Exam 2 - Wed, Oct 23
Lecture 5 - Lecture 10

More on Independence

- If X, Y are independent, Cov(X,Y) = 0 (always)
- But some dependent R.Vs also have Cov(X,Y) = 0

That is . . .

X, Y independent $\Rightarrow$ $\mathrm{Cov}(X,Y) = 0$
X, Y dependent $\Rightarrow$ $\mathrm{Cov}(X,Y) = 0$ or $\mathrm{Cov}(X,Y) \neq 0$

So, if we calculate Cov(X,Y) and get

- $\mathrm{Cov}(X,Y) \neq 0$, then X, Y <u>must</u> be dependent
- $\mathrm{Cov}(X,Y) = 0$, then X, Y could be dependent or independent

(<u>Requires additional steps</u>)

- If $P_{X,Y}(x,y) = P_X(x)\,P_Y(y)$ for <u>all</u> x,y pairs, then X, Y are <u>independent</u>.
- If $P_{X,Y}(x,y) \neq P_X(x)\,P_Y(y)$ for any x,y pair, then X, Y are <u>dependent</u>

Example: $Y = X^2$

X \ Y	0	1	
-1	0	0.2	0.2
0	0.6	0	0.6
1	0	0.2	0.2
	0.6	0.4	

Joint Probability Table

Get marginal probability tables

x	-1	0	1
$P_X(x)$	0.2	0.6	0.2

y	0	1
$P_Y(y)$	0.6	0.4

Are X and Y independent?

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$E(X) = \sum_x x P_X(x) = (-1)(0.2) + (0)(0.6) + (1)(0.2) = 0$$

$$E(Y) = \sum_y y P_Y(y) = (0)(0.6) + (1)(0.4) = 0.4$$

$$\begin{aligned} E(XY) = \sum_{x,y} xy P_{X,Y}(x,y) &= \overset{0}{\cancel{(-1)(0)(0)}} + (-1)(1)(0.2) \\ &+ \overset{0}{\cancel{(0)(0)(0.6)}} + \overset{0}{\cancel{(0)(1)(0)}} \\ &+ \overset{0}{\cancel{(1)(0)(0)}} + (1)(1)(0.2) \\ &= -0.2 + 0.2 = 0 \end{aligned}$$

$$\begin{aligned} Cov(X,Y) &= E(XY) - E(X)E(Y) \\ &= 0 - (0)(0.4) \\ &= 0 \end{aligned}$$

But just b/c $Cov = 0$, we can't say whether X & Y are dependent or independent.

implies independence

check whether $P_{X,Y}(x,y) = P_X(x) P_Y(y)$ for all x,y pairs.

$$P_{X,Y}(-1,0) = 0 \neq (0.2)(0.6) = P_X(-1) P_Y(0)$$

$\Rightarrow$ Hence X, Y are dependent in this case (not independent)

even though $Cov(X,Y) = 0$

Discrete Dist. Review

- R.V X can take on only discrete values (ie $0, 1, 2, 3, \ldots$)

For discrete R.Vs, we have:

- PMF: $P_X(x) = P(X = x)$ (the prob. X = some value)
- CDF: $F_X(t) = P(X \le t)$ (the prob $X \le$ some value)

use PMF/CDF to calculate probabilities of events

- $E(X)$: the mean/average value that X takes on
- $Var(X)$: gives idea of how spread apart the values of X are w.r.t the mean

Helps summarize and understand the values of X

Example

Jacob has a 0.6 probability of making a free throw (basketball)

p

In a game, he attempts 10 free throws. What is the probability he makes more than 5 FTs?

n

$$X = \text{\# of made FTs}$$
$$X \sim Bin(n, p) \equiv Bin(10, 0.6)$$

$$P(X > 5) = 1 - P(X \le 4)$$
$$= 1 - [P(X=0) + \cdots + P(X=4)]$$

or

$P(X > 5) = 1 - P(X \le 4)$ (or use cdf table)