Semantic Theory 2025: Exercise 2 Key

Question 1

Provide the derivations (type inferencing) of each of the following sentences. Brackets indicate the combinatorics, and subscripts indicate the types of (some of) the expressions—the rest must be deduced. Underlined expressions should be treated as a single term: for example, you can treat "the father of" as *the-father-of* in (a).

a. $\underline{\text{Darth Vader}}_e$ [$\text{is}_{\langle e,\langle e,t\rangle\rangle}$[$\underline{\text{the father of}}$ $[\text{Luke}]_e$]]

$$\dfrac{\dfrac{is :: \langle e,\langle e,t\rangle\rangle \qquad \dfrac{\textit{father-of} :: \langle e,e\rangle \quad luke :: e}{\textit{father-of}(luke) :: e}}{is(\textit{father-of}(luke)) :: \langle e,t\rangle} \qquad \textit{darth-vader} :: e}{is(\textit{father-of}(luke))(\textit{darth-vader}) :: t}$$

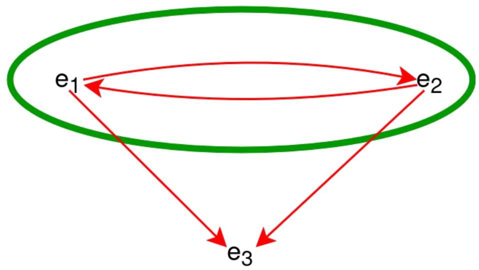
b. [Every $\text{student}_{\langle e,t\rangle}$] [reads $\underline{\text{the book}}_e$]

$$\dfrac{\dfrac{every :: \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle \quad student :: \langle e,t\rangle}{every(student) :: \langle\langle e,t\rangle,t\rangle} \qquad \dfrac{read :: \langle e,\langle e,t\rangle\rangle \quad \textit{the-book} :: e}{read(\textit{the-book}) :: \langle e,t\rangle}}{every(student)(read(\textit{the-book})) :: t}$$

c. Mark_e [is [$\text{the}_{\langle\langle e,t\rangle,e\rangle}$ [[most annoying] $\text{man}_{\langle e,t\rangle}$]]] [$\text{on}_{\langle e,\langle\langle e,t\rangle,\langle e,t\rangle\rangle\rangle}$ $\underline{\text{the planet}}_e$]

(ι = "the")

$$\dfrac{\dfrac{\dfrac{on :: \langle e,\langle et,et\rangle\rangle \quad p' :: e}{on(p') :: \langle et,et\rangle} \qquad \dfrac{is :: \langle e,et\rangle \qquad \dfrac{\iota :: \langle et,e\rangle \qquad \dfrac{\dfrac{most :: \langle\langle et,et\rangle,\langle et,et\rangle\rangle \quad a' : \langle et,et\rangle}{most(a') :: \langle et,et\rangle} \qquad man :: \langle e,t\rangle}{most(a')(man)}}{\iota(most(a')(man)) :: e}}{is(\iota(most(a')(man))) :: \langle e,t\rangle}}{on(p')(is(\iota(most(a')(man)))) :: \langle e,t\rangle} \qquad m' :: e}{on(p')(is(\iota(most(a')(man))))(m') :: t}$$



Question 2

The diagram above graphically represents a model structure $M = (U_M, V_M)$ with a universe consisting of three entities: $U_M = \{e_1, e_2, e_3\}$. The interpretation function V_M describes the first-order property "tall" (indicated by the green circle) and the two-place relation "to see" (indicated by the red arrows).

2.1 Give the type-theoretic denotation of the interpretation function V_M for the following non-logical constants, using functions (rather than sets) when appropriate:

a. *tall* $\in \text{CON}_{\langle e,t \rangle}$

$V_M(\mathit{tall}) = \{e_1, e_2\}$

b. *see* $\in \text{CON}_{\langle e,\langle e,t \rangle\rangle}$

$$V_M(\mathit{see}) = \left\{ \begin{array}{l} e_1 \mapsto \left\{ \begin{array}{l} e_1 \mapsto 0 \\ e_2 \mapsto 1 \\ e_3 \mapsto 0 \end{array} \right. \\ e_2 \mapsto \left\{ \begin{array}{l} e_1 \mapsto 1 \\ e_2 \mapsto 0 \\ e_3 \mapsto 0 \end{array} \right. \\ e_3 \mapsto \left\{ \begin{array}{l} e_1 \mapsto 1 \\ e_2 \mapsto 1 \\ e_3 \mapsto 0 \end{array} \right. \end{array} \right.$$

2.2 Compute the type-theoretic denotations of the following expressions relative to the given model structure M and some arbitrary variable assignment g. Here, x, y are variables of type e, and F is a variable of type $\langle e,t\rangle$.

a. $[\![see(y)]\!]^{M,g[y/e_2]} = ?$

$[\![see(y)]\!]^{M,g[y/e_2]}$
$= [\![see]\!]^{M,g[y/e_2]}([\![y]\!]^{M,g[y/e_2]})$
$= V_M(see)(g[y/e_2](y))$
$= V_M(see)(e_2)$
$= \{e_1\}$

b. $[\![\forall x(\neg\exists y(see(y)(x)) \rightarrow \neg tall(x))]\!]^{M,g} = 1$

iff for all $d \in D_e$: $[\![\neg\exists y(see(y)(x)) \rightarrow \neg tall(x)]\!]^{M,g[x/d]} = 1$
i.e.: $d \in D_e$: $[\![\neg\exists y(see(y)(x))]\!]^{M,g[x/d]} = 0$ or $[\![\neg tall(x)]\!]^{M,g[x/d]} = 1$
$[\![\neg\exists y(see(y)(x))]\!]^{M,g[x/d]} = 0$ iff $[\![\exists y(see(y)(x))]\!]^{M,g[x/d]} = 1$
iff there is some $b \in D_e$ s.t. $[\![\exists see(y)(x)]\!]^{M,g[x/d][y/b]} = 1$
iff $d \in V_M(see)(b)$
$[\![\neg tall(x)]\!]^{M,g[x/d]} = 1$ iff $[\![tall(x)]\!]^{M,g[x/d]} = 0$
iff $d \notin V_M(tall)$
applied to the model M:
$V_M(see)(e_3) = \{e_1, e_2\} \Rightarrow [\![\neg\exists y(see(y)(x))]\!]^{M,g[x/e_1]} = 0$ and $[\![\neg\exists y(see(y)(x))]\!]^{M,g[x/e_2]} = 0$
$\Rightarrow [\![\neg\exists y(see(y)(x)) \rightarrow \neg tall(x)]\!]^{M,g[x/e_1]} = 1$
and $[\![\neg\exists y(see(y)(x)) \rightarrow \neg tall(x)]\!]^{M,g[x/e_2]} = 1$
$e_3 \notin V_M(tall) \Rightarrow [\![\neg tall(x)]\!]^{M,g[x/e_3]} = 1$
$\Rightarrow [\![\neg\exists y(see(y)(x)) \rightarrow \neg tall(x)]\!]^{M,g[x/e_3]} = 1$
$\Rightarrow [\![\forall x(\neg\exists y(see(y)(x)) \rightarrow \neg tall(x))]\!]^{M,g} = 1$

c. $[\![\forall F\exists x(F(x))]\!]^{M,g} = 1$

iff for all $P \in D_{\langle e,t\rangle}$: $[\![\exists x(P(x))]\!]^{M,g[F/P]} = 1$
iff there is some $d \in D_e$ such that $[\![P(x)]\!]^{M,g[F/P][x/d]} = 1$
iff $d \in V_M(P)$
we only have one one-place predicate in M: *tall*, and $e_1, e_2 \in V_M(tall)$
$\Rightarrow [\![\forall F\exists x(F(x))]\!]^{M,g} = 1$