Semantic Theory 2025: Exercise 3 Key

Question 1

Translate the following into λ-expressions. Use subscripts to indicate the types of the λ-bound variables (e.g. $\lambda x_e.P(x)$ for an e-type x).

a. $pink_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}$ (as in "Jumbo is a pink elephant"; the expression should have $pink^*_{\langle e,t\rangle}$ as the underlying first-order predicate)

$pink_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle} = \lambda P_{\langle e,t\rangle}\lambda x_e[pink^*(x) \wedge P(x)]$

b. $and_{\langle e,\langle e,\langle\langle e,t\rangle,t\rangle\rangle\rangle}$ (as in "John and Suzy danced"; the expression should incorporate $\wedge$ as the underlying operator)

$and_{\langle e,\langle e,\langle\langle e,t\rangle,t\rangle\rangle\rangle} = \lambda x_e\lambda y_e\lambda P_{\langle e,t\rangle}[P(x) \wedge P(y)]$

c. $not_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}$ (as in "Mark did not like the party"; the expression should incorporate $\neg$ as the underlying operator)

$not_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle} = \lambda P_{\langle e,t\rangle}\lambda x_e[\neg P(x)]$

Question 2

Translate the following sentences into λ-expressions, assuming the syntactic structure indicated by the brackets. Then use lambda conversions (β-/η-/α-conversion) to reduce to λ-free terms.

Use the terms you derived for *pink*, *and*, and *not* in Question 1. If you weren't able to derive those terms, you can simply use the predicates $pink'$, and', and not' (with the types indicated in Question 1).

Ignore the contribution of past/plural morphology, "is"/"are"/"did", and "a".

a. Jumbo [is a [pink elephant]]

$$
\begin{aligned}
& pink(elephant)(jumbo) \\
&= \lambda P_{\langle e,t\rangle}\lambda x_e[pink^*(x) \wedge P(x)](elephant)(jumbo) \\
&\Leftrightarrow_\beta \lambda x_e[pink^*(x) \wedge elephant(x)](jumbo) \\
&\Leftrightarrow_\beta pink^*(jumbo) \wedge elephant(jumbo)
\end{aligned}
$$

b. [John and Suzy] danced

$$
\begin{aligned}
& and(suzy)(john)(dance) \\
&= \lambda x_e\lambda y_e\lambda P_{\langle e,t\rangle}[P(x) \wedge P(y)](suzy)(john)(dance) \\
&\Leftrightarrow_\beta \lambda y_e\lambda P_{\langle e,t\rangle}[P(suzy) \wedge P(y)](john)(dance) \\
&\Leftrightarrow_\beta \lambda P_{\langle e,t\rangle}[P(suzy) \wedge P(john)](dance) \\
&\Leftrightarrow_\beta dance(john) \wedge dance(john)
\end{aligned}
$$

c. Mark did [not [like $\underline{\text{the party}}_e$]]

$$\begin{aligned}
&\mathit{not}(\mathit{like}(\mathit{the\text{-}party}))(\mathit{mark})\\
&= \lambda P_{\langle e,t\rangle}\lambda x_e[\neg P(x)](\mathit{like}(\mathit{the\text{-}party}))(\mathit{mark})\\
&\Leftrightarrow_\beta \lambda x_e[\neg \mathit{like}(\mathit{the\text{-}party})(x)](\mathit{mark})\\
&\Leftrightarrow_\beta \neg \mathit{like}(\mathit{the\text{-}party})(\mathit{mark})
\end{aligned}$$

d. [Tim [and Mary]] are [not [pink elephants]]

$$\begin{aligned}
&\mathit{and}(m')(t')(\mathit{not}(\mathit{pink}(\mathit{elephant})))\\
&= \lambda x_e\lambda y_e\lambda P_{\langle e,t\rangle}[P(x)\wedge P(y)](m')(t')(\lambda R_{\langle e,t\rangle}\lambda z_e[\neg R(z)](\lambda Q_{\langle e,t\rangle}\lambda w_e[\mathit{pink}^*(w)\wedge Q(w)](\mathit{elephant})))\\
&\Leftrightarrow_\beta \lambda x_e\lambda y_e\lambda P_{\langle e,t\rangle}[P(x)\wedge P(y)](m')(t')(\lambda R_{\langle e,t\rangle}\lambda z_e[\neg R(z)](\lambda w_e[\mathit{pink}^*(w)\wedge \mathit{elephant}(w)]))\\
&\Leftrightarrow_\beta \lambda x_e\lambda y_e\lambda P_{\langle e,t\rangle}[P(x)\wedge P(y)](m')(t')(\lambda z_e[\neg\lambda w_e[\mathit{pink}^*(w)\wedge \mathit{elephant}(w)](z)])\\
&\Leftrightarrow_\beta \lambda x_e\lambda y_e\lambda P_{\langle e,t\rangle}[P(x)\wedge P(y)](m')(t')(\lambda z_e[\neg(\mathit{pink}^*(z)\wedge \mathit{elephant}(z))])\\
&\Leftrightarrow_\beta \lambda y_e\lambda P_{\langle e,t\rangle}[P(m')\wedge P(y)](t')(\lambda z_e[\neg(\mathit{pink}^*(z)\wedge \mathit{elephant}(z))])\\
&\Leftrightarrow_\beta \lambda P_{\langle e,t\rangle}[P(m')\wedge P(t')](\lambda z_e[\neg(\mathit{pink}^*(z)\wedge \mathit{elephant}(z))])\\
&\Leftrightarrow_{\beta,\alpha} \lambda z_e[\neg(\mathit{pink}^*(z)\wedge \mathit{elephant}(z))](m')\wedge\lambda u_e[\neg(\mathit{pink}^*(u)\wedge \mathit{elephant}(u))](t')\\
&\Leftrightarrow_\beta \neg(\mathit{pink}^*(m')\wedge \mathit{elephant}(m'))\wedge\lambda u_e[\neg(\mathit{pink}^*(u)\wedge \mathit{elephant}(u))](t')\\
&\Leftrightarrow_\beta \neg(\mathit{pink}^*(m')\wedge \mathit{elephant}(m'))\wedge\neg(\mathit{pink}^*(t')\wedge \mathit{elephant}(t'))
\end{aligned}$$