Updated schedule:

- you need to hand in ~~7 of 10~~ **6 of 9** exercises to be admitted to the exam

Week	Reading	Tuesday	Wednesday
Week 1: April 15-16	None	Introduction	No lecture
Week 2: April 22-23	1. *Logic in Action*, Ch. 4 (Sec. 4.5-4.6) 2. *Elements of Formal Semantics*, Ch. 2	Predicate Logic	Overflow (*if necessary*)
Week 3: April 29-30	*Elements of Formal Semantics*, Ch. 3 (Parts 1-2)	Type Theory	Exercise 1: Predicate Logic
Week 4: May 6-7	*Elements of Formal Semantics*, Ch. 3 (Part 3)	Lambda Calculus	Exercise 2: Type Theory
Week 5: May 13-14	Generalized Quantifiers (Stanford Encyclopedia of Philosophy)	Generalized Quantifiers	Exercise 3: Lambda Calculus
Week 6: May 20-21	Event-Based Semantics (Lasersohn, 2012)	Event Semantics	Exercise 4: Generalized Quantifiers
Week 7: May 27-28	None	Lexical Semantics	Exercise 5: Event Semantics
Week 8: June 3-4	Dynamic Semantics (Stanford Encyclopedia of Philosophy)	Dynamic Semantics	Exercise 6: Lexical Semantics
Week 9: June 10-11	Discourse Representation Theory (Stanford Encyclopedia of Philosophy)	DRT	Exercise 7: Dynamic Semantics
Week 10: June 17-18	None	Presuppositions in DRT	Exercise 8: DRT
Week 11: June 24-25	None	~~Implicature~~ Current Issues and Applications	Exercise 9: Presuppositions in DRT
Week 12: July 1-2	None	~~Current Issues and Applications~~ Exam Review	~~Exercise 10: Implicature~~ Take-home Practice Exam
Week 13: July 8-9	None	~~Exam Review~~ *No lecture*	~~Take-home Practice Exam~~ *No lecture*
Week 14: July 15-16	None	**Exam**	No lecture

Discourse Representation Theory (DRT)

Week 9

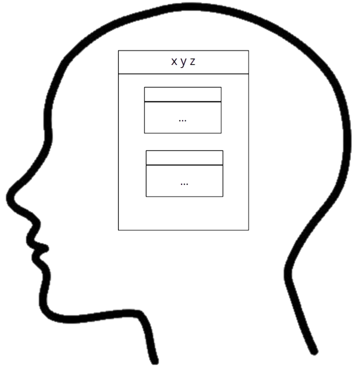
Slides and materials based on the courses by
Noortje Venhuizen and Mareike Hartmann

Recap: dynamic semantics

- Basic semantic value: context-change potential
- Existential quantification over: the discourse
- Quantification is: unselective

Discourse Representation Theory

- **Mentalist**, **representationalist**, and *dynamic* theory of the interpretation of discourse



- Ingredients:
  - Discourse Representation Structures (DRSs)
  - Construction procedure for DRSs
  - Model-theoretic interpretation (at the discourse level)

Basic features of DRT

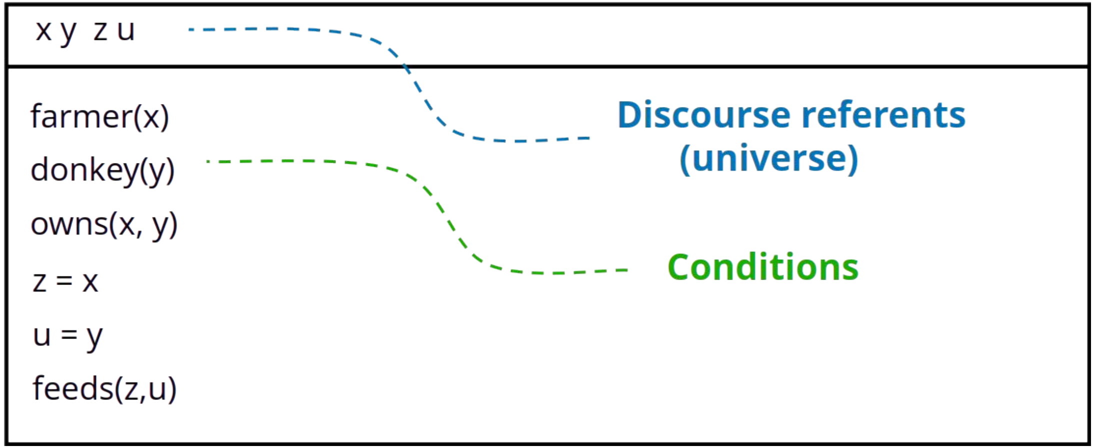
- DRT models linguistic meaning as **anaphoric potential** (through DRS construction) plus truth conditions (through model embedding)

- DRT explains the ambivalent character of indefinite noun phrases:
  - Indefinite NPs are expressions that introduce new reference objects into the context (like DPL)

Discourse Representation Structures

- A context is represented as a **Discourse Representation Structure** (DRS) consisting of a set of **discourse referents** and a set of **conditions**

"A farmer owns a donkey. He feeds it."



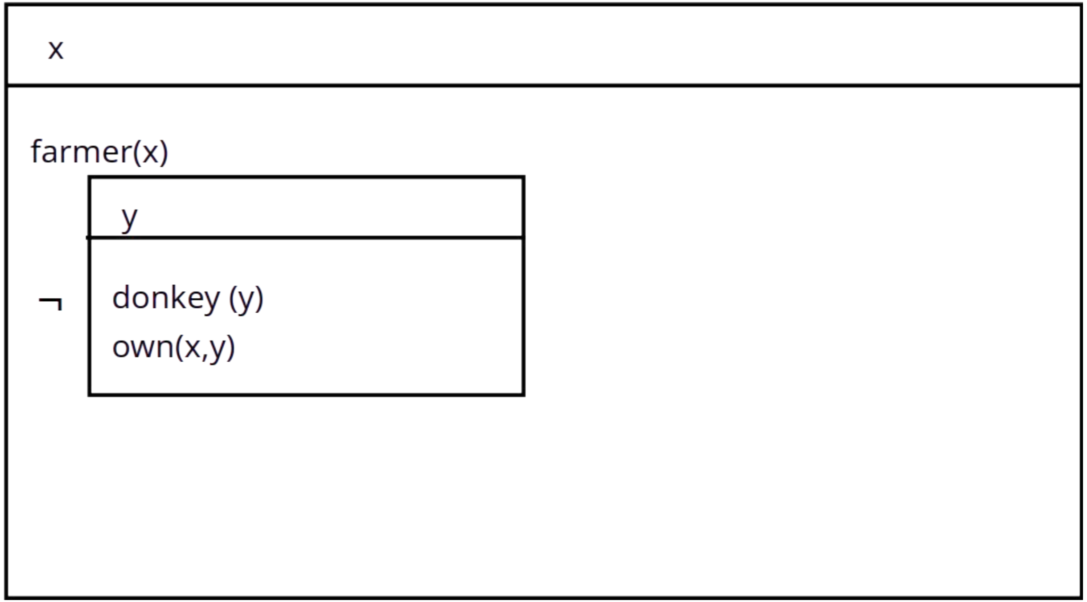
DRS syntax

- A discourse representation structure (DRS) K is a pair (U_K, C_K), where:
  - $U_K \subseteq U_D$ and U_D is a set of **discourse referents**
  - C_K is a set of well-formed **DRS conditions**

- Well-formed DRS conditions:
  - $R(u_1, \ldots, u_n)$ where: R is an n-place relation, $u_i \in U_D$
  - $u = v$ $u, v \in U_D$
  - $u = a$ $u \in U_D$, a is a constant
  - $\neg K_1$ K_1 is a DRS
  - $K_1 \Rightarrow K_2$ K_1 and K_2 are DRSs
  - $K_1 \vee K_2$ K_1 and K_2 are DRSs

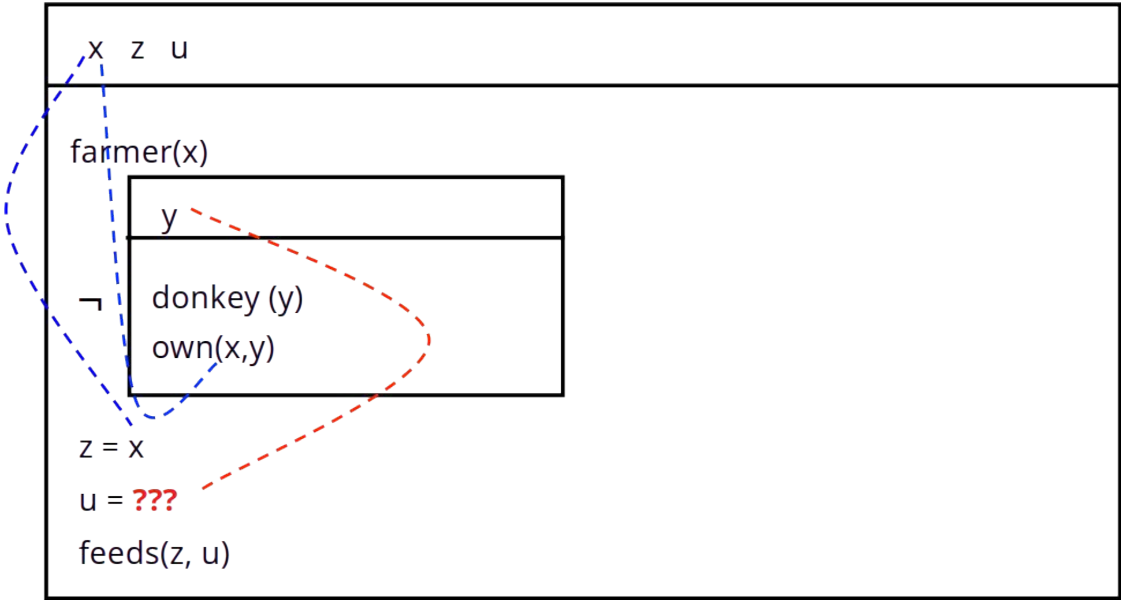
DRS example

"A farmer does not own a donkey."



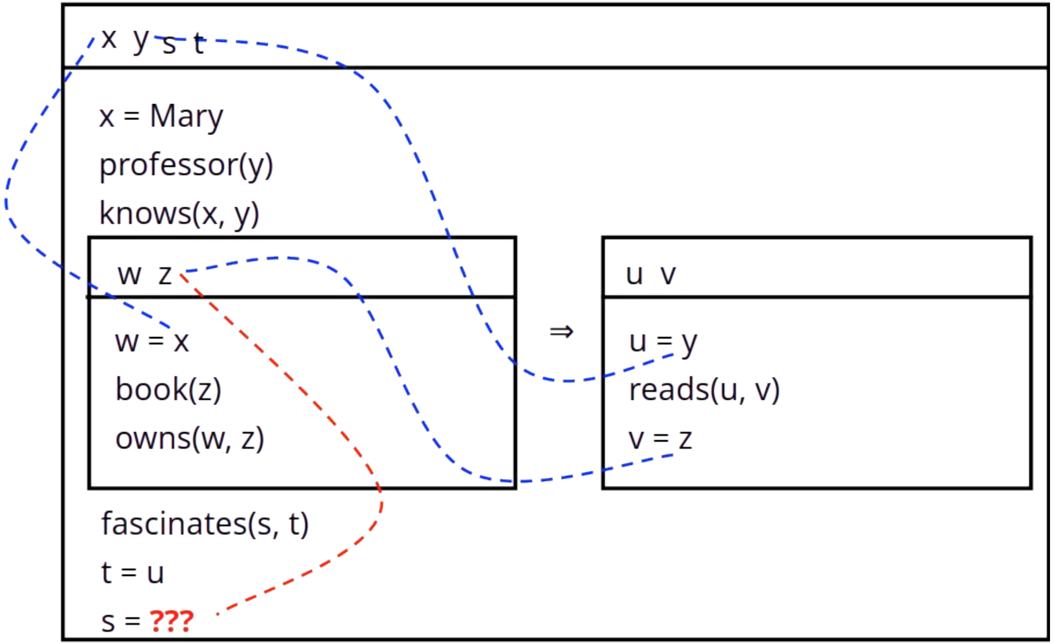
Anaphora and accessibility

"A farmer does not own a donkey. **#** *He feeds it."*



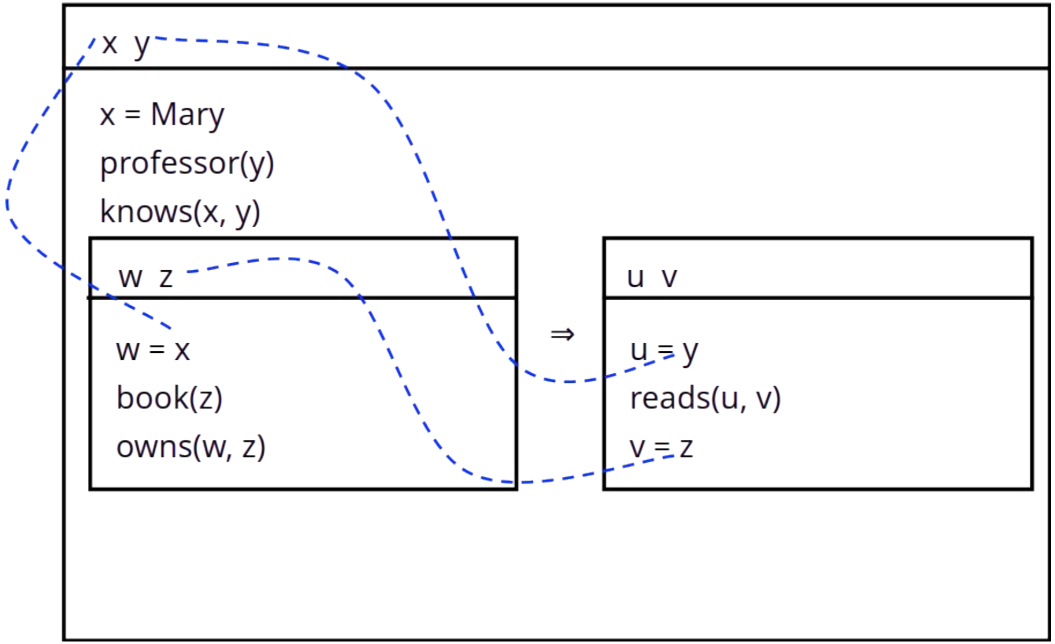
Anaphora and accessibility

"Mary knows a professor. If she owns a book, he reads it. **#** *It fascinates him."*



Anaphora and accessibility

"Mary knows a professor. If she owns a book, he reads it."



Non-accessible discourse referents

- *“If a professor owns a book, he reads it. It has 300 pages.”*
- *“It is not the case that a professor owns a book. He reads it.”*
- *“Every professor owns a book. He reads it.”*
- *“If every professor owns a book, he reads it.”*
- *“Peter owns a book, or Mary reads it.”*
- *“Peter reads a book, or Mary reads a newspaper article. It is interesting.”*

- To explain this pattern, we need to formalize accessibility of discourse referents!

Accessible discourse referents

- The following discourse referents are accessible from a DRS condition:
    - Referents in the same local DRS
    - Referents in a superordinate DRS
    - Referents in an antecedent DRS, if the condition occurs in the consequent DRS

- We need a formal notion of DRS subordination

Subordination

- DRS K_1 is an **immediate sub-DRS** of a DRS $K = (U_K, C_K)$ iff
  - C_K contains a condition of the form: $\neg K_1$, $K_1 \Rightarrow K_2$, $K_2 \Rightarrow K_1$, $K_1 \vee K_2$ or $K_2 \vee K_1$

- DRS K_1 is a **sub-DRS** of DRS K (notation: $K_1 \leq K$) iff
  - $K_1 = K$, or
  - K_1 is an immediate sub-DRS of K, or
  - there is a DRS K_2 such that $K_1 \leq K_2$ and K_2 is an immediate sub-DRS of K

- DRS K_1 is a **proper sub-DRS** of DRS K iff
  - $K_1 \leq K$ and $K_1 \neq K$

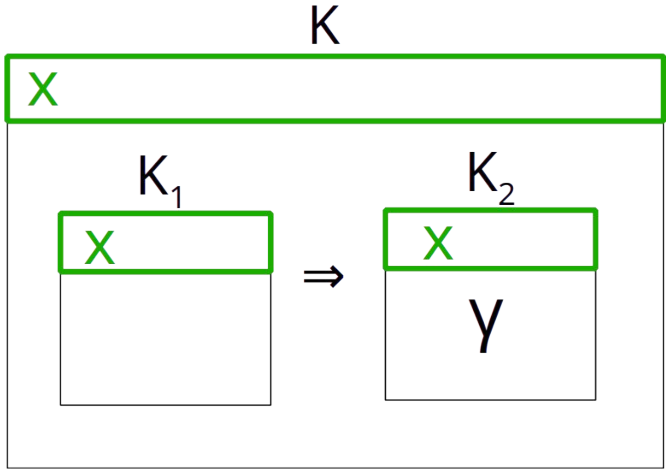
Accessible discourse referents: formal definition

- Let K, K_n, K_m be DRSs such that:

  $K_n, K_m \leq K$ and $x \in U_{K_\square}$ and $\gamma \in C_{K_\square}$

- Then, **x is accessible from γ in K** iff
  - $K_m \leq K_n$ or
  - there are $K_h, K_i \leq K$ such that:

    $K_n \Rightarrow K_h \in C_{K_i}$ and $K_m \leq K_h$



Free and bound variables in DRT

- A DRS variable x, introduced in the conditions of DRS K_1, is **bound** in global DRS K iff there exists a DRS $K_2 \leq K$, such that:
  - $x \in U_{K_2}$, and
  - K_2 is accessible from K_1 in K

- **Properness**: a DRS is proper iff it does not contain any free variables

- **Purity**: a DRS is pure iff it does not contain any otiose declarations of variables
  - i.e. $x \in U_{K_1}$ and $x \in U_{K_2}$ and $K_1 \leq K_2$

From text to DRS

Text

Syntactic analysis

DRS

$$
\begin{tikzcd}
\Sigma = \langle S_1, \arrow[d] & S_2, \arrow[d] & \dots, & S_n \rangle \arrow[d] \\
P(S_1) \arrow[d] & P(S_2) \arrow[d] & \dots & P(S_n) \arrow[d] \\
K_1 \arrow[r] & K_2 \arrow[r] & \dots \arrow[r] & K_n
\end{tikzcd}
$$

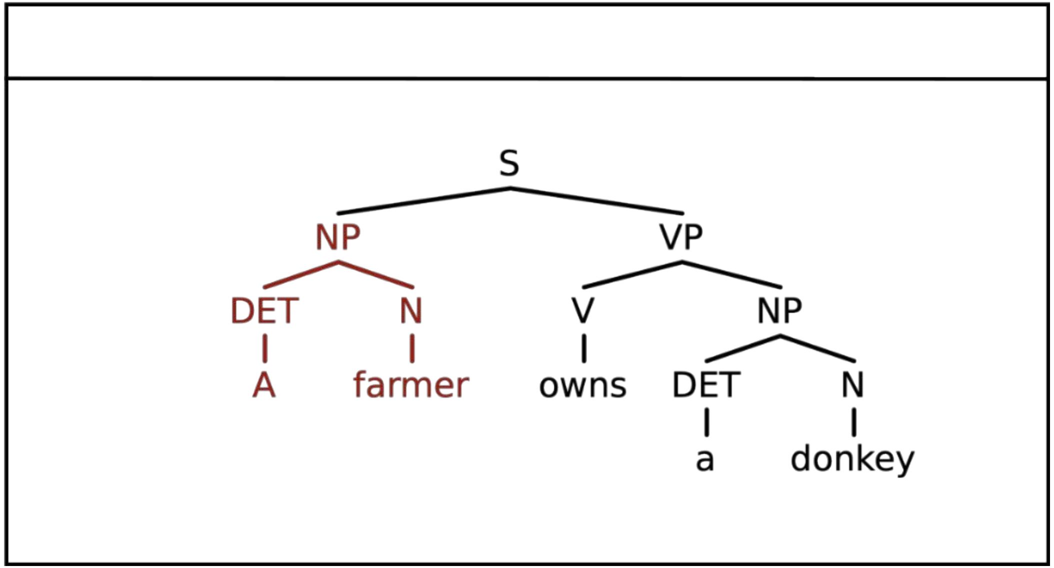
DRS construction algorithm

- Let the following be a well-formed, **reducible** DRS condition:
  - Conditions of form α or $\alpha(x_1, \ldots, x_n)$, where α is a context-free parse tree

- DRS construction algorithm:
  - Given a text $\Sigma = (S_1, \ldots, S_n)$, and a DRS K_0 (= (∅, ∅), by default)
  - Repeat for $i = 1, \ldots, n$:
    - Add parse tree $P(S_i)$ to the conditions of K_{i-1}
    - Apply DRS construction rules to reducible conditions of K_{i-1}, until no more reduction steps are possible
    - The resulting DRS is K_i , the discourse representation of text Σ

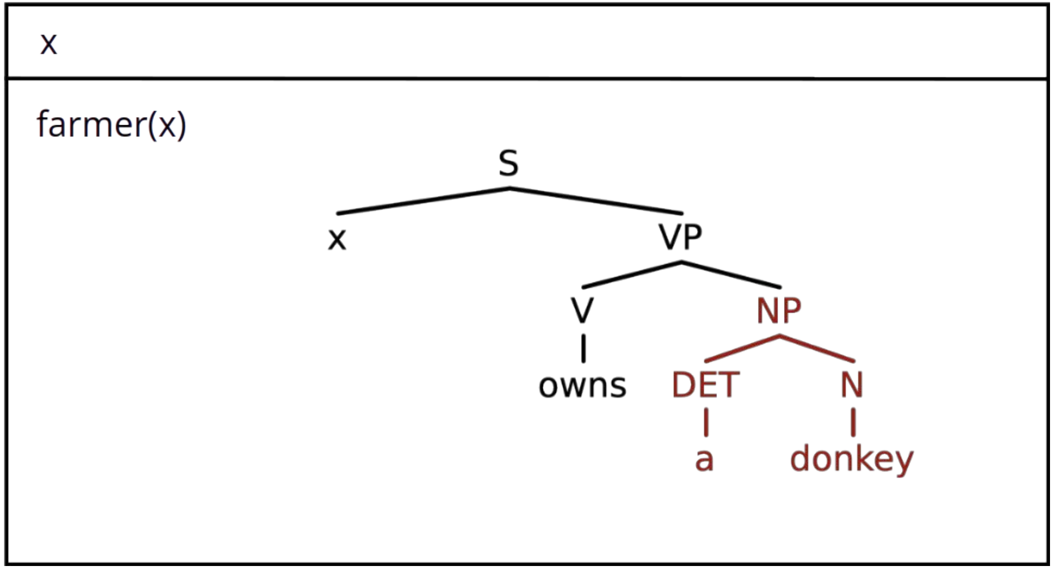
DRS construction: example

"A farmer owns a donkey. He beats it."



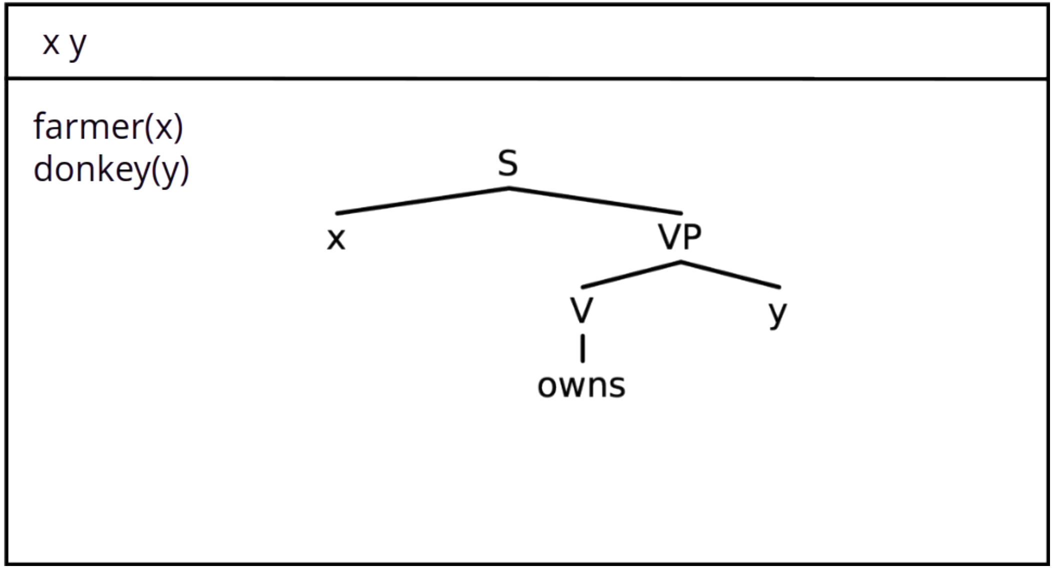
DRS construction: example

“A farmer owns a donkey. He beats it.”



DRS construction: example

"A farmer owns a donkey. He beats it."



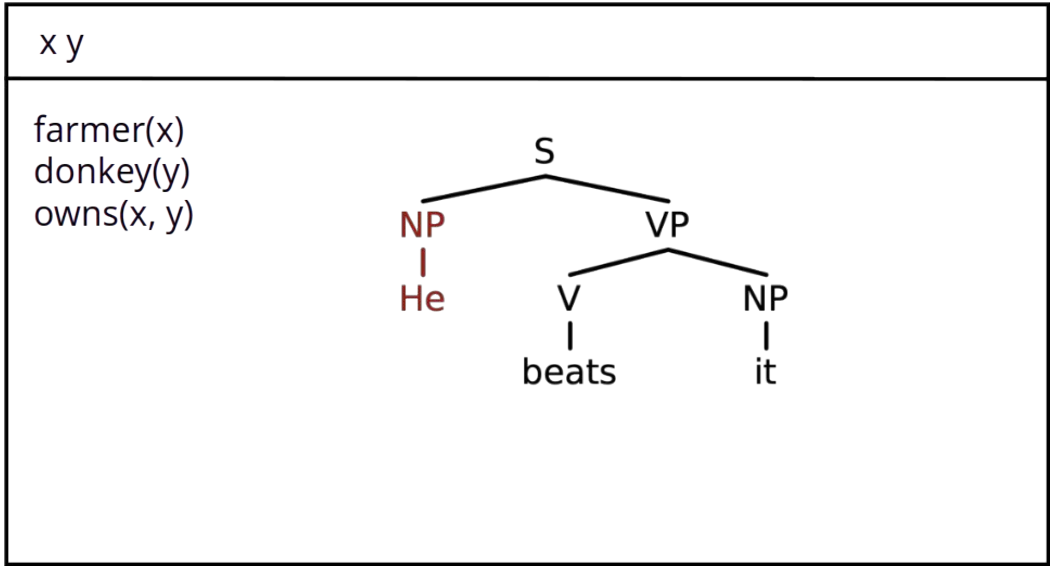
DRS construction: example

“A farmer owns a donkey. He beats it.”

x y
farmer(x) donkey(y) owns(x, y)

DRS construction: example

"A farmer owns a donkey. He beats it."

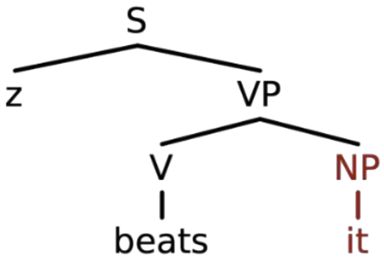


DRS construction: example

“A farmer owns a donkey. He beats it.”

x y z

farmer(x)
donkey(y)
owns(x, y)
z=x



DRS construction: example

"A farmer owns a donkey. He beats it."

x y z u
farmer(x) donkey(y) owns(x, y) z = x u = y beat(z, u)

Construction rules: indefinite NPs

- given a reducible condition α in DRS K, with [S [NP β] [VP γ]] or [VP [V γ] [NP β]] as a substructure, such that β = εδ, where ε is an indefinite article:

  (i) add a new discourse referent x to U_K

  (ii) replace β in α by x

  (iii) add δ(x) to C_K

Construction rules: proper names

- Given a global DRS K^*, and some $K \leq K^*$, such that α is a reducible condition in DRS K, with [S [NP β] [VP γ]] or [VP [V γ] [NP β]] as a substructure, such that β is a proper name

  (i) add a new discourse referent x to U_{K^*}

  (ii) replace β in α by x

  (iii) add $x = \beta$ to C_{K^*}

Construction rules: conditionals

- Given a reducible condition α in DRS K, with [S if [S β] (then) [S γ]] as a substructure:

  (i) remove α from C_K

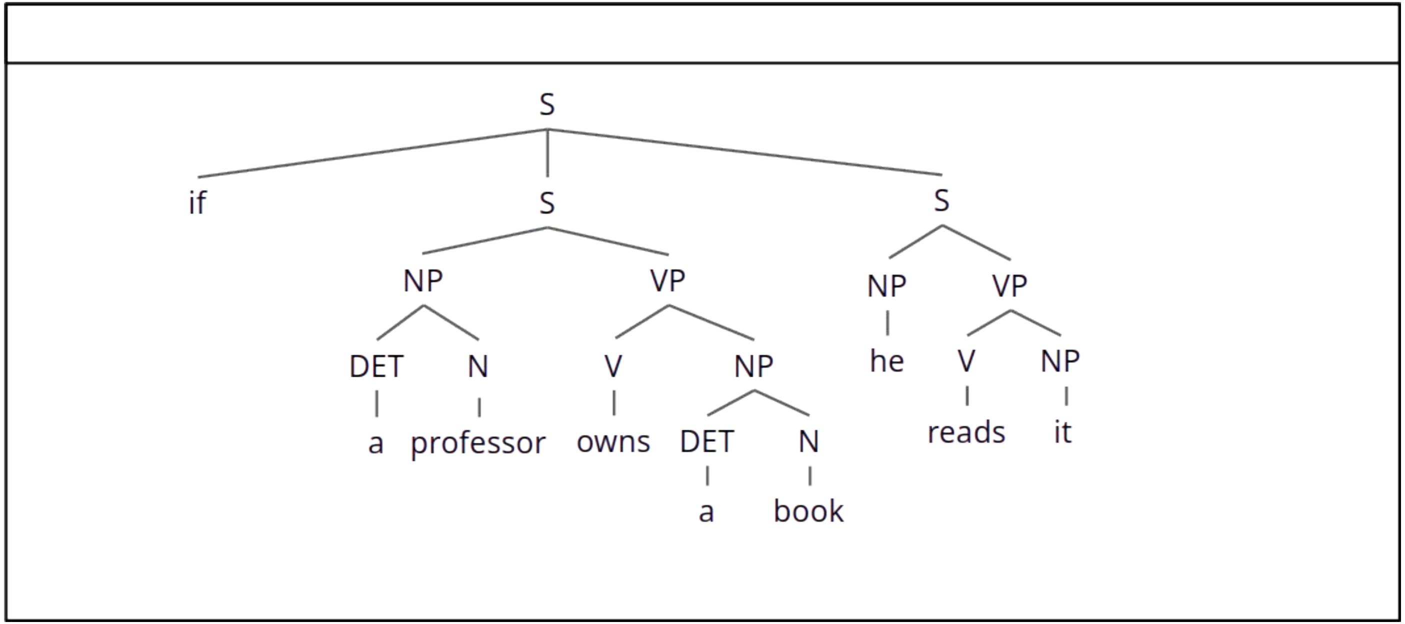
  (ii) add $K_1 \Rightarrow K_2$ to C_K, such that:

  - $K_1 = (\varnothing, \{\beta\})$
  - $K_2 = (\varnothing, \{\gamma\})$

Remark: $K_1 \Rightarrow K_2$ is called a **duplex condition**—K_1 is the **antecedent** DRS and K_2 is the **consequent** DRS

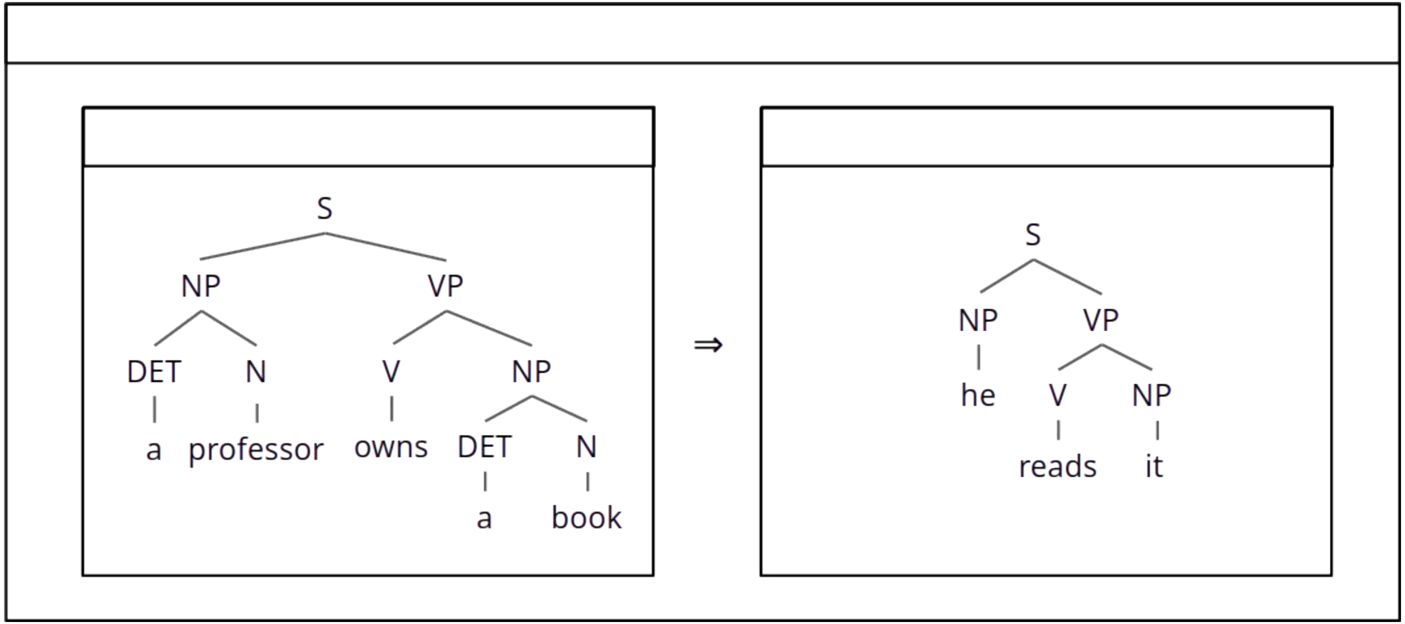
Construction rules: conditionals

"if a professor owns a book, he reads it"



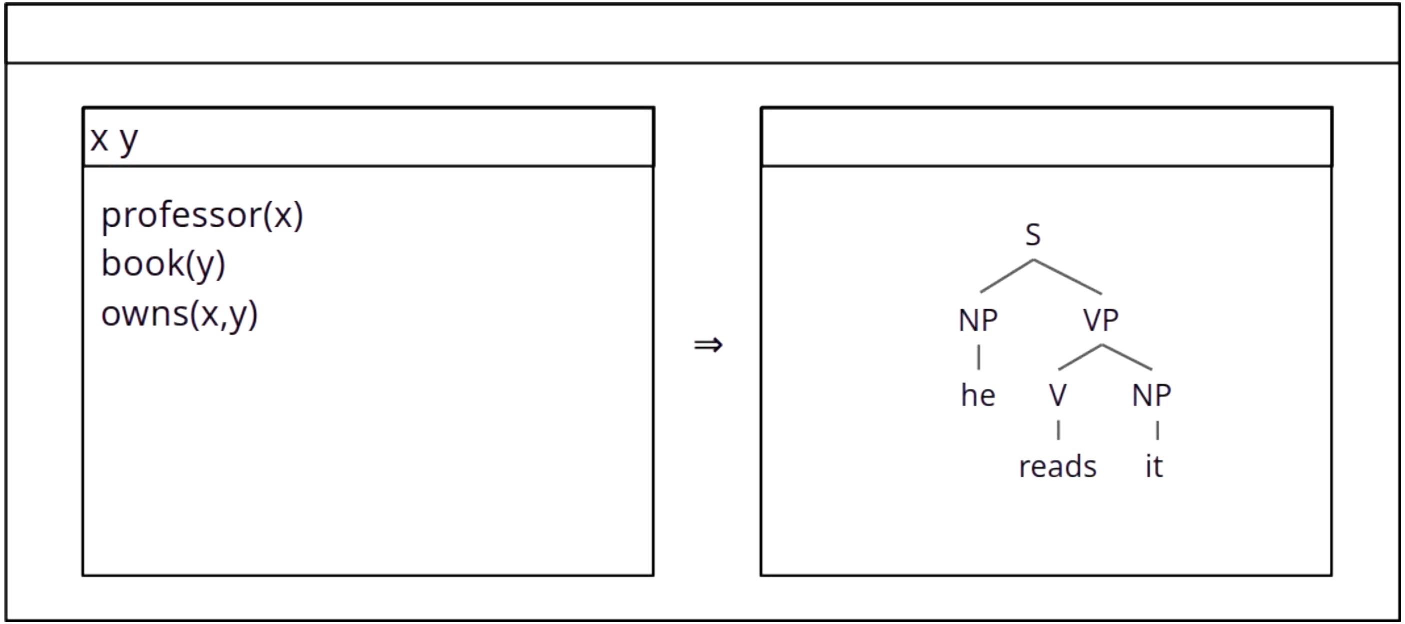
Construction rules: conditionals

“if a professor owns a book, he reads it”



Construction rules: conditionals

“if a professor owns a book, he reads it”



Construction rules: conditionals

“if a professor owns a book, he reads it”

x y		z v
professor(x) book(y) owns(x,y)	⇒	reads(z,v) z = x v = y

Construction rules: universal NPs

- given a reducible condition α in DRS K, with [S [NP β] [VP γ]] or [VP [V γ] [NP β]] as a substructure, such that $\beta = \varepsilon\delta$, where ε is a universal quantifier:

  (i) remove α from C_K

  (ii) add $K_1 \Rightarrow K_2$ to C_K, such that:

  - $K_1 = (\{x\}, \{\delta(x)\})$
  - $K_2 = (\varnothing, \{\alpha'\})$
    - where α' is obtained from α by replacing β with x

Construction rules: negation

- given a reducible condition α in DRS K, with [S β [VP doesn't [VP γ]]] as a substructure

  (i) remove α from C_K

  (ii) add $\neg K_1$ to C_K, such that:

    - $K_1 = (\varnothing, \{$ [S β [VP γ] $\})$

Construction rules: clausal disjunction

- given a reducible condition α in DRS K, with [[S β] or [S γ]] as a substructure

    (i) remove α from C_K

    (ii) add $K_1 \vee K_2$ to C_K, such that:

    - $K_1 = (\varnothing, \{\beta\})$
    - $K_2 = (\varnothing, \{\gamma\})$

From text to DRS

Text	$\Sigma = \langle\, S_1,$	$S_2,$	$\ldots,$	$S_n \,\rangle$
	$\downarrow$	$\downarrow$		$\downarrow$
Syntactic analysis	$P(S_1)$	$P(S_2)$	$\ldots$	$P(S_n)$
	$\downarrow$	$\downarrow$		$\downarrow$
DRS	$K_1 \longrightarrow$	$K_2 \longrightarrow$	$\ldots \longrightarrow$	K_n $\longrightarrow$ Interpretation by model embedding: truth conditions of Σ

DRS interpretation: model embedding

- Given a DRS $K = (U_K, C_K)$, with $U_K \subseteq U_D$:
  - Let $M = (U_M, V_M)$ be a FOL model structure that is **appropriate for** K, i.e. a model structure that provides interpretations for all predicates and relations in K

- K is true in M iff:
  - there exists an **embedding function** for K in M that verifies all conditions in K

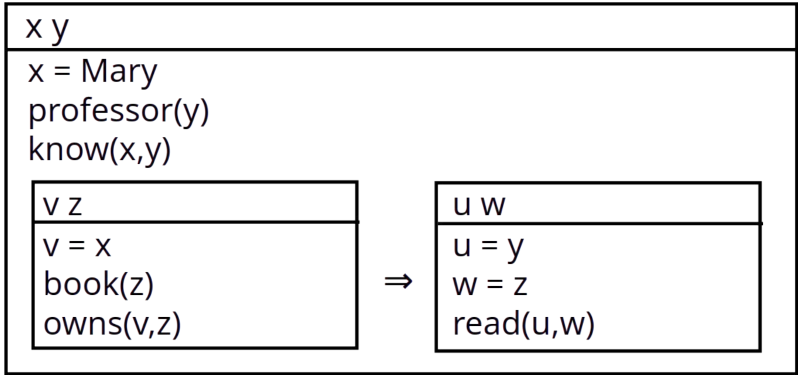
- An embedding function for DRS K in model M is defined as:
  - a (partial) function $f\colon U_D \nrightarrow U_M$ such that $U_K \subseteq dom(f)$

$g \supseteq_U f :=$
$(dom(g) = dom(f) \cup U) \wedge$
$\forall x[x \in dom(f) \rightarrow f(x) =$
$g(x)]$

Verifying by embedding

- An embedding f of K in M **verifies K in M** ($f \vDash_M K$) iff f verifies every condition $\alpha \in C_K$ ($f \vDash_M K$ for all $\alpha \in C_K$)
  - $f \vDash_M R(x_1, \ldots, x_n)$ iff $(f(x_1), \ldots, f(x_n)) \in V_M(R)$
  - $f \vDash_M x = y$ iff $f(x) = f(y)$
  - $f \vDash_M x = a$ iff $f(x) = V_M(a)$
  - $f \vDash_M \neg K_1$ iff there is no $g \supseteq_{U_{\square_1}} f$ such that $g \vDash_M K_1$
  - $f \vDash_M K_1 \vee K_2$ iff there is a $g_1 \supseteq_{U_{\square_1}} f$ such that $g_1 \vDash_M K_1$ or there is a $g_2 \supseteq_{U_{\square_2}} f$ such that $g_2 \vDash_M K_2$
  - $f \vDash_M K_1 \Rightarrow K_2$ iff for all $g_1 \supseteq_{U_{\square_1}} f$ such that $g_1 \vDash_M K_1$: there is a $g_2 \supseteq_{U_{\square_2}} g_1$ such that $g_2 \vDash_M K_2$

Verifying by embedding: example

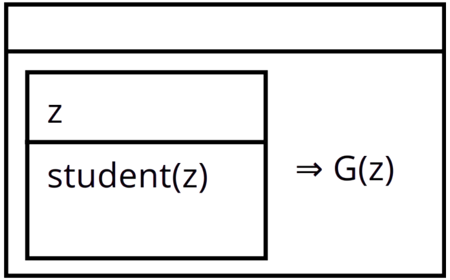


- *"Mary knows a professor. If she owns a book, he reads it."* is true in $M = (U_M, V_M)$ iff there is an $f: U_D \rightarrow U_M$, (with $\{x, y\} \subseteq dom(f)$) s.t.:
  - $f(x) = V_M(mary)$ and $f(y) \in V_M(professor)$
  - and $(f(x), f(y)) \in V_M(know)$
  - and for all $g \supseteq_{\{v, z\}} f$ such that $g(v) = g(x)$, $g(z) \in V_M(book)$ and $(g(v), g(z)) \in V_M(own)$:
    - there exists $h \supseteq_{\{u, w\}} g$ s.t. $h(u) = h(y)$, $h(w) = h(z)$, and $(h(u), h(w)) \in V_M(read)$

DRT and compositionality

- DRT is a **representational** theory of meaning
  - Structural information that cannot be reduced to truth conditions is required to compute the semantic value of discourses
- DRT is **non-compositional** on truth conditions (in the traditional sense)
  - The difference in discourse-semantic status of the text pairs is not predictable through the truth conditions of its component sentences
- But wait a minute… can't we just combine type theoretic semantics and DRT?
  - Use λ-abstraction and reduction as before, but where the target (type t) representations are DRSs, not formulas from type theory (or FOL)
  - This is called **λ-DRT**

λ-DRT

- An expression in λ-DRT consists of a lambda prefix and a partially instantiated DRS
  - *"every student"* :: $\langle\langle e, t\rangle, t\rangle \mapsto \lambda G.$ 

- Alternative notation: λG.[∅ | [z | *student*(z)] ⇒ G(z)]
  - *"works"* :: $\langle e, t\rangle \mapsto$ λx.[∅ | *work*(x)]

λ-DRT: β-reduction

- *"every student works"*

$$\mapsto \lambda G.[\ \varnothing \mid [\ z \mid \textit{student}(z)\ ] \Rightarrow G(z)\ ]]\ \ (\lambda x.[\ \varnothing \mid \textit{work}(x)\ ])$$

$$\Rightarrow_\beta [\ \varnothing \mid [\ z \mid \textit{student}(z)\ ] \Rightarrow (\lambda x.[\ \varnothing \mid \textit{work}(x)\ ])(z)\ ]$$

$$\Rightarrow_\beta [\ \varnothing \mid [\ z \mid \textit{student}(z)\ ] \Rightarrow [\ \varnothing \mid \textit{work}(z)\ ]]$$

- Question: how do we define conjunction on DRSs?

Simple DRS merge: first try

- The **merge** operation (notation: $K_1 + K_2$) on two DRSs combines the universes and conditions of both DRSs into a new DRS

- Let $K_1 = [\ U_1 \mid C_1\ ]$ and $K_2 = [\ U_2 \mid C_2\ ]$:
  - $K_1 + K_2 = [\ U_1 \cup U_2 \mid C_1 \cup C_2\ ]$

Compositional analysis with merge

- *"a student"* $\mapsto$ $\lambda G.([\ z \mid \mathit{student}(z)\ ] + G(z))$
- *"works"* $\mapsto$ $\lambda x.[\ \varnothing \mid \mathit{work}(x)\ ]$

- *"a student works"* $\mapsto$ $\lambda G.([\ z \mid \mathit{student}(z)\ ] + G(z))(\lambda x.[\ \varnothing \mid \mathit{work}(x)])$

$$\Rightarrow_\beta [\ z \mid \mathit{student}(z)\ ] + \lambda x.[\ \varnothing \mid \mathit{work}(x)](z)$$

$$\Rightarrow_\beta [\ z \mid \mathit{student}(z)\ ] + [\ \varnothing \mid \mathit{work}(z)]$$

$$\Rightarrow_\beta [\ z \mid \mathit{student}(z), \mathit{work}(z)]$$

Compositional analysis with merge

- *"Mary"* $\mapsto$ $\lambda G.([\ z \mid z = \mathit{mary}\ ] + G(z))$
- *"she"* $\mapsto$ $\lambda G.([\ v \mid v = z\ ] + G(v))$

- *"Mary works. She is successful."*

$$\mapsto \lambda K \lambda K'(K + K')([\ z \mid z = \mathit{mary}, \mathit{work}(z)])([\ v \mid v = z, \mathit{successful}(v)])$$

$$\Rightarrow_\beta \lambda K'([\ z \mid z = \mathit{mary}, \mathit{work}(z)] + K')([v \mid v = z, \mathit{successful}(v)])$$

$$\Rightarrow_\beta [\ z \mid z = \mathit{mary}, \mathit{work}(z)] + [\ v \mid v = z, \mathit{successful}(v)]$$

$$\Rightarrow [\ z\ v \mid z = \mathit{mary}, \mathit{work}(z), v = z, \mathit{successful}(v)]$$

DRS merge: second try (directional)

- The merge operation on two DRSs combines the universes and conditions of both DRSs into a new DRS

- Let $K_1 = [\ U_1 \mid C_1\ ]$ and $K_2 = [\ U_2 \mid C_2\ ]$:
  - $K_1 + K_2 = [\ U_1 \cup U_2 \mid C'_1 \cup C_2\ ]$
  - where: C'_1 is C_1 such that all free variables in the conditions $\gamma \in C_1$ that also occur as discourse referents $u \in U_2$ are α-converted to new variables

- under this definition, merge is directional:

$$K_1 + K_2 \not\Leftrightarrow K_2 + K_1$$

Variable capturing

- In λ-DRT, discourse referents are captured via the interaction of β-reduction and DRS-binding:

  $$\lambda K'([z \mid \mathit{student}(z), \mathit{work}(z)] + K')([\, v \mid v = z, \mathit{successful}(v)])$$
  $$\Rightarrow_\beta [z \mid \mathit{student}(z), \mathit{work}(z)] + [\, v \mid v = z, \mathit{successful}(v)]$$
  $$\Rightarrow_\beta [z\ v \mid \mathit{student}(z), \mathit{work}(z), v = z, \mathit{successful}(v)]$$

- But the β-reduced DRS must be equivalent to the original DRS!
  - This means that the potential for capturing discourse referents must be captured in the interpretation of λ-DRSs
    - Possible, but tricky