MOOD limiting for high-order discontinuous Galerkin methods

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1 Introduction

One of the current issues with high-order discontinuous Galerkin (DG) methods arises at discontinuities in the solution. The discontinuities cause unwanted oscillations and need to be dampened through classic limiting methods used finite volume (FV) methods or aritificial diffusion. Limiting methods tend to drop to first order accurate and disregard high-order accurate information.

When trying to find a stable solution there are two steps we need to take:

i Identify a troubled cell, and

ii compute a limited solution.

In [9] they present a comparison of different troubled cells indicators and are paired with a WENO limiter. In [8, 7] use a smoothness indicator denoted by

$$s_K = \log_{10}\left(\frac{\|\boldsymbol{u}_K - \hat{\boldsymbol{u}}_K\|_2^2}{\|\boldsymbol{u}_K\|_2^2}\right), \tag{1}$$

where $\boldsymbol{u}_K$ is the nodal solution in element K, and $\hat{\boldsymbol{u}}_K$ is a truncated modal solution in element K. Then based on s_K, a smoothing factor, ε_K, can be computed. Another indicator is presented in [10], where they present an artificial neural network to find troubled cells.

Biswas et al. [1] present a way to systematically cascade through the modes of the solution and [6] presents an updated version of the method. Another similar approach is to use multi-dimensional optimal order detection (MOOD) which is used to cascade through the desired polynomial orders. This method was first introduced in [3] and improved in [4]. Dumbser et al. [5] have introduced a subcell MOOD limiting method that uses the element nodes as cell-centered values and use a WENO reconstuction to obtain a high-order method.

Recently, [2] used a GP-MOOD method for posteriori limiting on a small amount of cells. The GP-MOOD method uses a series of criteria to identify troubled cells and use a GP reconstruction to compute a lower order method before reducing to a first order Godunov method. We plan on adopting the criteria check to identify troubled cells and then cascade through the modes of the solution until we identify an acceptable solution or reaching a constant (zeroth order polynomial) solution.

2 Basic Methodology

In this methodology we will use troubled cell indicators developed in [2] and use a truncation of modal solutions as the limiting method.

The criteria are as follows:

(a) Physical admissibility detection (PAD): this criteria is to ensure positivity on density and pressure variables.

(b) Computer science admissibility detection (CAD): criteria to identify whether density or pressure are `NaN` or `inf`.

(c) Compressibility-Shock detection (CSD): this detector measures the compressibility and the shock strength of the element. We check the following numerical quantities:

$$\nabla \cdot \bar{V} \geq -\sigma_v, \quad \tilde{\nabla}\bar{p} \leq \sigma_p, \tag{2}$$

where σ_v and σ_p are both tunable parameters.

(d) Relaxed DMP criteria: plateau + DMP + u2

Algorithm 1 DG MOOD limiting process.

Compute unlimited nodal solution $\boldsymbol{u}$.
Compute cell-centered average solution $\bar{\boldsymbol{u}}$.
Compute modal solution $\hat{\boldsymbol{u}} = V^{-1}\boldsymbol{u}$, where V is the Legendre polynomial Vandermonde matrix.
Current highest mode $i_m = n_n$.
while Unstable solution **do**
  if PAD or CAD Fail **then**
    Truncate modal solution $\hat{\boldsymbol{u}}(i_m) = 0$.
    Compute new cell-centered average solution.
    Update current highest mode, $i_m = i_m - 1$
  end if
  if Strong compressibility or shocks **then**
    if $\bar{\boldsymbol{u}}$ does not satisfy DMP criteria **then**
      Truncate modal solution $\hat{\boldsymbol{u}}(i_m) = 0$.
      Compute new cell-centered average solution.
      Update current highest mode, $i_m = i_m - 1$
    end if
  end if
end while
Truncated modal solution has been accepted.
Compute nodal solution $\boldsymbol{u} = V\hat{\boldsymbol{u}}$.

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