Generalized Quantifiers

Week 5

Slides and materials based on the courses by
Noortje Venhuizen and Mareike Hartmann

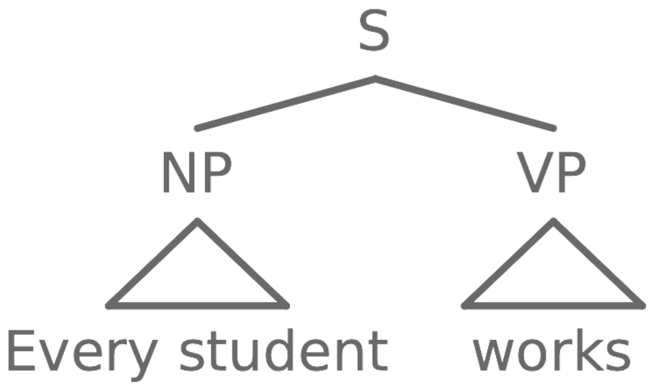
Back to Noun Phrases

- Natural language contains a wide variety of NPs, serving as **quantifiers**:
  - *all students, no woman, not every man, everything, nothing, three books, the ten professors, John, John and Mary, only John, firemen, at least five horses, most girls, all but ten marbles, less than half of the audience, John's car, some student's exercise, no student except Mary, more male than female cats, each other*

Aristotle: "all quantifiers are second-order relations between sets"

Frege: "all quantifiers can be defined in terms of logical quantifiers ($\forall$, $\exists$)"

NP interpretation



"every student" $\mapsto \lambda P \forall x(\mathit{student}(x) \rightarrow P(x))$

- $[\![\mathit{every\ student}]\!] \in D_{\langle\langle e,\, t\rangle,\, t\rangle}$
- $D_{\langle\langle e,\, t\rangle,\, t\rangle}$ is the set of functions from properties to truth values
  - In other words: *"every student"* denotes the set of properties that apply to every student (property = set of individuals)
  - $[\![\text{Every student}]\!]^{M} = \{ P \subseteq U_M \mid \text{every student has property } P \}$
    $= \{ P \subseteq U_M \mid [\![\mathit{student}]\!] \subseteq P\}$
  - $[\![\mathit{every\ student\ works}]\!]^{M} = 1 \text{ iff } [\![\mathit{work}]\!]^{M} \in [\![\mathit{every\ student}]\!]^{M}$

Generalised quantifiers

- Generalised quantifiers are sets of subsets of U_M
  - i.e. sets of properties
  - i.e. elements of $D_{\langle\langle e, t\rangle, t\rangle}$

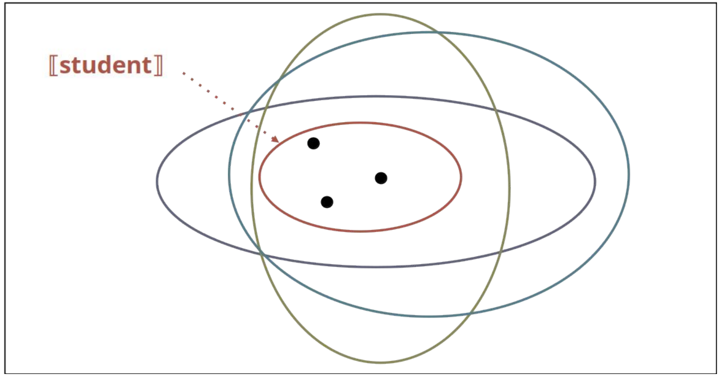
- *"every student"* $\mapsto \lambda P \forall x(\mathit{student}(x) \rightarrow P(x))$
  - $[\![\mathit{every\ student}]\!]^M = \{ P \subseteq U_M \mid [\![\mathit{student}]\!]^M \subseteq P \}$

- *"a student"* $\mapsto \lambda P \exists x(\mathrm{student'}(x) \land P(x))$
  - $[\![\mathit{a\ student}]\!]^M = \{ P \subseteq U_M \mid [\![\mathit{student}]\!]^M \cap P \neq \varnothing \}$

- *"Bill"* $\mapsto \lambda P.P(\mathit{bill^*})$
  - $[\![\mathit{bill}]\!]^M = \{ P \subseteq U_M \mid [\![\mathit{bill^*}]\!]^M \in P \}$

Universal quantifiers: denotation

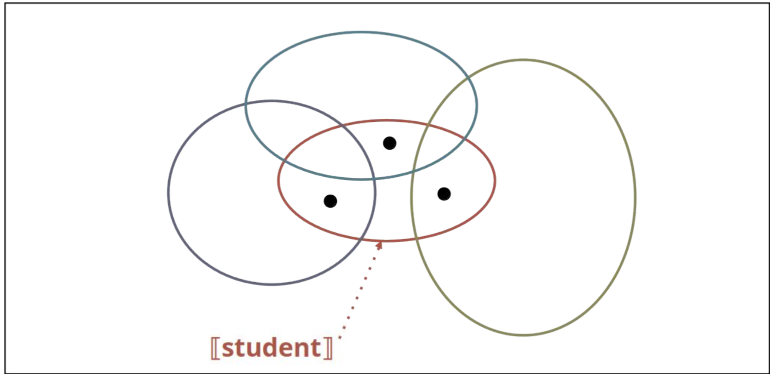
- *"every student"* denotes the set of properties that apply to every student
  - i.e., all supersets of [[*student*]]



- $[\![\textit{every } N]\!]^M = \{\, P \subseteq U_M \mid [\![N]\!]^M \subseteq P \,\}$

Existential quantifiers: denotation

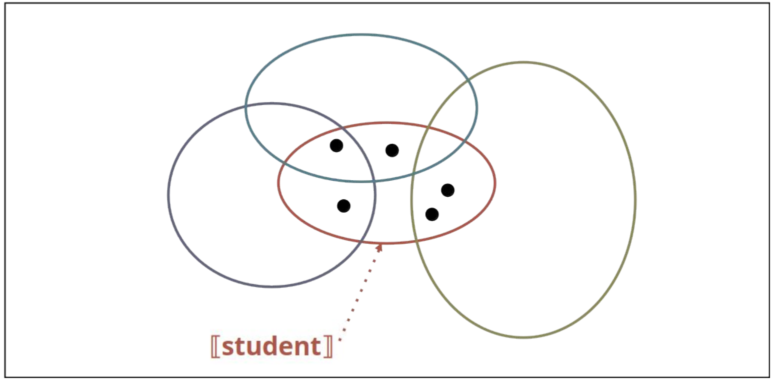
- *"a student"* denotes the set of properties that apply to at least one student
  - i.e., all sets that intersect with [[*student*]]



- $[\![a(n)\ N]\!]^M = \{\ \mathrm{P} \subseteq U_M \mid [\![N]\!]^M \cap \mathrm{P} \neq \varnothing\ \}$

Cardinal quantifiers: denotation

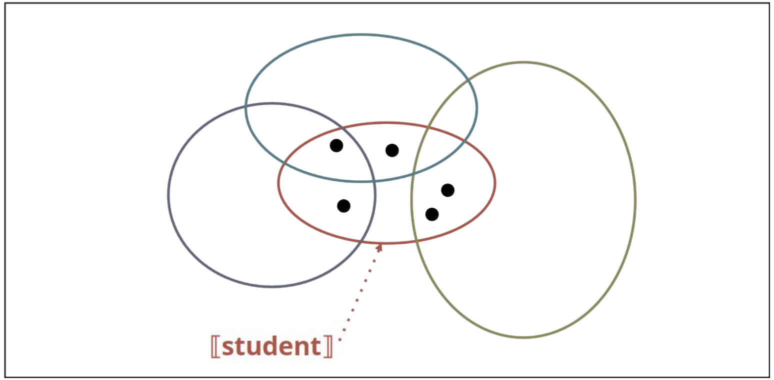
- “two students” denotes the set of properties that apply to at least two students
  - $\{ P \subseteq U_M \mid |[\![\textit{student}]\!]^M \cap P| \geq 2 \}$



- $[\![K\ N]\!]^M = \{ P \subseteq U_M \mid |[\![N]\!]^M \cap P| \geq K \}$ (probable exception: zero)

Named entities as (generalized) quantifiers

- *"Bill"* denotes the set of properties that apply to Bill
  - $[\![bill]\!]^M = \{ P \subseteq U_M \mid [\![bill^*]\!]^M \in P \} = \{ P \subseteq U_M \mid \{[\![bill^*]\!]^M\} \subseteq P \}$



- $[\![N_{named}]\!]^M = \{ P \subseteq U_M \mid [\![N_{named}{}^*]\!]^M \in P \}$

Exercise

1. $[\![\textit{not all N}]\!]^M$
2. $[\![\textit{no N}]\!]^M$
3. $[\![\textit{half of the N}]\!]^M$
4. $[\![\textit{most of the N}]\!]^M$
5. $[\![\textit{all but ten of the N}]\!]^M$

Generalised Quantifier Theory: central questions

1. How do generalised quantifiers *differ* in terms of their formal properties?
2. What *universal* regularities govern the meaning of terms?
3. Which *subclasses* represent meanings of natural language noun phrases?

Observation 1: inference patterns

- ***all men*** *walked rapidly* $\vDash$ ***all men*** *walked*
- ***all men*** *walked* $\nvDash$ ***all men*** *walked rapidly*

- ***no man*** *walked* $\vDash$ ***no man*** *walked rapidly*
- ***no man*** *walked rapidly* $\nvDash$ ***no man*** *walked*

- ***a girl*** *smoked a cigar* $\vDash$ ***a girl*** *smoked*
- ***a girl*** *smoked* $\nvDash$ ***a girl*** *smoked a cigar*

- ***few girls*** *smoked* $\vDash$ ***few girls*** *smoked a cigar*
- ***few girls*** *smoked a cigar* $\nvDash$ ***few girls*** *smoked*

Observation 2: negative polarity items

- **Negative polarity items** (NPIs: *"at all"*, *"anymore"*, etc.) typically occur only in contexts with negation:
  - "*she doesn't like driving <u>at all</u>*" vs.
  - **"*she likes driving <u>at all</u>*"

- What formally licenses Negative Polarity Items?
  - *"nobody saw <u>anything</u>"* vs. **"*somebody saw <u>anything</u>"*
  - *"no student has <u>ever</u> been to Saarbrücken"*
    vs. **"*a student has <u>ever</u> been to Saarbrücken"*
    - But: *"few students have <u>ever</u> been to Saarbrücken"*

Observation 3: coordination

- *“**no man** and **few women** walked”*
- *“**none of the girls** and **at most three boys** walked”*

- ***“**some man** and **few women** walked”*
- ***“**John** and **no woman** saw Jane”*

Explaining Observation 1: subsets and supersets

- “all men walked rapidly” ⊨ “all men walked”
  - [[*to walk rapidly*]] ⊆ [[*to walk*]]

- “a girl smoked a cigar” ⊨ “a girl smoked”
  - [[*to smoke a cigar*]] ⊆ [[*to smoke*]]

- Intuitively: For the given quantifiers, the sentence $[_S$ NP VP] remains true if the denotation of the VP is made “larger”

Upward monotonicity

- A quantifier Q is (*right*) **upward monotonic** (or: monotone increasing) in $M = (U, V)$ iff Q is “closed under supersets”, i.e.:
  - for all $X, Y \subseteq U$: if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$

- A noun phrase is (right) upward monotonic if it denotes a (right) upward monotonic quantifier

Upward monotonicity: entailment tests

- If $[\![VP_1]\!] \subseteq [\![VP_2]\!]$, then NP $VP_1 \vDash$ NP VP_2
  - $[\![$*to walk rapidly*$]\!] \subseteq [\![$*to walk*$]\!]$
  - “all men walked rapidly” $\vDash$ “all men walked”
  - “no man walked rapidly” $\nvDash$ “no man walked”

- NP (VP_1 and VP_2) $\vDash$ (NP VP_1) and (NP VP_2)
  where: $[\![VP1 \text{ and } VP2]\!] = [\![VP1]\!] \cap [\![VP2]\!]$
  - “all men smoked and drank” $\vDash$ “all men smoked and all men drank”
  - “no man smoked and drank” $\nvDash$ “no man smoked and no man drank”

Downward monotonicity

- A quantifier Q is (*right*) ***downward* monotonic** (or: monotone *decreasing*) in $M = (U, V)$ iff Q is "closed under subsets", i.e.:
  - for all X, Y $\subseteq$ U: if X $\in$ Q and Y $\subseteq$ X, then Y $\in$ Q

- "*no man walked*" $\vDash$ "*no man walked rapidly*"
  - [[*to walk rapidly*]] $\subseteq$ [[*to walk*]]

- "*few girls smoked*" $\vDash$ "*few girls smoked a cigar*"
  - [[*to smoke a cigar*]] $\subseteq$ [[*to smoke*]]

- A noun phrase is downward monotonic if it denotes a downward monotonic quantifier

Downward monotonicity: entailment tests

- If $[\![VP_2]\!] \subseteq [\![VP_1]\!]$, then NP $VP_1 \vDash$ NP VP_2
  - $[\![$*to walk rapidly*$]\!] \subseteq [\![$*to walk*$]\!]$
  - *“no man walked”* $\vDash$ *“no man walked rapidly”*
  - *“all men walked”* $\nvDash$ *“all men walked rapidly”*

- NP (VP_1 or VP_2) $\vDash$ (NP VP_1) and (NP VP_2)
  where: $[\![VP_1$ or $VP_2]\!] = [\![VP_1]\!] \cup [\![VP_2]\!]$
  and $[\![VP_1$ and $VP_2]\!] = [\![VP_1]\!] \cap [\![VP_2]\!]$
  - *“neither girl drank or smoked”* $\vDash$ *“neither girl drank and neither girl smoked”*
  - *“all boys sing or dance”* $\nvDash$ *“all boys sing and all boys dance”*

Explaining Observation 2

- *"nobody saw <u>anything</u>"* vs. ***"somebody saw <u>anything</u>"*
- *"no student has <u>ever</u> been to Saarbrücken"* vs. ***"a student has <u>ever</u> been to Saarbrücken"*
  - But: *"few students have <u>ever</u> been to Saarbrücken"*

- NPIs are licensed only in (right) downward monotonic contexts

Explaining Observation 3

- *"**no man** and **few women** walked"*
- *"**none of the girls** and **at most three boys** walked"*

- **"*some man* and *few women* walked"
- **"*John* and *no woman* saw Jane"

- NPs can only be coordinated when they have the same direction of (right) monotonicity

Monotonicity and logical operators

- (Right) monotonic quantifiers are ***closed under*** conjunction and disjunction:
    - "all boys and a girl walked rapidly" $\vDash$ "all boys and a girl walked"
    - "John or a student arrived late" $\vDash$ "John or a student arrived"
    - where: $[\![NP_1 \text{ and } NP_2]\!] = [\![NP_1]\!] \cap [\![NP_2]\!]$
      $[\![NP_1 \text{ or } NP_2]\!] = [\![NP_1]\!] \cup [\![NP_2]\!]$

- The intersection/union of two monotonic quantifiers is a quantifier with the same direction of monotonicity

Monotonicity and negation

- External negation: $\neg Q = \{ P \subseteq U_M \mid P \notin Q \}$
  - $[\![\textit{not all } N]\!] = \{ P \subseteq U_M \mid P \notin [\![\textit{all } N]\!] \}$.

    $= \{ P \subseteq U_M \mid [\![N]\!] \nsubseteq P \} = \neg[\![\textit{all } N]\!]$

- Internal negation: $\lambda R.Q(\neg R) = \{ P \subseteq U_M \mid (U_M - P) \in Q \}$
  - $[\![\textit{all N don't}]\!] = \{ P \subseteq U_M \mid [\![N]\!] \subseteq U_M - P \}$

    $= \{ P \subseteq U_M \mid [\![N]\!] \cap P = \varnothing \} = [\![\textit{no } N]\!]$

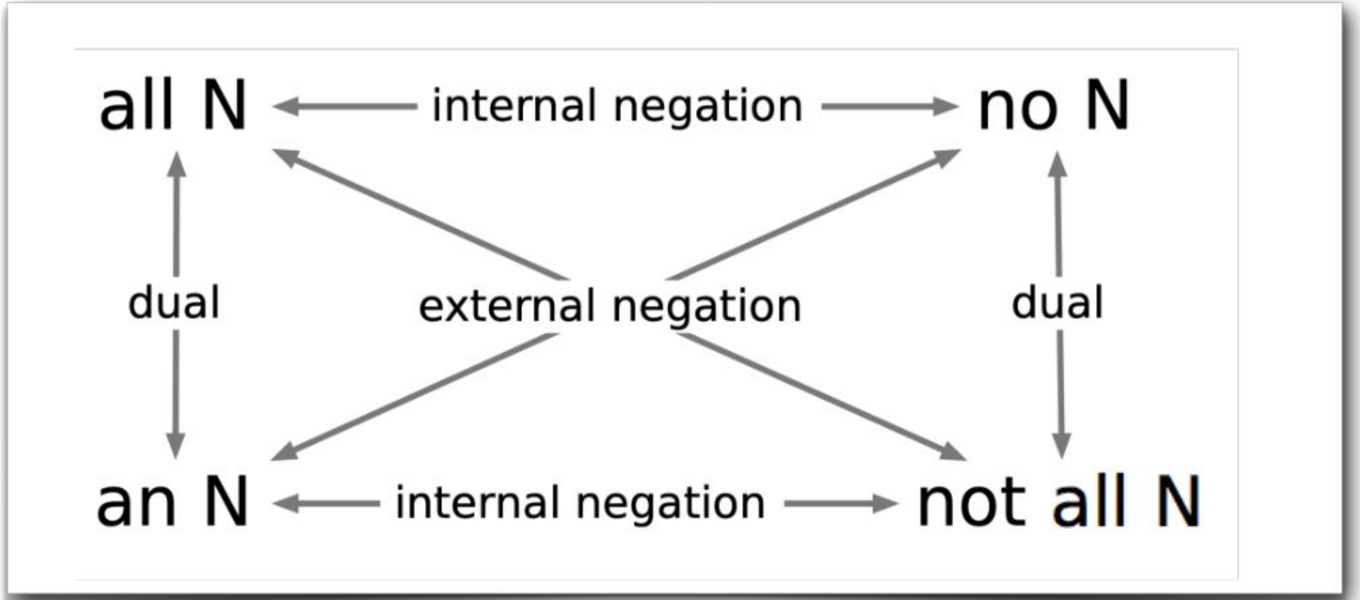
- Internal and external negation of a quantifier both flip the direction of monotonicity (*upward ⇒ downward* and *downward ⇒ upward*)

Duals

- The ***dual*** Q* of a quantifier Q in M is defined as the external *and* internal negation of Q: $Q^* = \lambda R.\neg Q(\neg R)$
  - Each quantifier is its own "double dual": $(Q^*)^* = Q$
  - $[\![Q^*]\!] = \{ P \subseteq U_M \mid U_M - P \notin Q \}$
  - $\forall = \exists^*$ (which means?)

- The dual preserves (right) monotonicity:
  - If Q is upward monotonic, then Q* is upward monotonic
  - If Q is downward monotonic, then Q* is downward monotonic

The “Square of Opposition”



BEHAVIORAL AND BRAIN SCIENCES (2009) **32**, 429–492
doi:10.1017/S0140525X0999094X

The myth
Language
importan

Universal grammar is dead

doi:10.1017/S0140525X09990744

Michael Tomasello
Max Planck Institute for Evolutionary Anthropology, D-04103 Leipzig, Germany.
tomas@eva.mpg.de

Abstract: The idea of a biologically evolved, universal grammar with linguistic content is a myth, perpetuated by three spurious explanatory strategies of generative linguists. To make progress in understanding human linguistic competence, cognitive scientists must abandon the idea of an innate universal grammar and instead try to build theories that explain both linguistic universals and diversity and how they emerge.

Universal grammar is, and has been for some time, a completely empty concept. Ask yourself: what exactly is in universal grammar? Oh, you don't know – but you are sure that the experts (generative linguists) do. Wrong; they don't. And not only that, they have no method for finding out. If there is a method, it would be looking carefully at all the world's thousands of languages to discern universals. But that is what linguistic typologists have been doing for the past several decades, and, as Evans & Levinson (E&L) report, they find no universal grammar.

The universal basis of l
exceptionality

doi:10.1017/S0140525X0999113(

Daniel Harbour
Departmen
United King
harbour@
http://web

The myth of la
of universal g

doi:10.1017/S01405

Morten H. Christia
[a]*Department of Psycho*
Fe Institute, Santa Fe,
Sciences, University College London, London, WC1E 6BT, United Kingdom.
christiansen@cornell.edu
http://www.psych.cornell.edu/people/Faculty/mhc27.htm
n.chater@ucl.ac.uk
http://www.psychol.ucl.ac.uk/people/profiles/chater_nick.htm

ersals: Abstract but not

25X09990604

, Rutgers University, New Brunswick, NJ 08901.
s.edu
edu/~mabaker/

sal language faculty

20

endoff[b,c]
University, Cambridge, MA 02138;
niversity, Medford, MA 02155; and
[c]*Santa Fe Institute, Santa Fe, NM 87501.*
pinker@wjh.harvard.edu
http://pinker.wjh.harvard.edu
Ray.jackendoff@tufts.edu
http://ase.tufts.edu/cogstud/incbios/RayJackendoff/index.htm

Looking for Universals I: monotonicity constraint

"The simple noun phrases of any natural language express monotone quantifiers or conjunctions of monotone quantifiers."

(Barwise & Cooper, 1981)

- **Simple noun phrase**: Proper names or NPs of the form $[\text{DET N}]_{\text{NP}}$
- **Monotone quantifiers**: quantifiers that are (right) upward or downward monotonic

From NPs to determiners

- *"every man walked"* $\mapsto$ $\forall x(\mathit{man}(x) \rightarrow \mathit{walk}(x))$
  - $[\![\mathit{every}]\!] = [\![\lambda P \lambda Q \forall x(P(x) \rightarrow Q(x))]\!] \in D_{\langle\langle e,\, t\rangle,\langle\langle e,\, t\rangle,\, t\rangle\rangle}$
  - $[\![\mathit{every}]\!](A)(B) = 1$ iff $A \subseteq B$

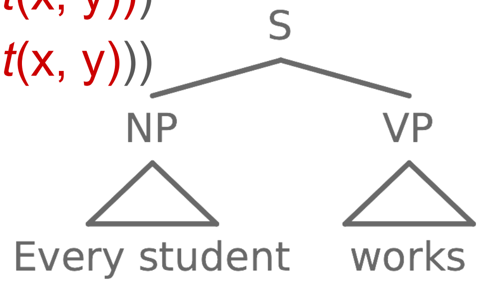
- *Syntactically*, determiners are expressions that take a noun and a verb phrase to form a sentence

- *Semantically* the interpretation of a determiner can be seen as:
  - a function from sets of entities to sets of properties: $\langle\langle e, t\rangle,\langle\langle e, t\rangle, t\rangle\rangle$
  - a relation between two sets A and B, denoted by the NP and VP, respectively

Quick aside: restriction and nuclear scope

- Logic: **scope** of a *quantifier*
  - $\underline{\forall x}((\mathit{person}(x) \wedge \mathit{hungry}(x)) \rightarrow \exists y(\mathit{banana}(y) \wedge \mathit{eat}(x, y)))$
  - $\forall x((\mathrm{person}(x) \wedge \mathrm{hungry}(x)) \rightarrow \underline{\exists y}(\mathrm{banana}(y) \wedge \mathrm{eat}(x, y))))$

- Linguistics: **restriction** (denotation of the noun the determiner combines with to form an NP) and **nuclear scope** (**scope** - **restriction**) of a *determiner*
  - $\underline{\forall x}((\mathit{person}(x) \wedge \mathit{hungry}(x)) \rightarrow \exists y(\mathit{banana}(y) \wedge \mathit{eat}(x, y)))$
  - $\forall x((\mathit{person}(x) \wedge \mathit{hungry}(x)) \rightarrow \underline{\exists y}(\mathit{banana}(y) \wedge \mathit{eat}(x, y)))$



A note on monotonicity

- We’ve been talking about *right* monotonicity (and will be for the remainder of the course): *right* monotonicity ⇒ nuclear scope
    - *left* monotonicity ⇒ *restriction*

	Decreasing	**Increasing**
Right	<u>¬∃:</u> “*no dog <u>moved</u>*” ⊨ “*no dog <u>walked</u>*” <u>¬∀:</u> “*not all dogs <u>moved</u>*” ⊨ “*not all dogs <u>walked</u>*”	<u>∃:</u> “*a dog walked*” ⊨ “*a dog <u>moved</u>*” <u>∀:</u> “*all dogs walked*” ⊨ “*all dogs <u>moved</u>*”
Left	<u>¬∃:</u> “*no <u>person</u> walked*” ⊨ “*no <u>man</u> walked*” <u>∀:</u> “*all <u>people</u> walked*” ⊨ “*all <u>men</u> walked*”	<u>∃:</u> “*a <u>woman</u> walked*” ⊨ “*a <u>person</u> walked*” <u>¬∀:</u> “*not all <u>women</u> walked*” ⊨ “*not all <u>people</u> walked*”

Persistence (left upward monotonicity)

- A determiner D is **persistent** in *M* iff: for all X, Y, Z:
  - if D(X, Z) and X ⊆ Y, then D(Y, Z)

- Persistence test: If $[[N_1]] \subseteq [[N_2]]$, then DET N_1 VP ⊨ DET N_2 VP
  - [[*woman*]] ⊆ [[*person*]]
    - *"some women walked"* ⊨ *"some people walked"*
  - [[*boy*]] ⊆ [[*child*]]
    - *"at least four boys were smoking"* ⊨ *"at least four children were smoking"*

Antipersistence (left downward monotonicity)

- A determiner D is **antipersistent** in *M* iff: for all X, Y, Z:
  - if D(X, Z) and Y ⊆ X, then D(Y, Z)

- Antipersistence test: If $[[N_2]] \subseteq [[N_1]]$, then DET N_1 VP ⊨ DET N_2 VP
  - [[*toddler*]] ⊆ [[*children*]]
    - *“all children walked”* ⊨ *“all toddlers walked”*
  - [[*girl*]] ⊆ [[*female*]]
    - *“no female was smoking”* ⊨ *“no girl was smoking”*

Looking for Universals II: conservativity constraint

“In every natural language, simple determiners together with an N yield an NP which ‘lives on ⟦*N*⟧*’.”* (Barwise & Cooper, 1981)

- A determiner D is **conservative** iff “D lives on A”: for every A, B $\subseteq$ *U*: D(A, B) $\Leftrightarrow$ D(A, A $\cap$ B)
  - “all students work” $\Leftrightarrow$ “all students are students that work”
  - “some girls are dancing” $\Leftrightarrow$ “some girls are girls that are dancing”
  - But: “only men smoke cigars” $\not\Leftrightarrow$ “only men are men that smoke cigars”