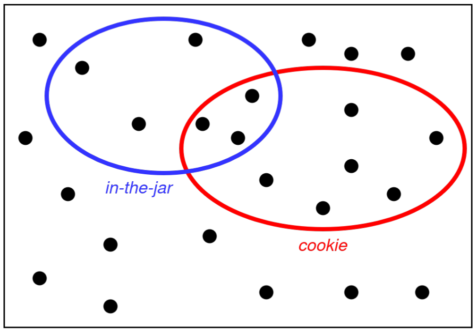
Semantic Theory 2025: Exercise 4 Key

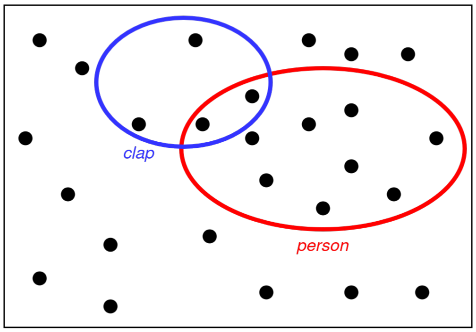
Question 1

Give the truth conditions for the following sentences using Generalized Quantifier Theory: interpret each VP as a property (i.e. a set of entities) and interpret quantified noun phrases (underlined) as sets of properties. Illustrate your answer with a graphical depiction of a model in which the sentence is true.

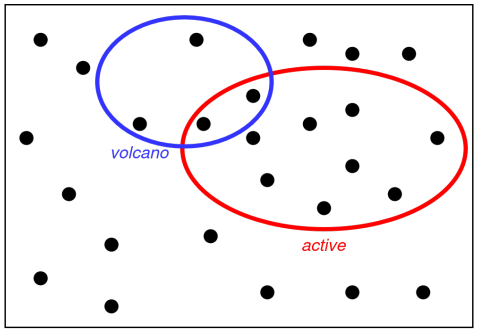
a. $[\![\underline{\textit{at least three cookies}}\ \textit{are in the jar}]\!] = 1$ iff
$[\![\textit{in-the-jar}]\!] \in \{P \subseteq U_M \mid |[\![\textit{cookie}]\!] \cap P| \geq 3\}$ iff
$|[\![\textit{cookie}]\!] \cap [\![\textit{in-the-jar}]\!]| \geq 3$



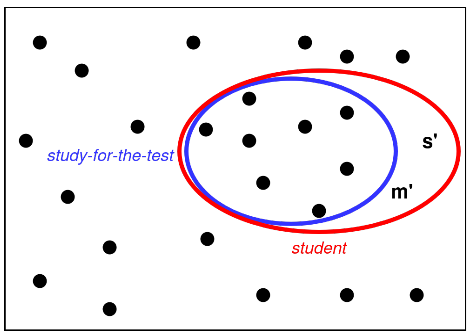
b. $[\![\underline{\textit{few people}}\ \textit{clapped}]\!] = 1$ iff
$[\![\textit{clap}]\!] \in \{P \subseteq U_M \mid |[\![\textit{person}]\!] \cap P| \leq \frac{|[\![\textit{person}]\!]|}{n}\}$ iff
$|[\![\textit{person}]\!] \cap [\![\textit{clap}]\!]| \leq \frac{|[\![\textit{person}]\!]|}{n}$



c. $[\![\underline{\textit{some, but not all, volcanoes}}\ \textit{are active}]\!] = 1$ iff
$[\![\textit{active}]\!] \in \{P \subseteq U_M \mid 1 \leq |[\![\textit{volcano}]\!] \cap P| < |[\![\textit{volcano}]\!]|\}$ iff
$1 \leq |[\![\textit{volcano}]\!] \cap [\![\textit{active}]\!]| < |[\![\textit{volcano}]\!]|$



d. $[\![\underline{\textit{every student except for Mark and Sally}}\ \textit{studied for the test}]\!] = 1$ iff
$[\![\textit{study-for-the-test}]\!] \in \{P \subseteq U_M \mid [\![\textit{student}]\!] - \{[\![m']\!], [\![s']\!]\} \subseteq P\}$ iff
$[\![\textit{student}]\!] - \{[\![m']\!], [\![s']\!]\} \subseteq [\![\textit{study-for-the-test}]\!]$



Question 2

Determine the monotonicity properties (left and right) of the following determiners. Show how you derived these monotonicity properties.

a. *half of the*
Right upward:
half of the students walked quickly $\vDash$ *half of the students walked*
Not left monotonic:
half of the students walked $\nvDash$ *half of the people walked*
half of the people walked $\nvDash$ *half of the students walked*

b. *no more than six*
Right downward:
no more than six students walked $\vDash$ *no more than six students walked quickly*
Left downward:
no more than six people walked $\vDash$ *no more than six students walked*

c. *most*

Right upward:

most people swam quickly $\vDash$ *most people swam*

Not left monotonic:

most people swam $\nvDash$ *most students swam*

most students swam $\nvDash$ *most people swam*

Question 3

Prove that the following statement holds:

The external negation of a right downward monotonic (monotone decreasing) quantifier is a right upward monotonic (monotone increasing) quantifier

$DM(Q) := \forall P, R\colon ([\![P]\!] \subseteq [\![R]\!] \wedge Q(R)) \rightarrow Q(P)$
$UM(Q) := \forall P, R\colon ([\![P]\!] \subseteq [\![R]\!] \wedge Q(P)) \rightarrow Q(R)$

(i) assume we have P, R such that $[\![P]\!] \subseteq [\![R]\!]$ and $\neg Q(P)$
(ii) assume $\neg Q(R)$ does not hold, i.e. $\neg(\neg Q(R))$, i.e. $Q(R)$
(iii) then by the assumption that $[\![P]\!] \subseteq [\![R]\!]$ and $DM(Q)$, we have $Q(P)$
(iv) by (i) and (iii), we have $Q(P) \wedge \neg Q(P)$. This is a contradiction.
Therefore, $DM(Q) \rightarrow UM(\lambda P.\neg Q(P))$