Error Analysis

Please write your solutions in LaTeX . Your LaTeX report will need to have one figure (from problem 5); you may use matplotlib or a scientific graphics package of your choice to create the plot. Save it as a .pdf or a .png file for inclusion in your LaTeX file. Please turn in a printed copy of your solutions.
Due: BEFORE class Tuesday, 5 Sept 2017

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1. Rewrite the following results in their *clearest forms*, with suitable numbers of significant figures:

   (a) measured height $= 2.091 \pm 0.04329$ m

   Answer: (2.09 ± 0.04) m.

   (b) measured time $= 1.5432 \pm 1.01$ s

   Answer: (1.5 ± 1.0) s.

   (c) measured charge $= -3.21 \times 10^{-18} \pm 2.67 \times 10^{-20}$ C

   Answer: $(-3.21 \pm 0.03) \times 10^{-18}$ C.

   (d) measured distance $= 0.000, 000, 563 \pm 0.000, 000, 07$ m

   Answer: $(5.6 \pm 0.7) \times 10^{-7}$ m.

   (e) measured momentum $= 3.267 \times 10^3 \pm 64$ g·cm/s

   Answer: $(3.27 \pm 0.06) \times 10^3$ g·cm/s.

2. A student measures the density of a liquid 6 times and gets the results (in units of g/cm^3) 1.8, 2.0, 2.0, 1.9, 1.7 and 1.8.

   (a) What would you suggest as the best estimate and uncertainty based on these measurements?

   Answer: The best estimate would be the mean value using the standard deviation as the measure of uncertainty.

$$\overline{x} = 1.8\overline{6} \approx 1.9$$

$$\sigma = \sqrt{\frac{\sum(x_i - \overline{x})^2}{N-1}} \approx 0.1211 \approx 0.1$$

$$(1.9 \pm 0.1)\text{g/cm}^3$$

   (b) The student is told that the accepted value is 1.85 g/cm^3. What is the discrepancy between the student's best estimate and the accepted value?

   Answer: $0.01\overline{6}$ or 0.5

   (c) Do you think the discrepancy it is significant?

   Answer: The discrepancy is within one standard deviation of the mean and so I do not believe it to be significant.

   (d) Calculate the rms deviation of these measurements.

   Answer:

$$\text{RMS} = \sqrt{\frac{\sum(x_i - \overline{x})^2}{N}} \approx 0.11055 \approx 0.1$$

3. If I measure the radius of a sphere as $r = 2.0 \pm 0.1$ m, what should I report for the sphere's volume?

   Answer: $\text{V} = \frac{4}{3}\pi r^3 \approx 33.5$

$$\delta V = \sqrt{(\frac{\partial V}{\partial r}\mid_{r_0} \delta r)^2} \approx 5.0265 \approx 5.0$$

$$V = (33.5 \pm 5.00)\text{m}^3.$$

4. Suppose that you measure two independent variables as

$$x = 6.0 \pm 0.2 \quad \text{and} \quad y = 3.0 \pm 0.1,$$

and use these values to calculate $q = xy + x^2/y$. What will be your answer and its uncertainty? Show your work!

Answer: $q = xy + x^2y^{-1} = 30$

$$\delta q = \sqrt{(\frac{\partial q}{\partial x}|_{x_0,y_0}\ \delta x)^2 + (\frac{\partial q}{\partial y}|_{x_0,y_0}\ \delta y)^2} = \sqrt{2}$$

$$q = 30 \pm 1.4$$

5. If a stone is thrown vertically upward with speed v, it should rise to a height h given by $v^2 = 2gh$. In particular, v^2 should be proportional to h. To test this proportionality, a student measures v^2 and h for seven different throws and obtains the results shown in Table I.

   (a) Make a plot of v^2 vs h, including vertical and horizontal error bars using a graphics package of your choosing. As usual, label your axes, and scale the axes suitably. Is your plot consistent with the prediction that $v^2 \propto h$?

   Answer: Yes, as seen in Fig. 1 there does appear to be a direct, linear relationship between v^2 and h.

   (b) The slope of your graph should be $2g$. To find the slope, draw the best fit straight line by performing a linear fit to the data. Your plotting program will likely spit out an estimate of the uncertainty of the slope; if so what is that uncertainty?

   Answer: The value obtained for slope of the best fit line is 18.35 ± 0.63 when allowing for a non-zero y-intercept and 18.44 ± 0.29 when setting the y-intercept to zero.

   (c) Estimate the uncertainty in the slope manually by drawing in the steepest and least steep lines that seem to fit the data reasonably. The slopes of these lines give the largest and smallest probable values of the slope. Are your results consistent with the accepted value $2g = 19.6$ m/s^2?

   Answer: The accepted value is outside of the bounds of error obtained in both fitting routines: one using a standard linear fit, Fig. 1, and one which does not allow for a non-zero y-intercept, Fig. 2. However, approximately twice the range or value of the error margin obtained in Fig. 1 would include the value, so it is close, therefore, especially given the expected direct proportionality to a good approximation, an unconsidered or outside source of error is probable. That being said, the numbers in Fig. 2 should be closer to reality and the accepted value is significantly outside the bounds of this fit.

TABLE I. Heights and speeds of a stone thrown vertically upward. For Problem 5.

h	v^2
($\pm$0.05 m)	(m^2/s^2)
0.4	7 ± 3
0.8	17 ± 3
1.4	25 ± 3
2.0	38 ± 3
2.6	45 ± 3
3.4	62 ± 3
3.8	72 ± 3

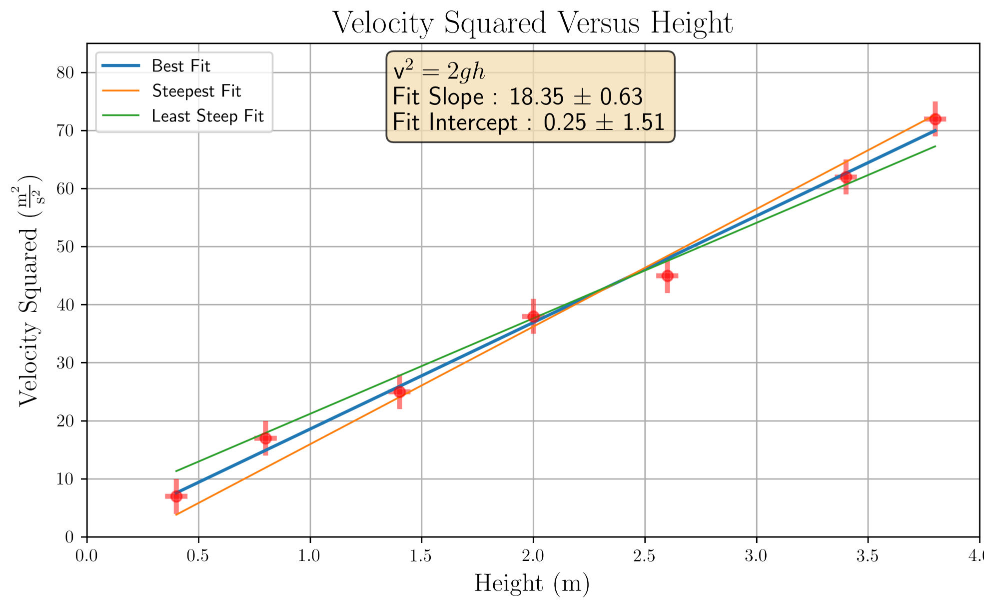

FIG. 1. This graph shows the data in Table I with the specified uncertainties displayed as error bars. This figure was created in a Jupyter Notebook, code for which can be seen here: `https://github.com/mjones98806/Intermediate-Lab-Repository/blob/master/Assignment1.ipynb`

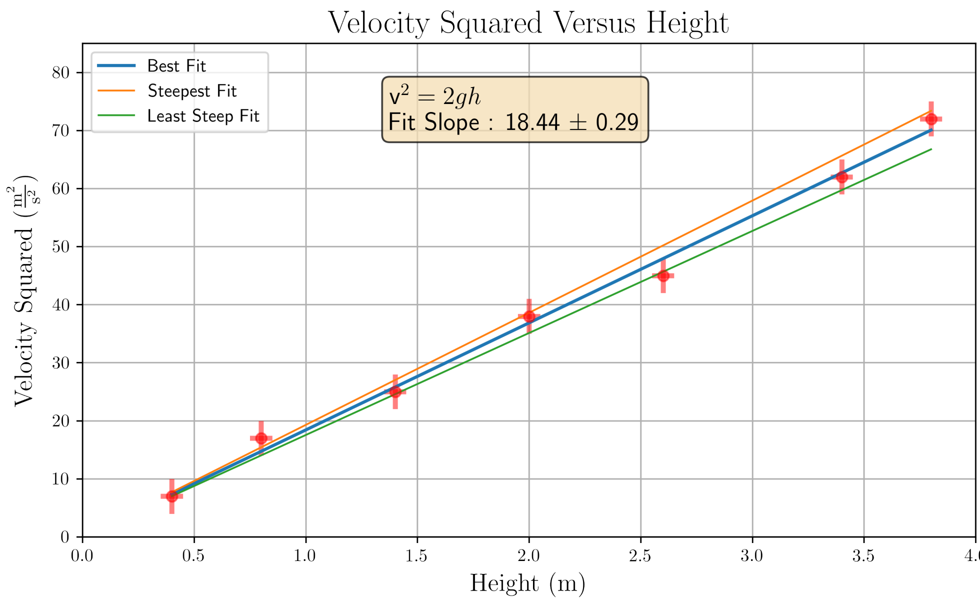

FIG. 2. As in Fig. 1, this graph shows the data in Table I with the specified uncertainties displayed as error bars, but does not allow for a non-zero y-intercept. The obtained slope is approximately the same, but with less allowance for error.