Assigned: 22Jan2018 Due: 29Jan2019, **4:30 PM**

Homework 02

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Unless specified otherwise, $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$ is the impulse train. If T_s is not mentioned, it is generic and you have to obtain your answers for a generic T_s.

1. Find the Fourier Transform (FT) of the following signals and plot both their manitude and phase of the FTs (a) $x(t) = -7\sin(1200\pi t)$ (b) $\cos(300\pi t)$

   (c) $x(t)$ is a periodic signal with period 20 milli-seconds (ms) given by

$$x(t) = \begin{cases} 1 & 0 \le t < 5\text{ms} \\ 0 & 5 \le t < 20\text{ms} \end{cases}$$

2. Plot the two signals $s_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - \frac{k}{400})$ and $s_2 = \sum_{k=-\infty}^{\infty} \delta(t - \frac{k}{1000})$ on the same time-domain plot (with possibly different colors). Plot their FTs on the same frequency domain plot (with possibly different colors). Identify the signals and state the observations that are important from the perspective of sampling.

3. Solve ANY TWO from the first four (i) - (iv) parts and ANY THREE from last six (v) - (x) parts. Plot $x_s(t) = x(t)s(t)$ and the magnitude of its Fourier Transfrom if $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - \frac{k}{1000})$ and (i) $x(t) = \cos(300\pi t)$, (ii) $x(t) = \sin(500\pi t)$, (iii) $x(t) = \cos(1000\pi t)$, (iv) $x(t) = \sin(1000\pi t)$, (v) $\frac{\sin(200\pi t)}{\pi t}$, (vi) $x(t) = \frac{\sin(500\pi t)}{\pi t}$, (vii) $x(t) = \frac{\sin(1000\pi t)}{\pi t}$ (viii) $x(t) = \frac{\sin(1500\pi t)}{\pi t}$, (ix) $x(t) = \frac{\sin(1000\pi t)}{\pi t}$ (x) $x(t) = \frac{\sin(1500\pi t)}{\pi t}$.

4. Let $x(t)$ be bandlimited to ω_M. Let $x_s(t) = x(t)s_\mu(t)$ where $s_\mu(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s - \mu)$ and $\mu > 0$ is some constant delay. Is it possible to reconstruct $x(t)$ from $x_s(t)$. Under what conditions of T_s, μ and ω_M will one be able to reconstruct $x(t)$ from $x_s(t)$? Describe how will you recover $x(t)$. You are permitted to use ideal filters.

5. Suppose $x_\rho(t) = x(t)r_p(t)$ where $r_p(t)$ is a periodic signal of period T_s whose one period is given by

$$r_p(t) = \begin{cases} 1 & 0 \le t \le \frac{T_s}{10} \\ 0 & \frac{T_s}{10} < t < T_s \end{cases}.$$

   Specify $X_\rho(j\omega)$ in terms of $X(j\omega)$.[1] Can $x(t)$ be recovered from $x_\rho(t)$ if you are allowed to use any LTI system (causal, non-causal or otherwise)? If yes, under what conditions can $x(t)$ be recovered from $x_\rho(t)$? Specify the LTI system that you would need in this case.

6. (You may solve any one from Problems 6 and 7). Let $x_s(t) = x(t)s(t)$ where $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$.

[1] You can prove and use the fact that $X(j\omega) * \delta(\omega - \omega_s) = X(j(\omega - \omega_s))$. This lets you write $X_\rho(j\omega)$ in terms of $X(j\omega)$ without having to leave your answer in terms of a convolution.

Let $x_r(t) = (x_s * r)(t)$ denote the convolution of $x_s(t)$ and $r(t)$, where

$$r(t) = \begin{cases} 1 & 0 \leq t \leq \frac{T_s}{10} \\ 0 & \text{otherwise.} \end{cases}.$$

Specify $X_r(j\omega)$ - Fourier Transform of $x_r(t)$. Can $x(t)$ be recovered from $x_r(t)$ if you are allowed to use any LTI system (causal, non-causal or otherwise)? If yes, under what conditions can $x(t)$ be recovered from $x_r(t)$? Specify the LTI system that you would need in this case.

7. (You may solve any one from Problems 6 and 7). Let $x(t) = \frac{\sin(1000\pi t)}{\pi t}$ and $h(t) = \sin(3000\pi t)$. Let $y(t) = x(t)h(t)$. Find the Nyquist rate of $y(t)$.

8. (You may solve any one from Problems 8 and 9). Let $x(t) = \left(\frac{\sin 200\pi t}{\pi t}\right)^2$. Let $x_s(t) = x(t)s(t)$ where $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$ What is the maximum radial frequency ω_0 for which we have

$$X_s(j\omega) = \frac{1}{T_s} X(j\omega) \quad |\omega| \leq \omega_0$$

if (i) $T_s = \frac{1}{500}$ (ii) $T_s = \frac{1}{300}$ (iii) $T_s = \frac{1}{1000}$. In other words, this question is asking what is the band $[-\omega_0, \omega_0]$ is which the FTs $X_s(j\omega)$ and $X(j\omega)$ are equal upto a scalar multiplication? Answer this same question considering $x(t) = \frac{\sin 200\pi t}{\pi t}$.

9. (You may solve any one from Problems 8 and 9). $x_1(t)$ is band-limited signal to ω_{M1} rad/sec and $x_2(t)$ is band-limited signal to ω_{M2} rad/sec. Find the smallest sampling frequency for $y(t) = x_1(t)x_2(t)$ that will guarantee reconstruction of $y(t)$ from its samples, no matter what the signals $x_1(t), x_2(t)$ are (so long as they are bandlimited as specified). Repeat for $z(t) = (x_1 * x_2)(t)$.