Semantic Theory 2025: Exercise 6 Key

Question 1

A model structure for plural terms is a tuple $M = ((U_M, \leq), V_M)$, where $(U_M, \leq)$ is an atomic join semi-lattice with universe U_M and individual part relation $\leq$, and V_M is an interpretation function mapping elements of the logical language to elements of the universe.

Consider the model M_1, where the universe U_{M_1} is generated by the following set of atoms: $\{a, b, j, m, s\}$.

a. Assume that $[\![\textit{John, Mary, and Bill sing}]\!]^{M_1} = 1$, $[\![\textit{Albert sings}]\!]^{M_1} = 1$, and $[\![X \ \textit{sing}(s)]\!]^{M_1} = 0$ for all other individuals (and proper sums) X for the predicate $\textit{sing} \in P_d$. Then:

$V_{M_1}(\textit{sing}) =$ the join semi-lattice generated by $\{j, m, b, a\}$
$= \{j, m, b, a, j \sqcup m, j \sqcup b, j \sqcup a, m \sqcup b, m \sqcup a, b \sqcup a, j \sqcup m \sqcup b, j \sqcup m \sqcup a, j \sqcup b \sqcup a, m \sqcup b \sqcup a, j \sqcup m \sqcup b \sqcup a\}$

b. Assume that $[\![\textit{John and Mary meet}]\!]^{M_1} = 1$, $[\![\textit{Albert and Sally meet}]\!]^{M_1} = 1$, $[\![\textit{Bill and Mary meet}]\!]^{M_1} = 1$, and $[\![X \ \textit{meet}]\!]^{M_1} = 0$ for all other individuals (and proper sums) X for the predicate $\textit{meet} \in P_c$. Then:

$V_{M_1}(\textit{meet}) = \{j \sqcup m, a \sqcup s, b \sqcup m\}$

Question 2

Consider the following sentence: $S =$ "*two students summarized three papers*"

a. How many readings does the sentence S have? List all possible readings in natural language.

Let:

D_s = the distributive reading of "two students"
C_s = the collective reading of "two students"
D_p = the distributive reading of "three papers"
C_p = the collective reading of "three papers"

We have the following readings:

i. $C_s > C_p$: two students together wrote one summary that summarizes all three papers
ii. $C_s > D_p$: two students together wrote one summary for each of the three papers
iii. $D_s > C_p$: two students each wrote one summary that summarizes all three papers (not necessarily the same three for both students)
iv. $D_s > D_p$: two students each wrote one summary for each of the three papers (not necessarily the same three for both students)
v. $C_p > C_s$: same as (i)
vi. $C_p > D_s$: two students each wrote one summary that summarizes all three papers (the same three for both students)
vii. $D_p > C_s$: for each of the three papers, there was a group of two students that summarized that paper together
viii. $D_p > D_s$: for each of the three papers, there were two students who individually summarized that paper

b. Translate each reading of S to the extended first-order logic for plural terms introduced in the lecture, which extends first-order logic with a $\oplus$ operator, a $\lhd$ operator, and variables $\mathrm{X}, \mathrm{Y}, \mathrm{Z}, \ldots$ ranging over proper sums: $\mathrm{X} \oplus \mathrm{Y}$ denotes the group consisting of X and Y, $\lhd$ denotes the part-of-relation.

You may (and should) also incorporate the function $N(\mathrm{X}) = |\{y \mid At(y) \wedge y \lhd \mathrm{X}\}|$ that takes a proper sum X and returns the number of atoms in X.

i. $(C_s > C_p)$:
$\exists \mathrm{X} \exists \mathrm{Y}[\mathit{student}_{PL}(\mathrm{X}) \wedge N(\mathrm{X}) = 2 \wedge \mathit{paper}_{PL}(\mathrm{Y}) \wedge N(\mathrm{Y}) = 3 \wedge \mathit{summarize}(\mathrm{X}, \mathrm{Y})]$

ii. $(C_s > D_p)$:
$\exists \mathrm{X} \exists \mathrm{Y}[\mathit{student}_{PL}(\mathrm{X}) \wedge N(\mathrm{X}) = 2 \wedge \mathit{paper}_{PL}(\mathrm{Y}) \wedge N(\mathrm{Y}) = 3 \wedge \forall w[(At(w) \wedge w \lhd Y) \rightarrow \mathit{summarize}(\mathrm{X}, w)]]$

iii. $(D_s > C_p)$:
$\exists \mathrm{X}[\mathit{student}_{PL}(\mathrm{X}) \wedge N(\mathrm{X}) = 2 \wedge \forall z[(At(z) \wedge z \lhd \mathrm{X}) \rightarrow \exists \mathrm{Y}[\mathit{paper}_{PL}(\mathrm{Y}) \wedge N(\mathrm{Y}) = 3 \wedge \mathit{summarize}(z, \mathrm{Y})]]]$

iv. $(D_s > D_p)$:
$\exists \mathrm{X}[\mathit{student}_{PL}(\mathrm{X}) \wedge N(\mathrm{X}) = 2 \wedge \forall z[(At(z) \wedge z \lhd \mathrm{X}) \rightarrow \exists \mathrm{Y}[\mathit{paper}_{PL}(\mathrm{Y}) \wedge N(\mathrm{Y}) = 3 \wedge \forall w[(At(w) \wedge w \lhd \mathrm{Y}) \rightarrow \mathit{summarize}(z, w)]]]]$

v. $(C_p > C_s)$: same as (i)

vi. $(C_p > D_s)$:
$\exists \mathrm{Y}[\mathit{paper}_{PL}(\mathrm{Y}) \wedge N(\mathrm{Y}) = 3 \wedge \exists \mathrm{X}[\mathit{student}_{PL}(\mathrm{X}) \wedge N(\mathrm{X}) = 2 \wedge \forall z[(At(z) \wedge z \lhd \mathrm{X}) \rightarrow \mathit{summarize}(z, \mathrm{Y})]]]$

vii. $(D_p > C_s)$:
$\exists \mathrm{Y}[\mathit{paper}_{PL}(\mathrm{Y}) \wedge N(\mathrm{Y}) = 3 \wedge \forall w[(At(w) \wedge w \lhd \mathrm{Y}) \rightarrow \exists \mathrm{X}[\mathit{student}_{PL}(\mathrm{X}) \wedge N(\mathrm{X}) = 2 \wedge \mathit{summarize}(\mathrm{X}, w)]]]$

viii. $(D_p > D_s)$:
$\exists \mathrm{Y}[\mathit{paper}_{PL}(\mathrm{Y}) \wedge N(\mathrm{Y}) = 3 \wedge \forall w[(At(w) \wedge w \lhd \mathrm{Y}) \rightarrow \exists \mathrm{X}[\mathit{student}_{PL}(\mathrm{X}) \wedge N(\mathrm{X}) = 2 \wedge \forall z[(At(z) \wedge z \lhd \mathrm{X}) \rightarrow \mathit{summarize}(z, w)]]]]$