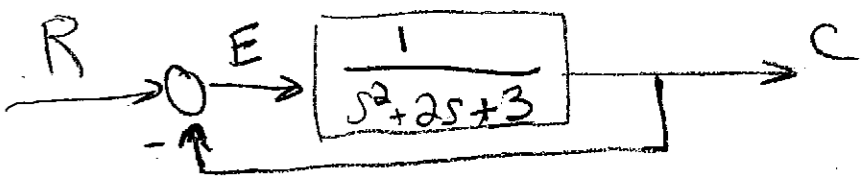
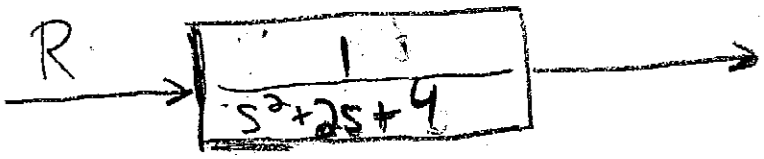
Let's look at the SS error of the following



Notice



Also, since $C = E\frac{1}{s^2+2s+3}$ AND $E = R-C$

then $R = E\left(\frac{1}{s^2+2s+3}+1\right)$

so that $\frac{E}{R} = \frac{s^2+2s+3}{s^2+2s+4}$

Lets consider an input of $r(t)=1$ so that $R = 1/s$

then $C = \frac{1}{s}\left(\frac{C}{R}\right)$

$= \frac{1/s}{s^2+2s+4}$

$= \frac{1}{s\left[(s+1)^2+(\sqrt{3})^2\right]}$

$= \frac{A}{s} + \frac{B(s+1)+C\sqrt{3}}{(s+1)^2+(\sqrt{3})^2}$ trying to fit "pattern" of items 10 & 11 in table 3.3.1

find A, B, C in partial fraction expansion

$1 = A((s+1)^2+(\sqrt{3})^2) + Bs^2+Bs+C\sqrt{3}$

$= As^2+As+3A+2As+Bs^2+Bs+C\sqrt{3}$

$\therefore$ $A = 1/4$

$B = -A = -1/4$

$C = \frac{1}{4\sqrt{3}}$

$$C(s) = \frac{1/4}{s} + \frac{-1/4(s+1)}{(s+1)^2+(\sqrt{3})^2} + \frac{\frac{1}{4\sqrt{3}}\sqrt{3}}{(s+1)^2+(\sqrt{3})^2}$$

$$\downarrow$$

$$c(t) = \underbrace{1/4}_{SS} \underbrace{-1/4\left(e^{-t}\cos\sqrt{3}t\right) + \frac{1}{4\sqrt{3}}\left(e^{-t}\sin\sqrt{3}t\right)}_{\text{transient}}$$

We see here that at steady state $C = 1/4$

Recall we ordered " $r = 1$, so that our

SS error $= R - C = 3/4$

that is significant, it is "off" by 75%!!

There is an easier way to find C_{ss} using the FVT

$$C = \frac{1/s}{s^2+2s+4}$$

recall $c(\infty) = \lim_{s\to 0} sC = \lim_{s\to 0} s\left(\frac{1/s}{s^2+2s+4}\right) = \lim_{s\to 0}\left(\frac{1}{s^2+2s+4}\right)$

$= 1/4$

as we see above!!

We could also do this with E/R

$$e(\infty) = \lim_{s\to 0} sE = \lim_{s\to 0} s\left[E\left(\frac{E}{R}\right)\right] = \lim_{s\to 0} \frac{s^2+2s+2}{s^2+2s+4} = 3/4$$

again, as we saw above!

finally, you can use the result derived for Step inputs for equation 7.12 where

$$e(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$ so that

$$= \frac{1}{1 + \lim_{s \to 0} \left(\frac{1}{s^2 + 2s + 3} \right)}$$

$$= \frac{1}{1 + 1/3}$$

$$= \frac{1}{4/3}$$

$= 3/4$ as expected!!