I. SUMMARY

This is a brief summary of what'd done and what's next to do for the project of CMB+Circular polarization. Inn this project we are considering the process Compton-like scattering: $eZ' \to e\gamma$. What we attempt to do is: Find the polarized squared amplitude in order to determine the cross section. Find the Boltzmann equation for polarization, to do so we need to follow what is already done for Compton scattering process, but adding an equation for the Z' (new interaction). We also *might add another scattering term in addition to Compton-like in the photon equation*-for this I am still not sure what additional scattering term. Here I summarize what has be done in the last month. Then A brief outlook.

- **Background review:**

  There are several mechanisms than can produce Circular Polarization (CP) in photons from the CMB. One of the most popular is the Faraday Conversion from Linear Polarization (LP) due to an external magnetic field. Also, the introduction of new physics, such as adding new interactions between photons from the SM and a external vector field can produce CP. For this work, we want to see/check/study the possibility of creating a circularly polarised photon through Z-prime boson scattering off electrons from the SM. In principle we can consider a scenario where Z-prime bosons and electrons are unpolarised which facilitate a ot the amplitude calculation of the process. Considering polarised Z-prime it is also factible but a bit more complicated. I think that for now we can try this unless in the narute ther's no way to get polarised photons from unpolarised particles(?).

  Done

  - Boltzmann equation and the necessary tools: Legendre Polynomials, spherical harmonics. In this part, it was followed the multi-pole expansion of the photon equation as an exercise in order to understand the physics of CMB. Some doubts at the beginning and discussed with Celine: As far as I understand, we could describe radiation, CDM, baryons and Lambda, either in, matter era, radiation era or Lambda era. Now, The power spectrum of CMB (matter era) gives us information of the early U, i.e. it provides information of its components (DM, baryons, dark energy). the question was: Is the power spectrum obtained only from the description of radiation? How do we obtain information about all the components of the U from the power spectrum? **need to add the discussion for these questions**

- **Based on 9501045:** in this paper is discussed the Boltzmann equation for polarization of photons from the CMB. The discussion of this Boltzmann equation is very different from the one discussed in Cosmology lectures and CMB power spectrum of temperature.

  - This Boltzmann equation is in terms of density matrix of the photon, which contains information of the Stokes parameters.
  - It is also in terms of the Hamiltonian of interaction in the RHS.
  - Regarding to the Collision term.
    - The term is not difficult but tedious. Maybe we can just take the result from this paper and apply it to our proses. However it may not be that simple.
    - The calculation of the polarized Compton scattering necessary, it is done. Now we need to check how is this modified if I interchange a photon for a Z'.
    - Also, because the density matrix is defined in terms of the amplitude of the process, we need to see how would it change for the new processes.

- **Based on 150603116, 1605.09382,0801.1345** In order to answer the following questions:

  1) Can you remind me of the Lagrangian we are working with in this model?
  2) Is the scattering cross section for DM + e- -> DM + e- larger or smaller than the Thompson cross section? If it is of comparable size or larger, then do we need to include this process in recombination?
  3) What is the mass of the DM? Is it relativistic or non-relativistic during recombination?

  I was checking these references and also discuss with Celine a bit about it. I already sent this by email but I guess it's better if we keep this in the document. First of all, these references show the Lagrangian and couplings of Z' to fermions: but we need to specify which model we will be working on. Second, as far as understanding the mass of the Z' should be small although several references claim it to be of O(Tev). the DM-e scattering can be dangerous if Z' is too light (since the cross section could blow up?), but probably smaller than Thomson scattering, though we need to check as this might depend on the DM spin. I'm not sure if I'm completely understanding this.

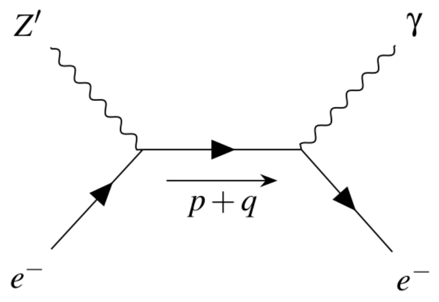

Figure 1: s-channel of $Z'\,e \to \gamma\,e$ scattering.

- **working on:** To start from somewhere, I decide to use the Lagrangian showed in reference 1605.09382 and determine the form of the (general) squared amplitude ($f\bar{f}Z'-$vertex $\sim g_f\gamma_\mu$).
- outlook:Regarding the Amplitude calculation, I think these are the first things we should do.
  - Review of physics of Z' (more deep review).
  - Finish the squared amplitude, this might be straightforward from Compton scattering.
  - Verify which another scattering process needs to be added (and see how this will modify the total amplitude) to the photon equation.
  - Check the unpolarized Z-prime+electron to unpolarised electron + photon? is it reasonable?

There are things I need to add besides the summary, and I will be adding as soon as I got some results from the calculations.

II. GENERAL OVERVIEW OF Z' PHISICS

III. POLARIZED MATRIX CALCULATION

In order to perform the matrix element we are using the general Feynman rule for the Neutral Current interactions coming from the Lagrangian [1]

$$\mathcal{L}^{int} = eJ^{\mu}_{em}A_\mu + \frac{1}{2}\sum_{\alpha=1}^{n+1}\sum_{i} g_\alpha \bar{f}_i[g_V^\alpha - g_A^\alpha\gamma^5]f_i \tag{1}$$

Where $g_{V,A}$ are the vector and axial couplings respectiblely, $\alpha = 1$ corresponds to the SM, while $\alpha = 2,\ldots,n+1$ correspond to the additional $\mathrm{U}(1)'$[I will add in the above section a brief discussion of this]. Taking up to take $\alpha = 2$ and the general coupling $(g_V + g_A)$, which will depend on the model we choose, we can calculate the total amplitude.
Following the Compton scattering calculations, we compute the s- and t- channer of the $Z'\,e \to \gamma\,e$ scattering in a very general way. Roughly the matrix element for the s-channel shown in fig. (1) is

$$iA_s = -\frac{ieg_2}{s-{m_e}^2}\bar{\mathrm{u}}(p_2)\gamma^\mu(\not{p}+\not{q}+m_e)\gamma^\nu[g_V+g_A\gamma^5]\mathrm{u}(p)\varepsilon^*_{\gamma_\mu}(k)\varepsilon_{z\nu}(q). \tag{2}$$

Similarly, the amplitude for t-channel is given by,

$$iA_t = -\frac{ieg_2}{t-{m_e}^2}\bar{\mathrm{u}}(p_2)\gamma^\nu[g_V+g_A\gamma^5](\not{p}-\not{k}+m_e)\gamma^\mu\mathrm{u}(p)\varepsilon^*_{\gamma_\mu}(k)\varepsilon_{z\nu}(q). \tag{3}$$

determaning the total amplitude and squaring, (this might change when we add the extra interaction)

$$|A_{\mathrm{Total}}|^2 = |A_s|^2 + |A_t|^2 + 2A_sA_t^* \tag{4}$$

Individually we determine the squared elements corresponging to each chanel and their interference, obtaining,

$$
\begin{aligned}
|A_s|^2 &= \frac{(eg_2)^2}{(s-m_e{}^2)^2} M_s^{\mu\nu\alpha\beta} \varepsilon^*_{\gamma_\mu} \varepsilon_{\gamma\beta} \varepsilon^*_{z\nu} \varepsilon_{z\alpha} \\
|A_t|^2 &= \frac{(eg_2)^2}{(t-m_e{}^2)^2} M_t^{\mu\nu\alpha\beta} \varepsilon^*_{\gamma_\mu} \varepsilon_{\gamma\beta} \varepsilon^*_{z\nu} \varepsilon_{z\alpha} \\
A_s A_t^* &= \frac{(eg_2)^2}{(s-m_e{}^2)(t-m_e{}^2)} M_{st}^{\mu\nu\alpha\beta} \varepsilon^*_{\gamma_\mu} \varepsilon_{\gamma\beta} \varepsilon^*_{z\nu} \varepsilon_{z\alpha}
\end{aligned}
$$

where, each M_i contain the resultant traces. In order to simplify the calculation of all these traces, we will rewrite the amplitude in the following form,

$$
|A_{\text{Total}}|^2 = (eg_2)^2[(g_V^2+g_A^2)(|A_{1s}|^2)+|A_{1t}|^2+A_{1s}A_{2t}^*)+(g_V^2-g_A^2)(|A_{2s}|^2)+|A_{2t}|^2+A_{2s}A_{2t}^*)+g_A g_V(|A_{3s}|^2)+|A_{3t}|^2+A_{3s}A_{3t}^*)] \tag{5}
$$

see Mathematica notebook for the explicit form of all treaces. Note: to preceed with the clculation of the amplitude, I need to be bery sure how to wrok with the Z' polarization vector in the initial state.

A. Unpolarised Z-prime + unpolarised electron to polarised photon + unpolarised electron(?)

Before continue, we will consider the processes where Z prime and electrons are unpolarised. The total amplitude is given by the s-channel and the t-channel. As in the case of Compton scattering,

$$
A_T^\lambda = A_s^\lambda + A_t^\lambda \tag{6}
$$

and in the case of the complex conjugate

$$
A_T^{*\lambda'} = A_s^{*\lambda'} + A_t^{*\lambda'}. \tag{7}
$$

$'\lambda$ and λ' are the polarization of the photon in the process and they are not necessary the same. Therefore the squared amplitude is:

$$
|A_T|^{2\lambda\lambda'} = |A_s|^{2\lambda\lambda'} + |A_2|^{2\lambda\lambda'} + A_s^\lambda A_t^{*\lambda'} + A_s^{*\lambda'} A_t^\lambda \tag{8}
$$

so, we have the cases: $\lambda = \lambda'$ and $\lambda \neq \lambda'$. However, if this is as in the case of the Compton scattering, the result for positive polarised photon and negative polarised photons will be the same. This means that we wouldn't be able to find a asymmetry, therefore NO circular polarization (or linear?) in this case.
For the case of unpolarised Z' boson, when we sum over the helicities we have

$$
\sum_{h_{z'}} \varepsilon_\nu \varepsilon_\alpha = -g_{\nu\alpha} + \frac{q_\nu q_\alpha}{M_{z'}^2}. \tag{9}
$$

For now, I will only determine the calculation in the limit $M_{z'} >> M_z$ (is this something we can assume for our calculations? So far in the literature I've found that the mass of the Z-prime bosons is larger than thao of Z bosons, but if necessary, I'll add a more complete calculation.), so the squared amplitude is simply to compute.

1. s-channel

Once we cume over Z' helicity, the general squared amplitude for the s-channel is

$$
\begin{aligned}
|A_s|^{\lambda\lambda' 2} =& \frac{(eg_2)^2}{(s-m^2)^2} \mathrm{Tr}[(\not{p}_2+m)\gamma^\mu(\not{p}+\not{q}+m)(g_{2V}+g_{2A}\gamma^5)(\not{p}-2m)(\not{p}+\not{q}+m)(g_{2V}-g_{2A}\gamma^5)\gamma^\beta]\varepsilon_\mu^{*\lambda}\varepsilon_\beta^{\lambda'} \\
=& \frac{(eg_2)^2}{(s-m^2)^2} \Big[(g_{2v}^2-g_{2A}^2)\Big(\mathrm{Tr}[\not{p}_2\gamma^\mu(\not{p}+\not{q})\not{p}(\not{p}+\not{q})\gamma^\beta] - 2m^2\mathrm{Tr}[\not{p}_2\gamma^\mu(\not{p}+\not{q})\gamma^\beta] + m^2\mathrm{Tr}[\gamma^\mu\not{p}(\not{p}+\not{q})\gamma^\beta] + 8m^4 g^{\mu\beta}\Big) \\
&+ (g_{2v}^2+g_{2A}^2)m^2\Big(\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}\gamma^\beta] - 2\mathrm{Tr}[\not{p}_2\gamma^\mu(\not{p}+\not{q})\gamma^\beta] + \mathrm{Tr}[\gamma^\mu(\not{p}+\not{q})\not{p}\gamma^\beta] - 8(p+q)^2 g^{\mu\beta}\Big) \\
&+ 2g_{2V}g_{2A}m^2\Big(\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}\gamma^\beta\gamma^5] - 2\mathrm{Tr}[\not{p}_2\gamma^\mu(\not{p}+\not{q})\gamma^\beta\gamma^5] + \mathrm{Tr}[\gamma^\mu(\not{p}+\not{q})\not{p}\gamma^\beta\gamma^5]\Big)\Big]\varepsilon_\mu^{*\lambda}\varepsilon_\beta^{\lambda'}
\end{aligned} \tag{10}
$$

2. t-channel

$$
\begin{aligned}
|A_t|^{\lambda\lambda' 2} =& \frac{(eg_2)^2}{(t-m^2)^2}\mathrm{Tr}[(\not{p}_2-2m)(g_{2V}-g_{2A}\gamma^5)(\not{p}-\not{k}+m)\gamma^\mu(\not{p}+m)\gamma^\beta(\not{p}-\not{k}+m)(g_{2V}+g_{2A}\gamma^5)]\varepsilon^{*\lambda}_\mu\varepsilon^{\lambda'}_\beta \\
=& \frac{(eg_2)^2}{(t-m^2)^2}\Big[(g^2_{2v}+g^2_{2A})\Big(\mathrm{Tr}[\not{p}_2(\not{p}-\not{k})\gamma^\mu\not{p}\gamma^\beta(\not{p}-\not{k})]+m^2\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}\gamma^\beta]+m^2\mathrm{Tr}[\not{p}_2(\not{p}-\not{k})\gamma^\mu\gamma^\beta]+m^2\mathrm{Tr}[\not{p}_2\gamma^\mu\gamma^\beta(\not{p}-\not{k})]\Big) \\
&+2(g^2_{2v}-g^2_{2A})m^2\Big(\mathrm{Tr}[(\not{p}-\not{k})\gamma^\mu\not{p}\gamma^\beta]+\mathrm{Tr}[\gamma^\mu\not{p}\gamma^\beta(\not{p}-\not{k})]+4(p-k)^2g^{\mu\beta}+4m^2g^{\mu\beta}\Big) \\
&+2g_{2V}g_{2A}\Big(\mathrm{Tr}[\not{p}_2(\not{p}-\not{k})\gamma^\mu\not{p}\gamma^\beta(\not{p}-\not{k})\gamma^5]+m^2\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}\gamma^\beta\gamma^5]+m^2\mathrm{Tr}[\not{p}_2(\not{p}-\not{k})\gamma^\mu\gamma^\beta\gamma^5] \\
&+\ m^2\mathrm{Tr}[\not{p}_2\gamma^\mu\gamma^\beta(\not{p}-\not{k})\gamma^5]\Big)\Big]\,\varepsilon^{*\lambda}_\mu\varepsilon^{\lambda'}_\beta
\end{aligned}
\tag{11}
$$

3. crossed-terms

$$
\begin{aligned}
A^\lambda_sA^{*\lambda'}_t =& -\frac{(eg_2)^2}{(s-m^2)(t-m^2)}\frac{1}{2}\mathrm{Tr}[\gamma^\nu(\not{p}_2+m)\gamma^\mu(\not{p}+\not{q}+m)\gamma^\nu(g_{2V}-g_{2A}\gamma^5)(\not{p}+m)\gamma^\beta(\not{p}-\not{k}+m)\gamma_\nu(g_{2V}-g_{2A}\gamma^5)]\varepsilon^{*\lambda}_\mu\varepsilon^{\lambda'}_\beta \\
=& \frac{(eg_2)^2}{(s-m^2)(t-m^2)}\Big[(g^2_{2V}+g^2_{2A})\Big(\mathrm{Tr}[(\not{p}+\not{q})\gamma^\mu\not{p}_2\not{p}\gamma^\beta(\not{p}-\not{k})]+m^2\mathrm{Tr}[(\not{p}+\not{q})\gamma^\mu\not{p}_2\gamma^\beta]+m^2\mathrm{Tr}[\gamma^\mu\not{p}\gamma^\beta(\not{p}-\not{k})]+4m^4g^{\mu\beta}\Big) \\
&-8m^2(g^2_{2V}-g^2_{2A})(2p-k)^\beta(p+p_2+q)^\mu+2g_{2V}g_{2A}\Big(\mathrm{Tr}[(\not{p}+\not{q})\gamma^\mu\not{p}_2\not{p}\gamma^\beta(\not{p}-\not{k})\gamma^5]+m^2\mathrm{Tr}[(\not{p}+\not{q})\gamma^\mu\not{p}_2\gamma^\beta\gamma^5] \\
&+m^2\mathrm{Tr}[\gamma^\mu\not{p}\gamma^\beta(\not{p}-\not{k})\gamma^5]\Big)\Big]\,\varepsilon^{*\lambda}_\mu\varepsilon^{\lambda'}_\beta
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
A^\lambda_tA^{*\lambda'}_s =& -\frac{(eg_2)^2}{(s-m^2)(t-m^2)}\frac{1}{2}\mathrm{Tr}[\gamma_\nu(\not{p}+\not{q}+m)\gamma^\beta(\not{p}_2+m)\gamma^\nu(g_{2V}-g_{2A}\gamma^5)(\not{p}-\not{k}+m)\gamma^\mu(\not{p}+m)(g_{2V}+g_{2A}\gamma^5)]\varepsilon^{*\lambda}_\mu\varepsilon^{\lambda'}_\beta \\
=& \frac{(eg_2)^2}{(s-m^2)(t-m^2)}\Big[(g^2_{2V}+g^2_{2A})\Big(\mathrm{Tr}[\not{p}_2\gamma^\beta(\not{p}+\not{q})(\not{p}-\not{k})\gamma^\mu\not{p}]+m^2\mathrm{Tr}[\not{p}_2\gamma^\beta(\not{p}+\not{q})\gamma^\mu]+m^2\mathrm{Tr}[\gamma^\beta(\not{p}-\not{k})\gamma^\mu\not{p}]+4m^4g^{\beta\mu}\Big) \\
&-8m^2(g^2_{2V}-g^2_{2A})(2p-k)^\mu(p_2+p+q)^\beta+2g_{2V}g_{2A}\Big(\mathrm{Tr}[\not{p}_2\gamma^\beta(\not{p}+\not{q})(\not{p}-\not{k})\gamma^\mu\not{p}\gamma^5] \\
&+m^2\mathrm{Tr}[\not{p}_2\gamma^\beta(\not{p}+\not{q})\gamma^\mu\gamma^5]+m^2\mathrm{Tr}[\gamma^\beta(\not{p}-\not{k})\gamma^\mu\not{p}\gamma^5]\Big)\Big]\,\varepsilon^{*\lambda}_\mu\varepsilon^{\lambda'}_\beta
\end{aligned}
\tag{13}
$$

4. Results in the rest frame

Working in the electron rest frame of reference (I'll add general expression later), we have the following momenta definition,

$$
\begin{aligned}
p^\mu &= (m_e,0,0,0) \\
q^\mu &= (E_z,0,0,E_z) \\
k^\mu &= (E_\gamma,E_\gamma\sin\theta,0,E_\gamma\cos\theta) \\
p^\mu_2 &= p^\mu+q^\mu-k^\mu
\end{aligned}
\tag{14}
$$

the squared of the matrix element it'll be presented in terms of the energies of z-prime and photon γ. To do so we need to define the $\cos\theta$ and $\sin\theta$ usingg energy conservstion. We find that

$$
\cos\theta = 1-\frac{2m(E_z-E_\gamma)-M^2_z}{2E_zE_\gamma}\ ,\quad \sin^2\theta = \frac{[2m(E_z-E_\gamma)-M^2_z][4E_zE_\gamma-2mE_z+2mE_\gamma+M^2_z]}{4E^2_zE^2_\gamma}
\tag{15}
$$

For the positive polarization at the final state $e+z \to e+\gamma_+$, the squared matrix element is:

$$|A(e+z\to e+\gamma_+)|^2 = -16e^2 g_2^2 \left(-\frac{m\left(E_\gamma\left(g_{2A}^2+g_{2V}^2\right)\left(2mE_z+M_z^2\right)+2E_z^2\sin^2\theta\left(g_{2A}^2\left(E_z+3m\right)+g_{2V}^2\left(E_z-m\right)\right)\right)}{\left(2mE_z+M_z^2\right)^2} - \frac{\left(g_{2A}^2+g_{2V}^2\right)\left(2mE_z-M_z^2\right)}{4mE_\gamma} + \frac{\left(g_{2A}^2+g_{2V}^2\right)\left(E_z^2\sin^2\theta+M_z^2\right)}{2mE_z+M_z^2} \right)$$

[1] P. Langacker, Rev. Mod. Phys. **81** (2009) 1199 doi:10.1103/RevModPhys.81.1199 [arXiv:0801.1345 [hep-ph]].