Assigned: 15Jan2018 Due: 22Jan2019, **4:30 PM**

Homework 01

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For all questions asked in this course, unless the question is simply asking about definitions, please justify all answers. Stating the correct answer without justification will earn no credit.

1. Represent the following complex numbers in polar **and** rectangular form. If the number is stated in polar form, convert it to rectangular form and vice versa. Next, plot these on the complex plane. You may solve **any two from each column**.

   (a) $3 + 15j$,
   (b) $10 - 10j$
   (c) $-4 + 6j$
   (d) $-6 - 4j$
   (e) $(-2 - 5j)(7 + 3j)$
   (f) $\frac{(1+5j)}{(-4+3j)}$
   (g) $(3 + 7j) + \frac{(5+6j)}{(2+6j)}$
   (h) $4e^{j\frac{7\pi}{6}} + 12e^{-j\frac{7\pi}{2}}$
   (i) $17e^{j\frac{2\pi}{3}}$
   (j) $je^{j\frac{\pi}{4}}$
   (k) $\frac{(5+6j)}{(2+6j)} + 7e^{-j\frac{5\pi}{3}}$
   (k) $\left(\frac{1}{(2+6j)} + 7je^{-j\frac{5\pi}{3}}\right)^*$

2. Consider the complex plane. We let $\mathbb{C}$ denote the set of complex numbers. Identify the following regions in the complex plane. You may solve **any two from each column**.

   (a) $\mathcal{A} = \{a \in \mathbb{C} : \text{Re}\{a\} \geq -2\}$,
   (b) $\mathcal{B} = \{a \in \mathbb{C} : \text{Im}\{a\} \leq 7\}$,
   (c) $\mathbb{C} = \{a \in \mathbb{C} : |a| \leq 1\}$
   (d) $\mathcal{D} = \{a \in \mathbb{C} : |a| < 1\}$
   (e) $\mathcal{E} = \{a \in \mathbb{C} : |a| \geq 1, -\frac{\pi}{4} \leq \angle a \leq \frac{\pi}{4}\}$,
   (f) $\mathcal{F} = \{a \in \mathbb{C} : |a| = 1, \frac{\pi}{3} \leq \angle a \leq \frac{7\pi}{8}\}$,

3. (**Will not be graded.**)$X(j\omega)$ denotes the Fourier Transform (FT) of a continuous-time signal $x(t)$. $X(e^{j\omega})$ denotes the Fourier Transform of a discrete-time signal $x(n)$. Find the inverse Fourier Transform of

   (a) $X(j\omega) = \frac{4+j2\omega}{13-\omega^2+6j\omega}$,
   (b) $X(j\omega) = \frac{\omega^2-4j\omega-6}{-j\omega^3-7\omega^2+14j\omega+8}$,
   (c) $X(e^{j\omega}) = \frac{100-23e^{-j\omega}}{20-9e^{-j\omega}+e^{-j2\omega}}$
   (d) $X(e^{j\omega}) = \frac{6-2e^{-j\omega}+\frac{1}{2}e^{-j2\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{4}e^{-j2\omega}+\frac{1}{16}e^{-j3\omega}}$,

4. Plot the magnitude and phase of the frequency responses $H(j\omega) = \frac{1}{1+10^{-3}j\omega}$ and $G(j\omega) = \frac{10^{-3}j\omega}{1+10^{-3}j\omega}$.

5. (a) State sifting and sampling properties of the impulse function $\delta(t)$.

   (b) Find $y(t) = x(t) * h(t)$ where $x(t) = \frac{\sin(2(t-3))}{\pi(t-3)}$ and $h(t) = \delta(t + 8)$. Here $*$ denotes convolution.

   (c) Let $x(t) = \sin(200\pi t)$ and $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - \frac{k}{800})$ be the periodic impulse train signal. Specify and plot the product signal $z(t) = x(t)s(t)$. The periodic impulse train signal is found in Ex. 4.8 (Pg 299) of SAndS (315 textbook).

   (d) Let $x_1(t) = \sin(200\pi t)$ and $x_2(t) = \cos(700\pi t)$. We define discrete-time signals by considering periodic samples of these CT signals. Let $z_1(n) = x(\frac{n}{800})$. In this case,[1] we have $z_1(0) = x_1(0)$, $z_1(1) = x_1(\frac{1}{800})$ and $z_1(-1) = x_1(\frac{-1}{800})$ and so on. Similarly, let $z_2(n) = x_2(\frac{n}{2800})$, $z_3(n) = x_1(\frac{n}{100})$ and $z_4(n) = x_1(\frac{n}{50})$ and $z_5(n) = x_1(\frac{n}{12.5})$. Specify and plot the DT periodic signal $z_1(n)$ and $z_3(n)$. Compare $z_1(n)$ and $z_2(n)$. Compare $z_3(n)$, $z_4(n)$ and $z_5(n)$.

[1] Here, we are not performing impulse train sampling. Instead, we are defining a DT signal $z_i(n)$ through the relations as specified. Note that impulse train sampling results in an impulse train - a CT signal.