

MOOD limiting for high-order discontinuous Galerkin methods

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1 Introduction

One of the current issues with high-order discontinuous Galerkin (DG) methods arises at discontinuities in the solution. The discontinuities cause unwanted oscillations and need to be dampened through classic limiting methods used finite volume (FV) methods or artificial diffusion. Limiting methods tend to drop to first order accurate and disregard high-order accurate information.

When trying to find a stable solution there are two steps we need to take:

- i Identify a troubled cell, and
- ii compute a limited solution.

In [9] they present a comparison of different troubled cells indicators and are paired with a WENO limiter. In [8, 7] use a smoothness indicator denoted by

$$s_K = \log_{10} \left(\frac{\|\mathbf{u}_K - \hat{\mathbf{u}}_K\|_2^2}{\|\mathbf{u}_K\|_2^2} \right), \quad (1)$$

where \mathbf{u}_K is the nodal solution in element K , and $\hat{\mathbf{u}}_K$ is a truncated modal solution in element K . Then based on s_K , a smoothing factor, ε_K , can be computed. Another indicator is presented in [10], where they present an artificial neural network to find troubled cells.

Biswas et al. [1] present a way to systematically cascade through the modes of the solution and [6] presents an updated version of the method. Another similar approach is to use multi-dimensional optimal order detection (MOOD) which is used to cascade through the desired polynomial orders. This method was first introduced in [3] and improved in [4]. Dumbser et al. [5] have introduced a subcell MOOD limiting method that uses the element nodes as cell-centered values and use a WENO reconstruction to obtain a high-order method.

Recently, [2] used a GP-MOOD method for posteriori limiting on a small amount of cells. The GP-MOOD method uses a series of criteria to identify troubled cells and use a GP reconstruction to compute a lower order method before reducing to a first order Godunov method. We plan on adopting the criteria check to identify troubled cells and then cascade through the modes of the solution until we identify an acceptable solution or reaching a constant (zeroth order polynomial) solution.

2 Basic Methodology

In this methodology we will use troubled cell indicators developed in [2] and use a truncation of modal solutions as the limiting method.

The criteria are as follows:

- (a) Physical admissibility detection (PAD): this criteria is to ensure positivity on density and pressure variables.
- (b) Computer science admissibility detection (CAD): criteria to identify whether density or pressure are NaN or \mathbf{inf} .
- (c) Compressibility-Shock detection (CSD): this detector measures the compressibility and the shock strength of the element. We check the

following numerical quantities:

$$\nabla \cdot \bar{V} \geq -\sigma_v, \quad \tilde{\nabla} \bar{p} \leq \sigma_p, \quad (2)$$

where σ_v and σ_p are both tunable parameters.

(d) Relaxed DMP criteria: plateau + DMP + u2

References

- [1] Rupak Biswas, Karen D. Devine, and Joseph E. Flaherty. Parallel, adaptive finite element methods for conservation laws. Applied Numerical Mathematics, 14(1-3):255–283, 1994.
- [2] Rémi Bourgeois and Dongwook Lee. GP-MOOD: A positive-preserving high-order finite volume method for hyperbolic conservation laws. arXiv preprint arXiv:2110.08683, 2021.
- [3] S Claina, S Diota, and R Louberea. A very high-order finite volume method for hyperbolic systems: Multi-dimensional optimal order detection (mood).
- [4] Steven Diot, Stéphane Clain, and Raphaël Loubère. Improved detection criteria for the multi-dimensional optimal order detection (mood) on unstructured meshes with very high-order polynomials. Computers & Fluids, 64:43–63, 2012.
- [5] Michael Dumbser, Olindo Zanotti, Raphaël Loubère, and Steven Diot. A posteriori subcell limiting of the discontinuous galerkin finite element method for hyperbolic conservation laws. Journal of Computational Physics, 278:47–75, 2014.
- [6] Lilia Krivodonova. Limiters for high-order discontinuous Galerkin methods. Journal of Computational Physics, 226(1):879–896, 2007.

- [7] Will Pazner. Sparse invariant domain preserving discontinuous galerkin methods with subcell convex limiting. Computer Methods in Applied Mechanics and Engineering, 382:113876, 2021.
- [8] Per-Olof Persson and Jaime Peraire. Sub-cell shock capturing for discontinuous galerkin methods. In 44th AIAA Aerospace Sciences Meeting and Exhibit, page 112, 2006.
- [9] Jianxian Qiu and Chi-Wang Shu. A comparison of troubled-cell indicators for runge–kutta discontinuous galerkin methods using weighted essentially nonoscillatory limiters. SIAM Journal on Scientific Computing, 27(3):995–1013, 2005.
- [10] Deep Ray and Jan S Hesthaven. An artificial neural network as a troubled-cell indicator. Journal of computational physics, 367:166–191, 2018.