Dynamic Semantics

Week 8

Slides and materials based on the courses by
Noortje Venhuizen and Mareike Hartmann

A problem* for type theory: context-dependent expressions

- **Deictic** expressions depend on the *physical* utterance situation:
  - *“I”*, *“you”*, *“now”*, *“here”*, *“this”*, …

- **Anaphoric** expressions refer to the *linguistic context*/previous discourse:
  - *“he”*, *“she”*, *“it”*, *“then”*, …

More context-dependent expressions

- Context dependence is a pervasive property of natural language:
  - *"every student must be familiar with the basic properties of FOL"*
  - *"it is rainy everywhere"*
  - *"John is always late"*
  - *"Bill bought an expensive car"*
  - *"another one, please!"*
  - *"the student is working"*

Context theory

- Natural-language expressions can vary their meaning with context: *“I”*, *“you”*, *“here”*, *“this”*, *“now”*, etc.

- A simple context theory (based on Lewis 1970/1972):
    - Model contexts as “vectors”: sequences of semantically relevant context data with fixed arity
    - Model meanings as functions from contexts to denotations—more specifically, as functions from specific context components to denotations

Defining a context “vector”

- **Context** c = (a, b, ℓ, t, r)
  - a = speaker
  - b = addressee
  - ℓ = utterance location
  - t = utterance time
  - r = referred object

$$[\![I]\!]^{M,g,c} = \mathrm{utt}(c) = a$$

$$[\![you]\!]^{M,g,c} = \mathrm{adr}(c) = b$$

$$[\![here]\!]^{M,g,c} = \mathrm{loc}(c) = l$$

$$[\![now]\!]^{M,g,c} = \mathrm{time}(c) = t$$

$$[\![this]\!]^{M,g,c} = \mathrm{ref}(c) = r$$

Type-theoretic context semantics

- Model structure: $M = (U, C, V)$, where U is the universe, C is the **context set**, and V is value assignment function that assigns non-logical constant functions from contexts to denotations of appropriate type

- Interpretation:
  - $[\![\alpha]\!]^{M,g,c} = V(\alpha)(c)$, if α is a non-logical constant
  - $[\![\alpha]\!]^{M,g,c} = \mathrm{g}(\alpha)$, if α is a variable
  - $[\![\alpha(\beta)]\!]^{M,g,c} = [\![\alpha]\!]^{M,g,c}([\![\beta]\!]^{M,g,c})$
  - etc.

Type-theoretic context semantics: an example

"I am reading this book" ↦ *read*(*this-book*)(*I'*)

- $[\![\mathit{read}(\mathit{this\text{-}book})(I')]\!]^{M,g,c} = 1$
  - iff $[\![\mathit{read}]\!]^{M,g,c}([\![\mathit{this\text{-}book}]\!]^{M,g,c})([\![I']\!]^{M,g,c}) = 1$
  - iff $[\![\mathit{read}]\!]^{M,g,c}(V_M(\mathit{this\text{-}book})(c))(V_M(I')(c)) = 1$
  - iff $V_M(\mathit{read})(\mathit{ref}(c))(\mathit{utt}(c)) = 1$

- Note: context-invariant expressions are constant functions
  - $V_M(\mathit{read})(\mathrm{c}) = \mathrm{V}(\mathit{read})(\mathrm{c}') \text{ for all c, c}' \in C$

Another problem for traditional type theory: donkeys

(Compositional derivation of “donkey sentences”)

- Indefinite noun phrases and conditionals interact strangely, e.g.:

  “if a farmer owns a donkey, he beats it”

- $\exists x \exists y[\mathit{farmer}(x) \wedge \mathit{donkey}(y) \wedge \mathit{own}(x, y)] \rightarrow \mathit{beat}(x, y)$?

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- $\forall x \forall y[(\mathit{farmer}(x) \land \mathit{donkey}(y) \land \mathit{own}(x, y)) \rightarrow \mathit{beat}(x, y)]$
  - Correct! But how can it be derived compositionally?

Anaphora and compositionality

- Consider the following sentence: S_1 = "*a person who works hard is happy*"
  - we can interpret S_1 as: $\exists x[\mathit{person}(x) \wedge \mathit{work\text{-}hard}(x) \wedge \mathit{happy}(x)]$

- Now consider S_2 = "*a person$_i$ works hard*", S_3 = "*they$_i$ are happy*"

- $S_2 S_3 = S_1$, but can we derive this compositionally?
  - $\exists x[\mathit{person}(x) \wedge \mathit{work\text{-}hard}(x)] \wedge \mathit{happy}(x)$

What are indefinites?

- Option I: Existential quantifiers? (cf. Russell, 1919)
  - No: donkey sentences

- Option II: Universal quantifiers?
  - No: *"I saw a donkey"* $\not\models$ *"I saw every donkey"*

- Option III: Ambiguous?

We need to re-think our logic

- We need a new way to think about quantification and free variables: ***Dynamic* Predicate Logic** (DPL; Groenendijk and Stokhof, 1991)

DPL: Syntax

- **Terms**: $TERM = VAR \cup CON$
- **Atomic formulas**:
  - $R(t_1, \dots, t_n)$ for $R \in PRED^n$ and $t_1, \dots, t_n \in TERM$
- **Well-formed formulas** (WFF):
  - All atomic formulas are WFFs
  - If $x \in VAR$, then $\exists x$ is a WFF
    - *Note*: there is *no* quantifier scope in DPL—*all* variables are free
  - If φ and ψ are WFFs, then $\sim\varphi$ and $(\varphi \cdot \psi)$ are WFFs
  - Nothing else is a WFF

(Rough) correspondences with FOL

- $\neg\varphi \approx {\sim}\varphi$
- $\varphi \wedge \psi \approx \varphi \cdot \psi$
- $\varphi \vee \psi \approx {\sim}({\sim}\varphi \cdot {\sim}\psi)$
- $\varphi \rightarrow \psi \approx {\sim}(\varphi \cdot {\sim}\psi)$
- $\exists x\varphi \approx \exists x \cdot \varphi$
- $\forall x\varphi \approx \exists x \rightarrow \varphi = {\sim}(\exists x \cdot {\sim}\varphi)$

DPL: examples

- "*A person who works hard is happy.*":
  - $\exists x \cdot \mathit{person}(x) \cdot \mathit{work\text{-}hard}(x) \cdot \mathit{happy}(x)$

- "*A person works hard. They are happy.*":
  - $(\exists x \cdot \mathit{person}(x) \cdot \mathit{work\text{-}hard}(x)) \cdot (\mathit{happy}(x))$
    $= \exists x \cdot \mathit{person}(x) \cdot \mathit{work\text{-}hard}(x) \cdot \mathit{happy}(x)$

- "*If a farmer owns a donkey, he beats it.*":
  - $(\exists x \cdot \mathit{farmer}(x) \cdot \exists y \cdot \mathit{donkey}(y) \cdot \mathit{own}(x, y)) \rightarrow \mathit{beat}(x, y)$
    $= \sim((\exists x \cdot \mathit{farmer}(x) \cdot \exists y \cdot \mathit{donkey}(y) \cdot \mathit{own}(x, y)) \cdot \sim\mathit{beat}(x, y))$

DPL: semantics (one way to do it)

- We can *map DPL formulas into FOL* for interpretation. For a DPL formula φ, define its corresponding FOL formula φ° as $\langle\varphi\rangle\top$, where:
    1. $\langle\bot\rangle\psi = \bot$
    2. $\langle\top\rangle\psi = \top$
    3. $\langle P(x_1, \ldots, x_n)\rangle\psi = P(x_1, \ldots, x_n) \wedge \psi$
    4. $\langle\exists x\rangle\psi = \exists x[\psi]$
    5. $\langle\varphi_1 \cdot \varphi_2\rangle\psi = \langle\varphi_1\rangle(\langle\varphi_2\rangle\psi)$
    6. $\langle{\sim}\varphi\rangle\psi = \neg(\langle\varphi\rangle\top) \wedge \psi$

(Rough) correspondences with FOL (again)

- $(\sim\varphi)^\circ = \langle\sim\varphi\rangle\top = \neg(\langle\varphi\rangle\top) \wedge \top = \neg(\varphi \wedge \top) \wedge \top = \neg\varphi$
- $(\varphi \cdot \psi)^\circ = \langle\varphi \cdot \psi\rangle\top = \langle\varphi\rangle(\langle\psi\rangle\top) = \langle\varphi\rangle(\psi \wedge \top) = \varphi \wedge \psi \wedge \top = \varphi \wedge \psi$
- $(\sim(\sim\varphi \cdot \sim\psi))^\circ = \neg(\neg\varphi \wedge \neg\psi) = \varphi \vee \psi$
- $(\varphi \rightarrow \psi)^\circ = (\sim(\varphi \cdot \sim\psi))^\circ = \neg(\varphi \wedge \neg\psi) = \varphi \rightarrow \psi$
- $(\exists x \cdot \varphi)^\circ = \langle\exists x\rangle(\langle\varphi\rangle\top) = \langle\exists x\rangle(\varphi \wedge \top) = \exists x[\varphi \wedge \top] = \exists x[\varphi]$
- $(\exists x \rightarrow \varphi)^\circ = (\sim(\exists x \cdot \sim\varphi))^\circ = \neg(\langle\exists x \cdot \sim\varphi\rangle\top) \wedge \top = \neg(\langle\exists x\rangle(\neg\varphi \wedge \top \wedge \top)) \wedge \top = \neg\exists x[\neg\varphi \wedge \top \wedge \top] \wedge \top = \neg\exists x[\neg\varphi] = \forall x[\varphi]$

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DPL semantics: another example

1. $\langle \bot \rangle \psi = \bot$
2. $\langle \top \rangle \psi = \top$
3. $\langle P(x_1, \ldots, x_n) \rangle \psi = P(x_1, \ldots, x_n) \wedge \psi$
4. $\langle \exists x \rangle \psi = \exists x[\psi]$
5. $\langle \varphi_1 \cdot \varphi_2 \rangle \psi = \langle \varphi_1 \rangle (\langle \varphi_2 \rangle \psi)$
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$((\exists x \cdot F(x) \cdot \exists y \cdot D(y) \cdot O(x, y)) \rightarrow B(x, y))^{\circ}$

$= \neg \exists x[F(x) \wedge \exists y[D(y) \wedge O(x, y) \wedge \neg B(x, y)]]$

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$((\exists x \cdot F(x) \cdot \exists y \cdot D(y) \cdot O(x, y)) \to B(x, y))^\circ$

$= \neg\exists x[F(x) \land \exists y[D(y) \land O(x, y) \land \neg B(x, y)]]$

$\leftrightarrow \forall x[F(x) \to \forall y[(D(y) \land O(x, y)) \to B(x, y)]$

Meanwhile at the philosophy department…

- Asking the big questions: what is meaning?
  - Truth-conditions vs. context-change
  - Sentence vs. discourse
  - Semantics vs. pragmatics

A new perspective on meaning

- Truth-conditional Semantics ⇒ ***Dynamic* Semantics**

- Basic semantic value:
  - truth-conditions ⇒ context-change potential
- (In)definite NPs are:
  - quantificational ⇒ variables
- Existential quantification over:
  - A sentence ⇒ the discourse
- Quantification is:
  - selective ⇒ unselective

Meaning as context-change potential

- In dynamic semantics, the meaning of an expression is the effect it has on its context: context ⇔ meaning
  - context changes meaning
  - meaning changes context

- Note: This is a *generalisation of* rather than an *alternative to* classical truth-conditional semantics

Discourse variables and quantification

- Division of labor between definite and indefinite NPs:
  - Indefinite NPs introduce discourse referents, which can serve as antecedents for anaphoric reference
  - Definite NPs refer to “old” or “familiar” discourse referents (which are already part of the meaning representation)

- E.g.: *“A dog came in. It barked.”* ↦ *dog*(x) ∧ *came-in*(x) ∧ *barked*(x)
  - … true iff there is a value for x that verifies the conditions

Unselective quantification

"*every* *farmer who owns a donkey* *beats it*"

(quantifier) (restriction) (nuclear scope)

- This is true iff for every value assignment to x and y:
  - if $[[\mathit{farmer}(x) \wedge \mathit{donkey}(y) \wedge \mathit{owns}(x, y)]]^{M,g} = 1$
    then $[[\mathit{beats}(x, y)]]^{M,g} = 1$
- Quantification is restricted to those individuals who satisfy the restriction
  - Quantification is ***unselective***, i.e. all free variables are bound.