Semantic Theory 2025: Exercise 7 Key

Question 1

Recall the syntax for well-formed formulas (WFFs) in Dynamic Predicate Logic (DPL):

- All atomic formulas ($R(t_1, \ldots, t_n)$ for $R \in PRED^n$, $t_1, \ldots, t_n \in TERM$) are WFFs
- If $x \in VAR$, then $\exists x$ is a WFF
- If ϕ and ψ are WFFs, then $\sim\phi$ and $(\phi \cdot \psi)$ are WFFs
- Nothing else is a WFF

Translate the following natural language utterances into DPL. You may treat underlined expressions as single terms (e.g. *"came to"* $\Rightarrow$ *come-to*$(\ldots)$):

a. "*If John runs, he will pull a muscle.*"
$run(j') \rightarrow (\exists x \cdot muscle(x) \cdot pull(j', x))$
$= \sim(run(j') \cdot \sim(\exists x \cdot muscle(x) \cdot pull(j', x)))$

b. "*There was a party. A man came to the party. He was hungry.*"
$(\exists x \cdot party(x)) \cdot (\exists y \cdot man(y) \cdot come\text{-}to(y, x)) \cdot hungry(y)$
$= \exists x \cdot party(x) \cdot \exists y \cdot man(y) \cdot come\text{-}to(y, x) \cdot hungry(y)$

c. "*There is a farmer. She owns a donkey. If the donkey is hungry, she feeds it.*"
$(\exists x \cdot farmer(x)) \cdot (\exists y \cdot donkey(y) \cdot own(x, y)) \cdot (hungry(y) \rightarrow feed(x, y))$
$= \exists x \cdot farmer(x) \cdot \exists y \cdot donkey(y) \cdot own(x, y) \cdot \sim(hungry(y) \cdot \sim feed(x, y))$

Question 2

Translate the DPL formulas ϕ you constructed for 1a-c to FOL formulas $\phi^\circ = \langle\phi\rangle\top$, using the rules introduced in the lecture.

i. $\langle\bot\rangle\psi = \bot$

ii. $\langle\top\rangle\psi = \psi$

iii. $\langle P(x_1, \ldots, x_n)\rangle\psi = P(x_1, \ldots, x_n) \wedge \psi$

iv. $\langle\exists x\rangle\psi = \exists x[\psi]$

v. $\langle\phi_1 \cdot \phi_2\rangle\psi = \langle\phi_1\rangle(\langle\phi_2\rangle\psi)$

vi. $\langle\sim\phi\rangle\psi = \neg(\langle\phi\rangle\top) \wedge \psi$

Show each step of the derivation, and indicate the rule applied ((i)-(vi)) at each step:

a.

$$(\sim(run(j') \cdot \sim(\exists x \cdot muscle(x) \cdot pull(j',x))))^\circ$$
$$= \langle \sim(run(j') \cdot \sim(\exists x \cdot muscle(x) \cdot pull(j',x)))\rangle \top$$
$$= \neg(\langle run(j') \cdot \sim(\exists x \cdot muscle(x) \cdot pull(j',x))\rangle \top) \wedge \top \qquad \text{(vi)}$$
$$= \neg(\langle run(j')\rangle(\langle \sim(\exists x \cdot muscle(x) \cdot pull(j',x))\rangle \top)) \wedge \top \qquad \text{(v)}$$
$$= \neg(\langle run(j')\rangle(\neg(\langle \exists x \cdot muscle(x) \cdot pull(j',x)\rangle \top) \wedge \top)) \wedge \top \qquad \text{(v)}$$
$$= \neg(\langle run(j')\rangle(\neg(\langle \exists x\rangle(\langle muscle(x) \cdot pull(j',x)\rangle \top)) \wedge \top)) \wedge \top \qquad \text{(v)}$$
$$= \neg(\langle run(j')\rangle(\neg(\langle \exists x\rangle(\langle muscle(x)\rangle(\langle pull(j',x)\rangle \top))) \wedge \top)) \wedge \top \qquad \text{(v)}$$
$$= \neg(\langle run(j')\rangle(\neg(\langle \exists x\rangle(\langle muscle(x)\rangle(pull(j',x) \wedge \top))) \wedge \top)) \wedge \top \qquad \text{(iii)}$$
$$= \neg(\langle run(j')\rangle(\neg(\langle \exists x\rangle(muscle(x) \wedge pull(j',x) \wedge \top)) \wedge \top)) \wedge \top \qquad \text{(iii)}$$
$$= \neg(\langle run(j')\rangle(\neg(\exists x[muscle(x) \wedge pull(j',x) \wedge \top]) \wedge \top)) \wedge \top \qquad \text{(iv)}$$
$$= \neg(run(j') \wedge \neg(\exists x[muscle(x) \wedge pull(j',x) \wedge \top]) \wedge \top) \wedge \top \qquad \text{(iv)}$$
$$= \neg(run(j') \wedge \neg\exists x[muscle(x) \wedge pull(j',x)])$$
$$= run(j') \rightarrow \exists x[muscle(x) \wedge pull(j',x)]$$

b.

$$(\exists x \cdot party(x) \cdot \exists y \cdot man(y) \cdot come\text{-}to(y,x) \cdot hungry(y))^\circ$$
$$= \langle \exists x \cdot party(x) \cdot \exists y \cdot man(y) \cdot come\text{-}to(y,x) \cdot hungry(y)\rangle \top$$
$$= \langle \exists x\rangle(\langle party(x) \cdot \exists y \cdot man(y) \cdot come\text{-}to(y,x) \cdot hungry(y)\rangle \top) \qquad \text{(v)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\langle \exists y \cdot man(y) \cdot come\text{-}to(y,x) \cdot hungry(y)\rangle \top)) \qquad \text{(v)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\langle \exists y\rangle(\langle man(y) \cdot come\text{-}to(y,x) \cdot hungry(y)\rangle \top))) \qquad \text{(v)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\langle \exists y\rangle(\langle man(y)\rangle(\langle come\text{-}to(y,x) \cdot hungry(y)\rangle \top)))) \qquad \text{(v)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\langle \exists y\rangle(\langle man(y)\rangle(\langle come\text{-}to(y,x)\rangle(\langle hungry(y)\rangle \top))))) \qquad \text{(v)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\langle \exists y\rangle(\langle man(y)\rangle(\langle come\text{-}to(y,x)\rangle(hungry(y) \wedge \top))))) \qquad \text{(iii)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\langle \exists y\rangle(\langle man(y)\rangle(come\text{-}to(y,x) \wedge hungry(y) \wedge \top)))) \qquad \text{(iii)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\langle \exists y\rangle(man(y) \wedge come\text{-}to(y,x) \wedge hungry(y) \wedge \top))) \qquad \text{(iii)}$$
$$= \langle \exists x\rangle(\langle party(x)\rangle(\exists y[man(y) \wedge come\text{-}to(y,x) \wedge hungry(y) \wedge \top])) \qquad \text{(iv)}$$
$$= \langle \exists x\rangle(party(x) \wedge \exists y[man(y) \wedge come\text{-}to(y,x) \wedge hungry(y) \wedge \top]) \qquad \text{(iii)}$$
$$= \langle \exists x\rangle(party(x) \wedge \exists y[man(y) \wedge come\text{-}to(y,x) \wedge hungry(y) \wedge \top]) \qquad \text{(iv)}$$
$$= \exists x[party(x) \wedge \exists y[man(y) \wedge come\text{-}to(y,x) \wedge hungry(y) \wedge \top]] \qquad \text{(iv)}$$
$$= \exists x[party(x) \wedge \exists y[man(y) \wedge come\text{-}to(y,x) \wedge hungry(y)]]$$

c.

$$(\exists x \cdot F(x) \cdot \exists y \cdot D(y) \cdot O(x,y) \cdot \sim(H(y) \cdot \sim \mathit{feed}(x,y)))^{\circ}$$

$$= \langle \exists x \cdot F(x) \cdot \exists y \cdot D(y) \cdot O(x,y) \cdot \sim(H(y) \cdot \sim \mathit{feed}(x,y))\rangle \top$$

$$= \langle \exists x\rangle(F(x) \cdot \exists y \cdot D(y) \cdot O(x,y) \cdot \sim(H(y) \cdot \sim \mathit{feed}(x,y))\rangle \top) \qquad \text{(v)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\exists y \cdot D(y) \cdot O(x,y) \cdot \sim(H(y) \cdot \sim \mathit{feed}(x,y))\rangle \top)) \qquad \text{(v)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(D(y) \cdot O(x,y) \cdot \sim(H(y) \cdot \sim \mathit{feed}(x,y))\rangle \top))) \qquad \text{(v)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(O(x,y) \cdot \sim(H(y) \cdot \sim \mathit{feed}(x,y))\rangle \top)))) \qquad \text{(v)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(\langle O(x,y)\rangle(\langle \sim(H(y) \cdot \sim \mathit{feed}(x,y))\rangle \top))))) \qquad \text{(v)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(\langle O(x,y)\rangle(\neg(\langle H(y) \cdot \sim \mathit{feed}(x,y)\rangle \top) \wedge \top))))) \qquad \text{(vi)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(\langle O(x,y)\rangle(\neg(\langle H(y)\rangle(\langle \sim \mathit{feed}(x,y)\rangle \top)) \wedge \top))))) \qquad \text{(v)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(\langle O(x,y)\rangle(\neg(\langle H(y)\rangle(\neg(\langle \mathit{feed}(x,y)\rangle \top) \wedge \top)) \wedge \top))))) \qquad \text{(vi)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(\langle O(x,y)\rangle(\neg(\langle H(y)\rangle(\neg(\mathit{feed}(x,y) \wedge \top) \wedge \top)) \wedge \top))))) \qquad \text{(iii)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(\langle O(x,y)\rangle(\neg(H(y) \wedge \neg(\mathit{feed}(x,y) \wedge \top) \wedge \top) \wedge \top))))) \qquad \text{(iii)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(O(x,y) \wedge \neg(H(y) \wedge \neg(\mathit{feed}(x,y) \wedge \top) \wedge \top) \wedge \top)))) \qquad \text{(iii)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(\langle D(y)\rangle(O(x,y) \wedge \neg(H(y) \wedge \neg(\mathit{feed}(x,y) \wedge \top) \wedge \top) \wedge \top)))) \qquad \text{(iii)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\langle \exists y\rangle(D(y) \wedge O(x,y) \wedge \neg(H(y) \wedge \neg(\mathit{feed}(x,y) \wedge \top) \wedge \top) \wedge \top))) \qquad \text{(iii)}$$

$$= \langle \exists x\rangle(\langle F(x)\rangle(\exists y[D(y) \wedge O(x,y) \wedge \neg(H(y) \wedge \neg(\mathit{feed}(x,y) \wedge \top) \wedge \top) \wedge \top])) \qquad \text{(iv)}$$

$$= \langle \exists x\rangle(F(x) \wedge \exists y[D(y) \wedge O(x,y) \wedge \neg(H(y) \wedge \neg(\mathit{feed}(x,y) \wedge \top) \wedge \top) \wedge \top]) \qquad \text{(iii)}$$

$$= \exists x[F(x) \wedge \exists y[D(y) \wedge O(x,y) \wedge \neg(H(y) \wedge \neg(\mathit{feed}(x,y) \wedge \top) \wedge \top) \wedge \top]] \qquad \text{(iv)}$$

$$= \exists x[F(x) \wedge \exists y[D(y) \wedge O(x,y) \wedge \neg(H(y) \wedge \neg \mathit{feed}(x,y))]]$$

$$= \exists x[F(x) \wedge \exists y[D(y) \wedge O(x,y) \wedge (H(y) \rightarrow \mathit{feed}(x,y))]]$$