Testing the Figure of Optical Surfaces

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An approach to designing theoretically sound methods and tests which will ultimately indicate the figure of delicate, curved optical surfaces to a precise degree, with a focus on parabolic mirrors such as those used in telescopes. Simplicity is considered for ease in reproduction by undergraduate lab students.

1. BACKGROUND AND HISTORY

Since the telescope was invented in the beginning of the 17th century, astronomers have ground the reflecting surfaces by hand. In order for the telescope to produce clear images, it is necessary for these hand-ground mirrors to be perfectly parabolic. As such, there are several methods for testing the curvature of the mirror, and identifying any localized aberrations. Such tests vary in complexity and accuracy, but one method widely used by amateur telescope makers today is the Foucault Knife Edge test. Leon Foucault introduced this in 1858 as a means of detecting imperfections in the shape of spherical mirrors. The simplicity and accuracy of the test (capable of detecting deviations smaller than one wavelength of the source light [1]) make it ideal for amateurs.

2. PROBLEM AND RELEVANT THEORY

Foucault found that if you put a point light source capable of illuminating an entire reflecting surface at the center of curvature of that surface and introduce a knife edge in the focal plane of the reflected rays, you can determine the amount of spherical aberration in the mirror by observing and interpreting the shadowing introduced on the resultant image. This is due to a property of spherical mirrors which can be verified by imagining the path of rays of light shone onto a spherical mirror from a particular point along the optical axis such that the rays travel back to that same source point in a single focal plane. The only point along the optical axis at which this is possible is the center of curvature, due to spherical symmetry, in the same way rays emitted from a point source at the center of a mirrored spherical shell would all travel back to the center point after reflecting off of its walls. Conversely, a parabolic mirror will have many focal points with a similar configuration; certain zones of a mirror with a parabolic figure will have a focal point relative to their own apparent center of curvature in a spherical sense. When the knife edge is introduced in the focal plane of the reflected rays, all of the regions of the mirror associated with that focal plane gray out evenly, as one might expect. By moving the knife edge and point source along the optical axis, the experimenter will observe one of two possibilities; either the entire resultant image will gray out at one focal point, indicating a consistent spherical figuring of the reflecting surface, or different regions of the image will gray out at different corresponding positions of the knife edge, indicating several focal points and thus deviation from an ideal spherical figuring. This mirrors the properties of these two distinctly different shapes when the light source being reflected comes from a point which can be approximated as infinitely far away, such as in the case of a telescope observing a distant star. This is why an ideal parabolic figure is critical for mirrors used in a telescope, as they must be parabolic to a precise degree in order to obtain a focused image of parallel rays.

When creating a parabolic mirror, the maker should first begin with an ideal spherical shape, then continue to figure the surface based on the distances at which they observe a light graying of certain zones of the mirror under a Foucault test. To simplify this process, a mask may be placed over the mirrored surface to help clearly define the zones of the mirror which are of interest when attempting to create an ideal parabolic surface. It is important to note the difference between a shadow cast by the knife edge, which is dark and occurs when certain rays are blocked, and the graying which occurs when the knife edge is introduced at a point where the rays converge. An experienced maker may be able to tell where aberrations lie and which parts of the surface should be adjusted in a very short amount of time given an appropriate apparatus.

3. PROJECT GOALS

Our main goal is to use this information to design a user-friendly apparatus which will be capable of testing the figure of any curved mirror of reasonable size and determine its proximity to the ideal shape for its intended use. We also hope to introduce or reinforce the concept of using light itself in various ways to determine the shape of delicate optical surfaces based on expected behavior. Because the aberrations we wish to measure are small and we hope to obtain a reasonable degree of precision, the process of fabricating an apparatus which satisfies our

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intentions requires a great deal of planning and precise machining.

4. GEOMETRIC ANALYSIS

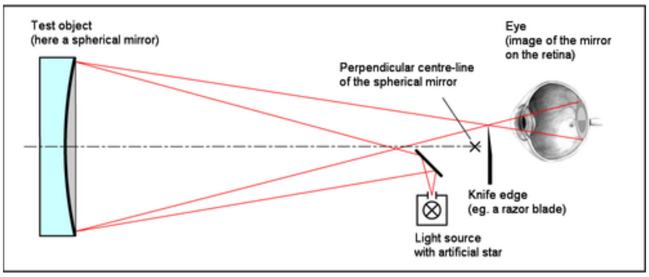

FIG. 1: This simple ray diagram shows the basic layout of the Foucault test.

Analyzing the proposed test via ray tracing shows its simplicity as shown in Fig 1. Because the test is null if the mirror is spherical, the aberration is of the most interest. The spacing of the focal planes which indicate ideal parabolic behavior of regions on the figured surface follows the formula,

$$d = \frac{r^2}{2R} \tag{1}$$

where d is the spacing on the optical axis of a zones center of curvature from that of the center zone of the mirror, r is the distance to the center of a particular zone from the center of the mirror, and R is the radius of curvature of the spherical mirror prior to figuring it as parabolic. This formula is based on the assumption that the knife edge and light source are kept in the same plane during the test. The variable r, or the spacing of the zones, is proportional to the mirror size and obtained by multiplying the full radius of the mirror by a series of decimal numbers which can be thought of as percentages along the mirror's radius at which we can expect the mid-point of a new zone or the ratio between the full flat radius of the mirror and the radius of a smaller concentric circle which represents the mid-point of a zone at all degrees of rotation, specifically $(0.316, 0.548, 0.707, 0.837, 0.945)$. [2]

Considering the parabolic shape of the reflecting surface as approximated by spheres, the ideal figure can be described in terms of the rate of change of the radius of curvature as we expect points at which the surface of a sphere of a particular radius, imagined relative to the figured reflecting surface nested in the same manner as that which describes center of curvature of a spherical mirror, will be tangent to the ideal figure at particular points along the figured surface in a manner that shows the desired parabolic behavior.[3]

Expectations for brightness can also be inferred through considering the relationship between the change in optical path length of a ray hitting the center and inner or outer portions of each zone and the half width of that zone. When creating a mask to help interpret the test, one can use this mathematical relationship to help decide the appropriate shape and area of each zone and exclude certain regions where particular terms become large and can create confusion in interpretation or irrelevant results.

5. APPARATUS

A large portion of this project was devoted to the design and construction of two pieces of apparatus; a mirror stand designed to support either a 6" or an 8" mirror, as well as a linear stage and mount for the light source and knife edge components. Both components required some adjustment, and some restriction of movement. The mirror stand was designed to allow the mirror to be adjusted both in yaw and in pitch (see figure 2), while the linear stage assembly was designed to be limited to translation forward and backward.

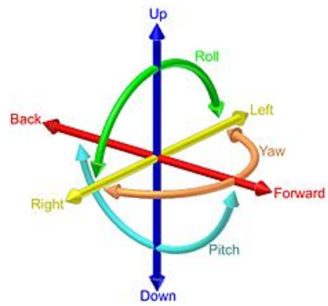

FIG. 2: 6 Degrees of Freedom for a Rigid Body

The plans for the mirror mount were taken from the Stellafane [3] website, and modified to accommodate only 6" and 8" mirrors. It was constructed out of plywood and assorted hardware (see figures 3, 6, 7, and 8).

The linear stage plans rely heavily on kinematic design theory [4]. The initial designs involved a square groove milled in an aluminum plate, upon which a second plate rested. The upper plate would slide along the groove upon two spherical feet, with a third foot acting as a tripod leg to keep the device level (see figure 9). This setup eliminates any possible rotation, and confines the plate to only one translational direction. The stage was outfitted with a threaded feed and dial indicator gauge allowing for precise, repeatable positioning of the knife edge. The preliminary plans were heavily modified in their implementation, although the main focus remains the same.

FIG. 3: Mirror Stand with 6” Mirror

6. NEWTON’S RINGS

Using Newton’s rings to verify the figure of a lens is a good representation of the level of simple precision we hope to bring to the similar problem of determining the shape of a reflecting surface. As part of this process we assembled optics apparatus in accordance with the Lambda Optics manual (see figure 4) in order to measure the radius of curvature of an unknown lens. For this experiment, a lens is touched to an optical flat and light is introduced to the system. The fringe spacing in the resultant interference pattern is related to the figuring of the surface by

$$R = \frac{r_m^2 - r_n^2}{(m-n)\lambda}, [5]$$

where R is the radius of curvature of the lens, $\lambda = (589.3 \pm 0.3)$ nm is the wavelength of sodium light, r_m and r_n are the radii of the m^{th} and n^{th} dark fringes, respectively. A visual scale inside of the apparatus helped to maintain consistency as a micrometer was used to determine spacing between fringes (see figure 5). We recorded the radius of each fringe up to m=15, and based on averaging the given value of R for 7 trials, found an experimental value of R= (0.871 ± 0.03) m. The value for R given by the LEOK-31 manual is R= 0.8685m. These results fall well within the uncertainty, confirming our hypothesis that we could determine the radius of curvature of a lens from an analysis of Newton’s Rings. The accuracy of this experiment, coupled with the simplicity and ability to be completed in an afternoon, exemplify the goal we hope to achieve with the Foucault test.

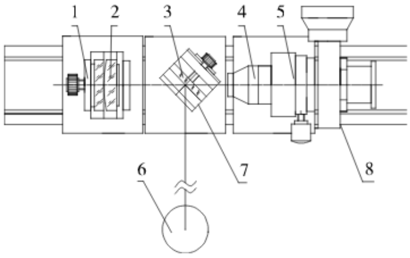

FIG. 4: Lambda Optics diagram for the assembly of equipment in the Newton’s Rings experiment

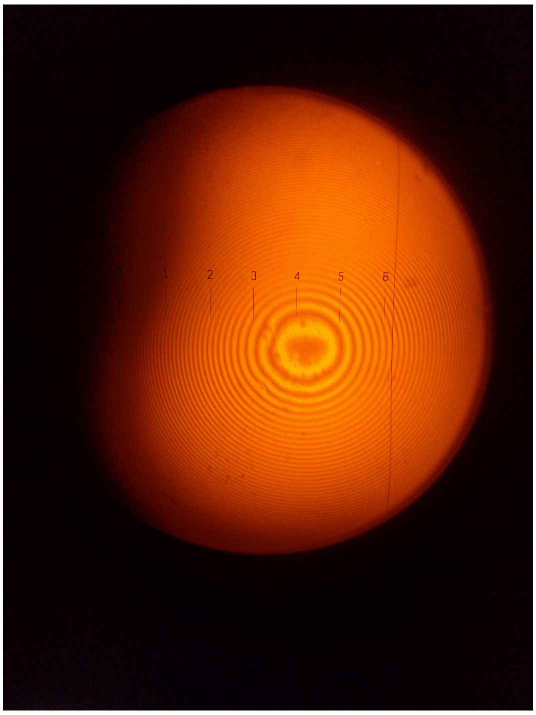

FIG. 5: Fringe Pattern in Newton’s Rings

7. CONCLUSIONS AND FUTURE WORK

The execution of a Foucault test (with the completion of both the mirror stand and the knife edge apparatus, along with the theoretical understanding of the Foucault test), will allow for straight forward data collection. A Foucault test will be easily assembled, and has many possible future applications. It can be used as an experiment for undergraduate lab students, allowing for accurate measurements of the curvature of a mirror’s surface. It can be used as a quick demonstration in an optics course, and allow students to see thermal distortion of the wave front due to body heat. Lastly, it provides a tool for students wishing to grind and test their own telescope mirror.

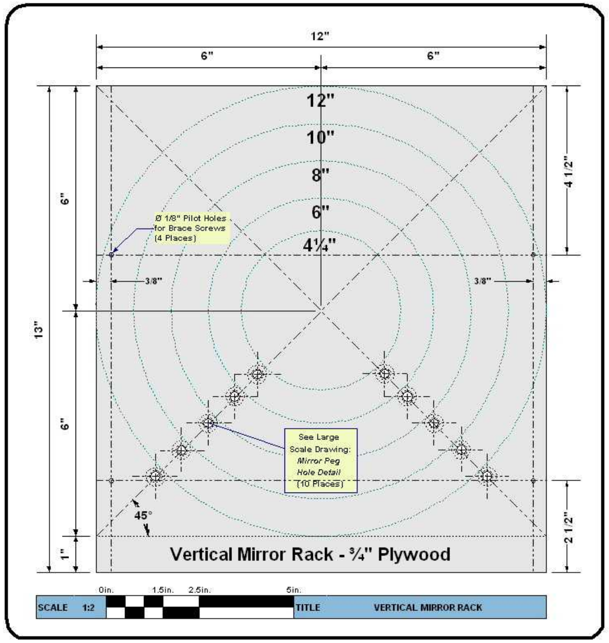

FIG. 6: Mirror Plans 1

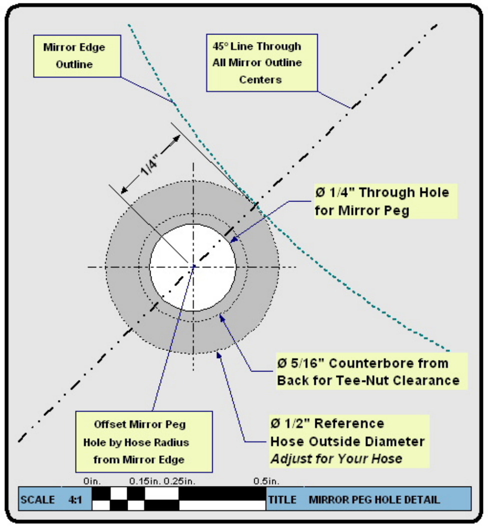

FIG. 7: Mirror Plans 2

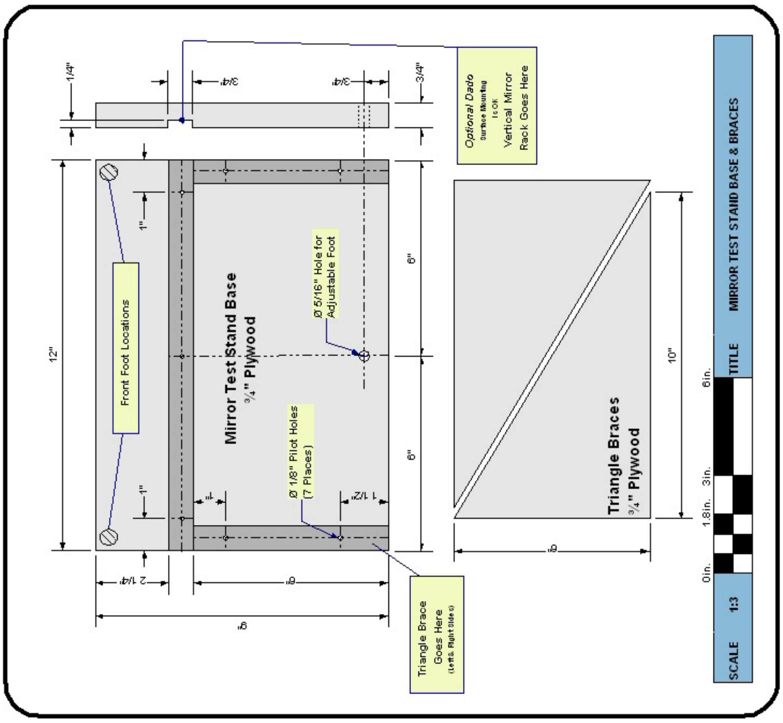

FIG. 8: Mirror Plans 3

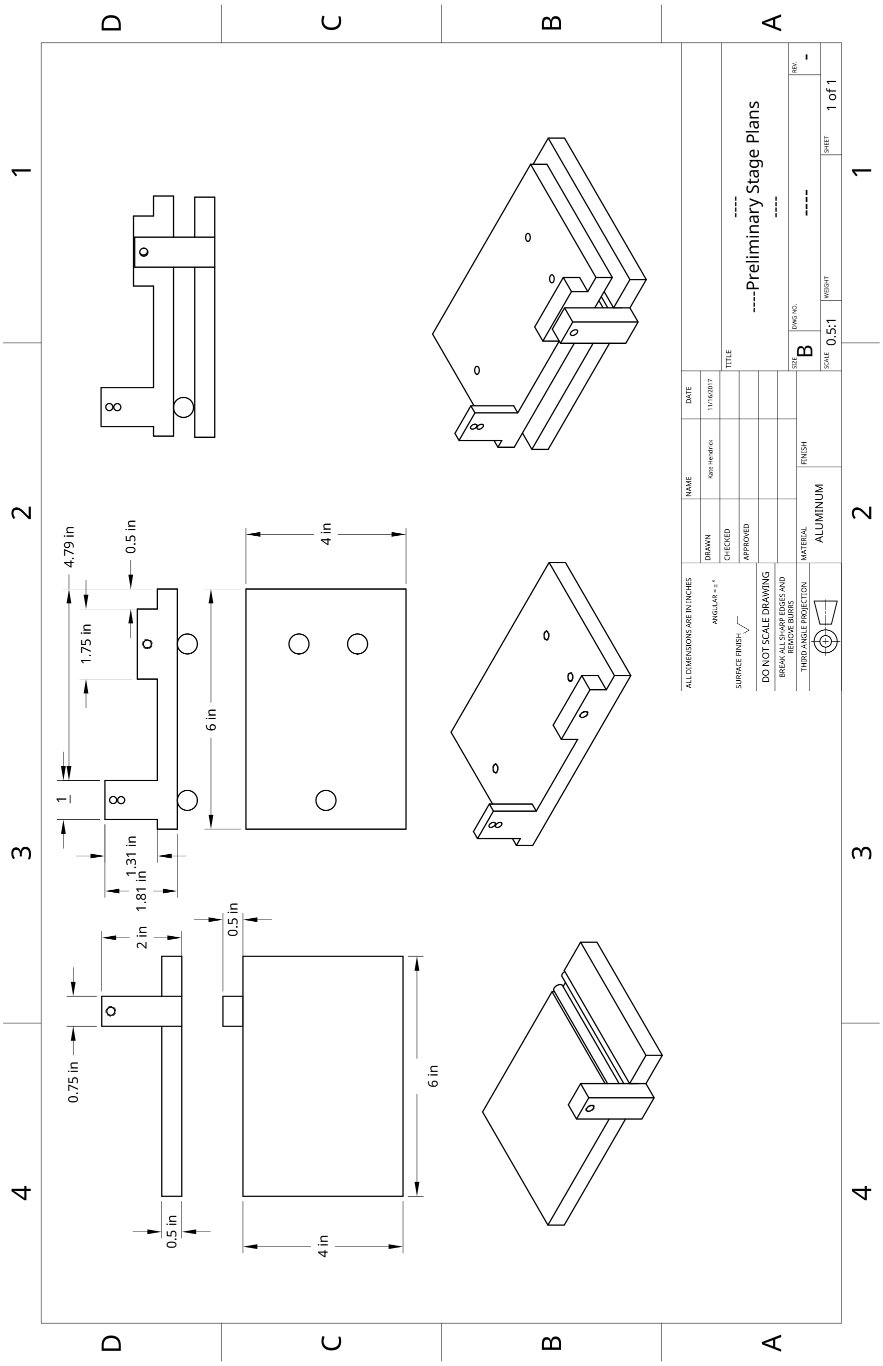

FIG. 9: Preliminary Stage Plans

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Acknowledgments

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