Lexical Semantics

Week 7

Slides and materials based on the courses by
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A closer look at plural NPs

- Entailment pattern (of predicates): **distributivity**

- *"Bill and Mary work"* ⊨ *"Bill works"*
  - $work(b) \land work(m) \vDash work(b)$
- *"Bill and Mary work"* ⊨ *"Mary works"*
  - $work(b) \land work(m) \vDash work(m)$
- *"all students work", "John is a student"* ⊨ *"John works"*
  - $\forall x(student(x) \rightarrow work(x)), student(j) \vDash work(j)$

Distributivity does not hold for all predicates

- *“Bill and Mary met”* ⊭ *“Bill met”*

- *“the students met”*, *“John is a student”* ⊭ *“John met”*

- *“the committee will dissolve”*, *“John is member of the committee”* ⊭ *“John will dissolve”*

- *“meet”* and *“dissolve”* are **collective** predicates

Distributive vs. collective predicates

- Distributive:
  - Applicable to singular and plural NPs
  - Predication with a plural NP *distributes* over the individual objects covered by the NP
  - Examples: *“work”*, *“sleep”*, *“eat”*, *“tall”*, …
- Collective:
  - Only applicable to plural or group NPs
  - Semantics cannot be reduced to atomic statements about single standard individuals
  - Examples: *“meet”*, *“gather”*, *“unite”*, *“disperse”*, *“dissolve”*, ...

- **Mixed** predicates (*“carry a piano”*, *“solve the exercise”*): predicates that are ambiguous between the distributive and collective reading

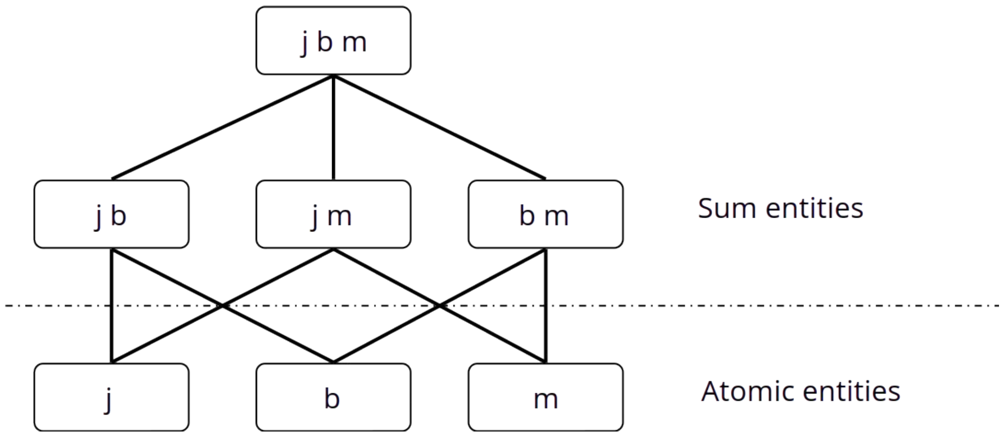
Modeling plural terms: desiderata

- A representation of plural terms that is not (only) defined in terms of atomic entities (to account for collective predicates)
  - We extend the universe of our model structures with **sums** (or: “groups”)

- A relation between atomic and plural entities (to account for the entailment pattern of distributive predicates)
  - We add a membership (or: “individual part”) relation to the model structure

- Denotations of types of predicates are restricted to particular parts of universe

Structured universe: entities and sums of entities



- Edges indicate the (individual) **part-of relation**

Algebraic detour: partial orders

- A **partial order** is an algebraic structure $(A, \leq)$ where $\leq$ is a *reflexive*, *transitive*, and *antisymmetric* relation over A.

- The **join** $(a \sqcup b)$ of $a, b \in A$ is the *lowest upper* bound for a and b
  - $a \sqcup b = x$ s.t. $a \leq x$ and $b \leq x$ and $\neg \exists y(y \neq x \land (a \leq y \leq x \lor b \leq y \leq x))$

- The **meet** $(a \sqcap b)$ of $a, b \in A$ is the *highest lower* bound for a and b
  - $a \sqcap b = x$ s.t. $x \leq a$ and $x \leq b$ and $\neg \exists y(y \neq x \land (x \leq y \leq a \lor x \leq y \leq b))$

Algebraic detour: lattices and semi-lattices

- A lattice is a partial order (A, ≤) that is **closed** under meet and join
  - i.e. for all $a, b \in A$: $(a \sqcup b) \in A$ and $(a \sqcap b) \in A$
  - Examples:
    - Boolean algebra
      - *complemented distributive* ***bounded*** lattice: $(A, \leq, \bot, \top, \neg)$
    - Powerset Boolean algebra: $(\wp(S), \subseteq, \varnothing, S, S - (–))$
    - Integers, real numbers

- A **join semi-lattice** is a partial order (A, ≤) that is closed under join

Algebraic detour: bounded lattices

- A **bounded lattice** has a maximal element (1) and a minimal element (0)

- An element $a \in A$ is an **atom**, if $a \neq 0$ and there is no $b \neq 0$ in A such that $b < a$

- A lattice is **atomic**, if for every $b \neq 0$ there is an atom a such that $a \leq b$

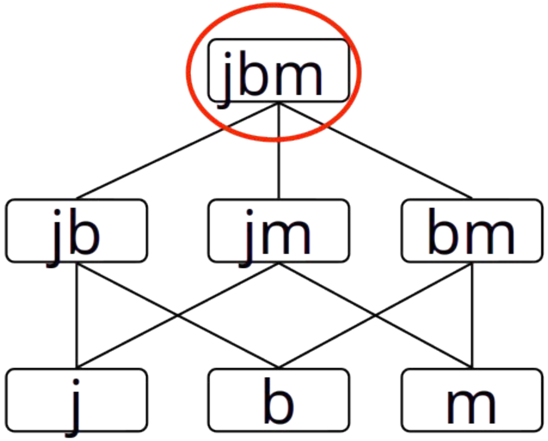
Model structures for plural terms

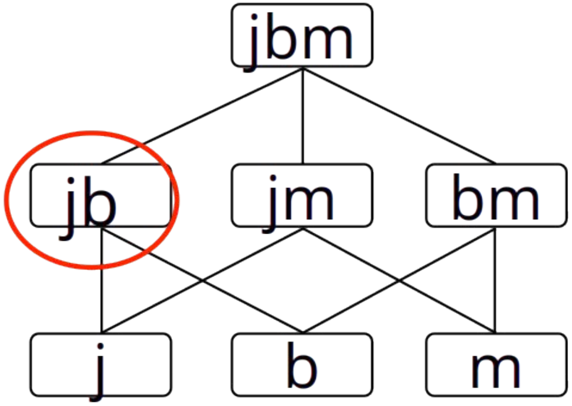
- A model structure is a pair $M = ((U, \leq), V)$, where
  - $(U, \leq)$ is an *atomic join semi-lattice* over the universe U, and $\leq$ is the **individual part relation**
  - V is an interpretation function

- In addition, we define:
  - $A \subseteq U$ is the set of atoms in $(U, \leq)$
  - $U - A$ is the set of non-atomic elements, i.e. the set of **proper sums** (or "groups") in U

Domain restrictions: collective predicates

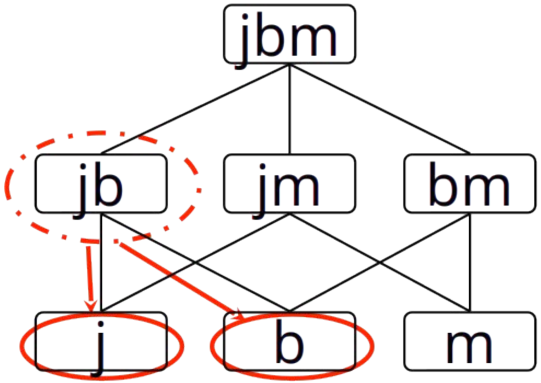
- Let P_c be the set of collective predicates (*"meet"*, *"dissolve"*, etc.)
  - The domain of P_c is restricted to non-atomic elements:
    for all $R \in P_c$, $V_M(R) \subseteq U - A$



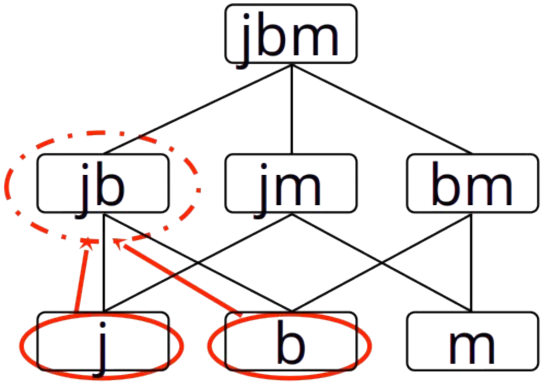


Domain restrictions: distributive predicates

- Let P_d be the set of distributive predicates (*"work"*, *"tall"*, *"student"*, etc.)
  - The domain of P_d is the universe of M: for all $R \in P_d$, $V_M(R) \subseteq U_M$, such that $a \in V_M(R)$ and $b \in V_M(R)$ iff $a \sqcup b \in V_M(R)$



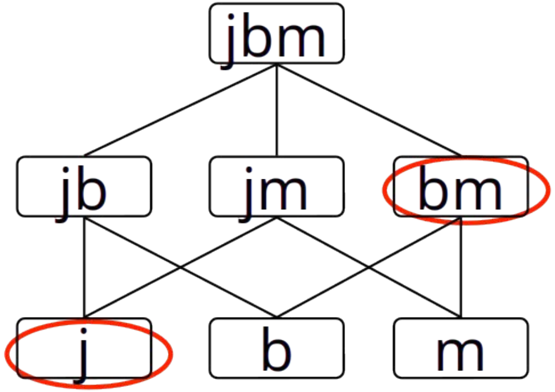
Distributivity



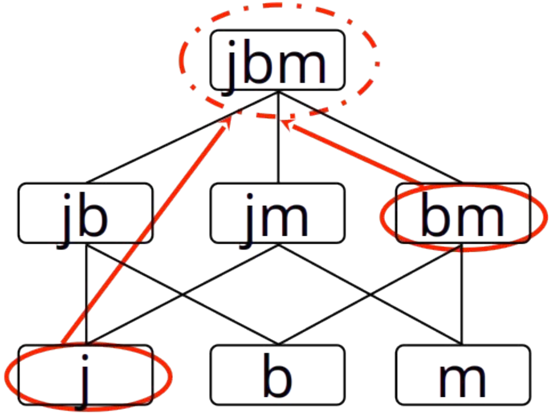
Closure under summation

Domain restrictions: mixed predicates

- Let P_m be the set of mixed predicates (*"carry a piano"*, *"solve the exercise"*, etc.)
  - The domain of P_m is the universe of M: for all $R \in P_m$, $V_M(R) \subseteq U$



Non-distributive



Closed under summation

Language for plural terms

- We extend our logical language with a **summation operator** $\oplus$, a **one-place predicate *At*** for "atom", and a **two-place relation** $\lhd$ for "(proper) individual part"
  - $s \oplus m$ *"the sum consisting of Sally and Mary"*
  - $s \lhd s \oplus m$ *"Sally is member of the sum consisting of Sally and Mary"*
  - $s \oplus m \lhd c$ *"Sally and Mary are members of the committee"*

- In addition, we introduce:
  - Variables ranging over proper sums: X,Y, Z, …
  - Number-specific (predicate) constants: $student_{SG}$, $student_{PL}$

Interpretation of plural terms

- $[\![a \oplus b]\!]^M \quad = [\![a]\!]^M \sqcup [\![b]\!]^M$
- $[\![a \lhd b]\!]^M \quad = 1$ iff $[\![a]\!]^M \leq [\![b]\!]^M$ and $[\![a]\!]^M \neq [\![b]\!]^M$
- $[\![At(a)]\!]^M \quad = 1$ iff $[\![a]\!]^M \in A$

- Individual constants denote either atoms ($x \in A$) or sums ($x \in U - A$)

- Predicate expressions satisfy specific constraints:
  - $V_M(student_{SG}) \subseteq A$
  - $V_M(student_{PL}) \subseteq U - A$

Interpretation of distributive predicates

- Meaning postulate for plural model structure
    - If a distributive predicate P applies to a set $X \subseteq A$, then the full denotation of P is the join semi-lattice **generated by X**

- The denotation of distributive predicates P is uniquely determined by their atomic members:
    - $\forall x[P(x) \leftrightarrow \forall y[At(y) \wedge y \triangleleft x \rightarrow P(y)]]$

Mixed predicates: examples of ambiguous interpretation

- *“every student summarized a paper”*
- *“John and Mary summarized a paper”*
- *“two students summarized a paper”*
- *“John summarized three papers”*

Mass nouns

- **Mass nouns** (*"water"*, *"gold"*, *"wood"*, *"money"*, *"soup"*, etc.) behave like plurals in different respects

- Closure under summation:
  - water + water = water
  - students + students = students
- Combination with cardinalities:
  - *"five liters of water"*
  - *"five bus-loads of students"*
- (Some) shared grammatical patterns:
  - ***"a students are hard workers"*
  - ***"a water is wet"*

Mass nouns vs. plurals

- Unlike plurals, mass nouns are divisive:
  - An amount of water can always be subdivided into proper parts, which are water again

- The denotation of mass nouns cannot be reduced to model theoretic atomic individuals
  - When talking about water, we are not talking about a collection of individual entities

Model structure for mass nouns

- Lets add another sort of entities, the “portions of matter” $\mathbb{M}$, to the model structure, and distinguish a part relation for individuals ($\leq_i$) and a part relation for materials ($\leq_m$):

- M = $((U, \leq_i), (\mathbb{M}, \leq_m), V)$
  - $U \cap \mathbb{M} = \varnothing$
  - $(U, \leq_i)$ is an atomic join semi-lattice
  - $(\mathbb{M}, \leq_m)$ is a non-atomic and **dense** join semi-lattice
  - V is an interpretation function

Materialization

- There is a close relation between the domain of material entities and the domain of (atomic and sum) individuals: *each individual consists of a specific portion of matter*

- Let $M = ((U, \leq_i), (\mathbb{M}, \leq_m), h, V)$ be a model structure in which h: $(U, \leq_i) \longrightarrow (\mathbb{M}, \leq_m)$ is a "materialization" function:
    - h is a **(join semi-)lattice homomorphism** that maps (atomic and plural) individuals to the matter they consist of
        - $a \leq_i b \longrightarrow h(a) \leq_m h(b)$
        - $h(a \sqcup_i b) = h(a) \sqcup_m h(b)$

Representation of mass nouns

- Additions to the logical representation language:
    - Variables referring to matter: **x**, **y**, **z**, …
    - A material fusion operation $\oplus_m$ and a material part relation $\triangleleft_m$ (to be distinguished from $\oplus_i$ and $\triangleleft_i$, respectively)

- A new logical operator *m* that expresses the materialization function:
    - $[\![m(\alpha)]\!]^M = h([\![\alpha]\!]^M)$, where $\alpha \in WE_e$ is a well-formed expression denoting an individual/group entity—i.e. $[\![\alpha]\!]^M \in U$

Examples

- *"the ring is made of gold"*

  $\mapsto \exists y(\mathit{ring}(y) \wedge \mathit{gold}(\mathit{m}(y)))$

- *"the ring contains gold"*

  $\mapsto \exists x \exists y(\mathit{ring}(x) \wedge y \lhd_{m} \mathit{m}(x) \wedge \mathit{gold}(y))$