DIRECT COMPARISON TESTS

Improper Integrals

Suppose that $f(x)$ and $g(x)$ are positive, continuous functions where $f(x) \geq g(x) \geq 0$ for $x \geq a$.

1. If $\int_a^\infty f(x)dx$ is **convergent**, then $\int_a^\infty g(x)dx$ is also **convergent**.
2. If $\int_a^\infty g(x)dx$ is **divergent**, then $\int_a^\infty f(x)dx$ is also **divergent**.

Series

Suppose that a_n and b_n are sequences where $b_n \geq a_n \geq 0$.

1. If the series $\sum_{n=1}^\infty b_n$ is **convergent**, then $\sum_{n=1}^\infty a_n$ is also **convergent**.
2. If the series $\sum_{n=1}^\infty a_n$ is **divergent**, then $\sum_{n=1}^\infty b_n$ is also **divergent**.

P-TESTS

Suppose that $f(x)$ is a positive function and that $f(x) = \frac{1}{x^P}$

The improper integral $\int_a^\infty \frac{1}{x^P}dx$ is **convergent** if $P > 1$ and **divergent** if $P \leq 1$

The integral $\int_0^a \frac{1}{x^P}dx$ is **convergent** if $P < 1$ and **divergent** if $P \geq 1$

The series $\sum_{n=1}^\infty \frac{1}{n^P}$ is **convergent** if $P > 1$ and **divergent** if $P \leq 1$

GEOMETRIC SERIES TEST

The series $\sum_{n=1}^\infty a(r^n)$ is **convergent** if $|r| < 1$ and **divergent** if $|r| \geq 1$

If $|r| < 1$, the series will converge to $\frac{a_1}{1-r}$

INTEGRAL TEST

Suppose that $f(x)$ is a positive, continuous, and decreasing function and that $f(n) = a_n$.

The series $\sum_{n=1}^\infty a_n$ and $\int_1^\infty f(x)dx$ will either **both converge** or **both diverge.**

Nth TERM TEST FOR DIVERGENCE

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^\infty a_n$ is **divergent**.

TELESCOPING SERIES TEST

A series in the form

$$\sum_{n=1}^{\infty}(\frac{1}{n} - \frac{1}{n+1})$$

can be written as

$$(\frac{1}{1} - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + (\frac{1}{5} - \frac{1}{6}) + ...(0)$$

which simplifies to

$$1 - 0 = 1$$

LIMIT COMPARISON TEST
Suppose that $a_n > 0$ and $b_n > 0$. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

where L is a positive finite value, the series

$$\sum_{n=1}^{\infty} a_n \text{ and } \sum_{n=1}^{\infty} b_n$$

either both converge or both diverge.

ALTERNATING SERIES TEST
Let $a_n > 0$. The series

$$\sum_{n=1}^{\infty} (-1)^n a_n \text{ and } \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converge if $\lim_{n\to\infty} a_n = 0$ and $a_{n+1} \leq a_n$ for all n.

RATIO TEST
1. $\sum a_n$ converges absolutely if $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| < 1$
2. $\sum a_n$ diverges if $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| > 1$
3. The ratio test is inconclusive if $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 1$

ROOT TEST
1. $\sum a_n$ converges absolutely if $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$
2. $\sum a_n$ diverges if $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$
3. The root test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$