Semantic Theory 2025: Exercise 7

Due by: Wednesday, June 11 at 10:00 am (before class)

Question 1

Recall the syntax for well-formed formulas (WFFs) in Dynamic Predicate Logic (DPL):

- All atomic formulas $(R(t_1,\ldots,t_n) \text{ for } R \in PRED^n, t_1,\ldots,t_n \in TERM)$ are WFFs
- If $x \in VAR$, then $\exists x \text{ is a WFF}$
- If ϕ and ψ are WFFs, then $\sim \phi$ and $(\phi \cdot \psi)$ are WFFs
- Nothing else is a WFF

Translate the following natural language utterances into DPL. You may treat <u>underlined</u> expressions as single terms (e.g. " $\underline{came\ to}$ " $\Rightarrow come\text{-}to(\dots)$):

- a. "If John runs, he will pull a muscle."
- b. "There was a party. A man came to the party. He was hungry."
- c. "There is a farmer. She owns a donkey. If the donkey is hungry, she feeds it."

Question 2

Translate the DPL formulas ϕ you constructed for 1a-c to FOL formulas $\phi^{\circ} = \langle \phi \rangle \top$, using the rules introduced in the lecture.

i.
$$\langle \perp \rangle \psi = \perp$$

ii.
$$\langle \top \rangle \psi = \psi$$

iii.
$$\langle P(x_1,\ldots,x_n)\rangle\psi=P(x_1,\ldots,x_n)\wedge\psi$$

iv.
$$\langle \exists x \rangle \psi = \exists x [\psi]$$

v.
$$\langle \phi_1 \cdot \phi_2 \rangle \psi = \langle \phi_1 \rangle (\langle \phi_2 \rangle \psi)$$

vi.
$$\langle \sim \phi \rangle \psi = \neg (\langle \phi \rangle \top) \wedge \psi$$

Show each step of the derivation, and indicate the rule applied ((i)-(vi)) at each step:

a.

b.

c.