Variabel Random Khusus

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Ir. I Nyoman Setiawan, MT.

Percobaan Bernoulli

Percobaan Bernoulli merupakan suatu percobaan yang mendapatkan dua hasil yaitu *sukses* dan *gagal*. Jika $X = 1$ menyatakan hasil sukses dan $X = 0$ menyatakan hasil gagal, maka probabilitas fungsi masa :

PMF is given by

$$P\{X = 0\} = 1 - p$$

$$P\{X = 1\} = p$$

$$p_X(x) = \begin{cases} 1 - p & x = 0 \\ p & x = 1 \end{cases}$$

dimana $p, 0 \leq p \leq 1,$ probabilitas kejadian sukses

The CDF is given by

Fungsi Dsitribusi

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1-p & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

The expected value of X is given by

$$E[X] = 0(1-p) + 1(p) = p$$

Similarly, the second moment of X is given by

$$E\left[X^2\right] = 0^2(1-p) + 1^2(p) = p$$

Thus, the variance of X is given by

$$\sigma_X^2 = E\left[X^2\right] - \{E[X]\}^2 = p - p^2 = p(1-p)$$

Distribusi Binomial

Distribusi Binomial yaitu suatu usaha Bernoulli yang dapat menghasilkan sukses dengan probabilitas *p* dan gagal dengan probabilitas $q = 1 - p$, maka didtribusi probabilitas variabel acak binomial, yaitu banyaknya sukses dalam n usaha bebas, adalah :

$$P(r\ \text{sukses dalam}\ n\ \text{percobaan}) = \frac{n!}{(n-r)!\,r!} q^{n-r} p^{r}$$

$$\text{Mean} : \mu = np$$

$$\text{Varians} : \sigma^2 = npq$$

$$\text{Simpangan baku} : \sigma = \sqrt{npq}$$

Contoh :

Sebuah dadu dilemparkan keatas sebanyak 5 kali. Hitunglah probabilitas mendapat tiga angka enam.

Penyelesaian :

$$n = 5,\ \ r = 3,\ \ P(\text{angka enam}) = p = \frac{1}{6},\ \ P(\text{bukan enam}) = q = \frac{5}{6}$$

$$P(3\,\text{angka}\ \ \text{enam}) = \frac{5!}{2!3!}\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3 = \frac{120}{12}\left(\frac{5^2}{6^5}\right) = \frac{250}{7776} = 0.0322$$

Contoh :

Dua belas persen dari suatu tumpukan transistor adalah cacat. Hitunglah distribusi probabilitas binomial bahwa sebuah paket berukuran lima transistor mengandung 5 cacat. Hitunglah mean dan simpangan baku distribusinya.

Penyelesaian :

P(cacat)= p= 0.12; q = 0.88; n = 5; x = banyaknya cacat

x	0	1	2	3	4	5
P	q^5 0·5277	$5q^4p$ 0·3598	$10q^3p^2$ 0·0981	$10q^2p^3$ 0·0134	$5qp^4$ 0·0009	p^5 0·0000

$$\text{Mean} \quad : \mu = np = 5x0.12 = 0.6$$

$$\text{Simpangan baku} : \sigma = \sqrt{npq} = \sqrt{5x0.12x0.88} = 0.528$$

Distribusi Poisson

$$P(x=r)=\frac{e^{-\mu}\mu^{r}}{r!}$$

$$\mu : \text{mean}$$

$$r : \text{banyaknya sukses dari kejadian}$$

$$\text{e} = 2.71828$$

Example 1

A machine produces on average 2 per cent defectives. In a random sample of 60 items, dertermine the probability of there being three defectives.

Penyelesaian :

$$n = 60; \quad p = \frac{2}{100} = 0{\cdot}02 \qquad \mu = \ldots\ldots\ldots\ldots$$

$$\boxed{\mu = np = 60 \times 0{\cdot}02 = 1{\cdot}2}$$

$$P(x = 3) = \frac{e^{-\mu}\mu^3}{3!} = \ldots\ldots\ldots\ldots\ldots$$

$$P = \frac{e^{-1\cdot 2}1{\cdot}2^3}{3!} = 0{\cdot}0867$$

Example 2

Items processed on a certain machine are found to be 1 per cent defective. Determine the probabilities of obtaining 0, 1, 2, 3, 4 defectives in a random sample batch of 80 such items.

Penyelesaian :

$$P(x = r) = \frac{e^{-\mu}\mu^r}{r!}$$

$$\mu = np; \quad n = 80; \quad p = 0{\cdot}01; \quad \therefore \quad \mu = 80 \times 0{\cdot}01 = 0{\cdot}8$$

For example, $P(x = 2) = \dfrac{e^{-0{\cdot}8}0{\cdot}8^2}{2!} = \dfrac{(0{\cdot}4493)(0{\cdot}64)}{2} = 0{\cdot}1438$,

x	0	1	2	3	4
P	0·4493	0·3595	0·1438	0·0383	0·0077

Distribusi Eksponensial

A continuous random variable X is defined to be an exponential random variable (or X has an exponential distribution) if for some parameter $\lambda > 0$ its PDF is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The CDF of X is given by

$$F_X(x) = P[X \leq x] = \int_0^x f_X(y)dy = \int_0^x \lambda e^{-\lambda y} dy$$
$$= \left[-e^{-\lambda y}\right]_0^x = 1 - e^{-\lambda x}$$

Expektasi dan Varians dari Distribusi Eksponensial

$$E[X] = \int_0^\infty x\lambda e^{-\lambda x} dx$$

$$= \left[-xe^{-\lambda x}\right]_0^\infty + \int_0^\infty e^{-\lambda x} dx = 0 - \left[\frac{1}{\lambda} e^{-\lambda x}\right]_0^\infty$$

$$= \frac{1}{\lambda}$$

$$E[X^n] = \int_0^\infty x^n \lambda e^{-\lambda x} dx = \frac{n!}{\lambda^n}$$

Thus, the variance of X is given by

$$\sigma_X^2 = E[X^2] - (E[X])^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Contoh :

Example 4.17 Assume that the length of phone calls made at a particular telephone booth is exponentially distributed with a mean of 3 minutes. If you arrive at the telephone booth just as Chris was about to make a call, find the following:

1. The probability that you will wait more than 5 minutes before Chris is done with the call.
2. The probability that Chris' call will last between 2 minutes and 6 minutes.

Solution Let X be a random variable that denotes the length of calls made at the telephone booth. Since the mean length of calls is $1/\lambda = 3$, we have that the PDF of X is given by

$$f_X(x) = \lambda e^{-\lambda x} = \frac{1}{3} e^{-x/3}$$

1. The probability that you will wait more than 5 minutes is the probability that X is greater than 5 minutes, which is given by

$$P[X > 5] = \int_5^\infty \frac{1}{3} e^{-x/3} dx = [-e^{-x/3}]_5^\infty = e^{-5/3} = 0.1889$$

2. The probability that the call lasts between 2 and 6 minutes is given by

$$P[2 \leq X \leq 6] = \int_2^6 \frac{1}{3} e^{-x/3} dx = [-e^{-x/3}]_2^6 = F_X(6) - F_X(2)$$
$$= e^{-2/3} - e^{-2} = 0.3781$$

Distribusi Erlang

A random variable X_k is referred to as a kth-order Erlang (or Erlang-k) random variable with parameter λ if its PDF is given by

$$f_{X_k}(x) = \begin{cases} \dfrac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} & k = 1, 2, 3, \ldots;\ x \geq 0 \\ 0 & x < 0 \end{cases}$$

The CDF of X_k is obtained through repeated application of integration by parts as

$$F_{X_k}(x) = P[X_k \leq x] = \int_0^x f_{X_k}(t)dt = 1 - \sum_{j=0}^{k-1} \frac{(\lambda x)^j e^{-\lambda x}}{j!}$$

Ekpektasi dan Varians dari Distribusi Erlang (Erlang –*k*)

$$E[X_k] = \frac{1}{\lambda(k-1)!}\int_0^\infty u^k e^{-u}du = \frac{\Gamma(k+1)}{\lambda(k-1)!} = \frac{k!}{\lambda(k-1)!} = \frac{k}{\lambda}$$

$$E[X_k^2] = \int_0^\infty x^2 f_{X_k}(x)dx = \int_0^\infty x^2 \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}dx$$

$$= \frac{1}{\lambda^2(k-1)!}\int_0^\infty u^{k+1}e^{-u}du = \frac{\Gamma(k+2)}{\lambda^2(k-1)!} = \frac{(k+1)!}{\lambda^2(k-1)!}$$

$$= \frac{k(k+1)}{\lambda^2}$$

$$\sigma_{X_k}^2 = E[X_k^2] - (E[X_k])^2 = \frac{k(k+1)}{\lambda^2} - \frac{k^2}{\lambda^2} = \frac{k}{\lambda^2}$$

Distribusi Uniform

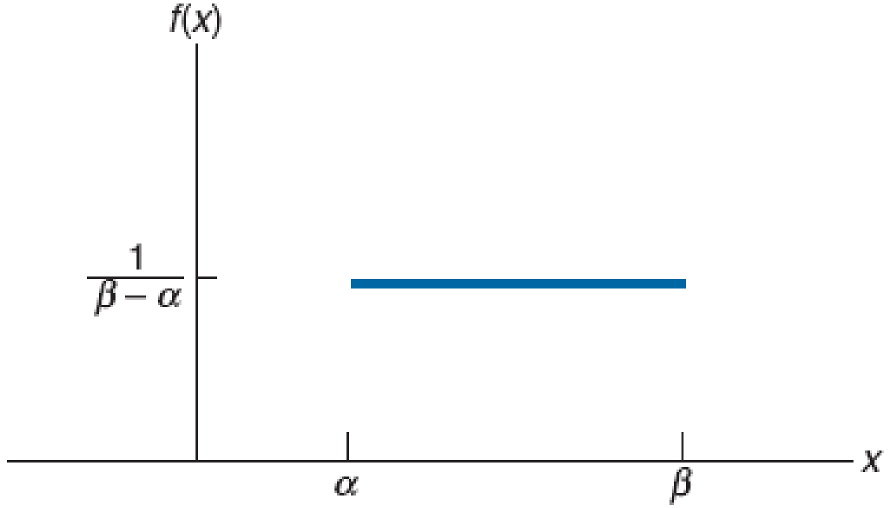
A random variable X is said to be uniformly distributed over the interval $[\alpha, \beta]$ if its probability density function is given by

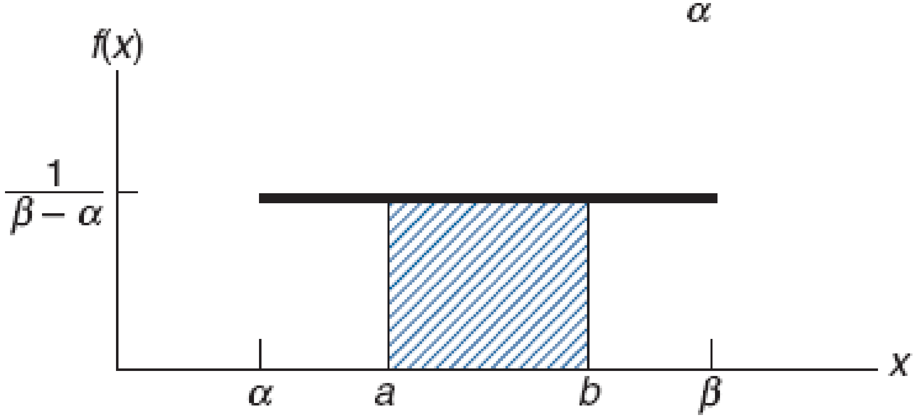
$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \text{if } \alpha \le x \le \beta \\ 0 & \text{otherwise} \end{cases}$$

A graph of this function is given in Figure 5.4. Note that the foregoing meets the requirements of being a probability density function since

$$\frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} dx = 1$$

Gambar Grafik Variabel Random Uniform





$$P\{a < X < b\} = \frac{1}{\beta - \alpha}\int_a^b dx$$

$$= \frac{b - a}{\beta - \alpha}$$

Contoh :

EXAMPLE 5.4a If X is uniformly distributed over the interval [0, 10], compute the probability that **(a)** $2 < X < 9$, **(b)** $1 < X < 4$, **(c)** $X < 5$, **(d)** $X > 6$.

SOLUTION The respective answers are **(a)** 7/10, **(b)** 3/10, **(c)** 5/10, **(d)** 4/10.

Mean/Expektasi dan Varians Variabel Random Uniform

$$E[X] = \int_\alpha^\beta \frac{x}{\beta - \alpha} dx$$

$$= \frac{\beta^2 - \alpha^2}{2(\beta - \alpha)}$$

$$= \frac{(\beta - \alpha)(\beta + \alpha)}{2(\beta - \alpha)}$$

$$E[X] = \frac{\alpha + \beta}{2}$$

$$E[X^2] = \frac{1}{\beta - \alpha} \int_\alpha^\beta x^2 dx$$

$$= \frac{\beta^3 - \alpha^3}{3(\beta - \alpha)}$$

$$= \frac{\beta^2 + \alpha\beta + \alpha^2}{3}$$

$$\mathrm{Var}(X) = E[X^2] - (E[X])^2$$

$$\mathrm{Var}(X) = \frac{\beta^2 + \alpha\beta + \alpha^2}{3} - \left(\frac{\alpha + \beta}{2}\right)^2$$

$$= \frac{\alpha^2 + \beta^2 - 2\alpha\beta}{12}$$

$$= \frac{(\beta - \alpha)^2}{12}$$

Distribusi Normal

From a more theoretical approach, the equation of the normal curve is, in fact, $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$ where μ = mean and σ = standard deviation of the

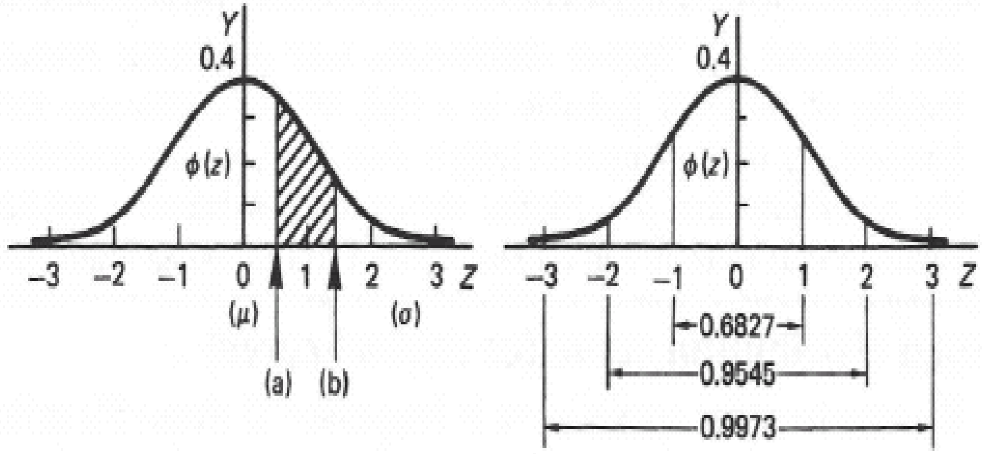
substitution $z = \frac{x-\mu}{\sigma}$ v

$$y = \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$z = \frac{x-\mu}{\sigma}$ is called the *standard normal variable*,

$\phi(z)$ is the *probability density function.*

Standard normal curve:



Note the following:

(a) Mean $\mu = 0$.

(b) z-values are in standard deviation units.

(c) Total area under the curve from $z = -\infty$ to $z = +\infty = 1$.

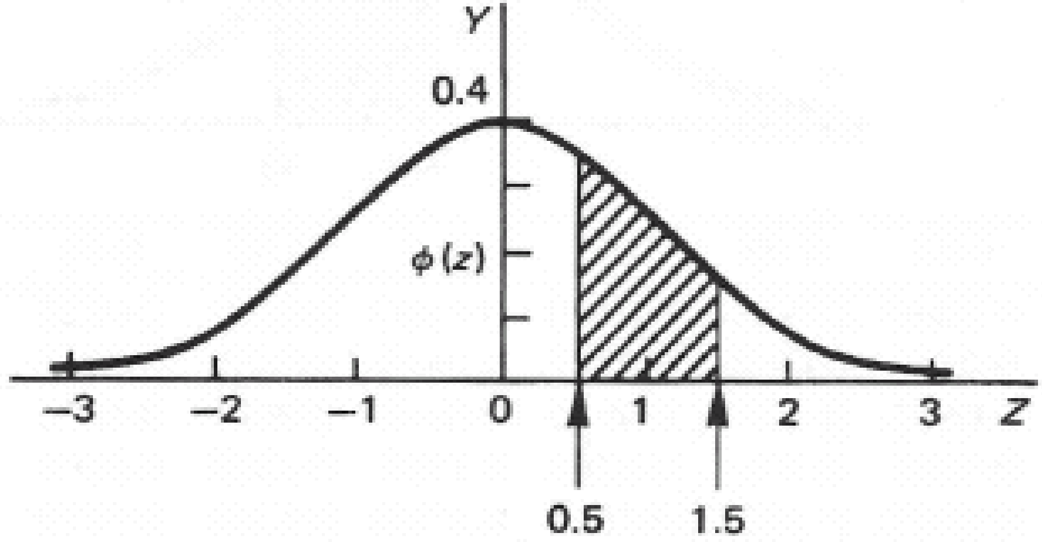
(d) Area between $z = a$ and $z = b$ represents the probability that z lies between the values $z = a$ and $z = b$, i.e. $P(a \leq z \leq b) =$ area shaded.

(e) The probability of a value of z being

between $z = -1$ and $z = 1$ is 68·27 per cent $= 0{\cdot}6827$

between $z = -2$ and $z = 2$ is 95·45 per cent $= 0{\cdot}9545$

between $z = -3$ and $z = 3$ is 99·73 per cent $= 0{\cdot}9973$

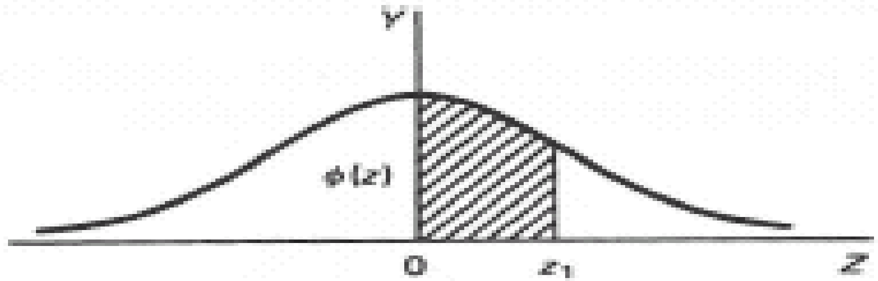


That is $P(0{\cdot}5 \le z \le 1{\cdot}5) = \int_{0\cdot5}^{1\cdot5} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \mathrm{d}z$

From the table:

$$\begin{aligned}
&\text{area from } z = 0 \text{ to } z = 1{\cdot}5 = 0{\cdot}4332 \\
&\text{area from } z = 0 \text{ to } z = 0{\cdot}5 = 0{\cdot}1915 \\
\therefore\quad &\text{area from } z = 0{\cdot}5 \text{ to } z = 1{\cdot}5 = 0{\cdot}2417 \\
\therefore\quad &P(0{\cdot}5 \le z \le 1{\cdot}5) = 0{\cdot}2417 = 24{\cdot}17 \text{ per cent}
\end{aligned}$$

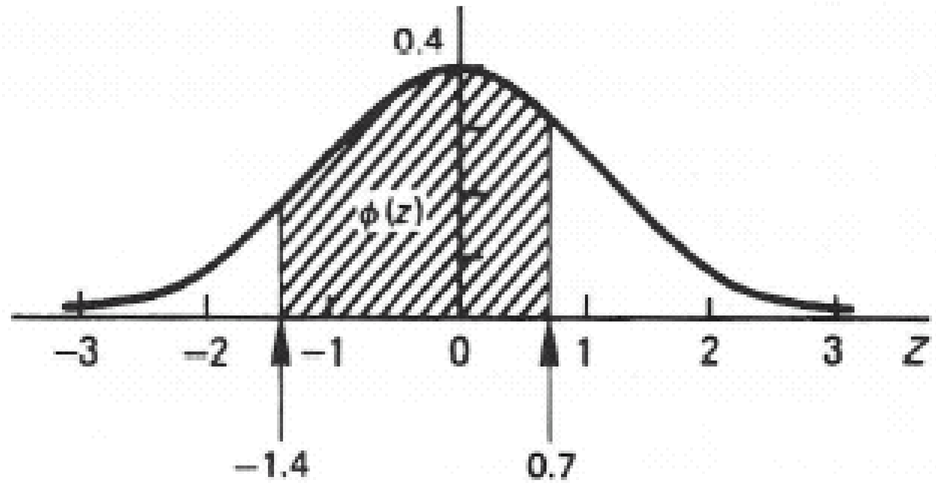
Area under the standard normal curve



z_1	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·0000	0·0040	0·0080	0·0120	0·0160	0·0199	0·0239	0·0279	0·0319	0·0359
0·1	0·0398	0·0438	0·0478	0·0517	0·0557	0·0596	0·0636	0·0675	0·0714	0·0753
0·2	0·0793	0·0832	0·0871	0·0910	0·0948	0·0987	0·1026	0·1064	0·1103	0·1141
0·3	0·1179	0·1217	0·1255	0·1293	0·1331	0·1368	0·1406	0·1443	0·1480	0·1517
0·4	0·1554	0·1591	0·1628	0·1664	0·1700	0·1736	0·1772	0·1808	0·1844	0·1879
0·5	0·1915	0·1950	0·1985	0·2019	0·2054	0·2088	0·2123	0·2157	0·2190	0·2224
0·6	0·2257	0·2291	0·2324	0·2357	0·2389	0·2422	0·2454	0·2486	0·2517	0·2549
0·7	0·2580	0·2611	0·2642	0·2673	0·2704	0·2734	0·2764	0·2794	0·2823	0·2852
0·8	0·2881	0·2910	0·2939	0·2967	0·2995	0·3023	0·3051	0·3078	0·3106	0·3133
0·9	0·3159	0·3186	0·3212	0·3238	0·3264	0·3289	0·3315	0·3340	0·3365	0·3389
1·0	0·3413	0·3438	0·3461	0·3485	0·3508	0·3531	0·3554	0·3577	0·3599	0·3621
1·1	0·3643	0·3665	0·3686	0·3708	0·3729	0·3749	0·3770	0·3790	0·3810	0·3830
1·2	0·3849	0·3869	0·3888	0·3907	0·3925	0·3944	0·3962	0·3980	0·3997	0·4015
1·3	0·4032	0·4049	0·4066	0·4082	0·4099	0·4115	0·4131	0·4147	0·4162	0·4177
1·4	0·4192	0·4207	0·4222	0·4236	0·4251	0·4265	0·4279	0·4292	0·4306	0·4319
1·5	0·4332	0·4345	0·4357	0·4370	0·4382	0·4394	0·4406	0·4418	0·4429	0·4441
1·6	0·4452	0·4463	0·4474	0·4484	0·4495	0·4505	0·4515	0·4525	0·4535	0·4545
1·7	0·4554	0·4564	0·4573	0·4582	0·4591	0·4599	0·4608	0·4616	0·4625	0·4633
1·8	0·4641	0·4649	0·4656	0·4664	0·4671	0·4678	0·4686	0·4693	0·4699	0·4706
1·9	0·4713	0·4719	0·4726	0·4732	0·4738	0·4744	0·4750	0·4756	0·4761	0·4767

Example 1

Determine the probability that a random value of z lies between $z = -1{\cdot}4$ and $z = 0{\cdot}7$.



$$\text{area from } z = -1{\cdot}4 \text{ to } z = 0 = \text{ area from } z = 0 \text{ to } z = 1{\cdot}4$$
$$= 0{\cdot}4192$$
$$\text{area from } z = 0 \text{ to } z = 0{\cdot}7 = 0{\cdot}2580 \quad \text{(from the table)}$$
$$\therefore \quad \text{area from } z = -1{\cdot}4 \text{ to } z = 0{\cdot}7 = \ldots\ldots\ldots\ldots$$

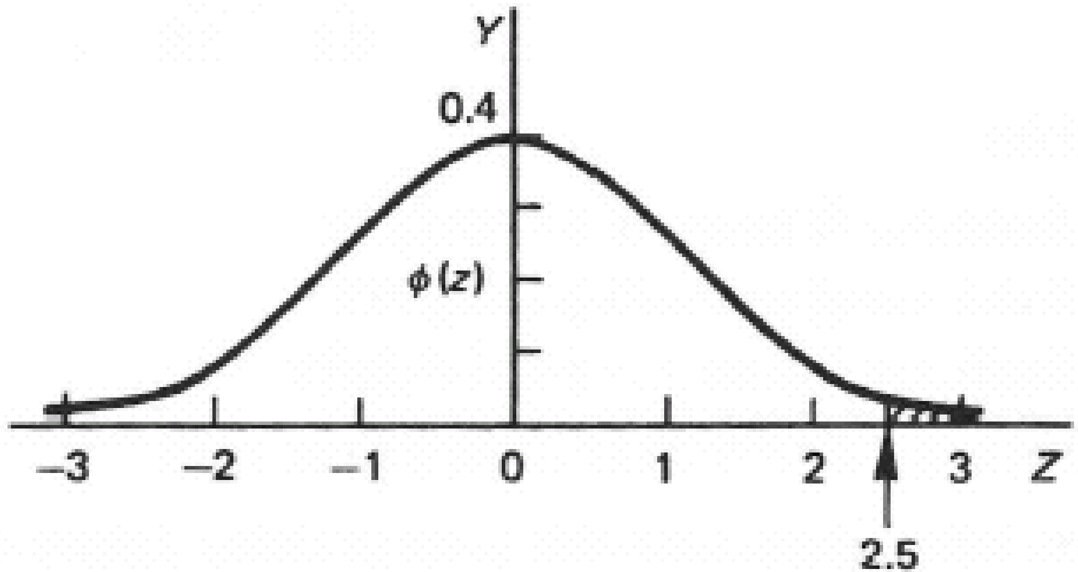
$$\boxed{0{\cdot}4192 + 0{\cdot}2580 = 0{\cdot}6772}$$

$$\therefore \quad P(-1{\cdot}4 \le z \le 0{\cdot}7) = 0{\cdot}6772 = 67{\cdot}72 \text{ per cent}$$

Example 2

Determine the probability that a value of z is greater than 2·5. Draw a diagram: then it is easy enough. Finish it off.

Penyelesaian :



The total area from $z = 0$ to $z = \infty$ is 0·5000.

The total area from $z = 0$ to $z = 2{\cdot}5 = 0{\cdot}4938$ (from the table).

$\therefore$ The area for $z \geq 2{\cdot}5 = 0{\cdot}5000 - 0{\cdot}4938 = 0{\cdot}0062$

$\therefore$ $P(z \geq 2{\cdot}5) = 0{\cdot}0062 = 0{\cdot}62$ per cent

Example 3

The mean diameter of a sample of 400 rollers is 22·50 mm and the standard deviation 0·50 mm. Rollers are acceptable with diameters $22{\cdot}36 \pm 0{\cdot}53$ mm. Determine the probability of any one roller being within the acceptable limits.

Penyelesaian :

We have $\mu = 22{\cdot}50$ mm; $\sigma = 0{\cdot}50$ mm

Limits of $x_1 = 22{\cdot}36 - 0{\cdot}53 = 21{\cdot}83$ mm

$x_2 = 22{\cdot}36 + 0{\cdot}53 = 22{\cdot}89$ mm

Using $z = \dfrac{x - \mu}{\sigma}$ we convert x_1 and x_2 into z_1 and z_2

$z_1 = \ldots\ldots\ldots\ldots$; $z_2 = \ldots\ldots\ldots\ldots$ $\boxed{z_1 = -1{\cdot}34; \quad z_2 = 0{\cdot}78}$

Now, using the table, we can find the area under the normal curve between $z = -1{\cdot}34$ and $z = 0{\cdot}78$ which gives us the required result.

$P(21{\cdot}83 \le x \le 22{\cdot}89) = P(-1{\cdot}34 \le z \le 0{\cdot}78) = .$ $\boxed{0{\cdot}6922}$

Example 4

A thermostat set to switch at 20°C operates at a range of temperatures having a mean of 20·4°C and a standard deviation of 1·3°C. Determine the probability of its opening at temperatures between 19·5°C and 20·5°C.

Penyelesaian :

$$x_1 = 19{\cdot}5 \quad \therefore \quad z_1 = \frac{19{\cdot}5 - 20{\cdot}4}{1{\cdot}3} = -\frac{0{\cdot}9}{1{\cdot}3} = -0{\cdot}692$$

$$x_2 = 20{\cdot}5 \quad \therefore \quad z_2 = \frac{20{\cdot}5 - 20{\cdot}4}{1{\cdot}3} = \frac{0{\cdot}1}{1{\cdot}3} = 0{\cdot}077$$

$$\text{Area } (z = -0{\cdot}69 \text{ to } z = 0) = \text{ area } (z = 0 \text{ to } z = 0{\cdot}69)$$
$$= 0{\cdot}2549$$
$$\text{Area } (z = 0 \text{ to } z = 0{\cdot}08) = 0{\cdot}0319$$
$$\therefore \quad \text{Area } (z = -0{\cdot}69 \text{ to } z = 0{\cdot}08) = 0{\cdot}2868$$
$$\therefore \quad P(19{\cdot}5 \leq x \leq 20{\cdot}5) = P(-0{\cdot}692 \leq z \leq 0{\cdot}077) = 0{\cdot}2868$$

Contoh :

- Suatu perusahan listrik menghasilkan bola lampu yang umurnya berdistribusi normal dengan mean/rataan 800 jam dan standar deviasi 40 jam. Hitunglah probabilitas suatu bola lampu dapat menyala antara 778 dan 834.

Penyelesaian :

Nilai z yang berpadanan dengan $x_1 = 778$ dan $x_2 = 834$

$$z_1 = \frac{778-800}{40} = -0.55 \qquad z_2 = \frac{834-800}{40} = 0.85$$

$$P(778 < X < 834) = P(-0.55 < Z < 0.85)$$

$$= 0.2088 + 0.3023 = 0.5111$$

Contoh :

4-39. Use Appendix Table II to determine the following probabilities for the standard normal random variable Z:

(a) $P(Z < 1.32)$ (b) $P(Z < 3.0)$

(c) $P(Z > 1.45)$ (d) $P(Z > -2.15)$

(e) $P(-2.34 < Z < 1.76)$

4-43. Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the following:

(a) $P(X < 13)$ (b) $P(X > 9)$

(c) $P(6 < X < 14)$ (d) $P(2 < X < 4)$

(e) $P(-2 < X < 8)$

4-39.	(a) 0.90658	(b) 0.99865
	(c) 0.07353	(d) 0.98422
	(e) 0.95116	
4-41.	(a) 1.28 (b) 0 (c) 1.28	
	(d) −1.28 (e) 1.33	
4-43.	(a) 0.93319	(b) 0.69146
	(c) 0.9545	(d) 0.00135
	(e) 0.15866	

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

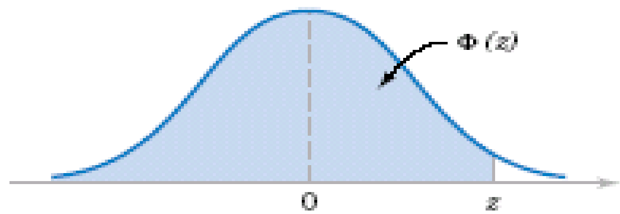


Table II Cumulative Standard Normal Distribution (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691

Distribusi *Chi-Square*

If $Z_1, Z_2, \ldots, Z_n$ are independent standard normal random variables, then X, defined by

$$X = Z_1^2 + Z_2^2 + \cdots + Z_n^2 \tag{5.8.1}$$

is said to have a *chi-square distribution with n degrees of freedom*. We will use the notation

$$X \sim \chi_n^2$$

to signify that X has a chi-square distribution with n degrees of freedom.

Distribusi - *T*

If Z and χ_n^2 are independent random variables, with Z having a standard normal distribution and χ_n^2 having a chi-square distribution with n degrees of freedom, then the random variable T_n defined by

$$T_n = \frac{Z}{\sqrt{\chi_n^2 / n}}$$

is said to have a *t-distribution with n degrees of freedom*. A graph of the density function of T_n is given in Figure 5.14 for $n = 1, 5,$ and 10.

Distribusi-*F*

If χ_n^2 and χ_m^2 are independent chi-square random variables with n and m degrees of freedom, respectively, then the random variable $F_{n,m}$ defined by

$$F_{n,m} = \frac{\chi_n^2/n}{\chi_m^2/m}$$

is said to have an *F-distribution with n and m degrees of freedom.*