A Functional Anova Approach to Detecting Changes in Soil Moisture and Temperature

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Abstract

Climate change poses significant challenges to the soil ecosystem. Green house effect and early snow melt were experimentally simulated, resulting in time series measurements of soil moisture and temperature. The identification of changes in daily minimum temperatures is of interest as it could have profound implications for plant phenology. However, due to inherent dependencies in temporal data, many of the classical statistical inferential methods are inadequate. To overcome this, we utilize a functional anova approach to identify differences in patterns between treatment groups, allowing us to account for temporal dependence. In this application, we study the effects of simulated climate change on daily minimum soil temperatures.

Objectives

- Compare daily minimum temperatures between 4 treatment groups.
- Understand interaction between heating treatment and snow removal treatment.
- Understand the effect of heating treatment, and snow removal treatment on daily minimum temperatures.

Method

The general framework of functional anova is presented. A modification of the below procedure using contrasts was used to test for interaction. Each time series was smoothed into functional form using Fourier basis. The hypotheses for the fanova, where $\mu_1(t)$ and $\mu_2(t)$ represent the mean curves for group 1 and group 2 as a function of time, t, are given by

$$H_0 : \mu_1(t) = \mu_2(t)$$
$$H_A : \mu_2(t) \neq \mu_2(t)$$

Obtain Test Statistic

1. Obtain the sample mean curve for each group, and calculate test statistic, T, as the L_2 distance between sample mean curves
2. For visualization, obtain a sample mean difference curve by subtracting sample mean curve of group 2 from group 1.

Obtain Bootstrap Resamples For $m = 1, \cdots, M$,

1. Generate resample curves from multivariate Gaussian distribution using mean $\underline{0}$, and sample covariance of each group
2. Obtain the mean resample curve for each group, and calculate T_m as the L_2 distance between the mean resample curves.
3. For visualization, obtain M mean resample difference curves by subtracting resample mean curve of group 2 from group 1.

Calculate p-value

1. $p\text{-value} = \sum_{m=1}^{M} I(T_m > T)$

Data

The experiment was setup at an elevation of 2100m in Grand Teton National Park, WY as a replicated block design in which 4 treatments were applied: (1) control, C; (2) heating, H, (3) heating + snow removal, HSR; (4) Snow Removal, S. Within each treatment, 3 replicates of soil moisture and temperature were obtained hourly from May, 27 - Sept. 27, 2011.

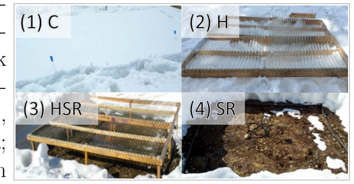

Figure 1: The 4 treatments are shown:(1) control, C; (2) heating, H, (3) heating + snow removal, HSR; (4) Snow Removal, S.

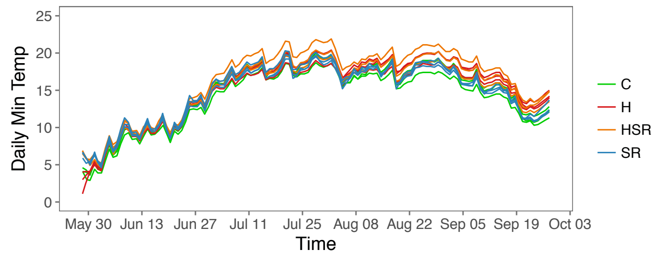

Figure 2: Raw daily minimum temperature time series for control (C), heating (H), heating + snow removal (HSR), and snow removal (SR) treatment groups

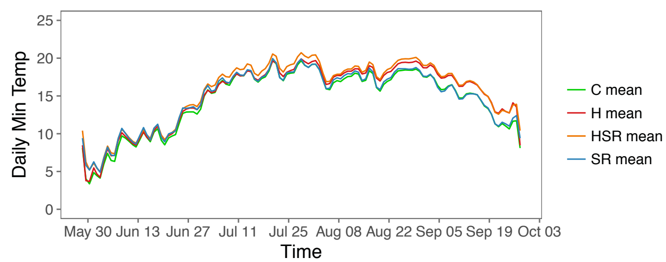

Figure 3: Functional mean temperature curves for control (C), heating (H), heating + snow removal (HSR), and snow removal (SR) treatment groups

Results

Test	p-value
Difference between 4 treatments	0.031
Interaction between heating and snow removal	0.635
Difference due to heating (main effect)	0.004
Difference due to snow removal (main effect)	0.268

Table 1: Summary table giving p-values for various functional anova tests

Fanova Visualizations

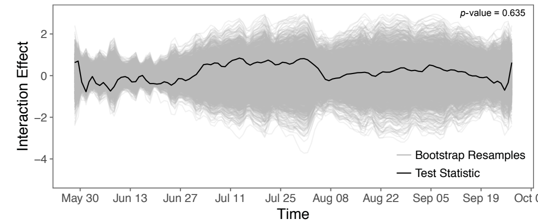

Figure 4: Visualization of test for interaction between heating and snow removal. Bootstrap resample curves under the assumption of no interaction are shown in gray. Test statistic (sample interaction) is shown in black.

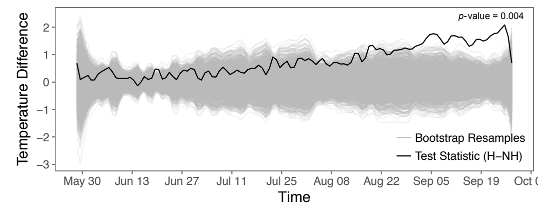

Figure 5: Visualization of test for heating main effect. Bootstrap resample curves under the assumption of no heating main effect shown in gray. Test statistic (sample mean heating - no heating) shown in black.

Conclusion

- Evidence of difference between 4 treatments groups, and difference due to heating for daily minimum temperatures
- Method useful when measurements are recorded at unequal times
- Visualizations useful for identification of when significant changes occur which may be of scientific interest.
- Method provides general framework for comparing groups of time series. Approach also applied to hourly soil moisture and temperature data.
- When number of sample curves is small, simulation study showed loss of power in identifying small differences but ability of identify large differences.

References

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