

Q1

$$y = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1^2 + \epsilon$$

$$\epsilon \sim \mathcal{N}(0, \sigma^2)$$

a) write down $P(y | x_1, x_2)$

$$p(y | x_1, x_2) = \mathcal{N}(w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1^2, \sigma^2)$$

$$p(y^L | x_1^L, x_2^L) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2\sigma^2} (y^L - (w_0 + w_1 x_1^L + w_2 x_2^L + w_3 (x_1^L)^2))^2\right\}$$

$$\begin{aligned} w_{MLE} &= \arg\max_w \left\{ \sum_{all} \left(\log \left\{ \frac{1}{\sqrt{2\pi}\sigma} \right\} + \log \left\{ \exp\left\{-\frac{1}{2\sigma^2} (y^L - (w_0 + w_1 x_1^L + w_2 x_2^L + w_3 (x_1^L)^2))^2\right\} \right\} \right) \right\} \\ &= \arg\max_w \left\{ \sum_L (y^L - w_0 + w_1 x_1^L + w_2 x_2^L + w_3 (x_1^L)^2)^2 \right\} \end{aligned}$$

$$= \arg\min_w \left\{ \sum_L (y^L - (w_0 + x_1^L w_1 + w_2 x_2^L + w_3 (x_1^L)^2))^2 \right\}$$

$$f(w, x^L, y^L) = \sum_L \left\{ (y^L - \underbrace{w_0 + w_1 x_1^L + w_2 x_2^L + w_3 (x_1^L)^2}_{F(x^L; w)})^2 \right\}$$

$$\nabla_w f(w, x^L, y^L) = \sum_L \begin{bmatrix} -2, & -2x_1^L, & -2x_2^L, & -2x_1^{2L} \end{bmatrix} (y^L - F(x^L; w))$$

$$w' = w - \eta \nabla_w f(w, x^L, y^L)$$