Assigned: 27Jan2019 Due: 21Feb2019

Project 01

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We recall some useful MATLAB 'help' commands. Any string that you wish to get information on can be looked up using the command `lookfor`. For example, try `lookfor record`. You can then zoom into a command using help. For example, try `help audiorecorder`.

Through out this project you will be defining vectors with shifted sincs. You might encounter the divide by 0 error, or MATLAB might store `NaN` whenever this occurs. You might want to take care of this if it happens. Using the inbuilt sinc function might circumvent this problem. **For all the problems, your reconstruction ideal LPF will have cut-off freq.** $\omega_c = \frac{\omega_s}{2} = \frac{\pi}{T_s}$ **where** T_s **is the sampling interval.**

1. In this problem, you are provided samples from a signal. Your goal is to reconstruct the original signal and identify the same. Download the `Project1Problem4.mat` from the Projects folder into your current work folder.[1] Execute `load Project1Problem4.mat` in MATLAB. You will have five vectors in your workspace : `CTTimeIndex`, `DTTimeIndex`, `Signa11Samples`, `Signal2Samples`, `Signal3Samples`. Except for `CTTimeIndex`, each of these vectors is of length 501. `CTTimeIndex` is of length 10001. `Signa11Samples` are samples from $x_1(t)$ obtained at $nT_s : n = -250, -249, \cdots, 250$. with a sampling interval of $T_s = \frac{1}{500}$. Use an ideal low pass filter with cut-off frequency $\omega_c = \frac{\omega_s}{2} = 500\pi$ to reconstruct $x_1(t)$. You should provide a vector `OriginalSignal1` with 10001 values, where `OriginalSignal1`$(i) = x_1(-0.5 + \frac{(i-1)}{10000})$ for all $i = 1, 2, \cdots, 10001$. You will then characterize $x_1(t)$ analytically, i.e., you will provide a formula for $x_1(t)$. Execute `plot(CTTimeIndex,OriginalSignal1)` and plot the reconstructed signal. Repeat all these steps for `Signal2Samples` and `Signal3Samples`.

2. In this problem you will verify sampling theorem and reconstruct band-limited signals from their samples using the ideal low-pass interpolation.[2] Sampling Thm. states that a CT signal can be reconstructed perfectly from its samples if we sample at greater than the Nyquist rate. Since you can only operate with DT signals in MATLAB, we refer to a CT signal as one that is sampled at a very high rate, and a sampled signal as one that is sampled at slightly above the Nyquist rate. To distinguish, we refer to sampling interval in the former as step-size.

   Choose a signal band-limited to $W_1 = 400\pi$ rad/sec. For example, you can choose $x_1(t) = \sin(400\pi t)$. Consider this signal in the time-interval $[-0.5, 0.5]$. To store this CT signal, choose a very small step-size, say $T_c = \frac{1}{10000}$ and define `OriginalSignal =` $\sin(400 * \texttt{pi} * [-0.5 : \frac{1}{10000} : 0.5])$. This is our CT signal. We now sample this 'CT signal' with a sampling interval of $T_s = \frac{1}{500}$ sec. We therefore have the sampled signal to be `SampledSignal =` $\sin(400 * \texttt{pi} * [-0.5 : \frac{1}{500} : 0.5])$. Now use the `SampledSignal` and Eqn (7.11) to reconstruct the original CT signal. Choose $\omega_c = \frac{\omega_s}{2} = \frac{\pi}{T_s}$ where T_s is the sampling interval. Your `ReconstructedSignal` will be similar to your `OriginalSignal`, in that it will be a vector with 10001 values, will be defined for the interval $[-0.5, 0.5]$ and values will be at a step-size of $T_c = \frac{1}{10000}$.

   On a single time-domain plot, plot a small segment of both the original signal and the reconstructed signal. Use different colors for both plots and in particular, plot the latter with a dotted line to enable

[1] Your current work folder is what MATLAB returns if you enter `pwd`.

[2] This is referred to as band-limited interpolation in the SAndS text in page 523, following Eqn 7.11

visibility of both lines. Finally, calculate the energy in `OriginalSignal-ReconstructedSignal`.

Choose two other band-limited signals $x_2(t)$ and $x_3(t)$. Identify their band-limits W_2 and W_3. Sample these signals at above the Nyquist rate. Reconstruct the signals and repeat the above questions for the same.[3]

3. We now study the effect of undersampling or aliasing. Choose a sampling rate T_s. Choose a signal $x(t)$ that is *not* bandlimited to $\frac{\pi}{T_s} = \frac{\omega_s}{2}$. Use Eqn (7.11) with $T = T_s$ and $\omega_c = \frac{\omega_s}{2}$ to reconstruct your signal from the samples. On a single time-domain plot, plot a small segment of both the original signal and the reconstructed signal. Use different colors for both plots. Finally, calculate the energy in `OriginalSignal-ReconstructedSignal`. Repeat this problem for two other choices of signals that do not meet the conditions in the Sampling Theorem.

4. In this problem your goal is to produce a figure analogous to Fig. 7.10(c) of SAndS book.[4]. See LowPassInterpolation.pdf (or LowPassInterpolation2.pdf) in the Projects folder generated through Matlab. Consider the signal $x(t) = \sin(40\pi t) - \sin(20\pi t) + \sin(30\pi t)$ in the interval $[-0.5, 0.5]$. Sample this signal with a sampling interval of $\frac{1}{500}$ sec. Reconstruct using an ideal low pass filter with cut-off frequency $\frac{\omega_s}{2} = 500\pi$. On the same time domain plot of width approximately 35 milli-sec, plot the original signal, the corresponding shifted and scaled sincs, i.e., the corresponding individual terms in Eqn. 7.11 (SAndS textbook)[5] to obtain a plot similar to LowPassInterpolation.pdf (or LowPassInterpolation2.pdf).

[3] It might be easier for you to sample all the signals at the same sampling rate and use the same shifted sincs to reconstruct the signals. For this, just choose the three signals for which the chosen sampling rate is above the corresponding Nyquist rate.

[4] Fig. 4.8c from DTSP book

[5] One of these terms is specified as the un-numbered equation on Pg 165 of 3rd edition of the DTSP book.