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Is the cosmic microwave background circularly polarized?

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Abstract

The primordial anisotropies of the cosmic microwave background (CMB) are linearly polarized via Compton-scattering. The Faraday conversion process during the propagation of polarized CMB photons through regions of the large-scale structure containing magnetized relativistic plasma, such as galaxy clusters, will lead to a circularly polarized contribution. Though the resulting Stokes-V parameter is of order 10^{-9} at frequencies of 10 GHz, the contribution can potentially reach the level of total Stokes-U at low frequencies due to the cubic dependence on the wavelength. In future, the detection of circular polarization of CMB can be used as a potential probe of the physical properties associated with relativistic particle populations in large-scale structures.

The CMB anisotropies are expected to be linearly polarized by anisotropic Compton scattering around the epoch of recombination [1]. This linear polarization field has been widely discussed in the literature [2] and its accurate measurement can ultimately shed new light on the thermal history of the universe and on the primordial gravitational-wave background. It is also well established that, in the standard scenario, no relevant circular polarization should be present. For these reasons, many of the present and near future CMB experiments like MAP and Planck Surveyor have not been designed for a detection of circular polarization.

It is, therefore, quite probable that in the near future, in spite of a continuous incremental knowledge on linear CMB polarization, the experimental bounds on the circular component will not drastically improve from those of early observations [3]. Since post-Planck CMB polarization experiments are already under study [4], it is extremely timely to address the question whether CMB is circularly polarized and what physical information can be extracted from its measurement. This Letter represents a discussion in this direction.

In order to understand the CMB polarization field, we make use of the Stokes parameters [5]. In the case of a propagating wave in the z direction, $\mathbf{E} = (E_x e^{i\phi_x}\hat{\mathbf{x}} + E_y e^{i\phi_y}\hat{\mathbf{y}})e^{-i\omega t}$, with amplitudes E_x and E_y in the x- and y-directions with phases ϕ_x and ϕ_y, respectively, we can write the Stokes parameters as time averaged quantities:

$$I \equiv \langle E_x^2 \rangle + \langle E_y^2 \rangle,$$

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$$Q \equiv \langle E_x^2 \rangle - \langle E_y^2 \rangle,$$
$$U \equiv \langle 2E_x E_y \cos(\phi_x - \phi_y) \rangle,$$
$$V \equiv \langle 2E_x E_y \sin(\phi_x - \phi_y) \rangle. \tag{1}$$

Note that the total intensity of the radiation is given by the Stokes-I parameter, while for unpolarized radiation $Q = U = V = 0$. The linearly polarized radiation is defined by non-zero values for Q and/or U. These latter two Stokes parameters form a spin-2 basis; a rotation of the coordinate system, by an angle θ, leads to a new set of parameters for the same radiation field given by $(\overline{Q} \pm i\overline{U}) = (Q \pm iU)e^{2i\theta}$. This coordinate dependence is avoided in the literature by introducing a new set of orthonormal basis with a part containing the gradient of a scalar field, called grad- or E-modes, and a part containing the curl of a vector field, called curl- or B-modes [6]. Note that the Stokes-V parameter, which is coordinate-independent similar to Stokes-I, defines the extent to which radiation is circularly polarized.

Though there may not be a physical mechanism to generate a Stokes-V contribution at the last scattering surface, the radiation detected today, however, is not exactly the field that last scattered. During the propagation from the last scattering surface to us, CMB photons encounter large-scale structures and undergo significant changes due to effects related to structure formation [7]. These modifications include a regeneration of new anisotropies, such as through the Sunyave–Zel'dovich (SZ, [8]) effect involving the inverse-Compton scattering of CMB photons via hot electrons in galaxy clusters. The transit of CMB photons also leads to modifications to the polarization signal. For example, the gravitational lensing deflection of the CMB propagation directions leads to a transfer of power from the dominant E-mode of polarization to the B-mode [9].

In the case of CMB, galaxy clusters present potential sources where interesting polarization modifications occur. In addition to the presence of thermal electrons, large-scale diffuse synchrotron emission towards galaxy clusters suggests the presence of magnetic fields [10]. The propagation of radiation through such magnetized plasma lead to the well-known modification involving the rotation of linear polarization between Stokes-Q and -U parameters via the Faraday rotation (FR) [11]. The FR comes about from the fact that normal modes of propagation in a magnetized plasma are circularly polarized. Instead of the description given in Eq. (1), then, it is useful to consider the linearly polarized radiation field, in terms of a superposition of equal left and right-hand circularly polarized contributions. We can write $\mathbf{E} = (E_R e^{i\phi_R}\widehat{\mathbf{R}} + E_L e^{i\phi_L}\hat{\mathbf{L}})e^{-i\omega t}$ with unit vectors for the right-, $(\widehat{\mathbf{R}})$, and left-, $(\hat{\mathbf{L}})$, hand polarized waves as $(\hat{\mathbf{x}} \mp i\hat{\mathbf{y}})/\sqrt{2}$, respectively. In terms of this redefinition of the radiation field, the Stokes-Q and -U parameters are

$$Q \equiv \langle 2E_R E_L \cos(\phi_R - \phi_L) \rangle,$$
$$U \equiv \langle 2E_R E_L \sin(\phi_R - \phi_L) \rangle. \tag{2}$$

The right- and left-hand circularly polarized waves travel through the magnetized medium with different phase velocities introducing an additional phase shift involving $(\phi_R - \phi_L)$. This phase shift mixes Stokes-Q and -U parameters such that

$$\dot{Q} = -2U \frac{d\Delta\phi_{\rm FR}}{dt} \quad \text{and} \quad \dot{U} = 2Q \frac{d\Delta\phi_{\rm FR}}{dt}, \tag{3}$$

where the overdot is the derivative with respect to time, t.

The associated mixing is described via the rotation measure angle

$$\begin{aligned} \Delta\phi_{\rm FR} &= \frac{e^3\lambda^2}{2\pi m_e^2 c^4} \int dl\, n_e(l)\mathbf{B}\mu, \\ &\approx 8 \times 10^{-2}\,\text{rad}\,(1+z)^{-2}\left(\frac{\lambda_0}{1\ \text{cm}}\right)^2 \\ &\quad \times \int \frac{dl}{1\ \text{kpc}}\left(\frac{n_e}{0.1\ \text{cm}^{-3}}\right)\left(\frac{\mathbf{B}}{10\ \mu\text{G}}\right)\mu. \end{aligned} \tag{4}$$

Here, μ is the cosine of the angle between the line of sight direction and the magnetic field, $\mathbf{B}$, in galaxy clusters, n_e is the number density of electrons, $\lambda = \lambda_0(1+z)^{-1}$ is the wavelength of radiation with wavelength today given by λ_0, while rest of the parameters have generally known values.

The FR effect on CMB anisotropies has been considered in the literature as a possible way to generate a Stokes-U contribution from the dominant Stokes-Q contribution both for a primordial [12] and galaxy cluster [13] magnetic fields. In the case of galaxy cluster magnetic fields, the observed rotation measures up to 250 radians m^{-2} in nearby massive clusters [10], suggest that the CMB linear polarization is rotated, on average, by an angle of order 10^{-1} radians at an observed frequency of few GHz. Thus, FR can potentially generate a Stokes-U contribution which is of order 10^{-7} in fractional temperature, $\Delta T_{\rm pol}/T_{\rm CMB}$,[1] from the dominant Stokes-Q contribution with an rms of order 10^{-6} at 10 GHz [13].

The propagation of radiation has already been discussed in the literature as an explanation for the circular polarization observed towards certain extragalactic radio sources [14]. A similar situation applies to the CMB. It is now well known that galaxy clusters also contain populations of relativistic particles; The observed hard X-ray and the extreme ultraviolet emission require their presence [15]. In a magnetized plasma containing highly relativistic electrons, the normal modes of propagation, however, are not perfectly circular but rather linear. The linear modes of propagation are also encountered when one is dealing with uniaxial crystals and so-called quarter wave plates which are configured to convert fully linear polarized radiation to a circularly polarized contribution. In the case of relativistic plasmas embedded in a magnetic field, the conversion of linear polarization to circular polarization can be described under the formalism of generalized Faraday rotation [14].

The effect is essentially same as the well known FR effect, except the propagation is considered in terms of the linear modes rather than the circular modes. Even if the plasma is not relativistic, under generalized Faraday rotation, the modes of propagation are linear when the radiation field is propagating more or less close to a perpendicular direction to the magnetic field. The conversion can be understood in terms of the description of the radiation field given in Eq. (1) for linear waves instead of the decomposition to circular states in Eq. (2). The difference in phase velocities now lead to a mixing between the Stokes-U and -V parameters with the addition of a phase shift to $(\delta_x - \delta_y)$. This mixing is similar to the ones involving Stokes-Q and -U under the normal FR with the introduction of a phase shift to $(\delta_R - \delta_L)$.

Since Stokes-V is effectively zero for the incoming radiation, the outgoing radiation contains a contribution to the Stokes-V and, therefore, the effect is generally described in the literature as the Faraday conversion (FC) [16]. We can write the converted Stokes-V contribution as

$$\dot{V} = 2U \frac{d\Delta\phi_{\rm FC}}{dt}. \tag{5}$$

The rotation measure angle related to FC in a magnetized relativistic plasma, analogous to the rotation measure associated with the FR effect is [14],

$$\begin{aligned}
\Delta\phi_{\rm FC} &= \frac{e^4\lambda^3}{\pi^2 m_e^3 c^5}\left(\frac{\beta-1}{\beta-2}\right) \\
&\quad\times \int dl\, n_r(l)\gamma_{\rm min}|\mathbf{B}|^2\left(1-\mu^2\right), \\
&\approx 3\times 10^{-7}\,{\rm rad}\,(1+z)^{-3}\left(\frac{\lambda_0}{1\ {\rm cm}}\right)^3 \\
&\quad\times \left(\frac{\beta-1}{\beta-2}\right)_{\beta=2.5} \\
&\quad\times \int \frac{dl}{1\ {\rm kpc}}\left(\frac{n_r}{0.1\ {\rm cm}^{-3}}\right)\left(\frac{\gamma_{\rm min}}{300}\right) \\
&\quad\quad\times \left(\frac{|\mathbf{B}|}{10\ \mu{\rm G}}\right)^2\left(1-\mu^2\right).
\end{aligned} \tag{6}$$

Here, n_r is the number density of relativistic particles and β defines the power-law distribution of the particles, in terms of the Lorentz-factor γ, such that

$$N(\gamma) = N_0\gamma^{-\beta}, \tag{7}$$

between $\gamma_{\rm min} < \gamma < \gamma_{\rm max}$. Other parameters are same as the ones defined in Eq. (4). A comparison of Eqs. (4) and (6) reveals that while the FR is proportional to the square of the observed wavelength, the FC scales as the cube of the wavelength. Thus, one finds a stronger wavelength dependence for the FC when compared to the rotation.

A quick estimate for the two suggests that the FC is at least 10^5 orders of magnitude smaller than the FR effect. Unlike FR, however, the FC simply depends on the number density of relativistic particles; a consequence of this is that an equal mixture of positive and negative particles will contribute to circular polarization conversion while there will be no rotation associated with linear polarization. Also, conversion depends on the square of the amplitude of the magnetic field and not the magnetic field itself. Thus, in certain favorable astrophysical conditions, the FC to circular polarization can be significant leading to a measurable

[1] We quote the polarization in terms of fractional difference in brightness temperature as both the temperature and polarization anisotropies have the same thermal spectrum.

contribution to the Stokes-V parameter. As a potential source of conversion between Stokes-U to -V, we will consider galaxy clusters, as there is some evidence for populations of relativistic particles in these massive objects.

The extent to which galaxy clusters convert Stokes-Q to a Stokes-U parameter under FR has already been discussed in the literature [13]. The rotation effect encountered here depends on the properties of the magnetic field and the distribution of thermal electrons, both of which are now well known for clusters through X-ray and synchrotron emission observations. The radio measurements of FR through intracluster gas indicate magnetic fields or order tens of microGauss towards nearby massive galaxy clusters [10]. The extreme ultraviolet and the hard X-ray emission observed towards certain clusters suggest the presence of relativistic electrons with bulk Lorentz factors of order ~ 300 and $\sim 10^4$, respectively [15]. The calculations that attempt to explain these observations generally suggest relativistic populations with a spectrum $N(\gamma) \propto \gamma^{-\beta}$ where $\beta \sim 2.3$ and as steep as ~ 3.3.

Assuming reasonable parameters for galaxy clusters with $B = 10$ μG, a path length of 1 Mpc, which is a typical size for a massive cluster, $\gamma_{\min} = 100$ for relativistic particles, and an observed frequency of 10 GHz, we estimate $\Delta\phi_{\rm FC} \sim \text{few} \times 10^{-3}$. With a typical rms contribution of order 10^{-6} to the incoming CMB polarization that propagate through galaxy clusters, the outgoing radiation should contain a circular polarization of order 10^{-9} at scales corresponding to galaxy clusters. Note that this estimate is highly uncertain by at least two orders of magnitude both due to the unknown number density of relativistic particles and the Lorentz-factor distribution of these particles. Since one expects a contribution to the Stokes-V parameter when the radiation is propagating nearly perpendicular to the magnetic field, the final contribution not only depends on the magnitude of the magnetic field, but also on detailed physical properties such as the spatial distribution. Due to the λ^3 dependence on the wavelength, the Stokes-V contribution can potentially reach the maximal Stokes-U contribution at low frequencies of 1 GHz and below.

We can extend the approach presented in Ref. [13], following the so-called halo model [17], to calculate the expected angular power spectrum of the Stokes-V contribution. The Stokes-V correlation is simply the product of correlation functions involving Stokes-U contribution and the FC rotation measure: $C_V(\theta) = C_U(\theta)C_{\rm FC}(\theta)$. The correlation function associated with the Stokes-U contribution can be written as [6]:

$$C_U(\theta) = \int \frac{l\,dl}{2\pi}\left\{\frac{C_l^{EE}}{2}\big[J_0(l\theta) - J_4(l\theta)\big] + \frac{C_l^{BB}}{2}\big[J_0(l\theta) + J_4(l\theta)\big]\right\}. \qquad (8)$$

We assume a total contribution to the B-mode power spectrum, C_l^{BB}, from both gravitational waves, with a tensor-to-scalar ratio of 0.1, and gravitational lensing conversion of E to B-mode. Since lensing, effectively, happens at redshifts greater than 1 [18], while FC happens in massive clusters at redshifts less than 1, it is unlikely that we have overestimated the total linear polarization contribution that can be converted to the circular polarization. To calculate $C_{\rm FC}(\theta)$, we use a halo distribution with masses greater than $10^{14}\ M_\odot$ with the assumption that the distribution of relativistic particles in these halos trace the gas distribution and the magnetic field in each cluster is constant, which in this case we set at 10 μG. We summarize our results in Fig. 1.

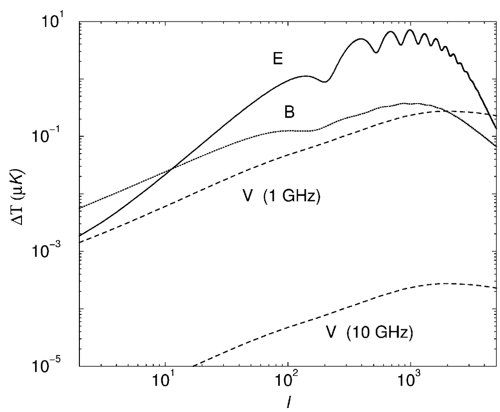

Fig. 1. The flat-band power spectra of CMB polarization ($\Delta T = T_{\rm CMB}\sqrt{l(l+1)/2\pi C_l}$). We show contributions to the E-mode (solid line), B-mode (dotted line) and estimates for the Stokes-V mode at 10 (dashed line) and 1 GHz (long-dashed line). The contribution to B-modes contains two parts involving gravitational-waves at large angular scales and gravitational lensing effect at small angular scales. The shown V-mode contributions should be considered reasonable given uncertainties associated with relativistic populations. Given the significant wavelength dependence, λ^3 in this plot, low-frequency observations are desirable to detect the circular polarization contribution.

In addition to galaxy clusters as discussed above, large-scale shocks involved with the formation of structures, including galaxy clusters, could be significant sources of magnetized plasma in which FC may be efficient. Due to the strong dependence on wavelength, the Faraday conversion effect can easily be identified and separated from other contaminant contributions such as radio point sources that may dominant the polarization signal at low frequencies. An additional, and possibly important, source of circular polarization is the conversion associated with a primordial magnetic field. Though current observations limit a primordial magnetic field to be at the level of 10^{-3} μG today [19], the evolution of the field as $(1+z)^2$ will lead to a significant contribution during the recombination era. A limit on the circular polarization at early times can be used as a way to put a reliable limit on the large scale primordial magnetic field. We will return to this subject in detail in the future.

Though our first estimate on the level of CMB circular polarization due to galaxy clusters is smaller than the contribution to the linear polarization, observational studies on circular polarization are clearly warranted. As discussed, the Faraday conversion involves the presence of relativistic particles and their detailed physical properties. Any detection of the circular polarization will allow a probe of these relativistic populations in the large scale structure and the associated magnetic fields. Furthermore, a detection of a V contribution comparable in signal to the Q- and U-modes would be nearly impossible to explain in the standard scenario, requiring the introduction of new physics or, again, providing an useful check for systematics.

Though the required sensitivity level to detect circular polarization is beyond what is allowed by current instrumental techniques, with new detector technologies and observational methods, it is likely that the required level of sensitivity will be reached in the future. As with Faraday rotation, the ultimate limitation will be contamination from foreground sources. At frequencies around 10 GHz and below, the synchrotron emission from galaxy is dominant with respect to the CMB. Measurements at 1 GHz shows a circular polarization from this foreground at level of 10^{-4} [20]. However, the circular polarization contribution associated with galaxy clusters could be separated out by cross-correlating with large-scale structure data and/or by the different frequency dependence.

Though most CMB experiments are not sensitive to the Stokes-V contribution, interferometric arrays detect this contribution. Therefore, the anticipated polarization data from interferometers such as DASI and CBI, at 30 GHz, will provide useful upper limits on the V-mode contribution at the order of a few μK. Given the strong wavelength dependence, however, any attempt to detect circular polarization should be considered at low frequencies; in this respect, the upcoming Square Kilometer Array (SKA) and the Low Frequency Array (LOFAR) may provide interesting results in this direction. The relevant astrophysical uses associated with CMB circular polarization clearly motivate future observational programs for a positive detection.

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