Assigned: 29Jan2019 Due: 05Feb2019, **4:30 PM**

Homework 03

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Unless specified otherwise, $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$ is the impulse train. If T_s is not mentioned, it is generic and you have to obtain your answers for a generic T_s.

1. (Among Problems 1 and 2, solve the one you did *not* solve for HW02). Let $x_s(t) = x(t)s(t)$ where $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$. Let $x_r(t) = (x_s * r)(t)$ denote the convolution of $x_s(t)$ and $r(t)$, where

$$r(t) = \begin{cases} 1 & 0 \leq t \leq \frac{T_s}{10} \\ 0 & \text{otherwise.} \end{cases}.$$

   Specify $X_r(j\omega)$ - Fourier Transform of $x_r(t)$. Can $x(t)$ be recovered from $x_r(t)$ if you are allowed to use any LTI system (causal, non-causal or otherwise)? If yes, under what conditions can $x(t)$ be recovered from $x_r(t)$? Specify the LTI system that you would need in this case.

2. (Among Problems 1 and 2, solve the one you did *not* solve for HW02). Let $x(t) = \frac{\sin(1000\pi t)}{\pi t}$ and $h(t) = \sin(3000\pi t)$. Let $y(t) = x(t)h(t)$. Find the Nyquist rate of $y(t)$.

3. (Among Problems 3 and 4, solve the one you did *not* solve for HW02). Let $x(t) = \left(\frac{\sin 200\pi t}{\pi t}\right)^2$. Let $x_s(t) = x(t)s(t)$ where $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$ What is the maximum radial frequency ω_0 for which we have

$$X_s(j\omega) = \frac{1}{T_s} X(j\omega) \quad |\omega| \leq \omega_0$$

   if (i) $T_s = \frac{1}{500}$ (ii) $T_s = \frac{1}{300}$ (iii) $T_s = \frac{1}{1000}$. In other words, this question is asking what is the band $[-\omega_0, \omega_0]$ is which the FTs $X_s(j\omega)$ and $X(j\omega)$ are equal upto a scalar multiplication? Answer this same question considering $x(t) = \frac{\sin 200\pi t}{\pi t}$.

4. (Among Problems 3 and 4, solve the one you did *not* solve for HW02). $x_1(t)$ is band-limited signal to ω_{M1} rad/sec and $x_2(t)$ is band-limited signal to ω_{M2} rad/sec. Find the smallest sampling frequency for $y(t) = x_1(t)x_2(t)$ that will guarantee reconstruction of $y(t)$ from its samples, no matter what the signals $x_1(t), x_2(t)$ are (so long as they are bandlimited as specified). Repeat for $z(t) = (x_1 * x_2)(t)$.

5. Let sampling interval $T_s = \frac{1}{500}$ and let $b_n(t) = \frac{\sin[\pi(t-nT_s)/T_s]}{[\pi(t-nT_s)/T_s]}$. Your goal is to store $b_n(t)$ and plot $b_n(t)$ using MATLAB for a few selected values of n. Since we cannot store a CT signal in MATLAB, we shall store values of $b_n(t)$ with a very small step-size, say $T_c = \frac{1}{20000}$ or $T_c = \frac{1}{10000}$. In addition, since we cannot store an infinite large vector, we store values of $b_n(t)$ with step-size T_c in the interval $[-\tau, \tau]$ for some choice of τ. Choose your τ and three values of n, with at least one value of $n < 0$ and at least one value of $n > 0$. Your choice must also be such that $nT_s \in [-\tau, \tau]$ for all your choices of n and your choice of τ. Now create a vector `b_n` that stores the values of $b_n(t)$ in the interval $[-\tau, \tau]$ with step-size T_c. Publish your code through Matlab and upload on canvas.

6. Solve any THREE of the parts (a)-(e). A CT signal $x(t)$ is converted to a DT signal through the two operations described in class. Firstly, $x_s(t) = x(t)s(t)$ is obtained by multiplying with a periodic

impulse train $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. Then a DT signal $x_d(n) = x_s(nT_s)$ is defined. Plot the magnitude of the CTFT $X_s(j\omega)$ and DTFT $X_d(e^{j\omega})$ if (a) $T_s = \frac{1}{500}$ sec and $x_c(t) = \sin(300\pi t)$, (b) $T_s = \frac{1}{250}$ sec and $x_c(t) = \cos(300\pi t)$, (c) $\frac{\sin(200\pi t)}{\pi t}$ and $T_s = \frac{1}{400}$ sec, (d) $\left(\frac{\sin(200\pi t)}{\pi t}\right)^2$ and $T_s = \frac{1}{1000}$ sec, (e) $\left(\frac{\sin(200\pi t)}{\pi t}\right)^2$ and $T_s = \frac{1}{300}$ sec.

7. Consider a DT signal $y_d(n)$. A CT signal $y_s(t) = \sum_{n=-\infty}^{\infty} y_d(n)\delta(t - nT_s)$ is defined. $y_s(t)$ is passed through an ideal low pass (reconstruction) filter with cut-off frequency ω_c rad/sec to obtain a CT signal $y(t)$. Specifically, the FT of this ideal low-pass filter is given by

$$H_{LP}(j\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Solve any THREE parts. Specify $y(t)$ if (i) $y_d(n) = \sin(\frac{\pi n}{4})$, $T_s = \frac{1}{500}$ sec, $\omega_c = \frac{\pi}{T_s}$, (ii) $y_d(n) = \sin(\frac{\pi n}{4})$, $T_s = \frac{1}{300}$ sec, $\omega_c = \frac{\pi}{T_s}$, (iii) $y_d(n) = \frac{\sin(\frac{\pi n}{4})}{\pi n}$, $T_s = \frac{1}{500}$ and $\omega_c = \frac{\pi}{T_s}$ rad/sec, (iv) $y_d(n) = \frac{\sin(\frac{\pi n}{4})}{\pi n}$, $T_s = \frac{1}{500}$ and $\omega_c = 250\pi$ rad/sec, (v) $y_d(n) = \frac{\sin(\frac{\pi n}{4})}{\pi n}$, $T_s = \frac{1}{500}$ and $\omega_c = 75\pi$ rad/sec.

8. A CT signal $x(t) = \frac{\sin(400\pi t)}{\pi t}$ is converted to a DT signal through the two operations described in class. Firstly, $x_s(t) = x(t)s(t)$ is obtained by multiplying with a periodic impulse train $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ where $T_s = \frac{1}{500}$ sec. Then, a DT signal $x_d(n) = x_s(nT_s)$ is defined.[1] (i) Specify and plot the DTFT $X_d(e^{j\omega})$ of $x_d(n)$. Next, $x_d(n)$ is passed through an LTI system with impulse response $h_d(n) = \frac{\sin(n\frac{\pi}{3})}{\pi n}$ and let $y_d(n)$ denote the output. (ii) Plot and specify the DTFT $Y_d(e^{j\omega})$ of $y_d(n)$. $y_d(n)$ is converted to a CT signal through the two operations described in class. Specifically, we let $y_s(t) = \sum_{n=\infty}^{\infty} y_d(n)\delta(t - nT_s)$. Next, we pass $y_s(t)$ through an ideal low pass filter with cut-off frequency $\omega_c = \frac{\omega_s}{2} = \frac{\pi}{T_s} = 500\pi$ and gain T_s to obtain a CT signal $y_c(t)$. Specifically, the frequency response of this ideal LPF is

$$H_{LP}(j\omega) = \begin{cases} \frac{1}{500} & |\omega| \le \omega_c \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Specify the continuous time signal $y_c(t)$.[2]

9. (Will NOT be Graded) SAndS Textbook problem 7.29.

10. (Will NOT be Graded) SAndS Textbook problem 7.31.

[1] This is exactly how we defined a DT signal in Lecture 06.

[2] Here, a step by step procedure to solve this problem is provided. I **strongly encourage** you to attempt/solve this problem before you look at the hints. These hints might be of use for Probs 6,7. Step 1: Calculate all the parameters ω_s, ω_M, T_s and check if the Nyquist Sampling Theorem holds. Step 2: Plot $X_s(j\omega)$ the CTFT of $x_s(t)$. Next, use the material taught in Lect. 6 to plot and specify $X_d(e^{j\omega})$ - the DTFT of $x_d(n)$. Step 3: Using the CTFT of $h_d(n)$ and the convolution property, specify and plot the DTFT $Y_d(e^{j\omega})$ of $y_d(n)$. Step 4: Now you go back from $Y_d(e^{j\omega})$ to $Y_s(j\omega)$. (For this you reverse the procedure that you took to come from $X_s(j\omega)$ to $X_d(e^{j\omega})$). Step 5: And then you finally find $Y_c(j\omega)$ in relation to $Y_s(j\omega)$ by recognizing that $y_c(t)$ is obtained as the output of an ideal low pass filter with cut-off frequency $\omega_c = \frac{\omega_s}{2} = \frac{\pi}{T_s} = 500\pi$. You can thus characterize $Y_c(j\omega)$. Step 6 : Take the inverse CTFT to obtain $y_c(t)$.