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Reinforcement Learning

Lecture 2 - Markov Decision Processes

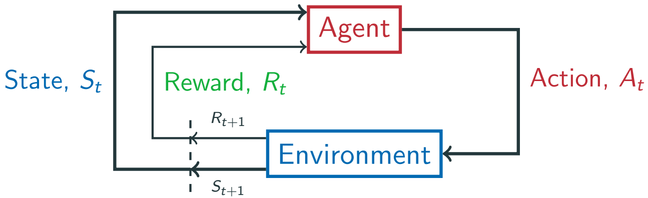
Per Mattsson

2022

Department of Information Technology

Repetition

Repetition



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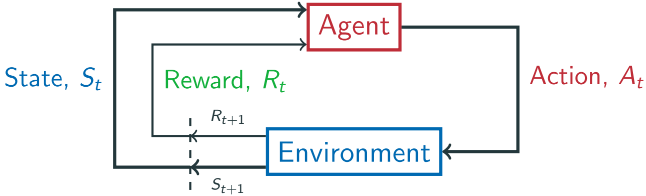
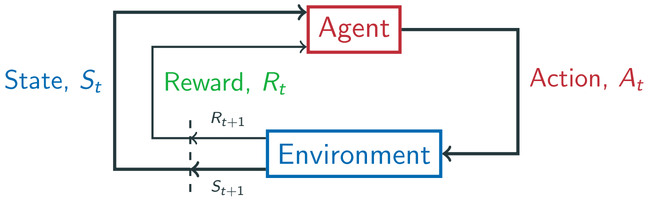


- **Markov property:** State contains all information that is useful to predict future:

$$p(S_{t+1}|S_t, A_t, S_{t-1}, A_{t-1}, \dots, S_0, A_0) = p(S_{t+1}|S_t, A_t).$$



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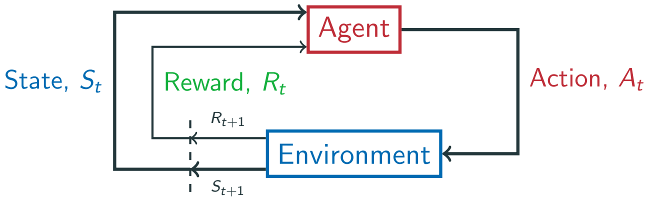

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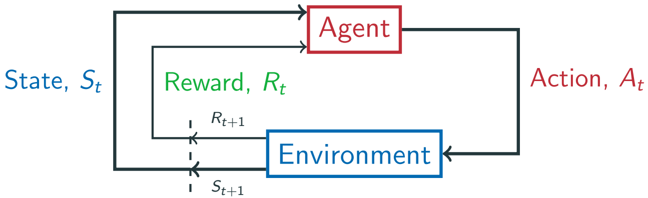

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- **Control:** Find the policy that maximize the cumulative future reward.

Markov Decision Process (MDP)

(Finite) Markov Decision Processes (MDP)

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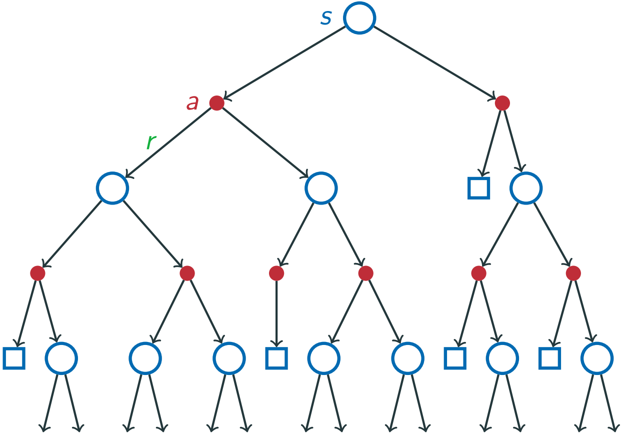
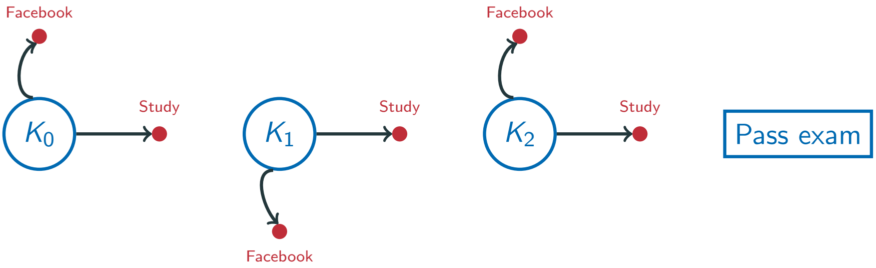
Markov Decision Process Tree



Example: Study or Facebook?



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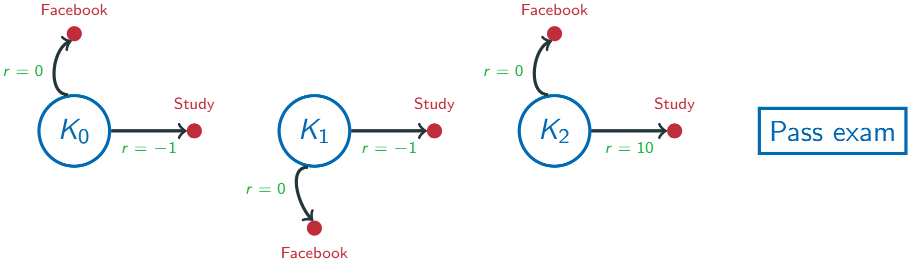

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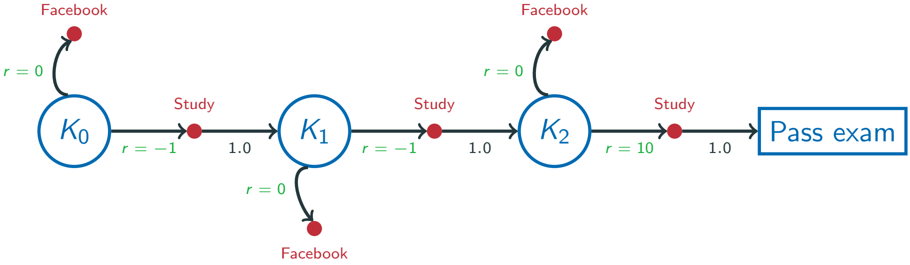

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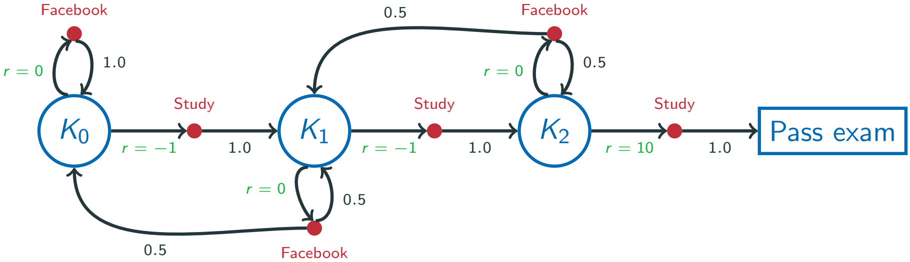

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Continuing tasks:

- Often not a clear way to divide up the task into independent episodes.
- **Example:** Keep balancing the pendulum. No state where the task is done.
- We have to take into account infinitely many future rewards.

The return

- In a given state we want to maximize future rewards $R_{t+1}, R_{t+2}, \ldots$

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The discounted return

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}, \text{ where } 0 < \gamma \leq 1.$$

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The discount rate γ:

- If $\gamma < 1$, we put less value on future rewards.
- If $\gamma < 1$, G_t will be finite as long as R_k are bounded!
- It is sometimes possible to use undiscounted returns ($\gamma = 1$), e.g., if the task always ends after a finite number of steps.

Value Functions

The state-value function

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The state-value function

The *state-value function* $v_\pi(s)$ of an MDP is the expected return starting from state s, and then following policy π:

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- **Prediction:** Compute $v_\pi(s)$.

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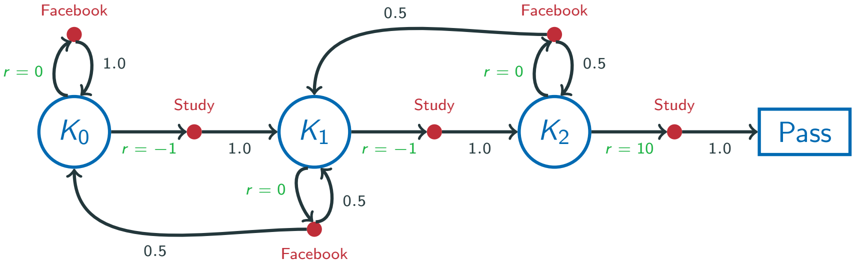
Discount: $\gamma = 0.9$.

Policy: $\pi(a|s) = 0.5$ for all a and s.



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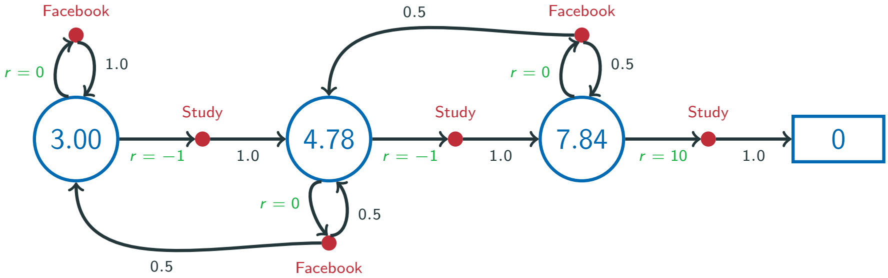
$v_\pi(s)$



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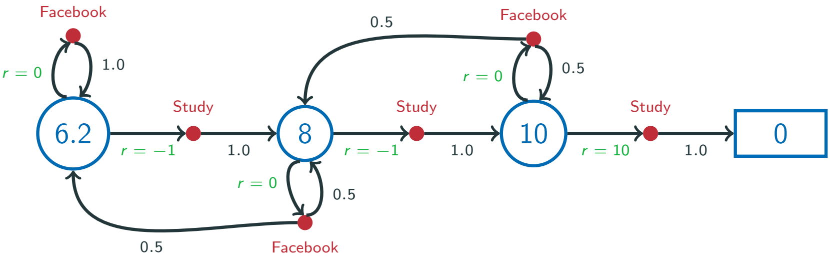
Discount: $\gamma = 0.9$.

Policy: Always choose study.



The action-value function

- Another important value function is the *action-value function*.

The action-value function

The *action-value function* $q_\pi(s, a)$ is the expected return starting from s, taking action a, and then following a policy π

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t | S_t = s, A_t = a].$$

- Often called the Q-function.

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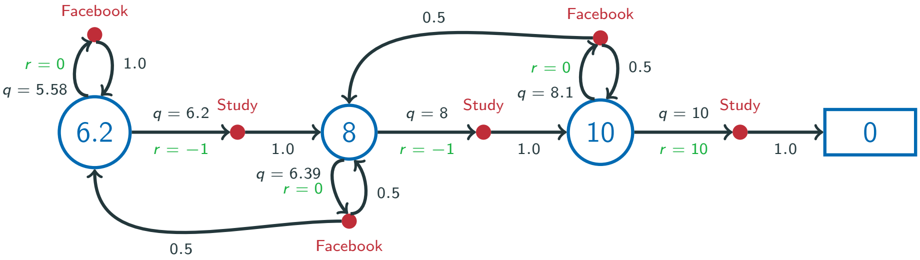
Action-values, q.



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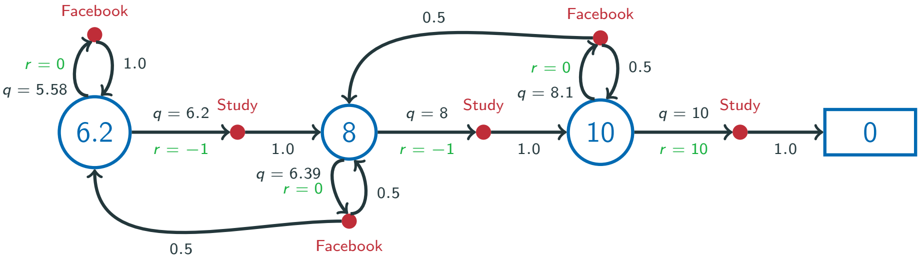

"If I just this one time choose Facebook, and after that follow the policy (always Study), what will my expected discounted return be?"

Bellman equations

- Return: Note that

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots =$$

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- "The value of s is the expected immediate reward plus the discounted expected value of the next state".
- In the same way

$$q_\pi(s, a) = \mathbb{E}_\pi[R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) | S_t = s, A_t = a].$$

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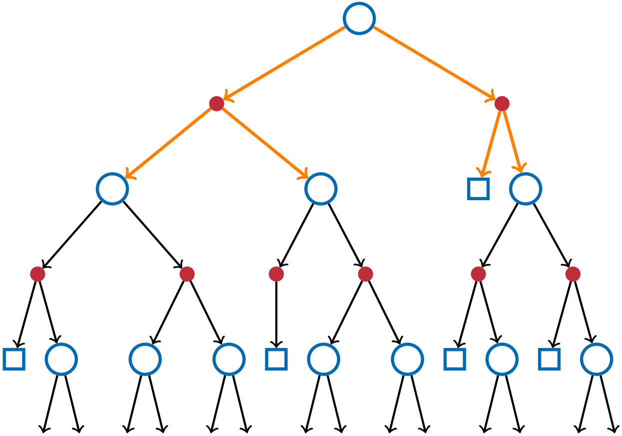
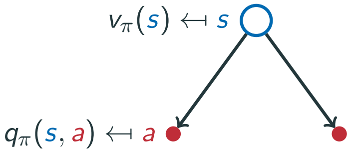
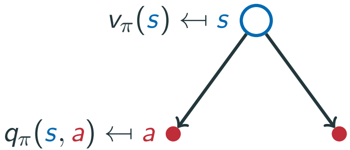




- The state-value of s is the expected action-value:

$$v_\pi(s) = \sum_a \pi(a|s) q_\pi(s, a).$$

The expectation



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- For a deterministic policy $a = \pi(s)$ we get $v_\pi(s) = q_\pi(s, \pi(s))$.

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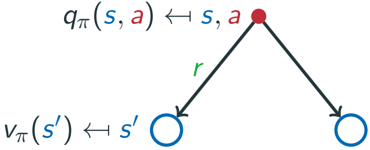

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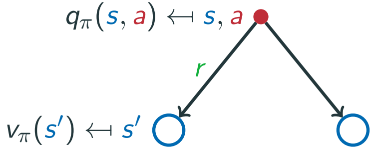

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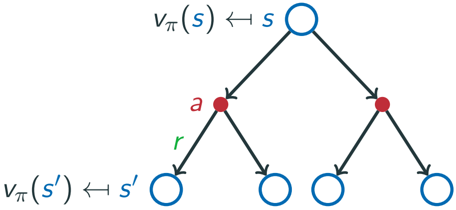
The Bellman equation for v_π



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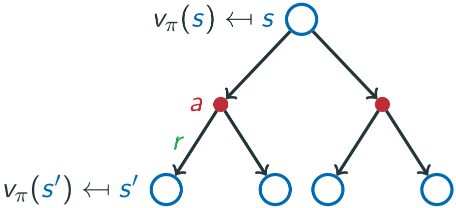

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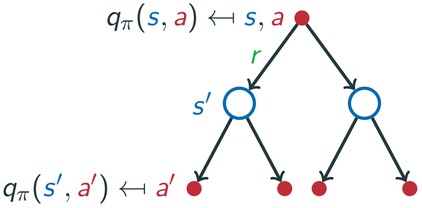
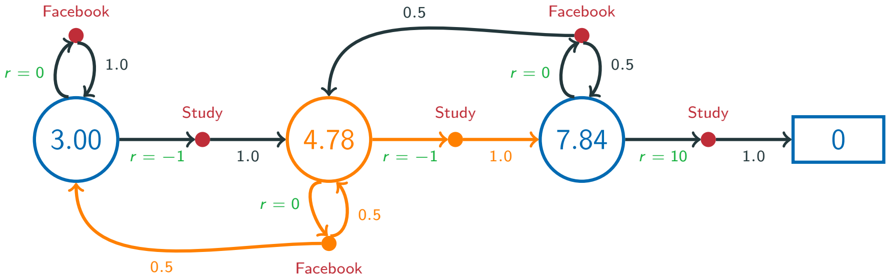
The Bellman equation for q_π



$$q_\pi(s, a) = \sum_{r, s'} p(s', r | s, a) \left[r + \gamma \sum_{a'} \pi(a' | s') q_\pi(s', a') \right]$$

Discount: $\gamma = 0.9$.

Policy: $\pi(a|s) = 0.5$ for all a and s.



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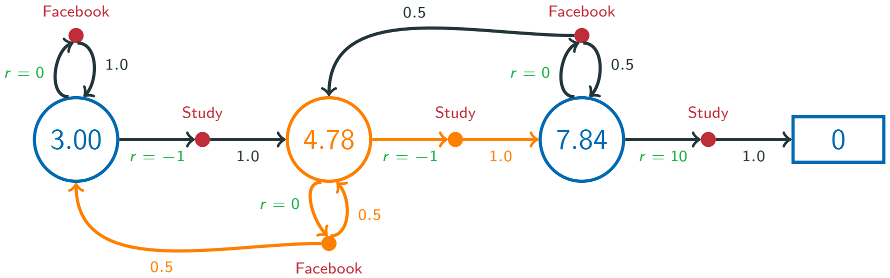

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$$4.78 = \underbrace{0.5}_{\pi(\text{facebook}|s)} \times \underbrace{\gamma[0.5 \times 3.00 + 0.5 \times 4.78]}_{q_\pi(s,\text{facebook})} + \underbrace{0.5}_{\pi(\text{study}|s)} \times \underbrace{[-1 + \gamma \times 7.84]}_{q_\pi(s,\text{study})}$$

Solving the Bellman equation

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- A system of linear equations in $v_\pi(s)$.

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- One equation for each $s \in \mathcal{S}$.

Solving the Bellman equation

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- If $p(s', r|s, a)$ is not known, we have to learn $v_\pi(s)$ from experience (Lecture 4).
- If $\mathcal{S}$ is infinite, we can't compute the value for each state individually, and instead have find some function $\hat{v}(s, \boldsymbol{w}) \approx v_\pi(s)$. (Second part of course)

Optimal Value Functions

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- **Optimal state-value function:**

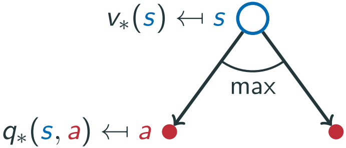
$$v_*(s) = \max_\pi v_\pi(s), \quad \text{for all } s \in \mathcal{S}.$$

- **Optimal action-value function:**

$$q_*(s,a) = \max_\pi q_\pi(s,a), \quad \text{for all } s \in \mathcal{S} \text{ and } a \in \mathcal{A}.$$

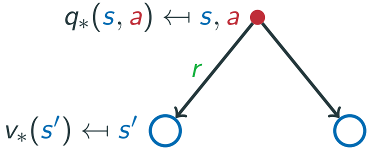
Bellman equation for v_*

The optimal $v_*(s)$ should be the maximum of $q_*(s, a)$.



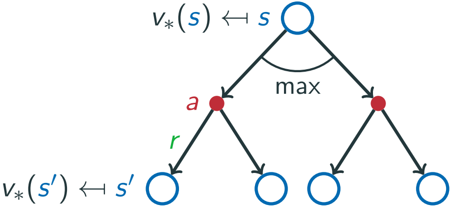
$$v_*(s) = \max_a q_*(s, a).$$

Bellman equation for q_*

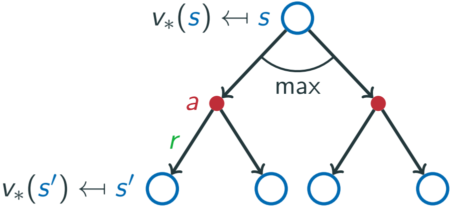


$$q_*(s, a) = \sum_{r, s'} p(s', r | s, a)(r + \gamma v_*(s')).$$

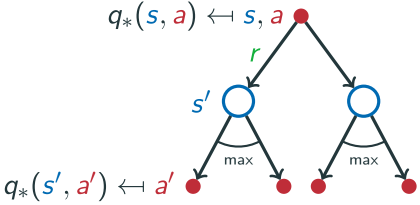
The Bellman equation for v_*



$$v_*(s) = \max_a \sum_{r,s'} p(s', r | s, a)[r + \gamma v_*(s')]$$

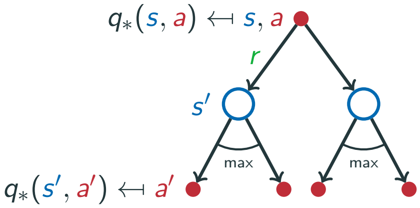


$$
\begin{aligned}
v_*(s) &= \max_a \sum_{r,s'} p(s', r|s, a)[r + \gamma v_*(s')] \\
&= \max_a \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1})|S_t = s, A_t = a]
\end{aligned}
$$



$$q_*(s, a) = \sum_{r, s'} p(s', r | s, a) \left[r + \gamma \max_{a'} q_*(s', a') \right]$$

The Bellman equation for q_*



$$q_*(s,a) = \sum_{r,s'} p(s', r | s, a) \left[r + \gamma \max_{a'} q_*(s', a') \right]$$

$$= \mathbb{E}[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a') | S_t = s, A_t = a]$$

Solving the Bellman optimality equation

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- A system of *non*-linear equations.

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- One equation for each s.

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- One equation for each s.
- In general no closed-form solution!
- But there are iterative solution methods (Lecture 3).

Optimal Policy

Optimal policy

Partial ordering over policies:

$$\pi \geq \pi' \text{ if } v_\pi(s) \geq v_{\pi'}(s), \text{ for all } s.$$

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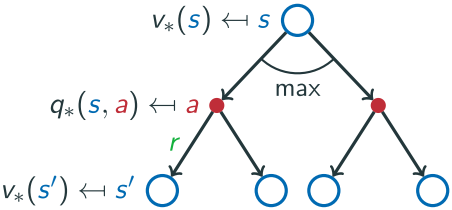
- All optimal policies achieve the optimal action-value function $q_{\pi_*}(s, a) = q_*(s, a)$.

Optimal policy



What to do in state s?

1. Choose an a that maximize the optimal action-value $q_*(s, a)$.
2. Then use an optimal policy from s'.

Finding an optimal policy

- **Control:** Find optimal policy.

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- The policy

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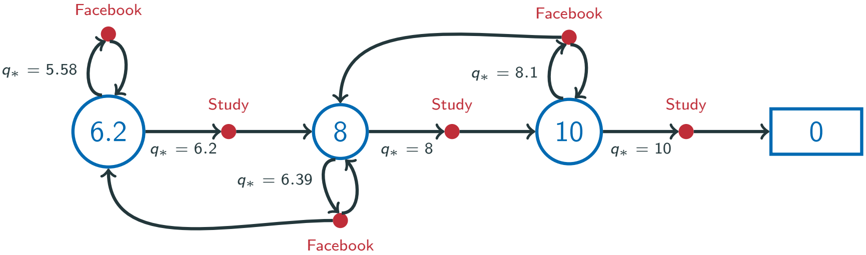
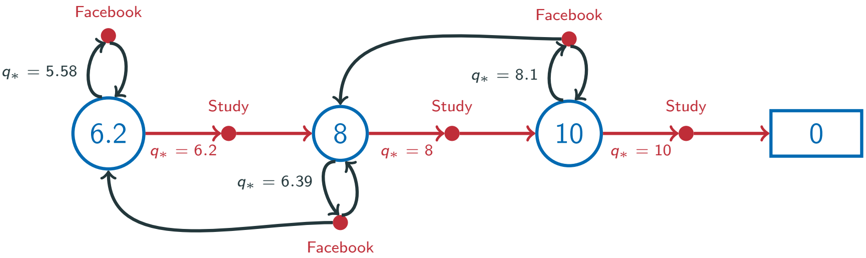
$$\pi_*(s) = \arg\max_a q_*(s, a) = \arg\max_a \sum_{r,s'} p(s', r|s, a)[r + \gamma v_*(s')]$$

  is optimal.
- **Note:** If we know $q_*(s, a)$ we don't need the dynamics to find an optimal policy!

Example: Optimal state-value and action-value

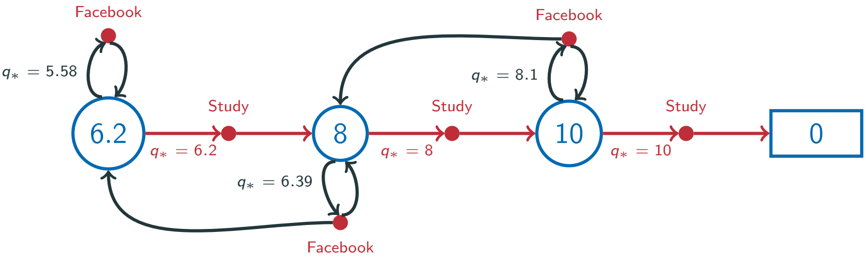


Example: Optimal policy



Optimal policy: Always study!

Example: Optimal policy



Optimal policy: Always study!

...according to this MDP. In real life it may be good to take a break once in a while.

Summary

- Markov Decision Processes
- Discounted return
- Value functions
- Bellman equations

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Next:

- Find value functions and optimal policy if $p(s', r|s, a)$ is known. (Lecture 3)
- What if $p(s', r|s, a)$ is not known? (Lecture 4-5)
- Tinkering Notebook 2: You can do Section 1-5 now.
- Assignment 1: You can look at Problem 1 – Problem 3a.