities are given in the absolute or CGS system of units.

Formula (2.2) may be used only in the case when

$$\left| \operatorname{ctg}\varphi + \frac{1}{\gamma+2} \frac{1}{Y} \frac{\mathrm{d}Y}{\mathrm{d}\varphi} \right| \left(\frac{B \sin\varphi}{B_\nu} \right)^{1/2} \ll 1 .$$

2.2. In real sources the magnetic field is, of course, non-uniform. Within the limits of model I we consider two simplest models of the distribution of the field in the source, introduced by Korchak and Syrovatsky (1961).

Ia. Superposition of Homogeneous and Random Fields

At each point of the source the total field $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_R ,$$

where $\mathbf{B}_0$ is homogeneous field and $\mathbf{B}_R$ is a random field of constant magnitude. The orientation of $\mathbf{B}_R$ changes, with passage from point to point in an arbitrary manner, however, the distance, over which $\mathbf{B}_R$ significantly changes its direction, is very much greater than the Larmor radius of the a electron.

Furthermore, we may consider that $B_0 \ll B_R$ so that the modulus of the total field $B = |\mathbf{B}_0 + \mathbf{B}_R| \simeq B_R$.

Ib. Non-isotropic Field of Constant Magnitude

The probability that at a given point of the source the direction of the field $\mathbf{B}$ lies within the solid angle $\mathrm{d}\Omega$ around the direction $\mathbf{n}$ is given by

$$\mathrm{d}w = \frac{\mathrm{d}\Omega}{4\pi} \left[1 + 3\varkappa \mathbf{nb} + \tfrac{5}{2}\delta \left(3\,(\mathbf{nb})^2 - 1\right)\right], \tag{2.3}$$

where $\mathbf{b}$ is a unit vector, defining the most probable direction of $\mathbf{B}$. The parameter $\varkappa$ gives the mean value of the vector B in the sources defined by the relation $\langle \mathbf{B} \rangle \equiv \equiv \int \mathrm{d}w \mathbf{B} = \varkappa \mathbf{b} B$. The parameter δ determines the anisotropy of the field, introduced by the relation $\langle B_\parallel^2 \rangle - \langle B_\perp^2 \rangle = \delta B^2$. Furthermore, we shall consider both $\varkappa$ and δ to be very much smaller than 1.

2.3. As is well known, the degrees of linear and circular polarization are given by

$$p_l = \frac{(Q^2 + U^2)^{1/2}}{F}, \qquad p_c = \frac{V}{F},$$

where F denotes the total flux of radiation from the sources, Q, U and V are the corresponding Stokes parameters. Averaging the Stokes parameters for the magnetic field, Korchak and Syrovatski (1961) showed that in the case of model Ia the degree of linear polarization is given by

$$p_l = L_a(\gamma)\left(\frac{B_0}{B}\right)^2 \sin^2\varphi. \tag{2.4}$$

In the case of model Ib,

$$p_l = L_b(\gamma)\,\delta\,\sin^2\varphi. \tag{2.5}$$

For the case of circular polarization, similar calculations yield, in the first case,

$$p_c = C_a(\gamma)\frac{B_0}{B}\left(\frac{B}{B\nu}\right)^{1/2} \cos\varphi, \tag{2.6}$$

and, in the second case,

$$p_c = C_b(\gamma)\,\varkappa\left(\frac{B}{B\nu}\right)^{1/2} \cos\varphi, \tag{2.7}$$

where φ is the angle between the line of sight and the homogeneous field component $\mathbf{B}_0$ or the direction of $\mathbf{b}$. The angular distribution of electrons was considered to be isotropic. The functions $L(\gamma)$ and $C(\gamma)$ for model Ia are given by

$$L_a(\gamma) = \frac{(\gamma+3)(\gamma+5)}{32}\,\frac{\gamma+1}{\gamma+\frac{7}{3}}, \tag{2.8}$$

TABLE I

γ	α	L_a	C_a	L_b	C_b
0.5	−0.25	0.318	0.299	0.728	0.276
1.0	0.00	0.450	0.686	0.844	0.588
1.5	+0.25	0.596	0.888	0.935	0.711
2.0	0.50	0.757	1.036	1.001	0.777
2.5	0.75	0.933	1.157	1.072	0.817
3.0	1.00	1.125	1.262	1.125	0.842
3.5	1.25	1.332	1.357	1.171	0.857
4.0	1.50	1.554	1.444	1.211	0.866
6.0	2.50	2.599	1.740	1.333	0.870
8.0	3.50	3.893	1.988	1.415	0.853

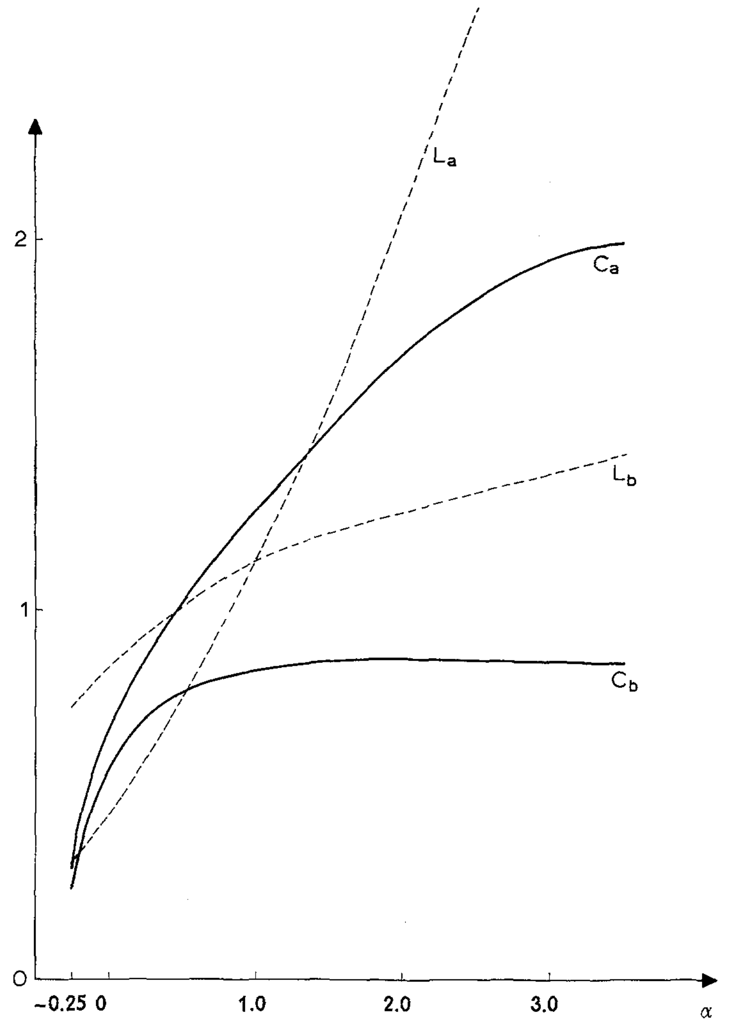

Fig. 1. The dependence of the coefficients L_a, L_b and C_a, C_b on the spectral index $\alpha = (\gamma - 1)/2$ see formulae (2.8)–(2.11)

For Figures 2 and 3 (General remarks)

The observed distribution of a certain group of sources in degree of linear and circular polarization in relation to the distribution of the same sources in terms of the field stress and degree of homogeneity of the magnetic field.

The quantities l and c are given by

$$l = \frac{p_l}{L}, \qquad c = \frac{p_c}{C},$$

where p_l and p_c are the degrees of linear and circular polarization, L and C are numerical coefficients of the order of unity. (see (2.8–(2.11), Table I and Figure 1).

The quantity β is equivalent to the magnetic field B, measured in units of B_ν:

$$\beta = \frac{B}{B_\nu},$$

where

$$B_\nu = \frac{2\pi}{3} \frac{mc}{e} \nu \simeq \frac{\nu}{10^6},$$

$$C_a(\gamma) = \frac{4}{3} \frac{(\gamma+1)(\gamma+2)}{\gamma(\gamma+\frac{7}{3})} \times \\ \times \frac{\Gamma\left(\frac{\gamma+4}{4}\right)\Gamma\left(\frac{\gamma+7}{4}\right)\Gamma\left(\frac{3\gamma+4}{12}\right)\Gamma\left(\frac{3\gamma+8}{12}\right)}{\Gamma\left(\frac{\gamma+5}{4}\right)\Gamma\left(\frac{\gamma+6}{4}\right)\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+7}{12}\right)}. \tag{2.9}$$

For model 1b

$$L_b(\gamma) = L_a(\gamma)\frac{60}{(\gamma+3)(\gamma+5)}, \tag{2.10}$$

$$C_b(\gamma) = C_a(\gamma)\frac{6}{\gamma+6}; \tag{2.11}$$

see Table I and Figure 1.

The expression (2.9) differs by a factor $\frac{4}{3}$ from the corresponding expression obtained by Melrose (1971). The formulae (2.4) to (2.11) determine the degrees of linear and circular polarization for a source with a non-uniform magnetic field and isotropic angular distribution of electrons. Note that with an increase of γ or the spectral index $\alpha = (\gamma - 1)/2$ the quantity p_c increases (see Figure 1).

2.4. Let us introduce notations

$$l = \frac{p_l}{L_a}, \qquad c = \frac{p_c}{C_a}, \qquad \varepsilon = \left(\frac{B_0}{B}\right)^2, \qquad \beta = \frac{B}{B_\nu}, \qquad x = |\cos\varphi|, \tag{2.12}$$

which permit us to rewrite Equations (2.4) and (2.6) of model Ia as

$$l = \varepsilon(1 - x^2), \tag{2.13}$$

$$c^2 = \varepsilon\beta x^2. \tag{2.14}$$

The quantity $\varepsilon = (B_0/B)^2$ characterizes degree of uniformity of the field in the source, the quantity $\beta = B/B_\nu$ represents the magnetic field, measured in units of B_ν.

Equations (2.5) and (2.7) of model Ib contain, not one, but two quantities characterizing the magnetic field: $\varkappa$ and δ. These quantities figure as coefficients in the expansion of the probability $\mathrm{d}W$ in terms of a series of Legendre polynomials (see

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ν is the frequency of observation. The quantity ε is the degree of homogeneity of the source field:

$$\varepsilon = \left(\frac{B_0}{B}\right)^2,$$

where B_0 is a homogeneous field. Each subset of sources with equal values of ε and β in the (l, c^2) plane correspond to the straight line

$$c^2 = -l\beta + \beta\varepsilon.$$

The density of points on the line is proportional to the number of sources. The angle between the direction of the uniform field B_0 in the source and the line of sight varies from 0 or π up to $\pi/2$ with movements along the line as shown by the arrows. The dotted line represents the mean value of c^2.

Equation (2.3)). For this reason, in the case of both $\varkappa$ and $\delta \ll 1$ we may substitute, without too much error, $\delta = \varkappa^2$; and if so, Equations (2.13), (2.14) retain their meaning also for the case Ib if we set

$$l = \frac{p_l}{L_b}, \qquad c = \frac{p_c}{C_b}, \qquad \varepsilon = \delta = \varkappa^2. \tag{2.15}$$

Moreover, we may apply formulae (2.13) and (2.14) equally to the models Ia and Ib and so drop the indices a, b from the coefficients. However in the text, for definitive purposes, we shall refer only to model Ia. Besides this, for brevity, we shall identify the degrees of linear and circular polarization both p_l, p_c and l, c which differ from p_l and p_c by numerical factors of order unity.

3. The Distribution of Magnetic Fields in an Aggregate of Sources

3.1. Let us consider such information as may be provided within the limits of model I knowing from observations the quantities p_l and p_c for a certain large number of sources. In order to describe this aggregation we introduce the function $F(x, \gamma, \varepsilon, \beta)$ in which the distribution of sources in terms of the exponent γ and parameters x, ε and β have been normalized to unity (see Equations (2.12), (2.15)).

Let us introduce the relationship which connects the function F with the observed quantities. Since all directions of the homogeneous field $\mathbf{B}_0$ in the different sources are equally probable, the quantity x is distributed uniformly in the interval (0, 1). In this case F does not depend on x, and the number $\mathrm{d}N$ of sources in the volume element $\mathrm{d}\gamma\,\mathrm{d}x\,\mathrm{d}\varepsilon\,\mathrm{d}\beta$ is given by

$$\mathrm{d}N = NF(\gamma, \varepsilon, \beta)\, \mathrm{d}\gamma\, \mathrm{d}x\, \mathrm{d}\varepsilon\, \mathrm{d}\beta. \tag{3.1}$$

Let us change over in (3.1) by means of (2.13) and (2.11) to the variables l and c^2, and integrate $\mathrm{d}N$ with respect to $\mathrm{d}\gamma$ and $\mathrm{d}x$. Since the dispersion of sources with respect to γ is comparatively weak and the coefficients $L(\gamma)$ and $C(\gamma)$ in (2.12), (2.15) depend but weakly on γ for γ in the range 1 to 3, it may be assumed that $F(\gamma, \varepsilon, \beta) = \delta(\gamma - \bar{\gamma})\, F(\varepsilon, \beta)$, where the mean value $\bar{\gamma} = 2$–2.5. In this case we obtain

$$\mathrm{d}N = N\, \mathrm{d}l\, \mathrm{d}c^2\, \frac{1}{l} \int_0^1 \frac{\mathrm{d}x}{x^2}\, F\left(\frac{l}{1 - x^2}, \frac{c^2}{l}\, \frac{1 - x^2}{x^2}\right), \tag{3.2}$$

where (and hereafter) we put $l = p_l / L(\bar{\gamma})$ and $c = p_c / C(\bar{\gamma})$. Let us rewrite the integral with respect to $\mathrm{d}x$ in a more convenient form, for which we introduce the integration element $\mathrm{d}y$, such that

$$\mathrm{d}N = N\, \mathrm{d}l\, \mathrm{d}c^2\, \frac{1}{l} \int_0^1 \frac{\mathrm{d}x}{x^2} \int_0^1 \mathrm{d}y\, 2y\, \delta(y^2 - x^2)\, F\left(\frac{l}{1 - x^2}, \frac{c^2}{l}\, \frac{1 - y^2}{y}\right),$$

and then we transfer from the integration increment $\mathrm{d}x\, \mathrm{d}y$ to

$$\mathrm{d}\varepsilon\, \mathrm{d}\beta = \left|\frac{D(\varepsilon, \beta)}{D(x, y)}\right| \mathrm{d}x\, \mathrm{d}y$$

with the aid of the equation

$$x = \left(1 - \frac{l}{\varepsilon}\right)^{1/2}, \qquad y = \left(1 + \frac{l\varepsilon}{c^2}\right)^{-1/2}.$$

From the relation

$$n(l, c^2) \equiv \frac{1}{N} \frac{\mathrm{d}N}{\mathrm{d}l\, \mathrm{d}c^2}$$

we obtain

$$n(l, c^2) = \tfrac{1}{2} \int\limits_l^1 \frac{\mathrm{d}\varepsilon}{(\varepsilon^2 - \varepsilon l)^{1/2}} \int\limits_0^1 \mathrm{d}\beta F(\varepsilon, \beta) \cdot \delta\left(c^2 - \beta(\varepsilon - l)\right). \tag{3.3}$$

This equation connects the function $F(\varepsilon, \beta)$ – the normalized distribution of sources in terms of the degree of uniformity of the magnetic field ε and the parameter β with the function $n(l, c^2)$ which expresses the normalized observed distribution of sources in terms of the degrees of linear and circular polarization* (see Equations (2.12) and (2.15)).

Equation (3.3) allows us to examine the distribution of sources $F(\varepsilon, \beta)$ with the function $n(l, c^2)$ which is known from the observations. In this section we found the solution of Equation (3.3) for a single particular case; in Section 6 we shall consider qualitatively the general case.

3.2. From (3.3) we obtain the function $F(\varepsilon) \equiv \int_0^1 \mathrm{d}\beta F(\varepsilon, \beta)$ which represents the normalized distribution of sources in terms of the degree of uniformity of the magnetic field $\bar{\varepsilon}$. Integrating (3.3) with respect to $\mathrm{d}c^2$ we obtain

$$n(l) = \tfrac{1}{2} \int\limits_l^1 \frac{\mathrm{d}\varepsilon}{(\varepsilon - l)^{1/2}} \frac{F(\varepsilon)}{\varepsilon^{1/2}}, \tag{3.4}$$

where $n(l) = \int_0^1 \mathrm{d}c^2 n(l, c^2)$ is the normalized distribution of sources in terms of the linear polarization. The solution of Equation (3.4), which leads to an Abel equation, is given by

$$F(\varepsilon) = -\frac{2}{\pi} \varepsilon^{1/2} \int\limits_\varepsilon^1 dl\,(l - \varepsilon)^{-1/2} \frac{\mathrm{d}n(l)}{\mathrm{d}l}. \tag{3.5}$$

* For the sake of definiteness we take the upper limits of the integration in (3.3) as unity. It is necessary to bear in mind, however, that in this paper we assume that there are no sources with large values of ε and β, so that $F(\varepsilon, \beta) > 0$ only for $\varepsilon, \beta \ll 1$. This corresponds to the case where the function $n(l, c^2)$ differs from zero only when $l, c^2 \ll 1$.

Let us observe that Equation (3.4) may be obtained with the aid of formula (2.13), regardless of problem of the circular polarization.

4. The Magnetic Field in a Separate Source

4.1. Let us consider what information may be provided within the limitations of model I if we know the observed degree of linear and circular polarization p_l and p_c for a single source.

Let a source with given p_l and p_c belong to a certain aggregate of sources, the distribution of which, in terms of the parameters ε and β is given by the function $F(\varepsilon, \beta)$. Let us find the function $f(x, \varepsilon, \beta)$ – the normalized distribution, in terms of the parameters x, ε and β, of those sources of the given aggregate, which have the given value p_l and p_c (remembering that $x = |\cos\varphi|$).

Let the parameters l and c^2 be contained within the limits

$$l_0 \leqslant l \leqslant l_0 + \Delta l, \qquad c_0^2 \leqslant c^2 \leqslant c_0^2 + \Delta c^2. \tag{4.1}$$

The number $\mathrm{d}N$ of sources for which the values of l and c^2 are contained within the limits (4.1) while the parameter x lies in the interval $\mathrm{d}x$, is given by

$$\mathrm{d}N = N\,\mathrm{d}x \iint_{\Omega} \mathrm{d}\varepsilon\,\mathrm{d}\beta F(\varepsilon, \beta). \tag{4.2}$$

The domain of integration Ω is determined from the conditions (4.1) and Equations (2.13), (2.14). We may change over, in (4.2), to integration with respect to $\mathrm{d}l$ and $\mathrm{d}c^2$ by nothing that

$$\mathrm{d}l\,\mathrm{d}c^2 = \left|\frac{D(l, c^2)}{D(\varepsilon, \beta)}\right| \mathrm{d}\varepsilon\,\mathrm{d}\beta.$$

Letting Δl and Δc^2 tend to zero and dropping the zero suffix from l and c^2, we obtain

$$\mathrm{d}N = N\,\mathrm{d}x\,\Delta l\,\Delta c^2\,\frac{F(\varepsilon(x), \beta(x))}{\left|\dfrac{D(l, c^2)}{D(\varepsilon, \beta)}\right|}.$$

All sources with given l and c^2 lie in the (x, ε, β) space on the line $(x, \varepsilon(x), \beta(x))$. The functions $\varepsilon(x)$ and $\beta(x)$ may be found from (2.13) and (2.14).

$$\varepsilon(x) = \frac{l}{1 - x^2}, \qquad \beta(x) = \frac{c^2}{l}\,\frac{1 - x^2}{x^2}.$$

The Jacobian $D(l, c^2)/D(\varepsilon, \beta)$ is equal to $l\,x^2$. Introducing

$$f(x, \varepsilon, \beta) \sim \frac{\mathrm{d}N}{N\,\Delta l\,\Delta c^2\,\mathrm{d}x\,\mathrm{d}\varepsilon\,\mathrm{d}\beta},$$

we obtain

$$f(x, \varepsilon, \beta) = A\delta(\varepsilon - \varepsilon(x))\,\delta(\beta - \beta(x))\,\frac{F(\varepsilon, \beta)}{lx^2}\,. \tag{4.3}$$

where A is a normalizing factor.

4.2. The mean value x for sources with given p_l and p_c may be found from the formula

$$\bar{x} = \int_0^1 \mathrm{d}x \cdot x \cdot p(x),$$

where

$$p(x) = \int_0^1 \mathrm{d}\varepsilon \int_0^1 \mathrm{d}\beta f(x, \varepsilon, \beta);$$

and similarly for $\bar{\varepsilon}$ and $\bar{\beta}$. For a separate source the most probable set of three values $(x_p, \varepsilon_p, \beta_p)$ is determined by the condition $f(x, \varepsilon, \beta) = \text{maximum}$.

As a qualitative example we may consider the case when the distribution of sources in terms of ε and β is uniform

$$F(\varepsilon, \beta) = \frac{1}{\varepsilon_m \beta_m}, \qquad \begin{matrix} 0 \leqslant \varepsilon \leqslant \varepsilon_m \\ 0 \leqslant \beta \leqslant \beta_m \end{matrix} \tag{4.4}$$

where ε_m and $\beta_m < 1$.

The probability density for the parameter x is than given by

$$p(x) = \frac{A}{l}\frac{1}{x^2}, \qquad \frac{c}{\sqrt{l\beta_m + c^2}} \leqslant x \leqslant \sqrt{\frac{\varepsilon_m - l}{\varepsilon_m}}; \tag{4.5}$$

and, similarly,

$$p(\varepsilon) = \frac{A}{2}\frac{1}{\varepsilon^{1/2}(\varepsilon - l)^{3/2}}, \qquad l + \frac{c^2}{\beta_m} \leqslant \varepsilon \leqslant \varepsilon_m; \tag{4.6}$$

$$p(\beta) = \frac{A}{2c^2}\frac{1}{\left(1 + \frac{l\beta}{c^2}\right)^{1/2}}, \qquad \frac{c^2}{\varepsilon_m - l} \leqslant \beta \leqslant \beta_m. \tag{4.7}$$

The normalizing factor A is given by

$$A = \frac{l}{\sqrt{1 + \frac{l\beta_m}{c^2}} - \sqrt{\frac{\varepsilon_m}{\varepsilon_m - l}}}.$$

The limits of the variables x, ε and β in (4.5)–(4.7) are defined by the condition that the line $(x, \varepsilon(x), \beta(x))$ lie inside the region $(0 < x < 1, 0 < \varepsilon < \varepsilon_m, 0 < \beta < \beta_m)$. Outside these limits the probability density is zero. Let us confine our attention to the subset of sources with such values of p_l and p_c that

$$l \ll \varepsilon_m, \qquad c^2 \ll l\beta_m. \tag{4.8}$$

Then $A=(lc^2/\beta_m)^{1/2}$ and from (4.5)–(4.7) we obtain the mean values

$$\bar{x}=\left(\frac{c^2}{l\beta_m}\right)^{1/2}\tfrac{1}{2}\ln\frac{l\beta_m}{c^2},\qquad \bar{\varepsilon}=\frac{l}{4},\qquad \bar{\beta}=\frac{\beta_m}{3}. \tag{4.9}$$

In order to find the most probable set of three values $(x_p, \varepsilon_p, \beta_p)$ we calculate $\mathrm{d}n(s)/\mathrm{d}s$, where $n(s)$ is the density of the number of sources on the line $(x, \varepsilon(x), \beta(x))$, $\mathrm{d}s^2=\mathrm{d}x^2+\mathrm{d}\varepsilon^2+\mathrm{d}\beta^2$. Of course,

$$n(s)=n(x)\frac{\mathrm{d}x}{\mathrm{d}s},$$

where

$$n(x)\sim\frac{F(\varepsilon(x),\beta(x))}{x^2}.$$

Now

$$\frac{\mathrm{d}n(s)}{\mathrm{d}s}=\frac{\mathrm{d}x}{\mathrm{d}s}\frac{\mathrm{d}}{\mathrm{d}x}\left[n(x)\frac{1}{\mathrm{d}s/\mathrm{d}x}\right].$$

From the condition $\mathrm{d}n/\mathrm{d}s=0$ we obtain

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{F(\varepsilon(x),\beta(x))}{x^2\sqrt{1+(\mathrm{d}\varepsilon/\mathrm{d}x)^2+(\mathrm{d}\beta/\mathrm{d}x)^2}}=0. \tag{4.10}$$

In the case of a uniform distribution $F(\varepsilon, \beta)=\text{const.}$, and from (4.10) it follows that

$$x_p^6(1-x_p^2)^3+6l^2x_p^8(1-x_p^2)+4l^2x_p^{10}=2\frac{c^4}{l^2}(1-x_p^2)^3. \tag{4.11}$$

Let the condition $c^2 \ll l$ be satisfied, then from (4.11) we find

$$x_p\simeq\left(\sqrt{2}\frac{c^2}{l}\right)^{1/3},\qquad \varepsilon_p=\varepsilon(x_p)\simeq l,\qquad \beta_p=\beta(x_p)\simeq\left(\frac{c^2}{2l}\right)^{1/3}. \tag{4.12}$$

The formulae (4.12) may be used if the point $(x_p, \varepsilon_p, \beta_p)$ lies inside the domain limited by $(0<x<1, 0<\varepsilon<\varepsilon_m, 0<\beta<\beta_m)$; for which it is necessary to fulfill the condition

$$\left(\frac{c^2}{2l}\right)^{1/3}<\beta_m.$$

5. The Circular Polarization of a Source with an Anisotropic Angular Distribution of Electrons

5.1. From the results of Equation (2.2) it may be understood that the distribution of relativistic electrons in energy and the angle θ was taken of the form

$$N(E,\theta)=KE^{-\gamma}Y(\theta). \tag{5.1}$$

However, Equation (5.1) is justifiable only when the angular distribution of electrons does not depend on their energies (model I). At the same time, for variable compact sources model II appears to be more realistic, in which the angular distribution is anisotropic and depends on the energies, so that the electrons with greater energies have an essentially greater anisotropy of angular distribution relative to the magnetic field, than the electrons with lesser energies (a similar situation may arise, for example, if the electrons escape off from an emitting region, 'breaking through' the magnetic field which would contain them).

5.2. In this section, for the purposes of qualitative illustration of the details of model II, we wish to calculate the circular polarization of a source with a homogeneous magnetic field in which the distribution of relativistic electrons takes the form

$$N(E,\theta)=\tilde{N}_e\left(\frac{E}{mc^2}\right)^{-\gamma}\left[Y(\theta)+\left(\frac{E}{mc^2}\right)^{\Delta\gamma}y(\theta)\right], \tag{5.2}$$

where $\Delta\gamma>0$. The function $y(\theta)$ describes a part of the angular distribution of electrons, the relative magnitude of which increases with the energy. If $y(\theta)=0$ the distribution (5.2) reverts to (5.1) with the coefficient $K=\tilde{N}_e(mc^2)^\gamma$. We consider in what follows, that the function $y(\theta)\ll 1$ and $Y(\theta)=1$. Substituting (5.2) in the general formulae for the Stokes' parameters I and V of synchrotron radiation (see, for example, Sazonov, 1969 we obtain

$$\begin{aligned}I={}&\tilde{N}_e\frac{\sqrt{3}e^2}{8c}\nu_B\sin\varphi\left[\frac{\gamma+\frac{7}{3}}{\gamma+1}\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+7}{12}\right)\left(\frac{3\nu_B\sin\varphi}{\nu}\right)^{(\gamma-1)/2}+\right.\\&\left.+\,y(\varphi)\frac{\gamma'+\frac{7}{3}}{\gamma'+1}\Gamma\left(\frac{3\gamma'-1}{12}\right)\Gamma\left(\frac{3\gamma'+7}{12}\right)\left(\frac{3\nu_B\sin\varphi}{\nu}\right)^{(\gamma'-1)/2}\right],\end{aligned} \tag{5.3}$$

$$\begin{aligned}V={}&-\tilde{N}_e\frac{e^2}{2\sqrt{3}c}\nu_B\sin\varphi\left[\frac{\gamma+2}{\gamma}\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+8}{12}\right)\times\right.\\&\times\operatorname{ctg}\varphi\left(\frac{3\nu_B\sin\varphi}{\nu}\right)^{\gamma/2}+\frac{\gamma'+2}{\gamma'}\Gamma\left(\frac{3\gamma'+4}{12}\right)\Gamma\left(\frac{3\gamma'+8}{12}\right)\times\\&\left.\times\left(\operatorname{ctg}\varphi\cdot y(\varphi)+\frac{1}{\gamma'+2}\frac{dy}{d\varphi}\right)\left(\frac{3\nu_B\sin\varphi}{\nu}\right)^{\gamma'/2}\right],\end{aligned} \tag{5.4}$$

where

$$\nu_B=\frac{eB}{2\pi mc},\qquad \gamma'=\gamma-\Delta\gamma.$$

For reasons which should be clear (see paragraph 6.2) we shall be interested in the region of intermediate frequencies, for which

$$\frac{(\gamma+2)\,|\operatorname{ctg}\varphi|}{\left|\frac{dy}{d\varphi}\right|}\ll\left(\frac{\nu}{3\nu_B\sin\varphi}\right)^{\Delta\gamma/2}\ll\frac{1}{|y(\varphi)|}. \tag{5.5}$$

In order to satisfy (5.5) in the region where the angle $\varphi \sim 1$, it is obviously necessary to have a sharp anisotropy of the variable component of the angular distribution

$$\left|\frac{\mathrm{d}y}{\mathrm{d}\varphi}\right| \gg |y(\varphi)|.$$

It is easy to see that in the region of intermediate frequencies, (5.5), the degree of circular polarization is given by

$$p_c \equiv \frac{V}{I} = -\frac{4(\gamma+1)}{3\gamma'(\gamma+\frac{7}{3})} \frac{\Gamma\left(\frac{3\gamma'+4}{12}\right)\Gamma\left(\frac{3\gamma'+8}{12}\right)}{\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+7}{12}\right)} \frac{\mathrm{d}y}{\mathrm{d}\varphi} \left(\frac{3\nu_B \sin\varphi}{\nu}\right)^{(1-\Delta\gamma)/2}. \tag{5.6}$$

In the domain of lower frequencies, where the first condition of (5.5) is violated, the quantity p_c is given by Equation (2.2) with $Y=1$. In the domain of higher frequencies, where the second condition of (5.5) is not fulfilled, the quantity p_c is given by (2.2) with $Y(\varphi)=y(\varphi)$ and $\gamma=\gamma'$.

Let us note two details of the considered model. First, the dependence of the quantity p_c on the frequency ν in the domain of intermediate frequencies takes the form $p_c \sim \nu^{-(1-\Delta\gamma)/2}$. For $\Delta\gamma > 0$, such a dependence departs from the law $p_c \sim \nu^{-1/2}$ both at high and low frequencies. Second, the sign of p_c, with the passage from higher or lower frequencies to the intermediate region must be changed if $\mathrm{d}y/\mathrm{d}\varphi < 0$.

It seems plausible that the qualitative details of model II which have just been indicated, are not tied to a specific form of distribution such as (5.2), but affect in general the properties of sources for which the anisotropy of the relativistic electrons distribution increases considerably with the electron energies.

A more detailed investigation of the properties of such sources will be given in a subsequent paper.

6. Discussion of Results Comparison with Observations

6.1. At the present time there is not a sufficiently exact and full knowledge of the circular polarization of radio-sources. Nevertheless the use of obtained measurements may allow one to make the following provisional conclusions:

(a) The degree of circular polarization of quasars has a tendency to decrease with growth of the spectral index $\alpha = (\gamma - 1)/2$ of the sources in the interval $-0.2 < \alpha < 0.5$ (Gilbert and Conway, 1970).

(b) The dependence of the quantity p_c on frequency differs from the law $p_c \sim \nu^{-1/2}$ (Seielstad, 1969).

(c) With a change of frequency in certain sources a change in the sign of p_c occurs. (Conway *et al.*, 1971; Seielstad, 1969).

(d) In the source VRO 42.22.01 there exists a correlation between the variation of the radiation flux and the variation of the degree of circular polarization (Biraud, 1969).

(e) There is no correlation between the quantities p_l, and p_c (Gilbert and Conway, 1970).

6.2. The conclusions (a), (b), and (c) are found to contradict Model I with a transparent source, isotropic angular distribution of electrons and inhomogeneous magnetic field; see (2.6) and (2.7), Table I and Figure 1*. In our opinion the most important cause of the disagreement is that in the derivation of Equations (2.6) and (2.7) the angular distribution of the electrons was considered to be isotropic.

It may be thought that in compact radio-sources the circular polarization is caused mainly by anisotropic angular distribution of relativistic electrons, which are ejected from the source (Model II). In this case the following situation should be expected:

(I) A variation of radio-sources having a greater degree of circular polarization in comparison to other sources.

(II) A faster relative variation in time of the circular polarization in comparison wish the relative variation of the total flux, since the time for a 'bunch' electrons in a plasma to become isotropic should be less than the time of variation of other parameters of the 'bunch'.

(III) The increase (a mean for many sources) of the degree of circular polarization with increase of the spectral index $\alpha=(\gamma-1)/2$. Actually, the continuous spectrum (i.e., small value of α) is usually associated with variable sources with powerful energy production as a result of which there may be produced a 'bunch' of relativistic electrons.

(IV) The change of sign of p_c and violation of the law $p_c \sim \nu^{-1/2}$ in the region of intermediate frequencies (see Equation (3.6)), if the anisotropy of the angular distribution of electrons increases with an increase of their energy.

The existing observations furnish certain evidence which would confirm the expectations I–IV.

It should be pointed out, however, that observational confirmation of any of the points in I–IV may not be considered as a sufficient verification of model II, since the separate attributes in I–IV could be explained by entirely different models. Thus, for example, Pacholczyk and Swihart (1971), and Melrose (1971a, b) showed that a change of sign of p_c could be explained by re-absorption of the emission within the source.

Nevertheless, the observational confirmation of all of the results in I–IV could, in our view, provide a convincing argument in favour of the hypothesis that within variable, compact radio sources acceleration of relativistic electrons occurs in the form of sharply anisotropic 'bunches', that is as in model II.

6.3. Let us consider what sort of qualitative results about the distribution of extended and 'quiescent' compact radio sources, in terms of the parameters ξ and β may be obtained on the basis of model I, with the aid of Equation (3.3), when a form of the function $n(l, c^2)$ may be known from observations. Remembering that $\varepsilon=(B_0/B)^2$ is a measure of the field homogeneity in the source and $\beta=B/B_\nu$ gives the

* Strictly speaking the growth of the coefficient $C(\gamma)$ with increase of γ contradicts the conclusion (a) only if the quantities B_0/B, B/B_ν and $\varkappa$ do not show a strong dependence on γ. There has been no indication whatever of such a dependence so far.

magnetic field B measured in units of $B_\nu = \nu/10^6$, where ν is the frequency of observation; $l = p_l/L_a$ and $c = p_c/C_a$, where p_l and p_c are the degrees of linear and circular polarization respectively, L_a and C_a being numerical coefficients of the order of unity (see Equations (2.8)–(2.11)).

Let a point on the plane (l, c^2) correspond to each source of given values of p_l and p_c. The subset of sources with equal values of ε and β constitute a straight line in the plane (l, c^2) see Equation (3.3) given by

$$c^2 = -l\beta + \varepsilon\beta, \tag{6.1}$$

which intersects the l-axis at the point $l = \varepsilon$, and subtends an angle arctgβ with this axis in clockwise direction. The density of sources on the line (6.1) is proportional to $((\varepsilon - l)^2 + C^4)^{-1/4}$.

The collection of sources having the same value of ε, but different values of β give a group of straight lines in the (l, c^2)-plane, emanating from the point $(\varepsilon, 0)$ as may be seen in Figure 2. The group of sources having the same value of β but different values of ε furnish a family of parallel lines in the $(l, c^?)$-plane forming an angle arctgβ with the l-axis, as may be seen from Figure 3.

If we compare diagrams of the type 2 or 3 with the observed distribution of points in the (l, c^2) plane it becomes possible to determine the approximate form of the function $F(\varepsilon, \beta)$ – in particular, to find the dispersion of the quantities ε and β about their mean values. Thus, for example, if the observations show the absence of a correlation between the quantities p_l and p_c, then it may be possible to conclude that the

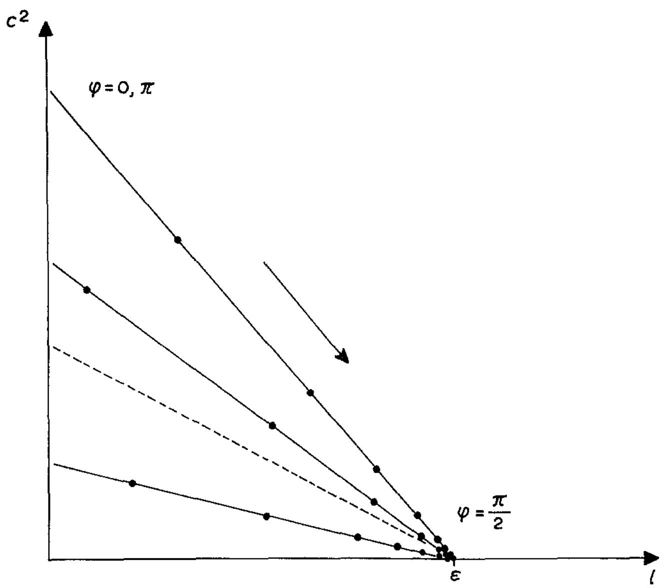

Fig. 2. Distribution in the (l, c^2) plane of a group of sources with the same value of ε but different values of β.

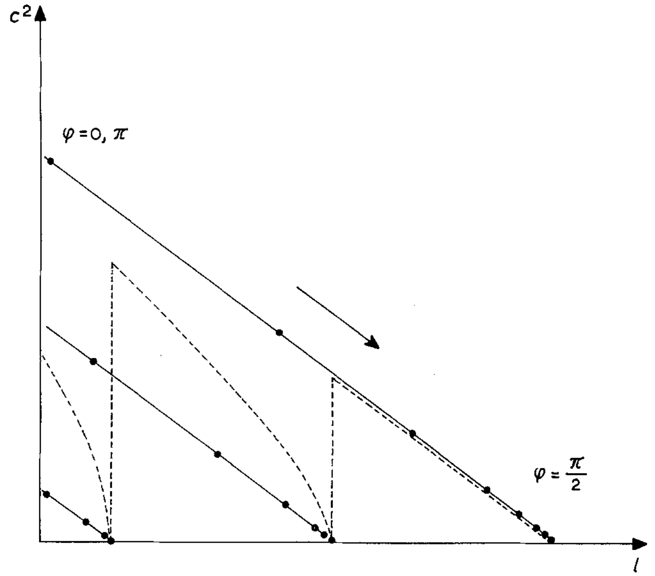

Fig. 3. Distribution in the (l, c^2) plane of a group of sources with the same value of β and different values of ε. See general remarks on pp. 30–31.
For ease of visual representation Figures 2 and 3 have a larger scale in the axes of c^2.

distribution of sources $F(\varepsilon, \beta)$ possesses a large dispersion $\Delta\varepsilon \gtrsim \varepsilon$ (for this case see also paragraph 6.6 below).

6.4. At the present time sufficient observational data exist to determine the distribution of different kinds of source in terms of their degree of linear polarization p_l and to construct the function $n(l)$. On this basis a substitution of $n(l)$ in (3.5) will allow the distribution of these kinds of source in terms of their homogeneous magnetic field $F(\varepsilon)$ to be discovered*.

To carry out this program appears to us to constitute an important and urgent problem.

6.5. Let the observed values of the degrees of linear and circular polarization be p_l and p_c for a certain source. At first glance, the solution of the problem of the determination of the parameters x, ε and β for this source could be obtained in the following manner.

* Let us observe one apparently paradoxical circumstance. According to (3.5), the number of sources with given ε_0 is determined from the observed number of sources $n(l)$, with linear polarization $l > \varepsilon_0$. But, of course, for a source satisfying $l \geqslant \varepsilon_0$ it is necessary that $\varepsilon \geqslant l \geqslant \varepsilon_0$. The question then arises: why does the number of sources $F(\varepsilon)$ within a small uniformity of the magnetic field ε_0 depend on the number of observations of sources with a greater linear polarization $l > \varepsilon_0$, although for all such sources the parameter $\varepsilon > \varepsilon_0$.

The answer may be given as follows. Let there be a certain function $n(l)$, which corresponds to the distribution $F(\varepsilon)$. Let us change $n(l)$ in the domain where $l > l_1$. It is evident that in the domain $\varepsilon > l_1$ the distribution $F(\varepsilon)$ must change. But variation of $F(\varepsilon)$ in this domain necessarily involves a variation of the function $n(l)$ for $l < l_1$. Therefore, in order to compensate for the change of $n(l)$ with $l < l_1$, it is necessary to alter the distribution $F(\varepsilon)$ for $\varepsilon < l_1$.

Let us take the parameter $x = |\cos\varphi|$ of our source to be close to its mean value of $\frac{1}{2}$. Then from (2.13) and (2.14) it follows that

$$x = \tfrac{1}{2}, \qquad \varepsilon = \tfrac{4}{3} l, \qquad \beta = 3 \frac{c^2}{l} \tag{6.2}$$

and it may be thought that the most probable values of x, ε and β are close to those given by (6.2).

Such an approach would be incorrect, since we do not consider an arbitrary source, but one in which the values of p_l and p_c have been fixed: under these conditions the distribution of x in the interval (0,1), will not generally speaking be uniform.

This problem was solved in Section 4 on the assumption that our source was arbitrarily selected from a certain multiplicity of sources having a distribution function defined by $F(\varepsilon, \beta)$ in terms of the parameters ε and β. Without such an assumption it would not be clear how the problem could even be formulated. The solution is given by Equation (4.3). The equation may however, be utilized only under the conditions imposed by the distribution $F(\varepsilon, \beta)$. In principle, $F(\varepsilon, \beta)$ can be found from Equation (3.3). Since there are no sufficient observational (and theoretical) data for the determination of $F(\varepsilon, \beta)$, in order to use formula (4.3) it is necessary to assume a form for the distribution $F(\varepsilon, \beta)$.

In Section 4, in a qualitative example, we considered an aggregate of sources characterized by a uniform distribution in terms of ε and β in the domain $0 < \varepsilon < \varepsilon_m$ and $0 < \beta < \beta_m$.* Let there be a source with definite values p_l and p_c, and for ahich $c^2 \ll l$. Let, moreover, this source be located in a given aggregate. Then, according to (4.12), it is most probable, that the homogeneous field in our source subtends an angle close to $\pi/2$ with ths line of sight – i.e., $x = |\cos\varphi| \ll 1$. The most probable value of the parameter β appears to be considerably larger than that given by (6.2), and only the value of the parameter ε from (4.12) and (6.2) agrees in the order of magnitude.

Hence, the determination of the most probable values of the parameters x, ε and β of the given source in terms of the known values p_l and p_c requires a knowledge of the distribution $F(\varepsilon, \beta)$ for the objects among which the given source is situated. If, however, the distribution $F(\varepsilon, \beta)$ is unknown, then in our view a qualitative first approximation may be provided by the uniform distribution (4.4). In this case the solution is provided by Equation (4.12).

It is however, necessary, to bear in mind that although the values from (4.12) are the most probable, ones the probability that the source possesses values of x, ε and β differing from those of (4.12) may be understood to be non-zero. Let us note in this connection that as $p_c \to 0$ it follows from (4.7) that in the domain $\beta > c^2/l$ the probability density for the parameter β is given by $p(\beta) \simeq \frac{1}{2}(\beta_m/\beta)^{1/2}$. From this it is

* The quantities ε_m and β_m may be estimated as follows. If the maximum value of the magnetic field in the sources, is so large that $\beta_m = B_m/B_\nu \gtrsim 1$, the radiation spectrum in the frequency-domain ν becomes discrete. This means that for sources with a continuous spectrum the quantity β_m has an upper limit, provided by the inequality $\beta_m \ll 1$ (or, in practice $\beta_m < 0.1 \pm 0.3$). The quantity ε_m may be found from the condition $\varepsilon_m = p_{l_{\max}}$ from which it follows that $\varepsilon_m \simeq 0.2 \pm 0.3$.

clear that even as $p_c \to 0$, the main contribution to the integral $\int_0^{\beta_m} \mathrm{d}\beta\, p(\beta)$ comes from the domain $\beta \lesssim \beta_m$. Thus even if a satisfactorily precise estimate of the upper limit of the degree of circular polarization for a separate source can be made, it does not allow an upper limit to the magnetic field $B = \beta\, B_\nu$ to be given with a high probability Only the trivial estimate $B < \beta_m\, B_\nu$, where $B_\nu \simeq \nu/10^6$, or $B < (0.1 \pm 0.3) \times 10^{-6} \nu$ can be offered.

A better estimate of the magnetic field B can be provided if an upper limit of the degree of circular polarization – not for a separate source, but for an aggregate of sources – is known, and when a value of the linear polarization p_l is known for each source. In this case, from the condition that $n(l, c^2) = 0$ as $c^2 > c_0^2$ from Equation (3.3) we obtain $F(\varepsilon, \beta) = 0$ for all $\beta > c_0^2/(\varepsilon - l)$. From this it follows that in the aggregate under consideration, for all the sources the parameter $\beta < c_0^2/(\varepsilon_{\max} - l_{\min})$. If we set $l_{\min} = 0$ and, from (3.5), set $\varepsilon_{\max} = l_{\max}$. We obtain

$$\beta < \frac{c_0^2}{l_{\max}}. \tag{6.3}$$

The estimate (6.3) may be used only if the distribution of sources in terms of l in the domain $l < l_{\max}$ may be considered to be continuous. For this, evidently, it is sufficient that

$$N_{\max} \gg 1\,,$$

where $N_{\max}$ is the number of sources with $l = l_{\max}$. Equation (6.3) may be persisted in the form

$$B < \frac{\nu}{10^6} \frac{L(\gamma)}{C^2(\gamma)} \frac{p_{c_0}^2}{p_{l_{\max}}}. \tag{6.4}$$

As a example of the use of (6.4) we estimate, for model I, the maximum upper limit to the magnetic field for 32 extragalactic sources as observed by Berge and Seielstad (1969). In these sources at the frequency $\nu = 9.6 \times 10^9$ (i.e., $\lambda = 3.12$ cm) the maximum value of the linear polarization $p_{l_{\max}} = 8\%$, and an upper limit of the quantity p_{c_0} was 1%. Let us take the mean value of the spectral index $\gamma = 2$. Substituting numerical values for L and C in (6.4) we have, for the model Ia, $B < 8$ G. For the model Ib formula (6.4) gives $B < 20$ G. A larger estimate for the magnetic field in model Ib in comparison to model Ia is obtained because the coefficient $L_0(2)/C^2\, \beta(2)$ is noticeably larger than $L_a(2)/C_0^2(2)$ (see (2.6), (2.7) and Table I).

Let us note that the estimate for the upper limit of the magnetic field, which follows from Equation (2.1) with $\varphi \sim 1$ as

$$B < \frac{\nu}{10^6} p_c^2 \simeq 1\mathrm{G} \tag{6.5}$$

allowed only such values of B which are very small in comparison with (6.4).

Thus, the limiting values of the magnetic field, which may be found from observations of circular polarization do not seem to be as large as is sometimes supposed. This circumstance should be borne in mind in the interpretations of the observations of radio-sources with strong magnetic fields. The conclusion that a field $B = 10$–20 G is incompatible with the small observed degree of circular polarization may be made only after the observations have been processed through Equation (6.4) and the condition that $N_{\max} \gg 1$ used.

Let us also note, that the inequality (6.3) may be rewritten in the form

$$\left(\frac{E_v}{mc^2}\right)^2 > \frac{l_{\max}}{c_0^2},$$

where $E_\nu = mc^2 (B_\nu/B)^{1/2}$ is the electron energy giving the basic contribution to the radiation at frequency ν in magnetic field B.

6.6. Let us consider, finally, the matter of a possible correlation between the linear and circular polarization. Let there be an aggregate of sources with known linear polarizations. For which sources may maximum circular polarization be expected?

Within the limits of model I to provide an answer it is necessary that:

(1) the function $n(l)$ be obtained;

(2) the function $n(l)$ be substituted into (3.5) to find $F(\varepsilon)$;

(3) the dispersion $\Delta\varepsilon$ around the most probable value ε' be determined (the value ε' is defined by the condition $F(\varepsilon') = \max$.)

If $F(\varepsilon)$ has a δ-function character $\Delta\varepsilon \ll \varepsilon'$, the search for circular polarization in discrete sources may be assisted by the approximate rule that small values of p_l should correspond to large values of p_c. The connectness of this anticipation should be clear from Figure 2.

If, however, it seems that the function $F(\varepsilon)$ is characterized by a broad maximum, then the search for circular polarization should be guided by the expectation that small or moderate values of p_l should correspond to large values of p_c. This situation is amply borne out by Figure 3.

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