# Polarization of photons emitted by decaying dark matter

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#### Abstract

Radiatively decaying dark matter may be searched through investigating the photon spectrum of galaxies and galaxy clusters. We explore whether the properties of dark matter can be constrained through the study of a polarization state of emitted photons. Starting from the basic principles of quantum mechanics we show that the models of symmetric dark matter are indiscernible by the photon polarization. However, we find that the asymmetric dark matter consisted of Dirac fermions is a source of circularly polarized photons, calling for the experimental determination of the photon state.

# 1 Introduction

Dark matter (DM) particles can be unstable with a life-time exceeding that of the Universe. If these particles have a radiative decay mode, the emission of an almost monochromatic photon is a specific signature allowing to search for them in astrophysical observations. Similar imprint can also come from two-photon annihilation of DM states. The examples of DM particles producing photon(s) include sterile neutrino with the mass in the keV range [1–5] (see also [6,7] for reviews), axions and axionlike particles [8–10], sgoldstinos [11], majorons [12], axinos [13], gravitinos [14], self-annihilating particles [15] and transitions between two dark matter states [16]. Some (controversial) indications in favour of these types of DM particles came recently from the analysis of photon spectra emitted by different

astrophysical objects in X-ray region (an unidentified 3.5 keV line) and were reported in  $[17, 18]^1$ .

This unidentified X-ray line may have no connection with the dark sector and come from some atomic transition, see the discussion [21]. Namely, there are several atomic lines near the 3.5 keV feature: potassium K XVIII lines at 3.48 and 3.52 keV, and charge exchange induced line of sulfur, S XVI at 3.47 keV. The situation remains unclear because it is hard to estimate accurately the flux of photons coming from these transitions, given the lack of precise knowledge of the chemical composition and temperature of the cosmic plasma. However, future observations with enhanced spectral resolution should be able to distinguish dark matter and atomic lines.

But one can address a question: even if the dark matter nature of the line would be proved, how can we decide what type of dark matter particle produces this line? For example, can we distinguish between a boson and a fermion? Of course, if we had a possibility to catch and study all decay products of hypothetical dark matter particle then we would be able to determine the spin and parity of the original particle. Unfortunately, we can register only one particle—photon—because another decay product flies in the opposite direction.

Clearly, besides the energy photons may carry another information encoded in their polarization state. The aim of this paper is to address the question whether the polarization measurements can help to constrain the properties of the dark matter<sup>2</sup>. We will show that the quantum mechanical state of each arriving individual photon may be different for the different types of dark matter particles. Still, we will demonstrate that it is in principle not possible to make such measurements that could determine the spin of dark matter particle if it is symmetric (i.e. contains equal numbers of particles and antiparticles).

We will show, however, that the case of asymmetric dark matter consisting of Dirac fermions is different. Namely, it provides a circularly polarized photons when decaying to the photon and the fermion (e.g. neutrino). This circular polarization, in principle, could be detected though it may be a challenge for the real experiment. We will discuss shortly a few models which can lead to radiatively decaying asymmetric dark matter.

The paper is organised as follows. In Section 2, we obtain the single photon polarization state for decaying scalar, fermion (with spin 1/2 and 3/2) and for the atomic line and prove that it is impossible to distinguish between them if dark matter is symmetric. In Section 3

<sup>&</sup>lt;sup>1</sup>For the current status of this line and future prospects see [7, 19, 20].

<sup>&</sup>lt;sup>2</sup>For discussion of photon polarizations in collider experiments and in indirect searches see [22] and [23,24], respectively.

we show that in a case of asymmetric dark matter consisted of Dirac fermions one obtains the circularly polarized emission line. In Section 4 more complicated models including the transition between dark states are briefly discussed and the common criteria for the polarized line are formulated. The last section is conclusions.

## 2 Symmetric dark matter

In this section we determine the quantum state of the photon emitted in dark matter decay. We assume that DM is CP-symmetric and that it can disintegrate directly (i.e. without any intermediate states) into final states with photon(s). In order to analyse different dark matter models we use the language of the effective field theory. We order our account by the spin of DM particle: starting from the scalar we go to the spin 3/2 fermion and complete with a discussion of atomic transitions. Then we consider possibilities to distinguish between different spins of dark matter particle.

### 2.1 Scalar and pseudoscalar

The neutral scalar  $\phi$  and pseudoscalar a particles may interact with the photon through the lowest order operators

$$L = \frac{1}{\Lambda} \phi F_{\mu\nu} F^{\mu\nu}, \quad L = \frac{1}{\Lambda} a F_{\mu\nu} \tilde{F}^{\mu\nu}, \tag{1}$$

respectively. Here  $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}/2$  is the dual electromagnetic field tensor and  $\Lambda$  is a mass parameter. In order to obtain the reasonable decay width providing the observable X-ray line intensity one needs  $\Lambda$  to be of order Planck scale [25] which looks natural for the variety of axion and axionlike particle models [8–10].

The spin-0 boson decays to two photons which polarization state is the maximally entangled Bell state [26]:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|LL\rangle \pm |RR\rangle).$$
 (2)

Hereafter we use the notation  $|L\rangle$  for the left-handed photon (with the spin projection -1 on the momentum direction) and  $|R\rangle$  for the right-handed photon.  $|RR\rangle$  means, for example, the state of two right-handed photons. The sign in (2) is determined by the parity of initial particle: plus for the scalar and minus for the pseudoscalar.

We can register only one photon from each pair because the other one flies in the opposite direction. In order to obtain the reduced density matrix for one of the photons, we need to take a trace of the density matrix  $|\Psi\rangle\langle\Psi|$  over all states of the unobservable photon. The result is the unity density matrix corresponding to the maximally mixed and unpolarized state for every coming photon in the flux [26]:

$$\rho = \frac{1}{2}(|R\rangle\langle R| + |L\rangle\langle L|). \tag{3}$$

### 2.2 Fermion with spin 1/2

An extra singlet Majorana fermion N (sterile neutrino) may be added to the Standard Model (SM) in order to explain dark matter. The only renormalizable interaction between N and SM particles allowed by symmetries may be written as

$$L = f \,\bar{l}_L N \,\tilde{\mathcal{H}} + h.c. \,\,. \tag{4}$$

Here  $\tilde{\mathcal{H}} = \epsilon \mathcal{H}^*$ , where  $\epsilon$  is  $2 \times 2$  antisymmetric unit matrix and  $\mathcal{H}$  is the Higgs doublet,  $l_L$  the left SM lepton doublet, f is a small dimensionless coupling. This interaction at one loop level leads to the effective coupling between the sterile neutrino and the field strength  $B_{\mu\nu}$  of the  $U(1)_Y$  gauge boson:

$$L \propto \frac{\tilde{\mathcal{H}}}{\Lambda^2} \bar{l}_L \sigma_{\mu\nu} N B^{\mu\nu} + h.c. . \tag{5}$$

Here  $\sigma_{\mu\nu} = (\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu})/2$ . The scale  $\Lambda$  is connected with the parameter f in (4) and Higgs vacuum expectation value v as  $\Lambda \sim v/\sqrt{f}$ . After spontaneous symmetry breaking one obtains the operator describing the interaction between the extra fermion N, SM neutrino  $\nu_L$  and photon  $F_{\mu\nu}$ :

$$L \propto \frac{v}{\Lambda^2} \bar{\nu}_L \sigma_{\mu\nu} N F^{\mu\nu} + h.c. \tag{6}$$

Since the SM (active) neutrinos are always in a pure spin state (left-handed for neutrinos and right-handed for antineutrinos) we expect that the photon state is determined by the initial state of sterile neutrino. Clearly, it depends on the mechanism of sterile neutrino production in the early Universe. If they were produced in the process of the scalar (inflaton, majoron) decay [27,28] then each particle is expected to be in the maximally mixed state and the same holds for the photon. So we obtain the density matrix (3). However, if the Majorana particles were produced by oscillations in lepton symmetric [1] or lepton asymmetric plasma [2] then we expect that each particle is in the pure spin state but the spin vectors of different particles are distributed randomly. Each particle may decay to the SM neutrino and antineutrino with equal probability (neglecting the possible CP-violation

in (4)). Then, the state of photon  $|\gamma\rangle$  may be described as follows:

$$|\gamma\rangle = \begin{cases} \sqrt{1-\beta}|L\rangle + \sqrt{\beta}e^{i\alpha}|R\rangle, & \text{with probability } \frac{1}{2}, \\ \sqrt{1-\beta}|R\rangle + \sqrt{\beta}e^{i\alpha}|L\rangle, & \text{with probability } \frac{1}{2}. \end{cases}$$
 (7)

Here the parameter  $\beta$  depends on the direction of the Majorana particle's spin and, therefore, is a random parameter. The same is for the phase  $\alpha$  since we expect no spin correlations for the dark matter particles. We see that the polarization state of each individual photon is a pure state in the quantum mechanical sense.

If the DM fermion  $\psi$  is of the Dirac type (we use this notation for the Dirac particle instead of N left for the Majorana case) then the particle decays only to the left-handed neutrino and antiparticle decays to the right-handed antineutrino, correspondingly. If s stands for the momentum projection of the photon spin (s = 1 for the right-handed photon and s = -1 for the left-handed one) then we find the fermion and antifermion decay width to be

$$\Gamma_{\psi} = \Gamma_0(1-s), \quad \Gamma_{\bar{\psi}} = \Gamma_0(1+s), \quad \Gamma_0 \propto \frac{v^2 m_{\psi}^3}{\Lambda^4}.$$
 (8)

We see that the fermion provides only right-handed photons while the antifermion gives only left-handed ones. Every photon is in the pure polarization state. If the number of fermions equals the number of antifermions, the photon flux will consist of equal numbers of left-handed and right-handed states:

$$|\gamma\rangle = \begin{cases} |R\rangle, \text{ with probability } \frac{1}{2}, \\ |L\rangle, \text{ with probability } \frac{1}{2}. \end{cases}$$
 (9)

## 2.3 Fermion with spin 3/2

We omit the discussion on the vector dark matter particles with spin 1 because the vector particle can not decay to two photons. Therefore we go further and consider spin 3/2 fermion. This particle of Majorana type naturally arises in supergravity models as a graviton superpartner – gravitino. The effective interaction of spin 3/2 fermion  $\psi_{\rho}$  with photon may be written as

$$L = \frac{v}{\Lambda^2} \bar{\psi}_{\rho} \sigma_{\mu\nu} \gamma^{\rho} \nu_L F^{\mu\nu} + h.c. . \qquad (10)$$

For the Majorana particle, all statements of the previous section related to the spin 1/2 remain unchanged. Namely, if the gravitino was produced in the mixed state then we register each photon in the state (3) while for the pure states we have (7).

If the spin 3/2 particle is Dirac (the corresponding model is worked out in [29]) then we again obtain results which are independent of the production mechanism. Namely, the spin-averaged decay width depends on the photon polarization as

$$\Gamma_{\psi_{\mu}} = \Gamma_{3/2}(1+s), \quad \Gamma_{\bar{\psi}_{\mu}} = \Gamma_{3/2}(1-s), \quad \Gamma_{3/2} \propto \frac{v^2 m_{\psi}^3}{\Lambda^4}.$$
 (11)

We see that the particle provides only right-handed photons while antiparticle gives only left-handed ones. Then the photon state in the symmetric case is the same as for spin 1/2 fermions (9).

#### 2.4 Atomic transition

The 3.5 keV line received many interpretations in terms of dark matter particle decay, but it may well be a result of usual atomic transitions [21]. Though this case may be distinguished with the enhanced spectral resolution we also discuss a polarization state of photons in the case of atomic transitions.

In a case of the line corresponding to the dipole atomic transition (i. e. the transition when the orbital quantum number of electron changes by unity) the polarization of emitted photon depends on the change of magnetic quantum number which may be  $\Delta m = -1, 1, 0$ . These cases correspond to the emission of left, right and linearly polarized photon, correspondingly. The textbook knowledge (see, for example, [30]) predicts that all three cases  $\Delta m = 0, \pm 1$  happen with equal probability if initial atoms are unpolarized (this condition holds in the interstellar medium). Therefore, the state of photon may be described as

$$|\gamma\rangle = \begin{cases} |R\rangle, & \text{with probability } \frac{1}{3}, \\ |L\rangle, & \text{with probability } \frac{1}{3}, \\ \frac{1}{\sqrt{2}}(|R\rangle + e^{i\alpha}|L\rangle), & \text{with probability } \frac{1}{3}. \end{cases}$$
(12)

Here  $\Psi = (|R\rangle + e^{i\alpha}|L\rangle)/2$  is the linearly polarized state and the phase  $\alpha$  is a random number characterising the direction of linear polarization. We see that the state of photons coming from the dipole atomic transition differs from all cases listed above<sup>3</sup>. In the next section we discuss whether this difference is measurable in any possible experiment.

<sup>&</sup>lt;sup>3</sup>Multipole transitions are expected to provide more complicated state distribution but, as we will show in Section 2.5, only the density matrix averaged over all the photon states really makes sense. Thus, the final result would be the same as in dipole transitions.

### 2.5 How to distinguish?

In the real experiment, we register a flux of photons. It can be described by the average polarization density matrix (see, for example, [31] for the definition) even when each individual photon is in the pure state. One can see that for all cases described in the previous sections the averaged density matrix for the flux of photons is the same and takes the form (3). But we deal with different states of individual photons: in the case of fermion, we have the pure state while for the scalar we have the completely mixed state. For the atomic transition, some photons have the linear polarization. If we were dealing with a *known* pure state, one can easily discern it from the mixed state. But in the case under consideration the structure of the pure state is *unknown* as it depends on the spin projection of the DM particle. Is it possible to distinguish between the described cases in any type of experiment?

Single photon measurements. The only thing that we can measure for a single photon is its projection on the basic state. After the first measurement, the state collapses to its projection and the initial state is lost. In order to reconstruct the full density matrix one need to make at least three projection measurements [32] which is impossible to do for a single photon.

Multiple photon measurements. Potentially, we can imagine to collect many photons from the coming flux and hold them in a box. Then, any possible measurement may be reduced to finding the correlation function for the corresponding product of operators:  $A = O(x_1)O(x_2)...O(x_n)$ . Here O stands for any gauge invariant operator containing photon field and  $x_1...x_n$  are the different points of the space and time. These correlators would be different for the pure and mixed many-particle states but each pure state correlator has its own dispersion defined by the usual formula

$$D_A(\Psi) = \langle \Psi | (A - \langle A \rangle)^2 | \Psi \rangle = \langle \Psi | A^2 | \Psi \rangle - (\langle A \rangle)^2, \tag{13}$$

where  $\langle A \rangle = \langle \Psi | A | \Psi \rangle$ . The dispersion of the correlator for the mixed state with density matrix  $\rho$  is

$$D_A(\rho) = \text{Tr}(\rho A^2) - (\text{Tr}(\rho A))^2. \tag{14}$$

Then, the theorem proven in [33] provides the answer to the question about the physical difference between the pure and mixed state for our case when density matrices averaged over all photons are the same.

Theorem.

Let A be an hermitian operator corresponding to the measurement of some quantity and N

be the number of particles (i.e. the number of photons),  $|\Psi\rangle$  be a pure state,  $\rho = 2^{-N} \times 1$  be the density matrix describing the completely mixed state <sup>4</sup>. Then the difference between the correlators over the pure state and over the state described by the density matrix  $\rho$  is always smaller (by factor  $1/\sqrt{N+1}$ ) than the intrinsic dispersion of this correlator (14), calculated over the density matrix  $\rho$ :

$$\left(\langle \Psi | A | \Psi \rangle_{|\Psi\rangle} - \text{Tr}(\rho A)\right)^2 = \frac{D_A(\rho)}{N+1}.$$
 (15)

The left-hand side means the averaged over all possible pure states  $|\Psi\rangle$  squared difference between the correlators for the pure and mixed state. In other words, this theorem reflects the fact that each correlator for the mixed state has a relatively large variance. So, one can not decide whether the measured correlation function corresponds to the pure or mixed state because the results for the pure states lie in the band of possible values for the mixed state. This implies that one can never distinguish between the indefinite pure state and mixed state of photons.

Summarising, we have no chance to determine if the photon coming from dark matter decay is in the pure state. So we can't distinguish between cases leading to the equal density matrices when averaging over all photon flux. By this reason, the polarization would not help us to detect even the case of the line provided by the usual atomic transition: the distribution (12) leads again to the unit density matrix when averaged over the photon flux.

# 3 Asymmetric dark matter

In this section, we show that the study of the dark matter photons polarization is still important because it allows detecting the asymmetry between the number of fermions and anti-fermions constituting the dark matter if it has a Dirac fermionic nature. As an example, we consider models connected with the sterile neutrino dark matter [1–6]. Usually, sterile neutrinos are treated as Majorana particles but it is also possible to consider them to be Dirac fermions. Then, the asymmetry in the dark sector strongly depends on the production mechanism in the early Universe. In case of thermal production in plasma no significant asymmetry is expected.

However, the asymmetry may arise in the processes sharing some features of leptogenesis. An example is provided by the resonant production of sterile neutrinos [2] originally proposed

<sup>&</sup>lt;sup>4</sup>This means that each collected photon is equally likely left-handed and right-handed.

for the Majorana neutrinos. The latter process would be still valid for the Dirac case as well. This scenario works as follows. Due to the presence of the SM lepton asymmetry the dispersion relation of the sterile neutrino is modified in such a way that at some temperature the level crossing with SM neutrinos happens. At this moment the large part of SM neutrinos presented in the Universe converts into sterile ones. Since in the Dirac case fermions are mixed only with fermions, but not with antifermions, one obtains that the major part of lepton asymmetry in the SM neutrino sector directly converts to the asymmetry in the dark sector. The described mechanism works only for large lepton asymmetry  $\eta_L = (n_L - \bar{n}_L)/(n_L + \bar{n}_L) > 10^{-4}$  and it can naturally provide  $\eta_{\psi} = (n_{\psi} - \bar{n}_{\psi})/(n_{\psi} + \bar{n}_{\psi}) \sim 1$  (where  $n_{\psi}$  and  $\bar{n}_{\psi}$  are the density numbers of DM particles and antiparticles, correspondingly), depending on the choice of parameters.

Since the particle decay may provide only left-handed photons while antiparticle give only right-handed photons (see eq. (8)) the asymmetric case leads to the polarization density matrix of the flux in the basis ( $|R\rangle$ ,  $|L\rangle$ ) of the form:

$$\rho = \frac{1}{n+\bar{n}} \begin{pmatrix} \bar{n} & 0 \\ 0 & n \end{pmatrix} = \frac{1}{2} \left( 1 - \eta_{\psi} \sigma_3 \right), \tag{16}$$

where  $\sigma_3 = |R\rangle\langle R| - |L\rangle\langle L|$  is the third Pauli matrix. If  $\eta_{\psi} \neq 1$  the flux is partially polarized corresponding to the set of Stokes parameters  $S_1 = S_2 = 0$ ,  $S_3 = -\eta_{\psi}$  (for parametrisation of polarized light see, for example, [32]).

To complete the consideration let us study the case of the Dirac fermion with spin 3/2. To the best of our knowledge, no dark matter models assuming asymmetry of spin-3/2 fermions were suggested in the literature but this does not look to be impossible. In Section 2 we found that decaying fermions of spin 3/2 yield only right-handed photons while antifermions yield only left-handed ones. Therefore, if there is an asymmetry  $\eta_{\psi}$  one can detect the polarization density matrix for coming flux of photons to be

$$\rho = \frac{1}{2} \left( 1 + \eta_{\psi} \sigma_3 \right). \tag{17}$$

The difference in signs in (16) and (17) is due to the fact that particles with spin 3/2 decay to the right-handed photons while spin-1/2 particles yield the left-handed ones.

While it is clear how to detect the circular polarization of the visible light, for many other wave-ranges, in particular for the X-ray bandwidth, it is not so obvious. The Compton scattering, which detects the linear polarization of X-rays, does not distinguish between right and left circular polarizations. However, there are some attempts to measure such kind of

X-ray polarization in laboratory [34] that use circular dichroism in two photon ionisation of helium. To the best of our knowledge, no methods of detecting circular polarization of gamma-rays have been suggested yet. However, in this bandwidth there are more possibilities for indirect searches due to the appearance of other decay channels (see for example [23,24]). In any case, detection of the circular polarization of the dark matter photons in future experiments may be a very clear signature of the fermionic nature and asymmetry in the dark matter sector.

### 4 Dark state transition models

Besides the DM particle decay, the line-like feature may also be produced in transitions between two dark states [16]. In models of such type, the dark matter goes to the excited state due to the plasma collisions or background emission and then decays back emitting the photon. If the transition happens between the two Dirac fermions  $\psi$  and  $\chi$  the photons would be polarized only when the following conditions are satisfied:

- 1. The number density of heavier state,  $\psi$ , differs from the number of  $\bar{\psi}$ .
- 2. The interaction with the lighter state  $\chi$  is P-asymmetric.

Obviously, for decays of  $\psi$  to SM neutrino and photon the second condition is satisfied automatically providing the results for the photons polarization obtained in the previous sections.

If transition happens between the two bosonic states (scalar and pseudoscalar or vector [35]) then, clearly, no circular polarization is expected because the corresponding process is P- and C- symmetric. Although, as far as we know, the models satisfying both conditions listed above are not discussed in the literature it looks not impossible to construct them. But for the most cases yet considered the emission would be unpolarized.

# 5 Conclusions

Unfortunately, the most important symmetric dark matter models exhibiting the photon line – fermion (sterile neutrino) and scalar (axion, ALP) – are indistinguishable in observations of the line polarization. So, in order to decide what kind of dark matter particle produces the X-ray line supplementary experiments will be needed. For example, sterile neutrino

dark matter might be searched in the tritium decay due to its small mixing with the SM neutrinos [36]. The axions and axionlike particles may be probed in a variety of special laboratory experiments connected with the axion-photon oscillations in the magnetic field, see [9] for a review and references.

However, if the experiment will show the circularly polarized line it would be a smoking gun of its dark matter origin because there is no way to obtain such polarization in usual atomic transitions. Moreover, it would be a signature of asymmetric fermionic nature of dark matter and the amount of polarization would be directly connected with the asymmetry in the dark sector. By this reason, we underline the importance of polarisation studies in the future observations.

In this paper we concentrated mostly on the keV-scale photons, motivated by the recent indications of 3.5 keV line. However, all the considerations are valid for any photon energy range.

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