Semantic Theory 2025: Exercise 1 Key

Question 1

Translate the following sentences into first-order predicate logic. You can freely introduce predicates, but try to retain as much of the structure as possible. Also provide the key to the translation.

a. Every student uses their computer.

$\forall x(student(x) \rightarrow \exists y(computer(y) \wedge own(x,y) \wedge use(x,y)))$

b. John doesn't see anything.

$\neg\exists x(see(j,x))$ (equivalently: $\forall x(\neg see(j,x))$)

c. The pen is the only instrument that is mightier than the sword.

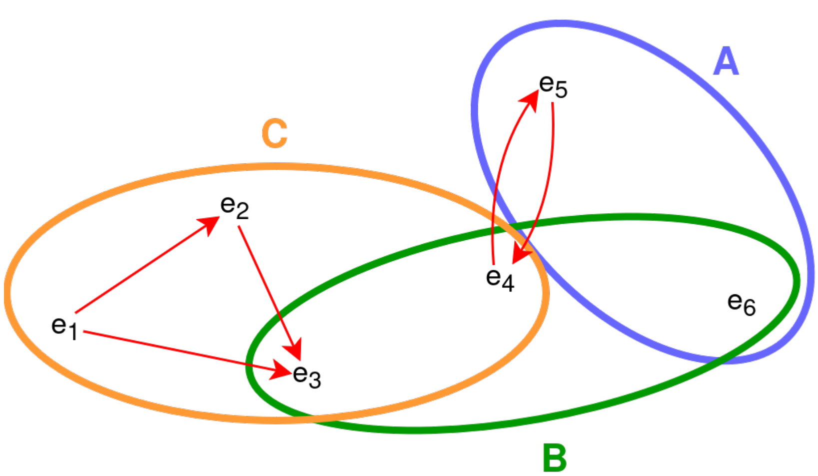
$\forall x((instrument(x) \wedge \forall y(sword(y) \rightarrow mightier\text{-}than(x,s))) \rightarrow pen(x))$

d. If the Prime Minister of the country is defeated, the city will celebrate.

$defeated(pm') \rightarrow celebrate(c')$

Question 2

Consider the following model $M_1 = (U_1, V_1)$, with $U_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.



The interpretation function V_1 is defined as follows:

- $V_1(j) = e_1$
- $V_1(m) = e_4$
- $V_1(b) = e_6$
- $V_1(A) = \{e_5, e_6\}$
- $V_1(B) = \{e_3, e_4, e_6\}$
- $V_1(C) = \{e_1, e_2, e_3, e_4\}$
- $V_1(R) = \{(e_1, e_2), (e_1, e_3), (e_2, e_3), (e_4, e_5), (e_5, e_4)\}$

Let the assignment function g_1 be defined as follows:
$g_1(x) = e_4$, $g_1(x') = e_2$, $g_1(x'') = e_3$ and for all other variables x'^*: $g_1(x'^*) = e_5$.

2.1 Evaluate the following formulas in model M_1, with respect to assignment function g_1. First, derive the truth conditions (showing all relevant steps of the derivation) and then evaluate these truth conditions with respect to M_1 and g_1.

a. $[\![R(x', x'') \wedge R(x''', b)]\!]^{M_1,g_1} = 1$
iff $[\![R(x', x'')]\!]^{M_1,g_1} = 1$ and $[\![R(x''', b)]\!]^{M_1,g_1} = 1$
iff $[\![(x', x'')]\!]^{M_1,g_1} \in [\![R]\!]^{M_1,g_1}$ and $[\![(x''', b)]\!]^{M_1,g_1} \in [\![R]\!]^{M_1,g_1}$
iff $(g_1(x'), g_1(x'')) \in V_1(R)$ and $(g_1(x'''), V_1(b)) \in V_1(R)$
$\Rightarrow (e_2, e_3) \in \{(e_1, e_2), (e_1, e_3), (e_2, e_3), (e_4, e_5), (e_5, e_4)\}$ (true)
and $(e_5, e_6) \in \{(e_1, e_2), (e_1, e_3), (e_2, e_3), (e_4, e_5), (e_5, e_4)\}$ (false)
$\Rightarrow [\![R(x', x'') \wedge R(x''', b)]\!]^{M_1,g_1} = 0$

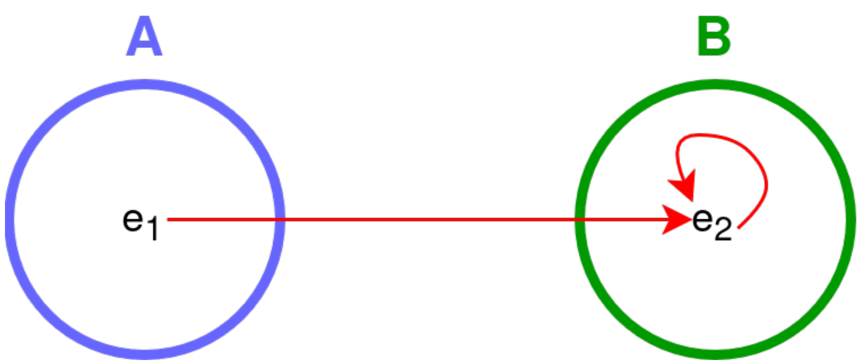
b. $[\![\exists x(A(x) \rightarrow R(x'', j))]\!]^{M_1,g_1} = 1$
iff $[\![\exists y(A(y) \rightarrow R(x'', j))]\!]^{M_1,g_1} = 1$
iff there is a $d \in U_1$ such that $[\![(A(y) \rightarrow R(x'', j))]\!]^{M_1,g_1[y/d]} = 1$
iff $[\![A(y)]\!]^{M_1,g_1[y/d]} = 0$ or $[\![R(x'', j)]\!]^{M_1,g_1[y/d]} = 1$
iff $[\![y]\!]^{M_1,g_1[y/d]} \notin [\![A]\!]$ or $[\![(x'', j)]\!]^{M_1,g_1[y/d]} \in [\![R]\!]^{M_1,g_1[y/d]}$
iff $g_1[y/d](y) \notin V_1(A)$ or $(g_1[y/d](x''), V_1(j)) \notin V_1(R)$
Pick any $d \in U_1 - V_1(A)$: let's set $d = e_1$
$e_1 \notin V_1(A) = \{e_5, e_6\} \Rightarrow [\![\exists x(A(x) \rightarrow R(x'', j))]\!]^{M_1,g_1} = 1$

c. $[\![\forall x(B(x) \rightarrow (A(x) \vee \exists x'(R(x', x))))]\!]^{M_1,g_1} = 1$
iff for all $d \in U_1$: $[\![B(x) \rightarrow (A(x) \vee \exists x'(R(x', x)))]\!]^{M_1,g_1[x/d]} = 1$
iff $[\![B(x)]\!]^{M_1,g_1[x/d]} = 0$ or $[\![A(x) \vee \exists x'(R(x', x))]\!]^{M_1,g_1[x/d]} = 1$
... we know how to do the rest from (a) and (b)
Evaluation w.r.t. M_1:
for all $d \in U_1 - V_1(B) = \{e_1, e_2, e_5\}$: $[\![B(x)]\!]^{M_1,g_1[x/d]} = 0$, so
$[\![B(x) \rightarrow (A(x) \vee \exists x'(R(x', x)))]\!]^{M_1,g_1[x/d]} = 1$
for all $d \in \{e_3, e_4\}$: $[\![\exists x'(R(x', x)]\!]^{M_1,g_1[x/d]} = 1$, so
$[\![B(x) \rightarrow (A(x) \vee \exists x'(R(x', x)))]\!]^{M_1,g_1[x/d]} = 1$
for $d = e_6$: $[\![A(x)]\!]^{M_1,g_1[x/d]} = 1$, so
$[\![B(x) \rightarrow (A(x) \vee \exists x'(R(x', x)))]\!]^{M_1,g_1[x/d]} = 1$
$\Rightarrow [\![\forall x(B(x) \rightarrow (A(x) \vee \exists x'(R(x', x))))]\!]^{M_1,g_1} = 1$

2.2 Provide a graphical representation of a model that satisfies the following formulas (NB: c_1 and c_2 are constants):

- $R(x, x')$
- $\forall x(A(x) \vee \exists x'(R(x, x')))$
- $\neg\exists x(R(x, c_1))$
- $\exists x''(B(x'') \wedge \neg\exists x'(A(x') \wedge R(x'', x')))$
- $\forall x'(A(x') \rightarrow (B(x') \vee R(x', c_2)))$

The simplest model I could think of:



Where:

$V_M(c_1) = e_1$, $V_M(c_2) = e_2$
$g(x) = e_1$, $g(x') = e_2$

2.3 (Bonus) Can you think of a sensible (or: funny) interpretation for the predicates A, B and R, and the constants c_1 and c_2 in your model of the previous exercise? Given this interpretation, what is the natural language translation of the formulas given in exercise 2.2?

Up to you!