

# Pragmatic small world uncertainty

SIADS 542: Presenting uncertainty – Week 1, Lecture 2

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School of Information

University of Michigan

# **Today**

A pragmatic look at **small world uncertainty**

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A pragmatic look at **small world uncertainty**

**Parameter uncertainty** and predictive uncertainty

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A pragmatic look at **small world uncertainty**

**Parameter uncertainty** and predictive uncertainty

**Bayesian** and **frequentist** approaches

# **Today**

A pragmatic look at **small world uncertainty**

**Parameter uncertainty** and **predictive uncertainty**

**Bayesian** and **frequentist** approaches

**Pragmatic reporting** of small world uncertainty

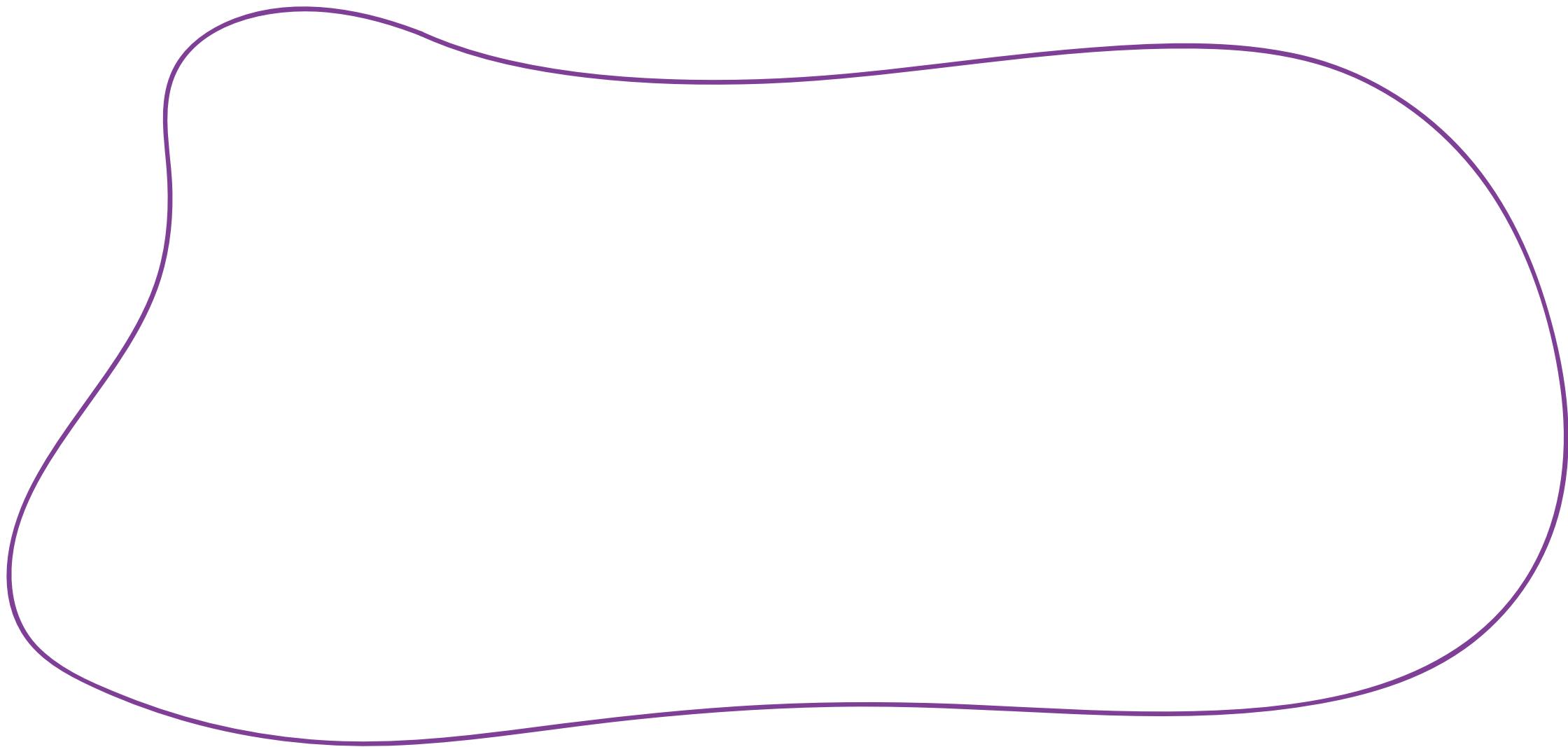
# **Before we proceed**

Make sure you've read this paper:

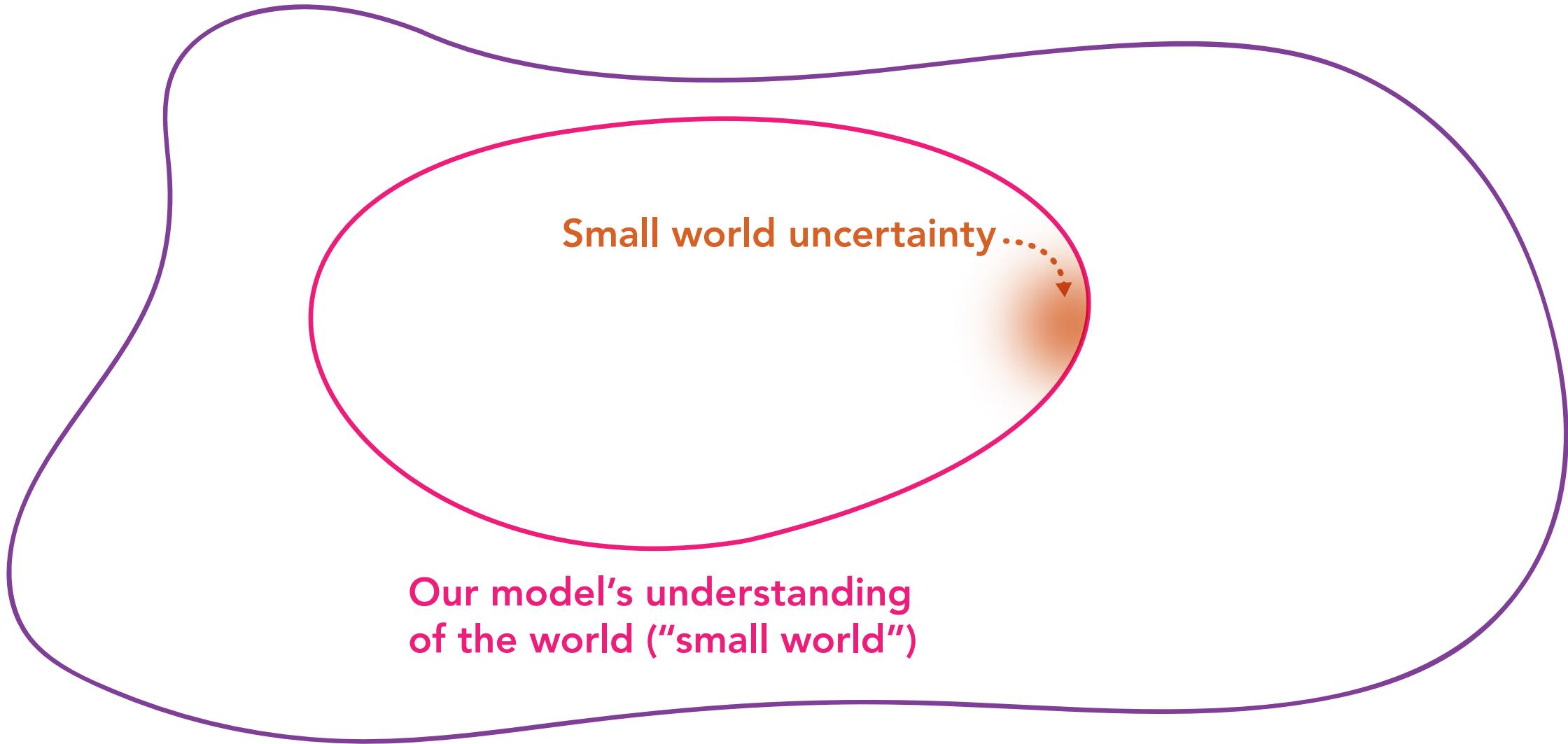
Kass (2011). [Statistical Inference: The Big Picture.](#)  
*Statistical Science*. 26, no. 1, 1–9. doi: 10.1214/10-STS337

# A pragmatic view of uncertainty & statistics

**Reality ("large world")**



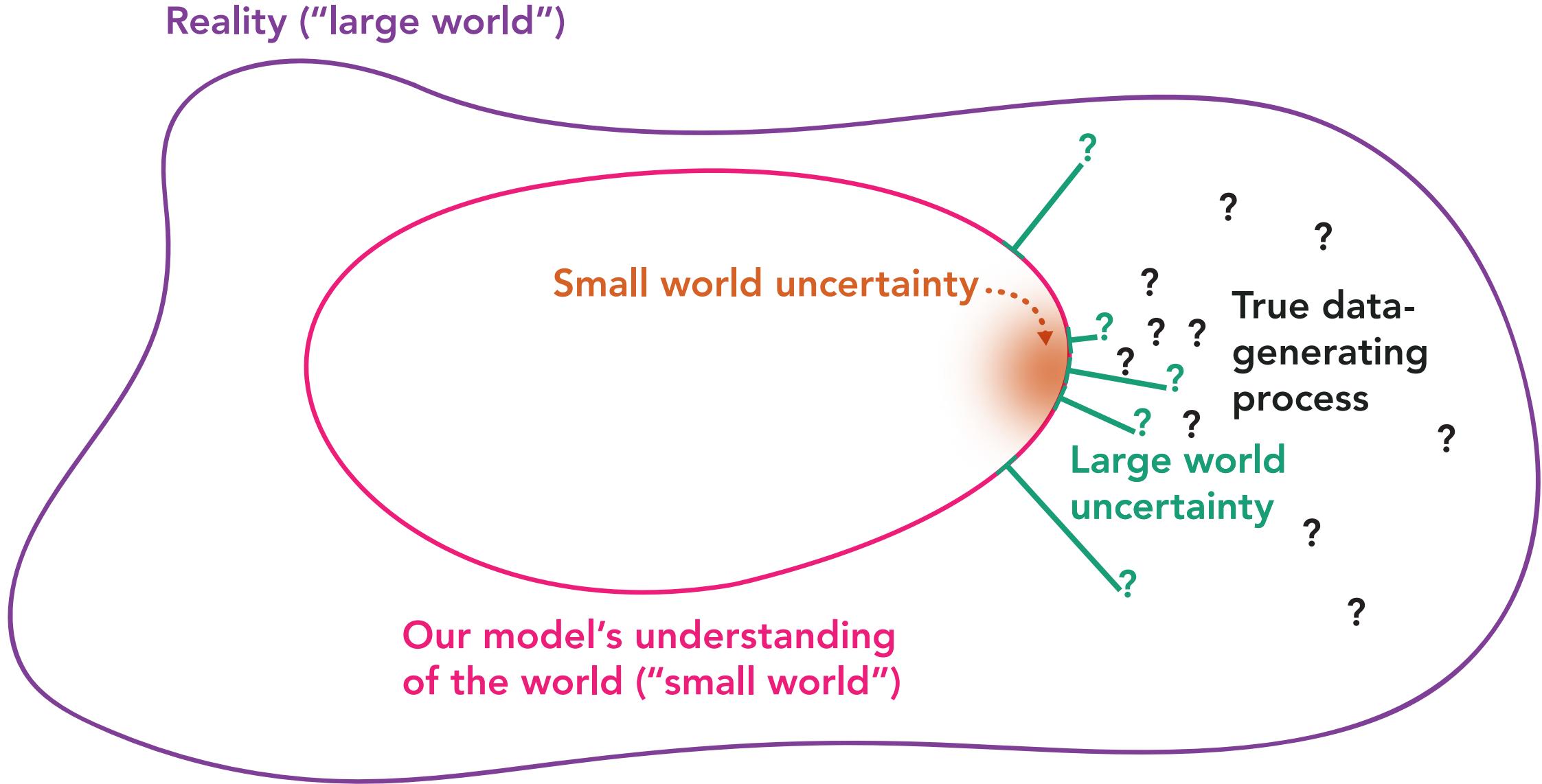
Reality ("large world")



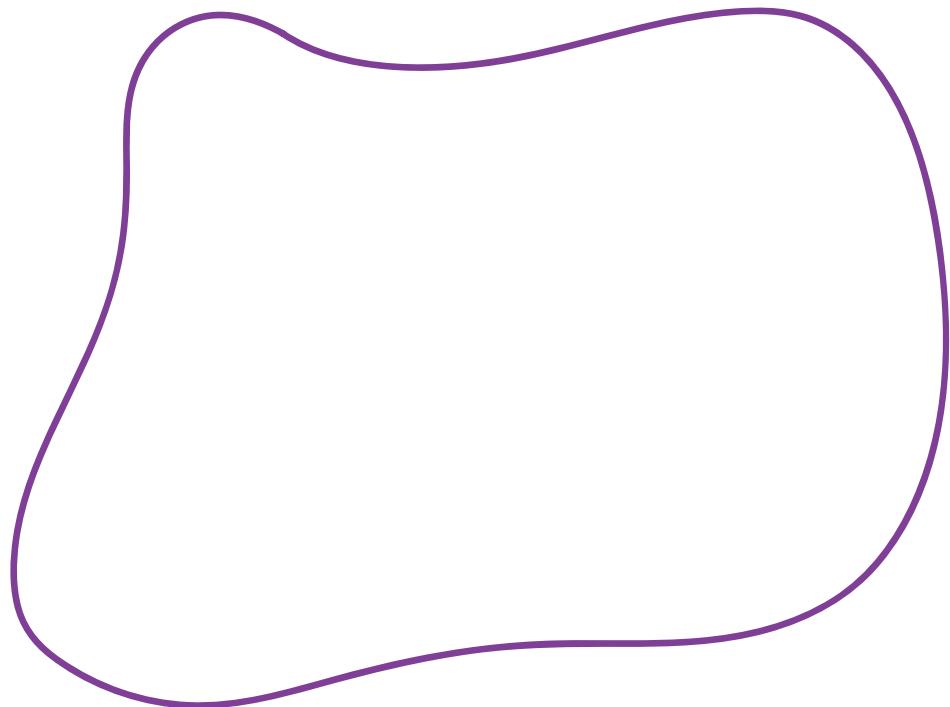
## Reality ("large world")

# Small world uncertainty.

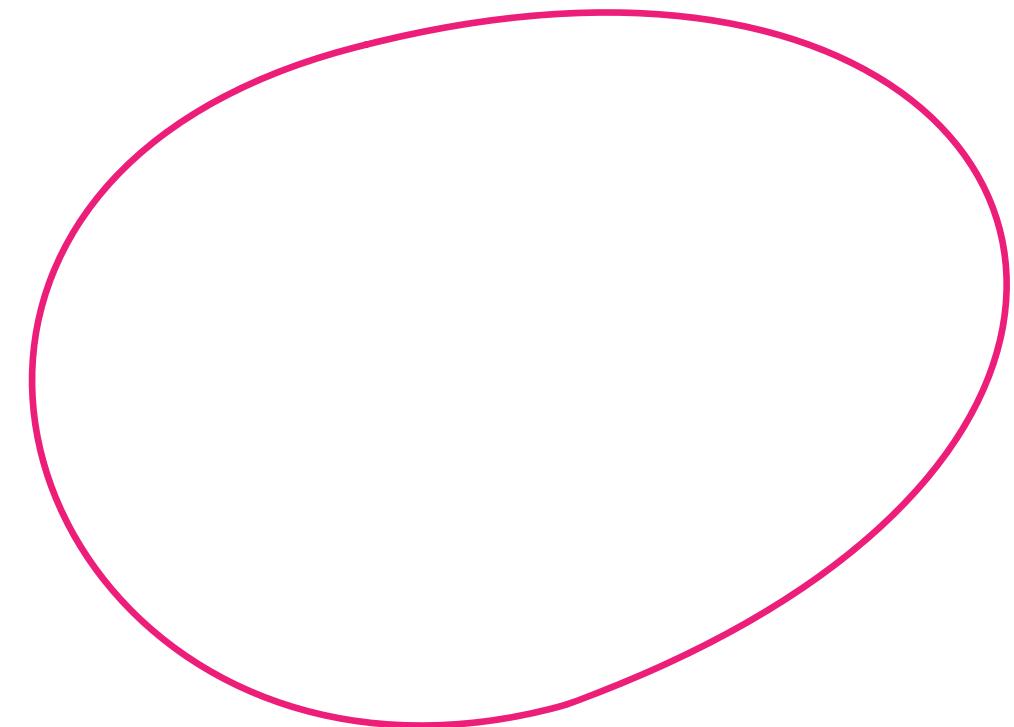
## Our model's understanding of the world ("small world")



Real world  
("large world")

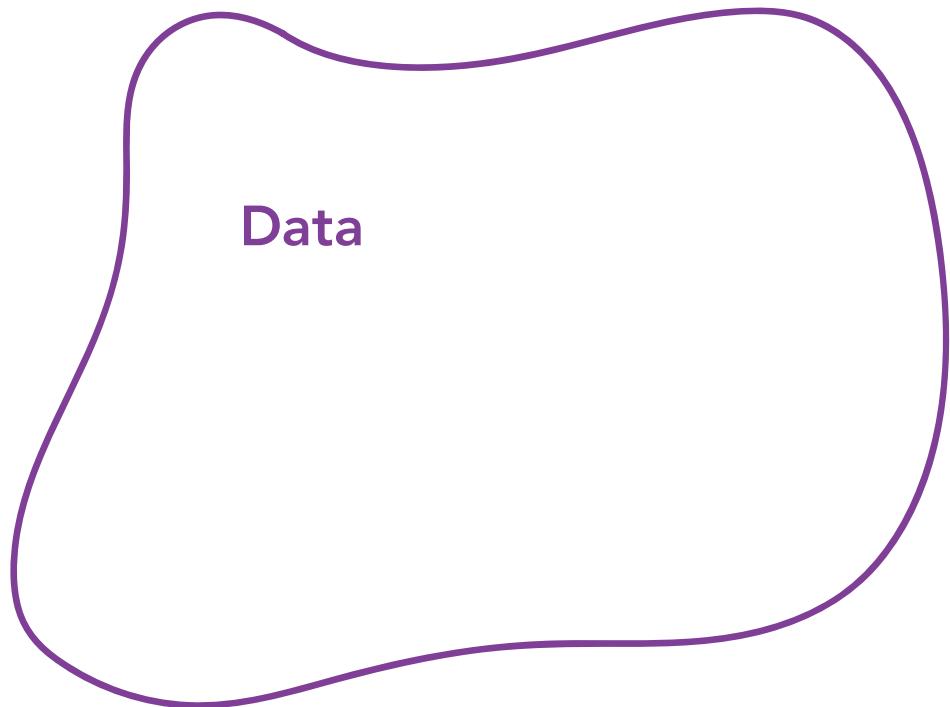


Theoretical world  
("small world")

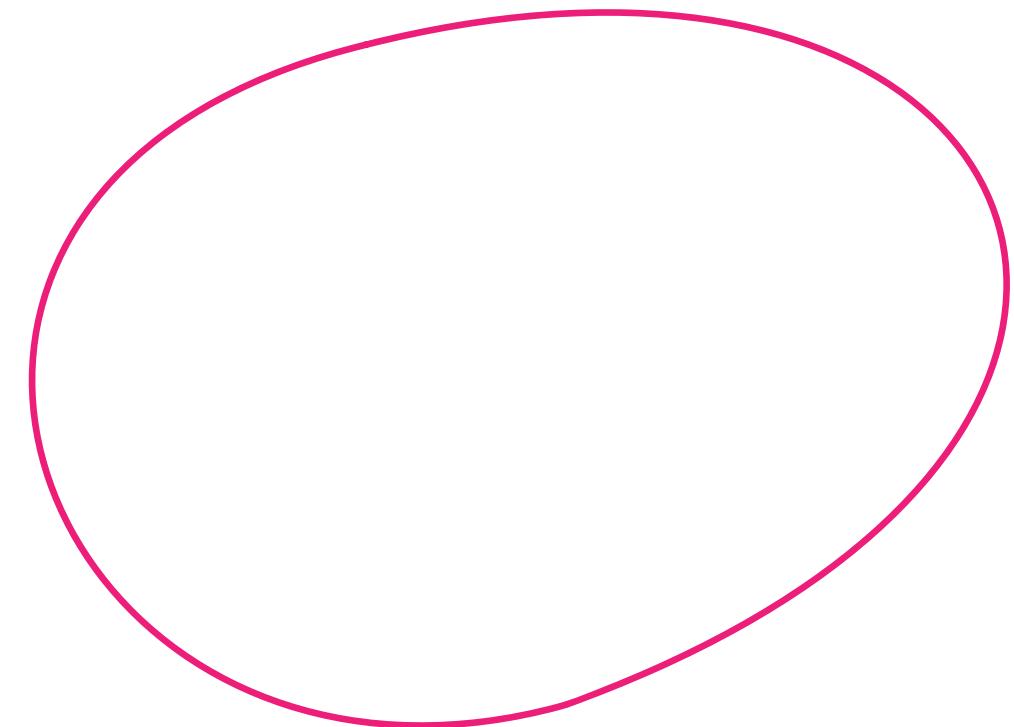


[adapted from Kass 2011]

Real world  
("large world")

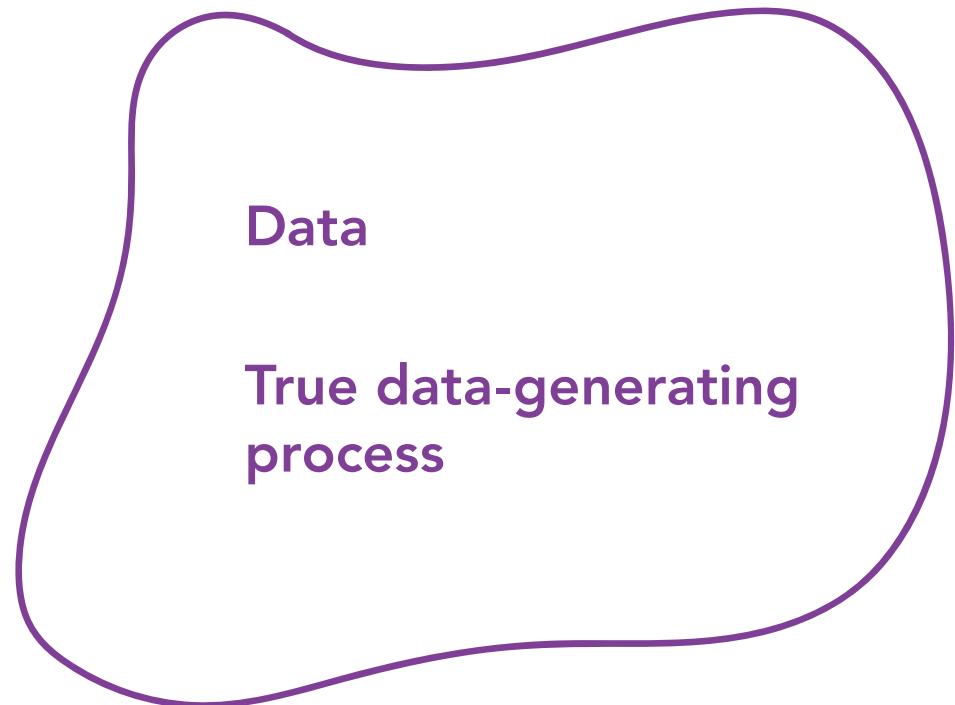


Theoretical world  
("small world")

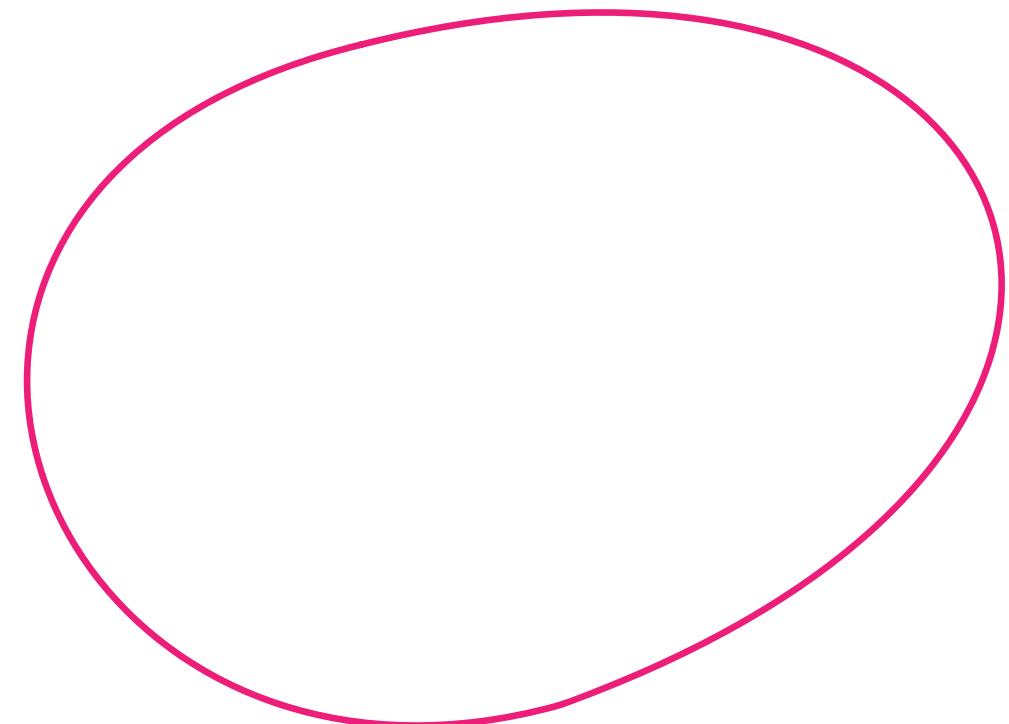


[adapted from Kass 2011]

Real world  
("large world")

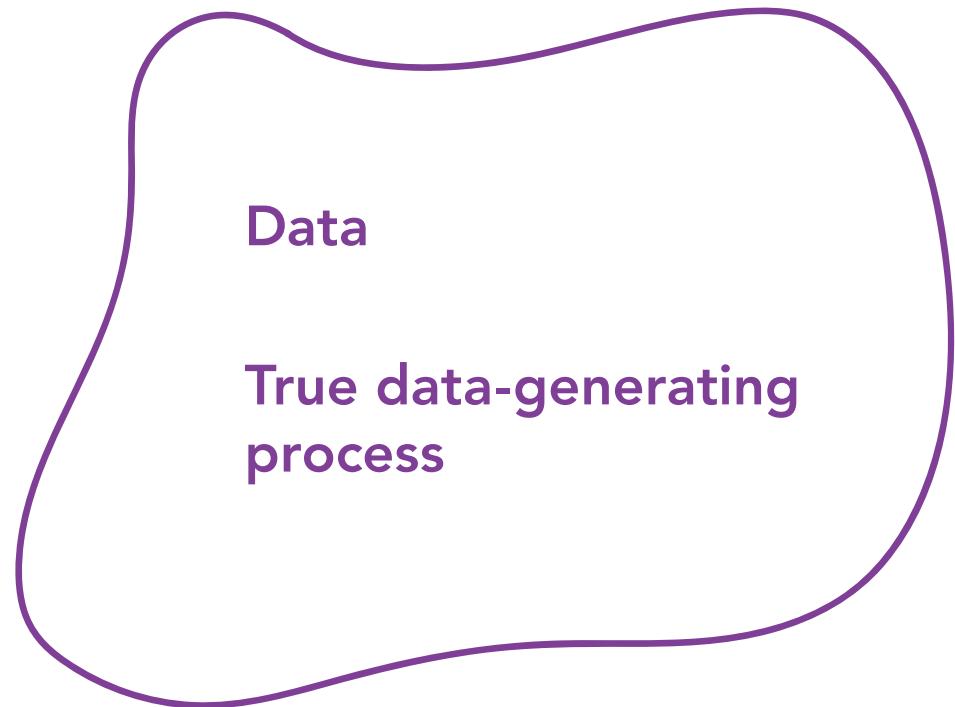


Theoretical world  
("small world")



[adapted from Kass 2011]

Real world  
("large world")



Theoretical world  
("small world")



[adapted from Kass 2011]

**Real world**  
("large world")

**Data**

**True data-generating  
process**

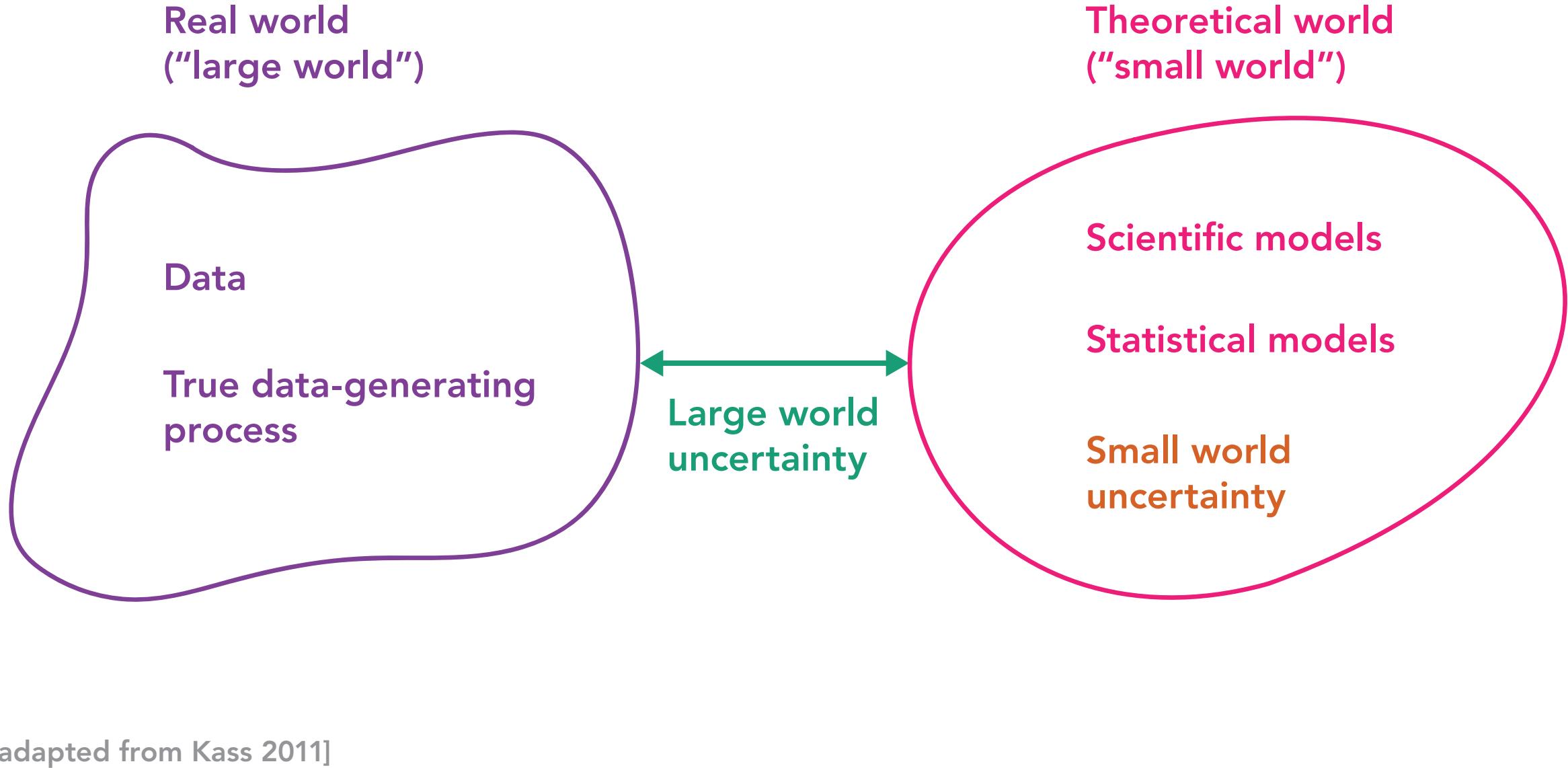
**Theoretical world**  
("small world")

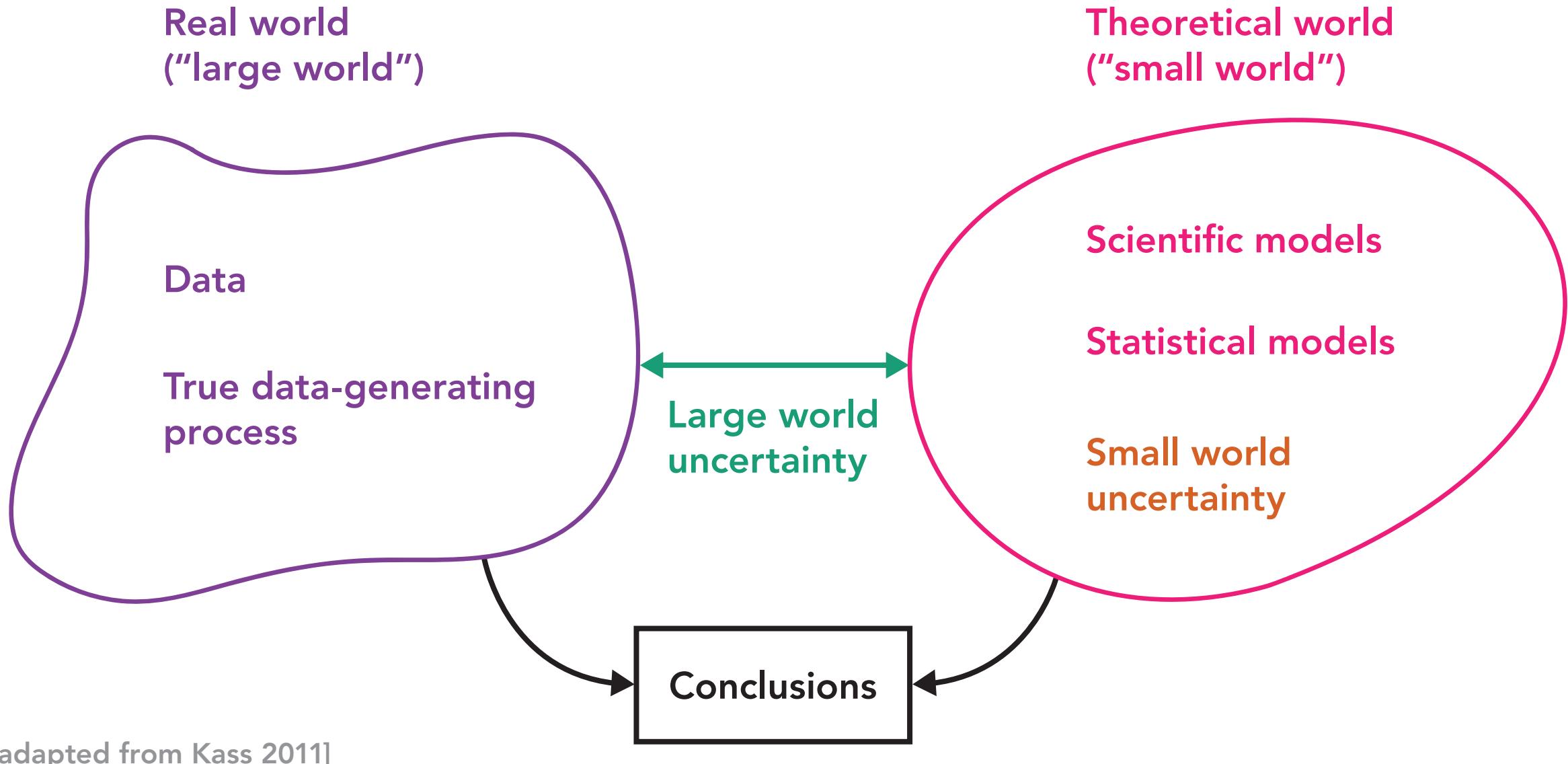
**Scientific models**

**Statistical models**

**Small world  
uncertainty**

[adapted from Kass 2011]



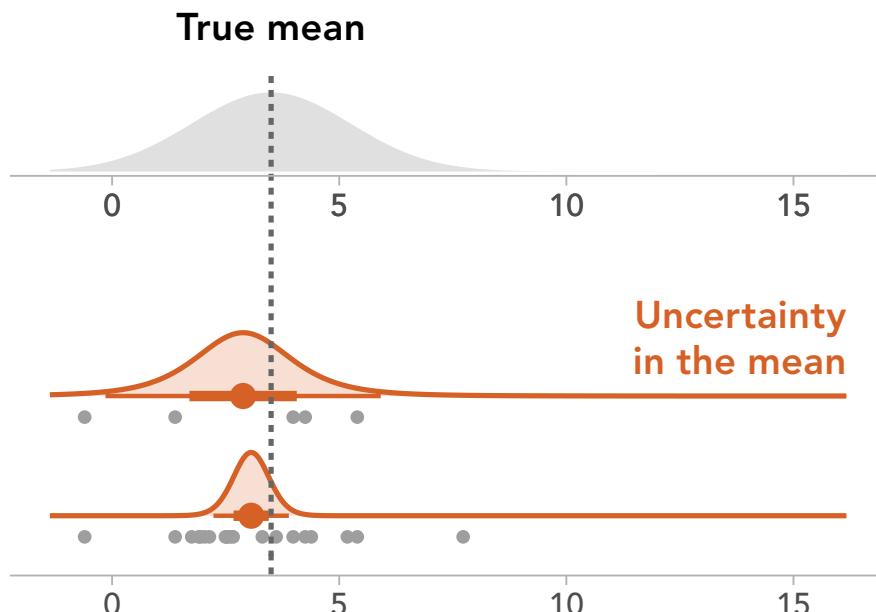


**Everything** we say about the **small world** is contingent upon **large world** assumptions

**Everything** we say about the **small world** is contingent upon **large world** assumptions

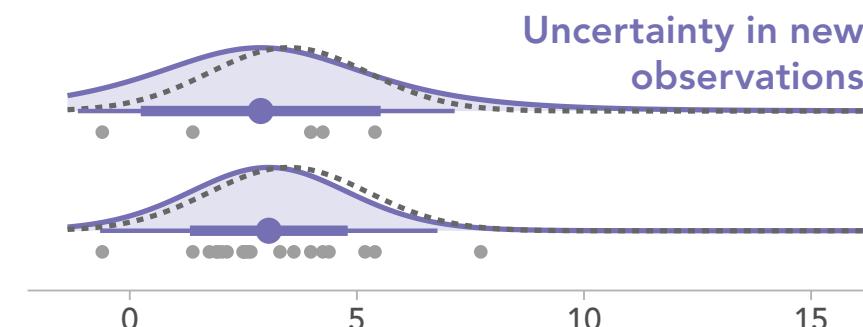
It is **overconfident** to assume otherwise

## Parameter uncertainty

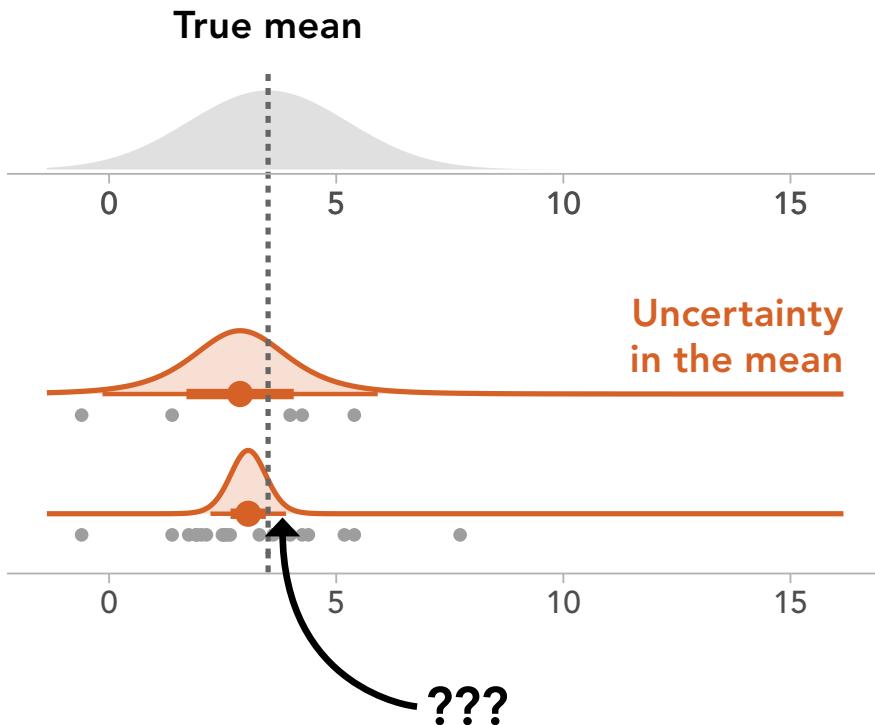


## Predictive uncertainty

True population distribution

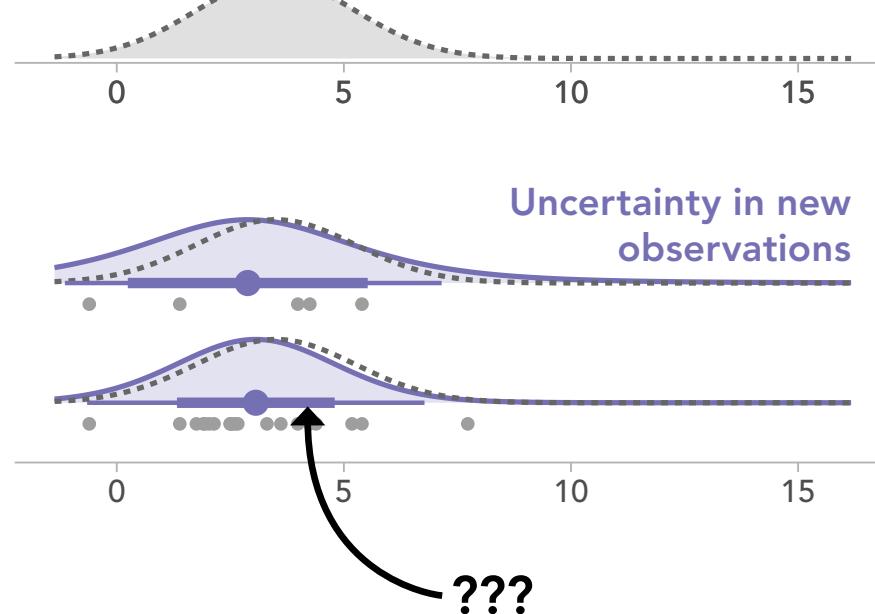


## Parameter uncertainty



## Predictive uncertainty

True population distribution



True (unknown) population

B – A (difference in seconds)

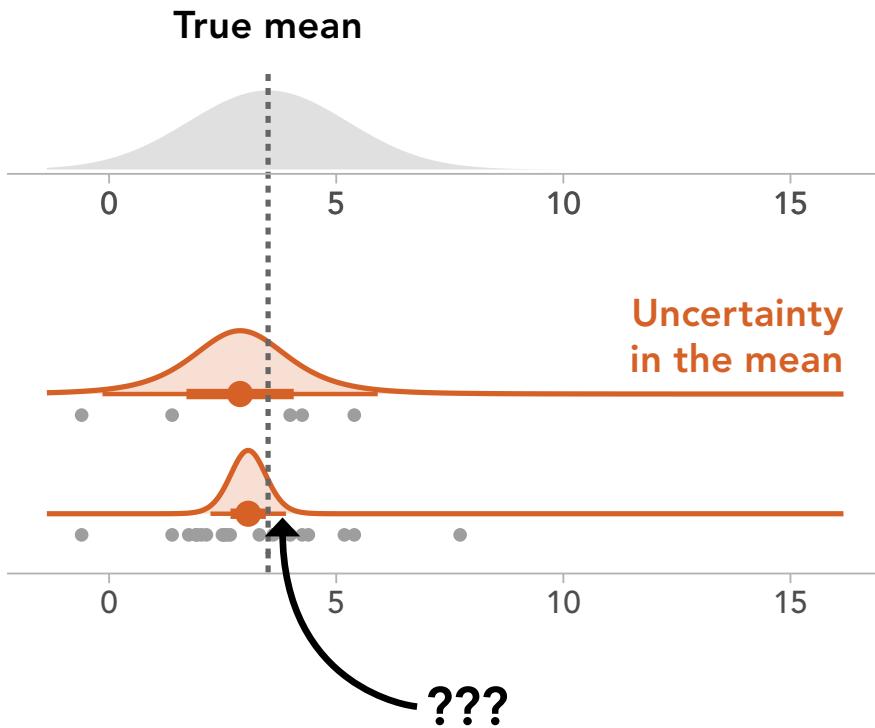
5 samples

20 samples

B – A (difference in seconds)

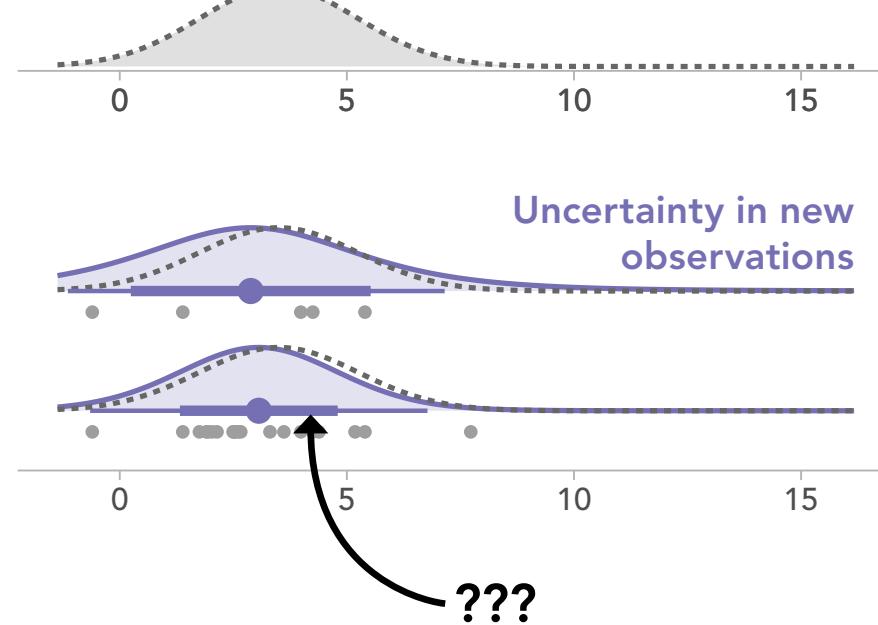
How do we talk about **small world uncertainty**

## Parameter uncertainty



## Predictive uncertainty

### True population distribution



### True (unknown) population

$B - A$  (difference in seconds)

5 samples

20 samples

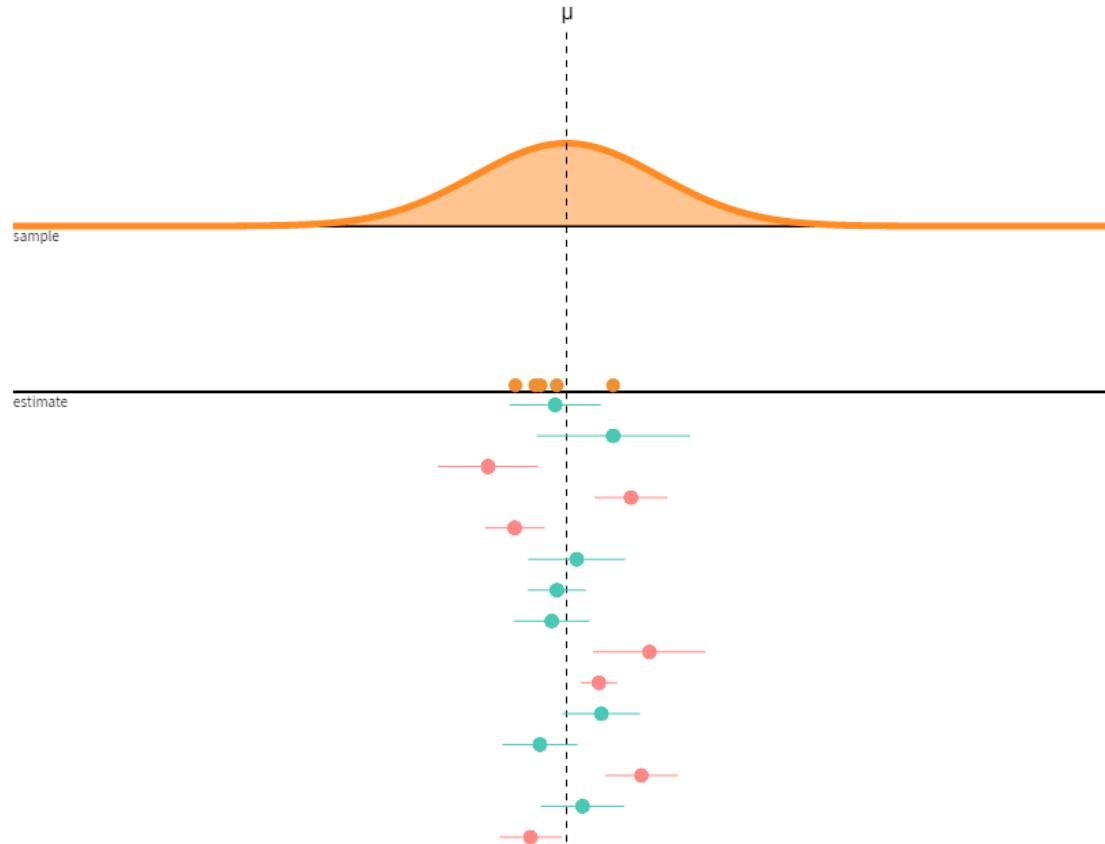
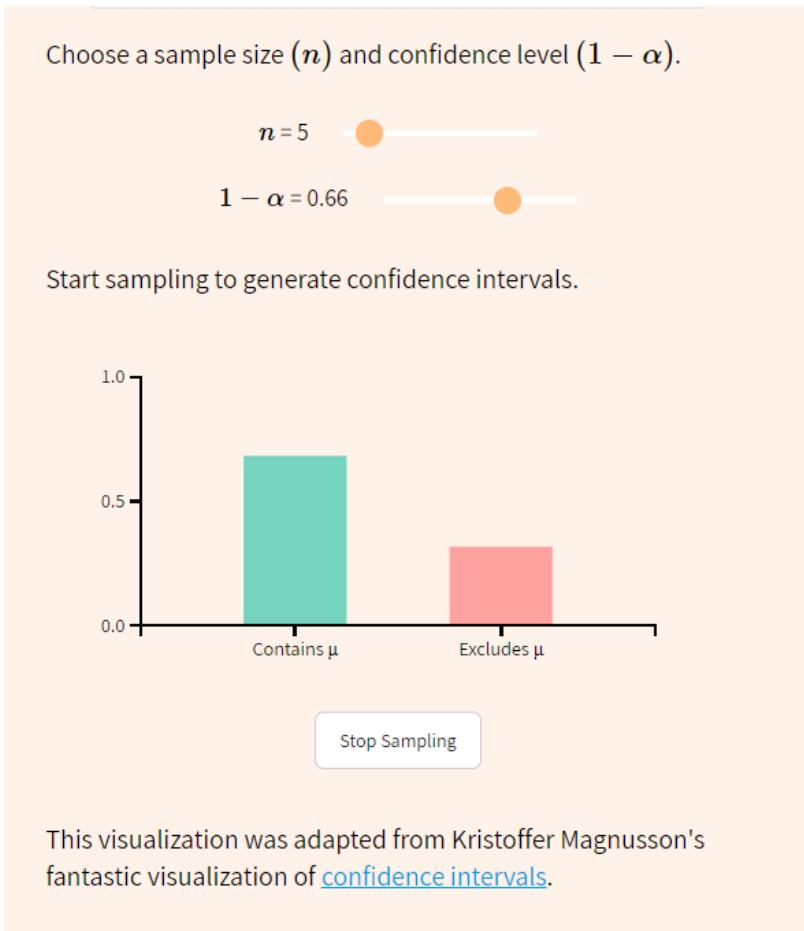
$B - A$  (difference in seconds)

How do we talk about **small world uncertainty** in frequentist and Bayesian paradigms?

# Frequentist small world

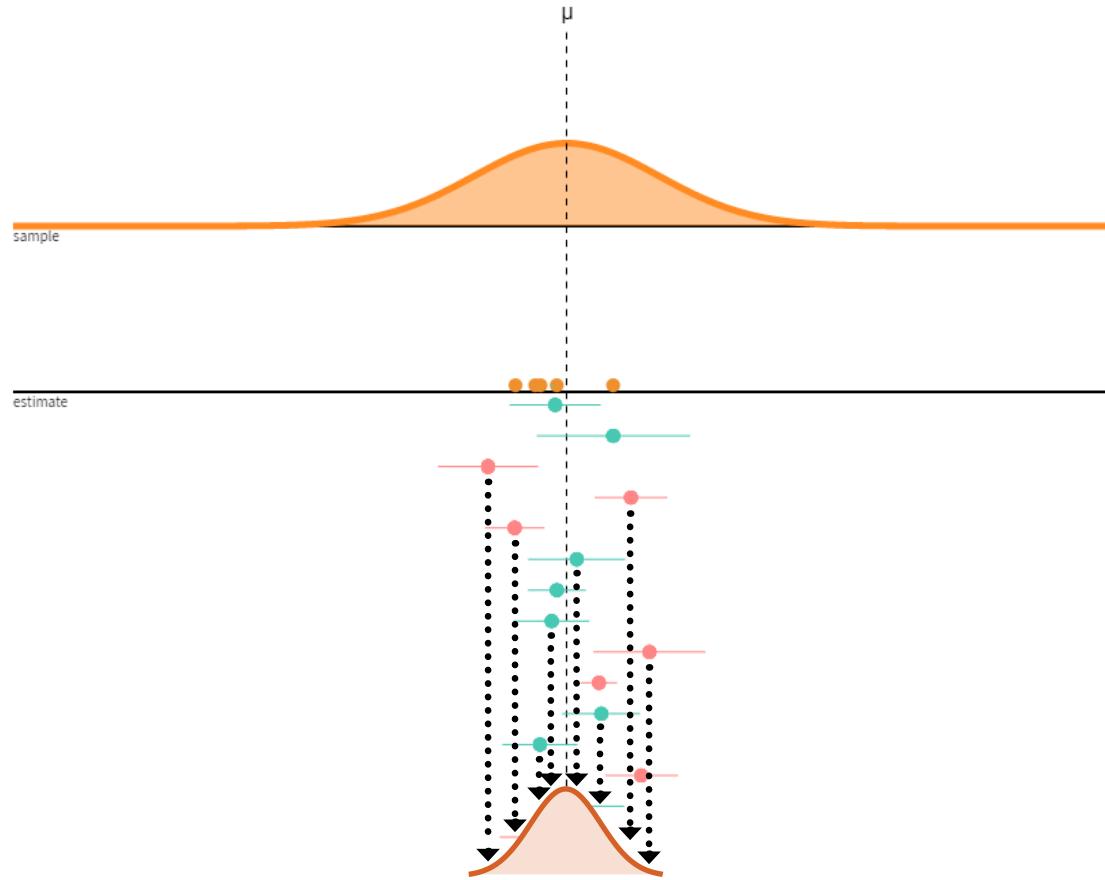
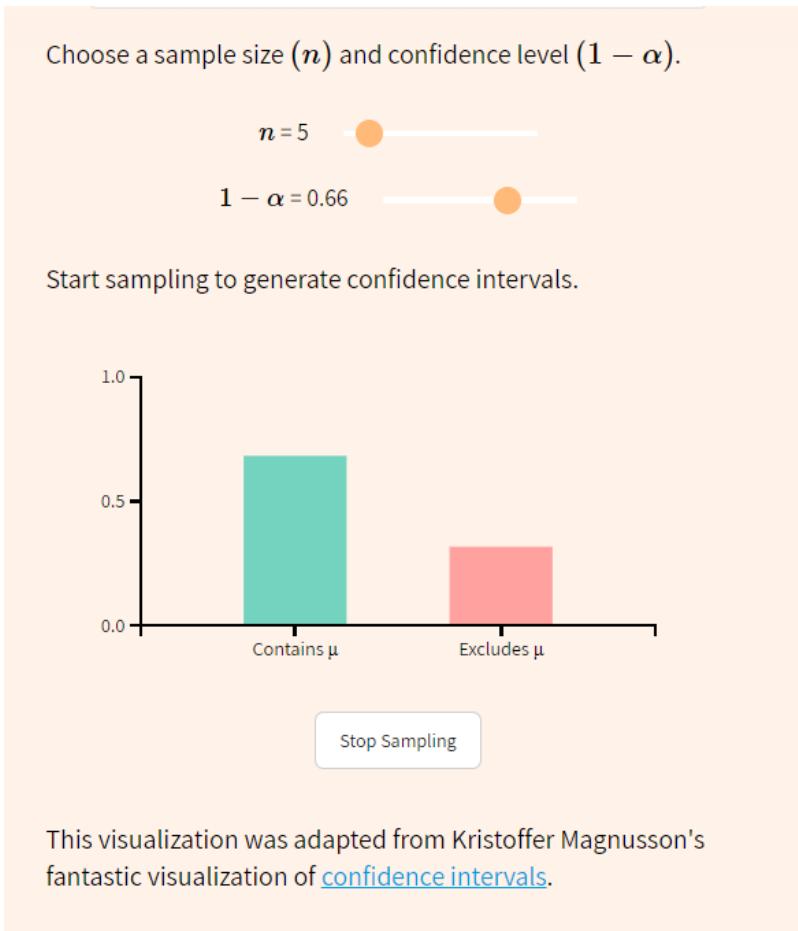
# Confidence intervals

[Seeing Theory: <https://seeing-theory.brown.edu/frequentist-inference/index.html#section2>]



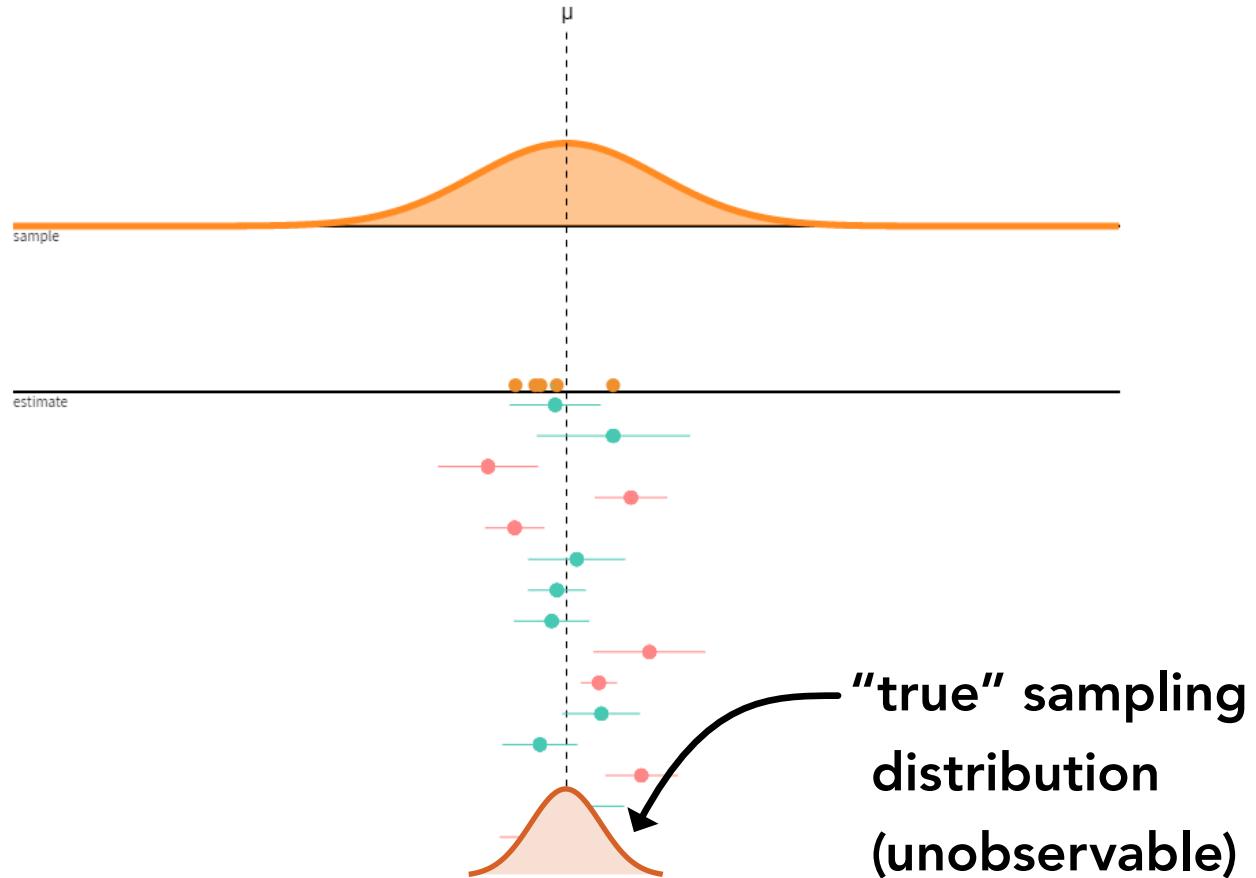
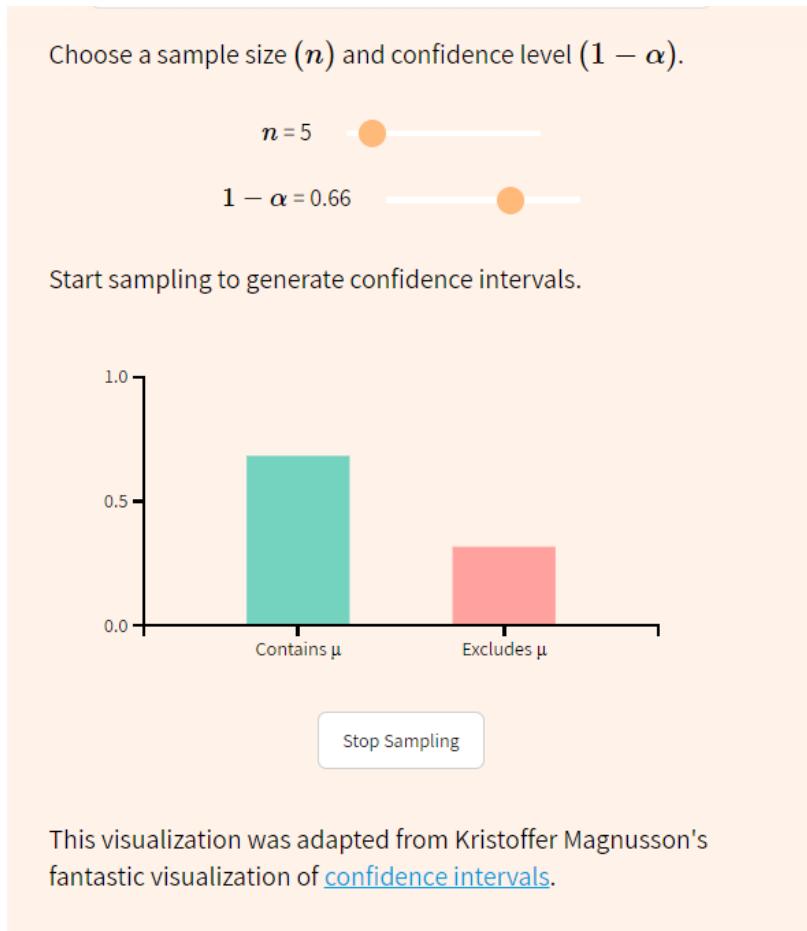
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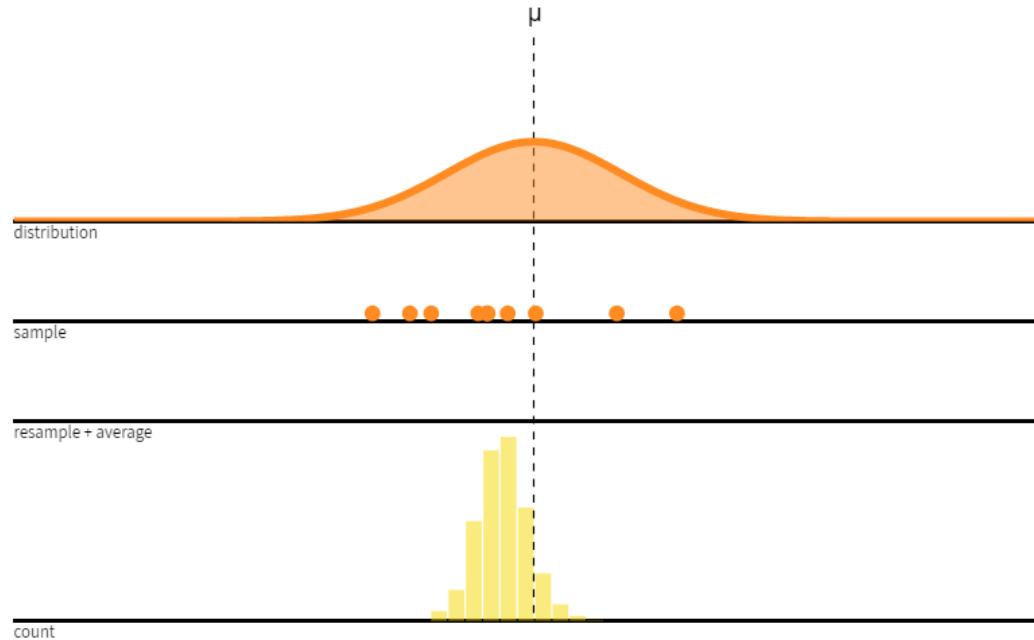
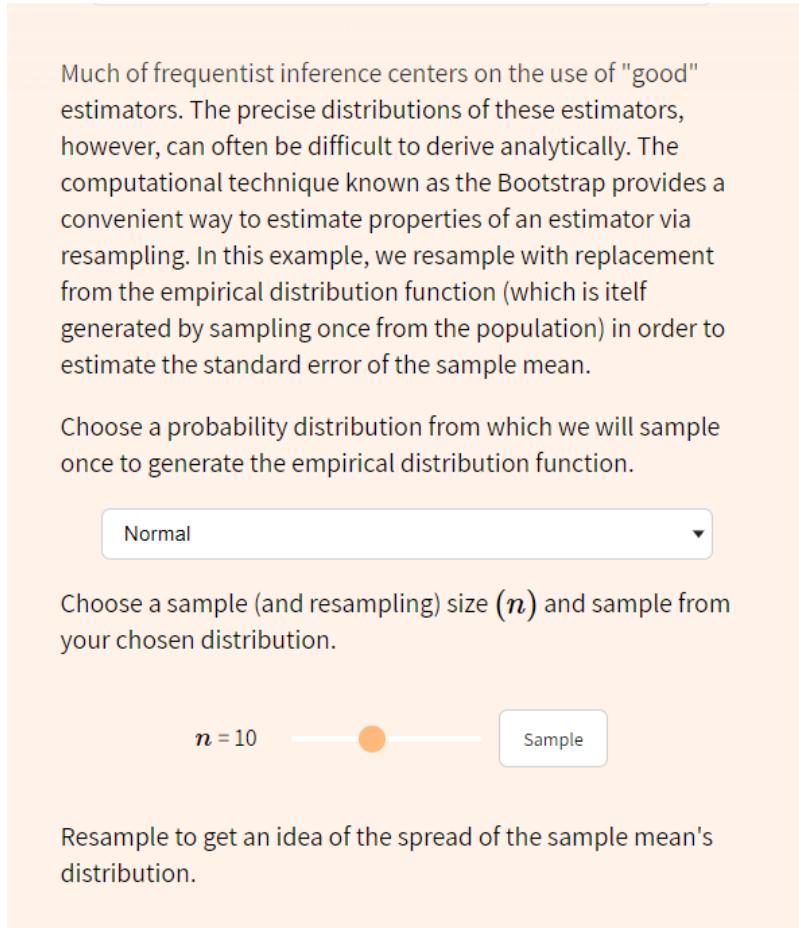
# Confidence intervals

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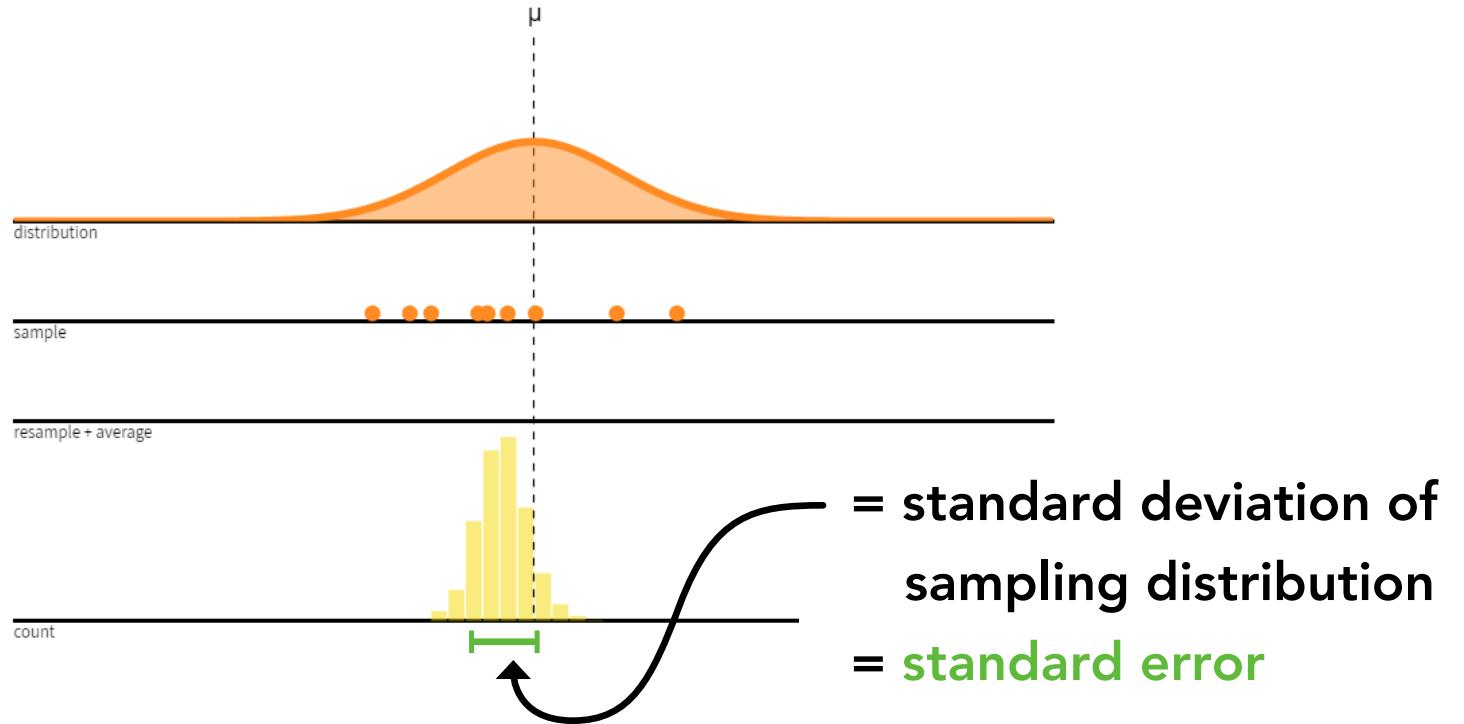
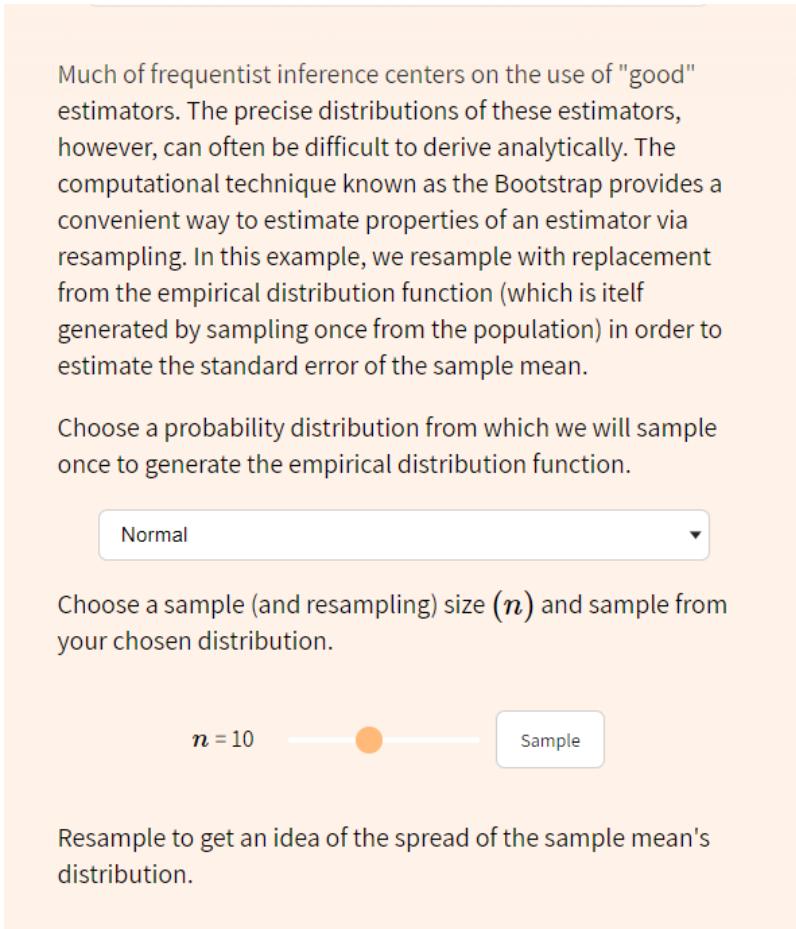
# Sampling distributions

[Seeing Theory: <https://seeing-theory.brown.edu/frequentist-inference/index.html#section2>]

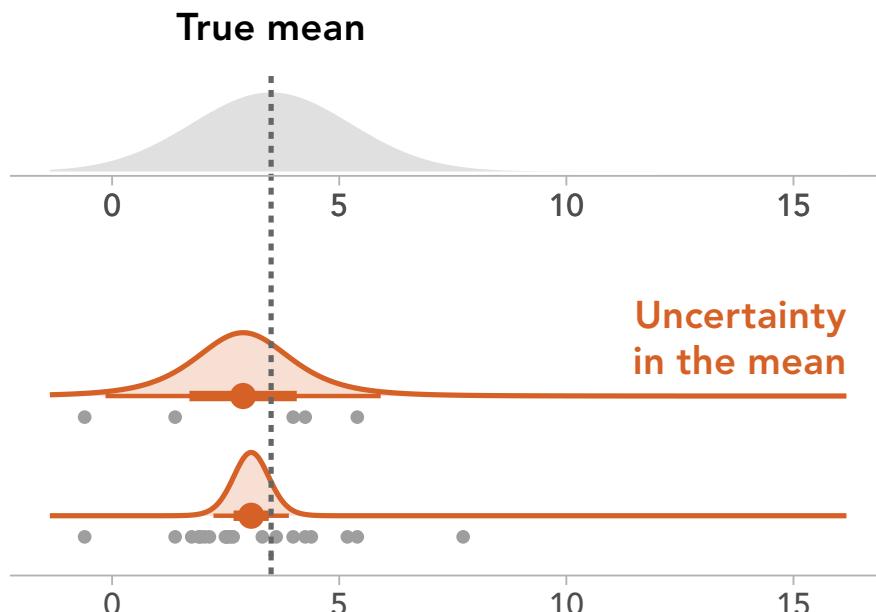


# Sampling distributions

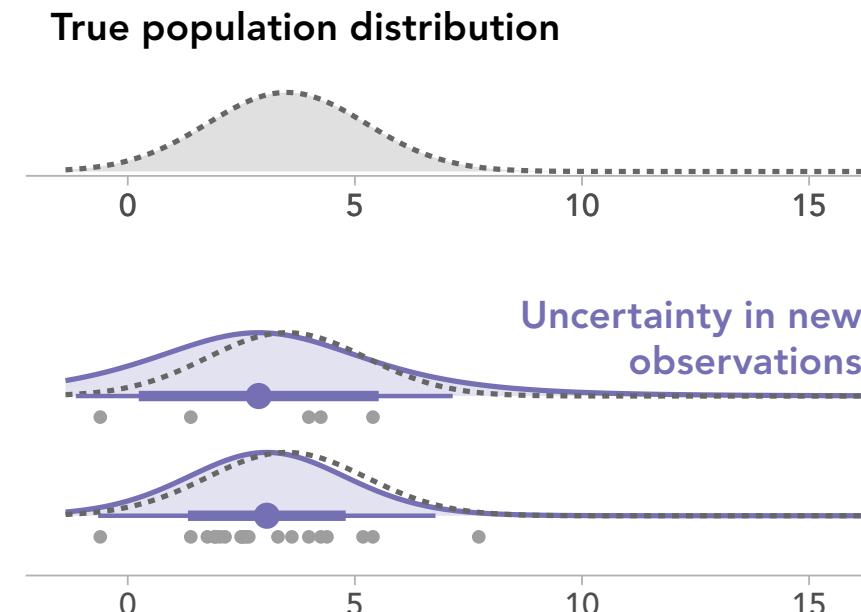
[Seeing Theory: <https://seeing-theory.brown.edu/frequentist-inference/index.html#section2>]



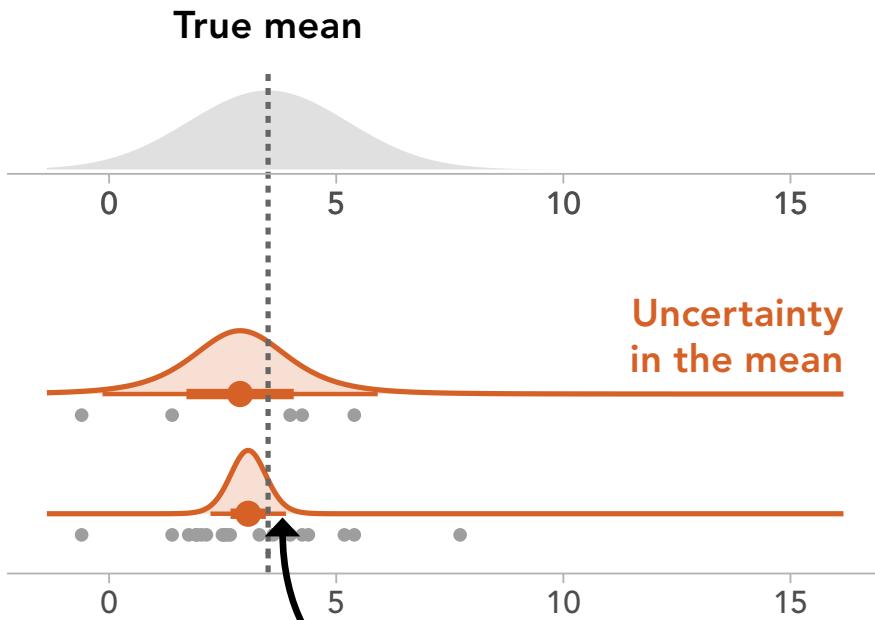
## Parameter uncertainty



## Predictive uncertainty

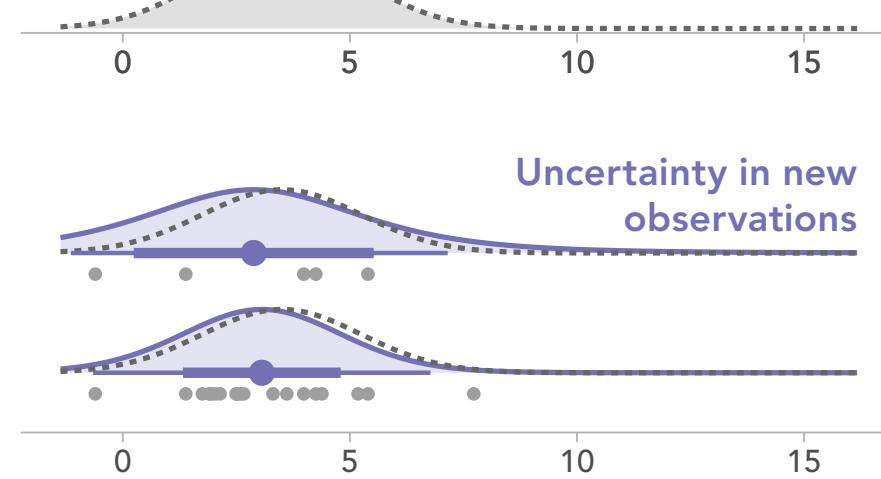


## Parameter uncertainty



## Predictive uncertainty

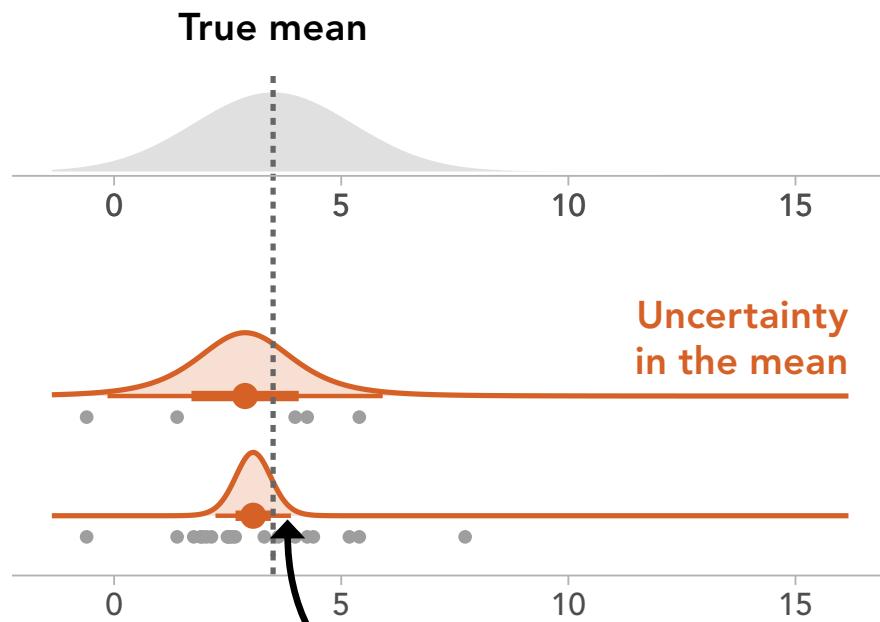
True population distribution



True (unknown) population  
 $B - A$  (difference in seconds)

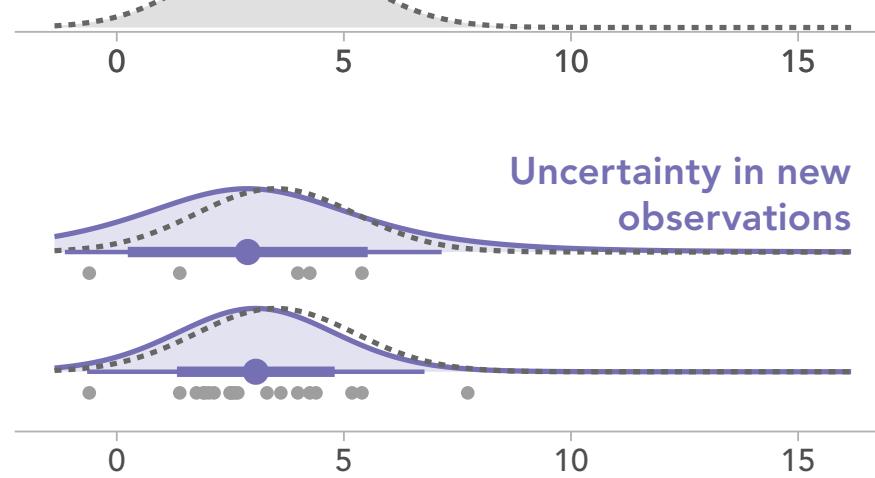
5 samples  
20 samples  
 $B - A$  (difference in seconds)

## Parameter uncertainty

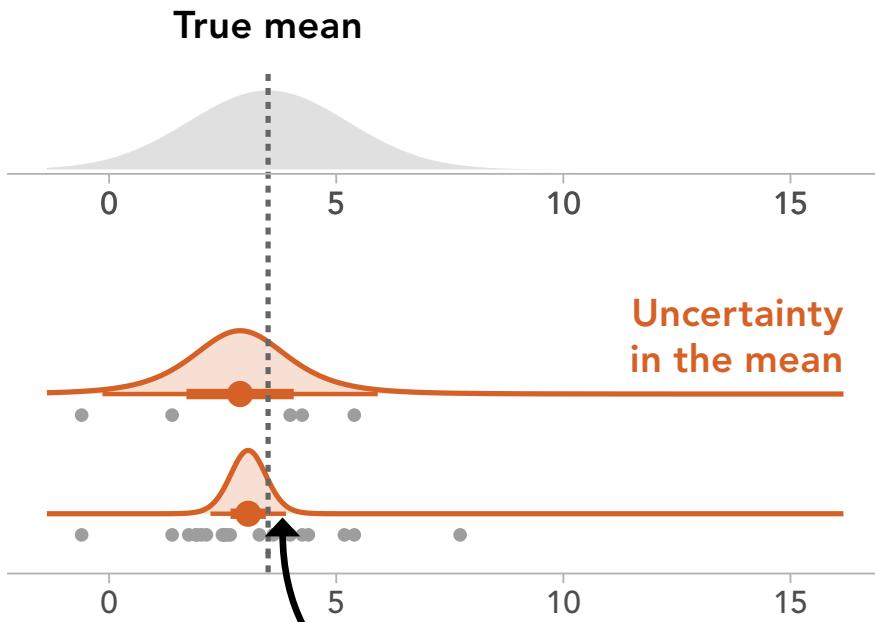


## Predictive uncertainty

True population distribution

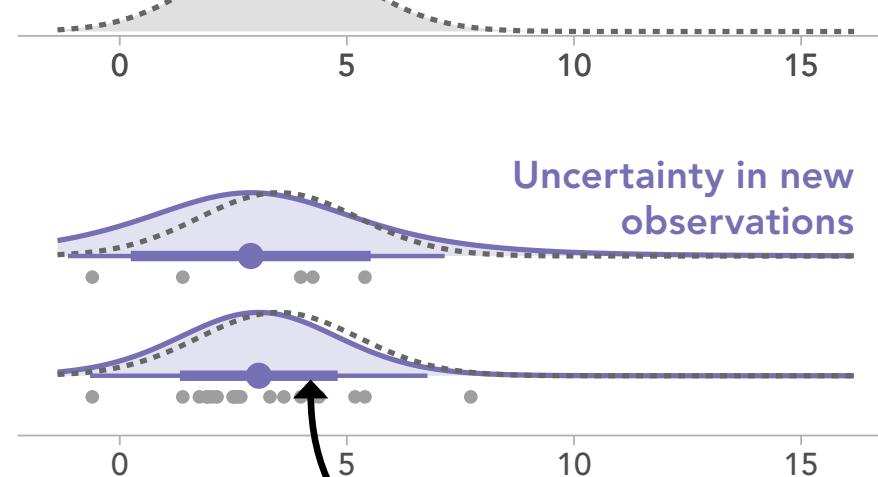


## Parameter uncertainty



## Predictive uncertainty

True population distribution



True (unknown) population

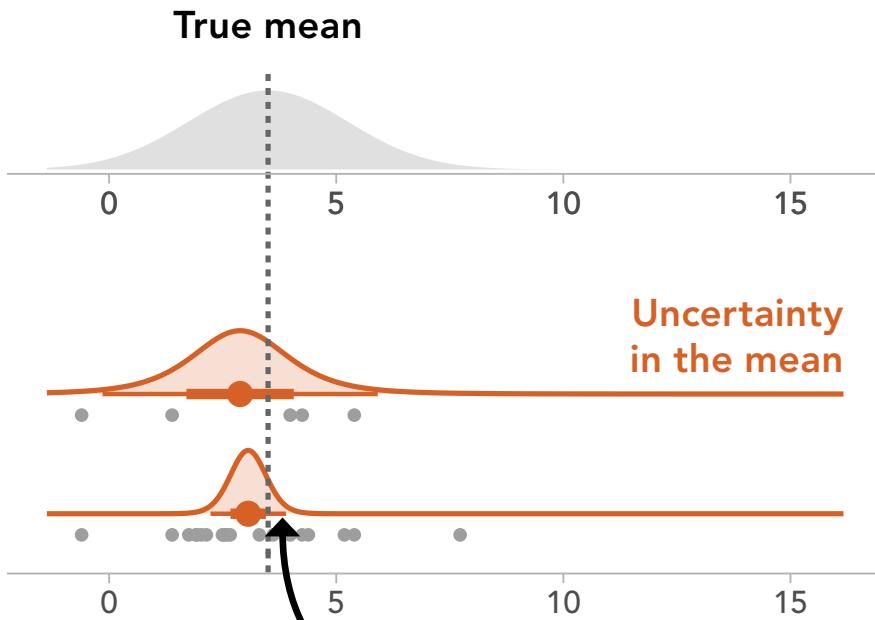
B - A (difference in seconds)

5 samples

20 samples

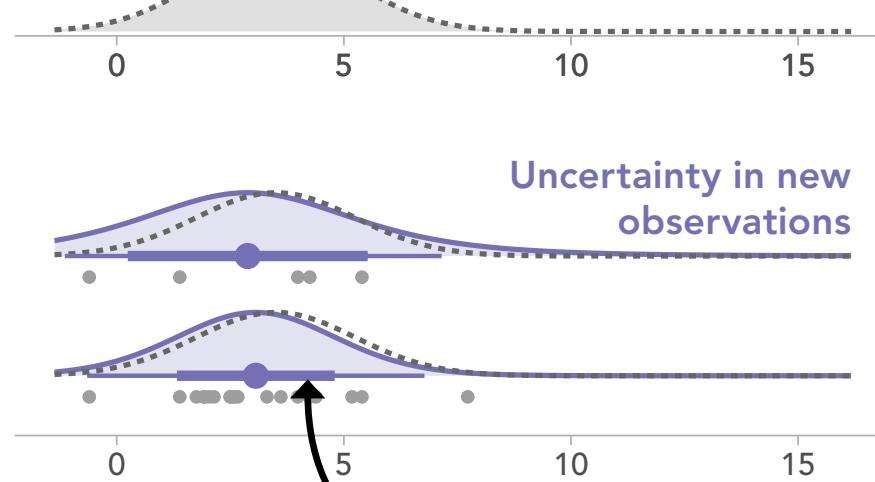
B - A (difference in seconds)

## Parameter uncertainty

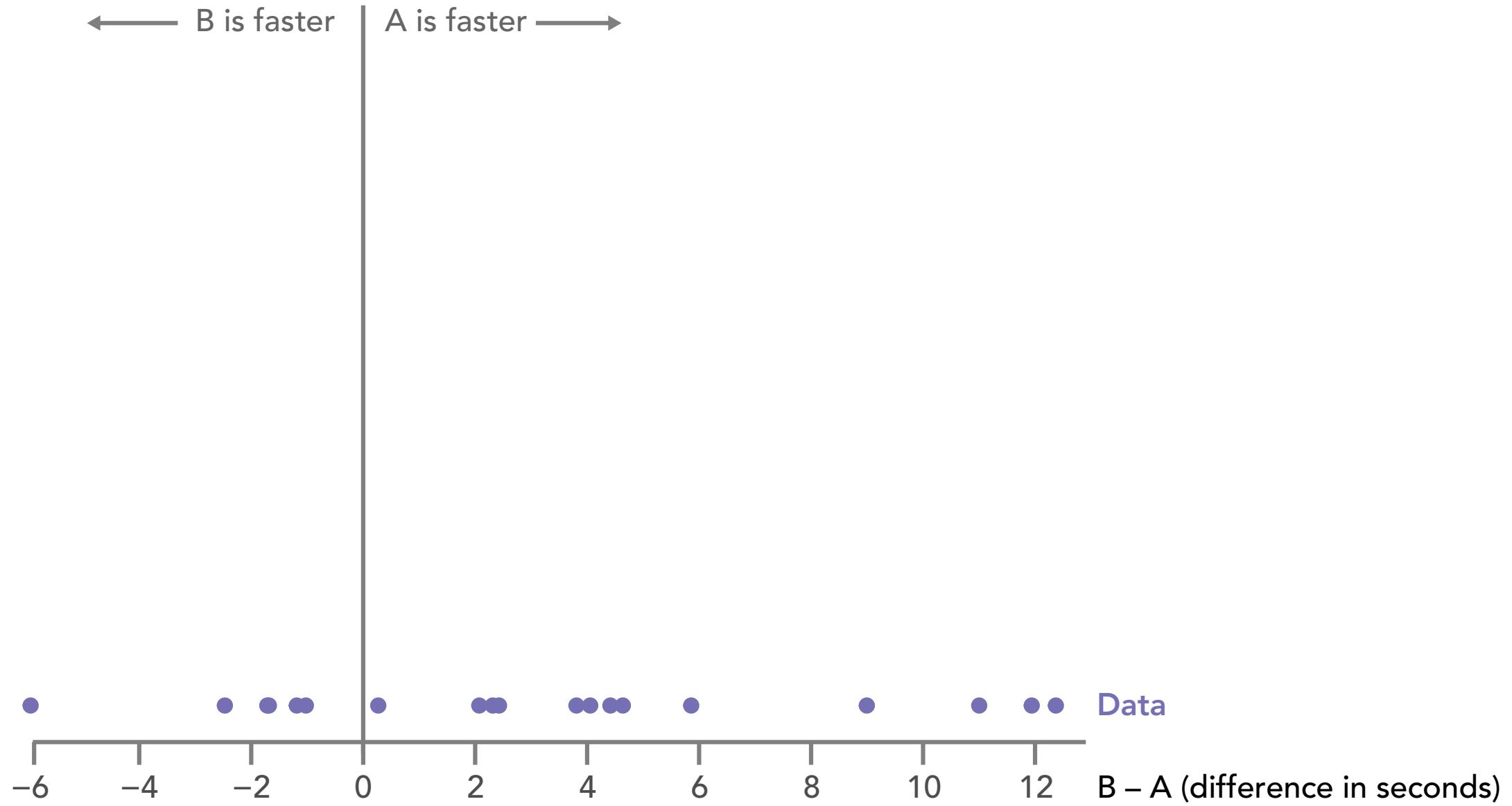


## Predictive uncertainty

True population distribution



# Bayesian small world



← B is faster

A is faster →

I want:

$P(\text{mean difference} \mid \text{data})$



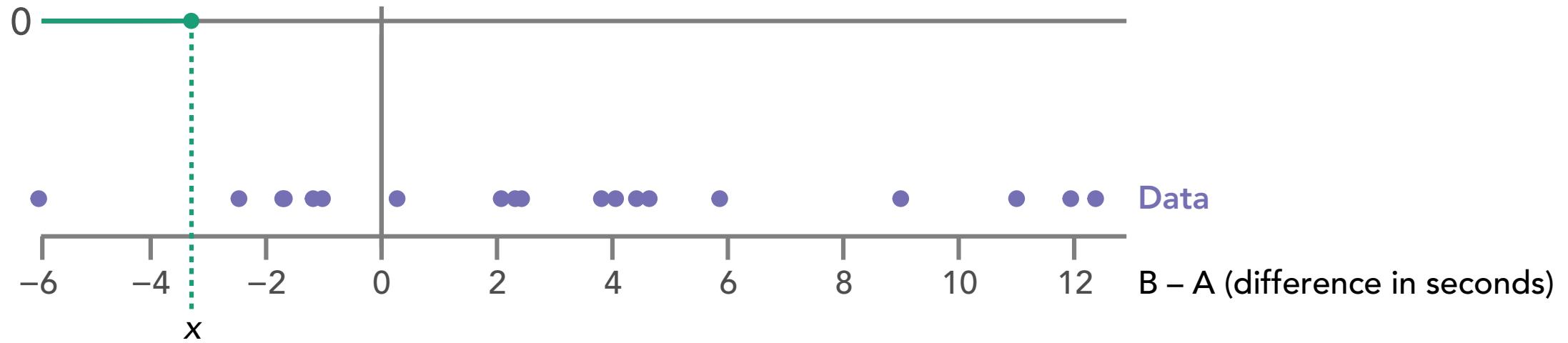
$\leftarrow$  B is faster

A is faster  $\rightarrow$

I want:

$P(\text{mean difference} \mid \text{data})$

$P(\text{data} \mid \text{mean difference} = x)$



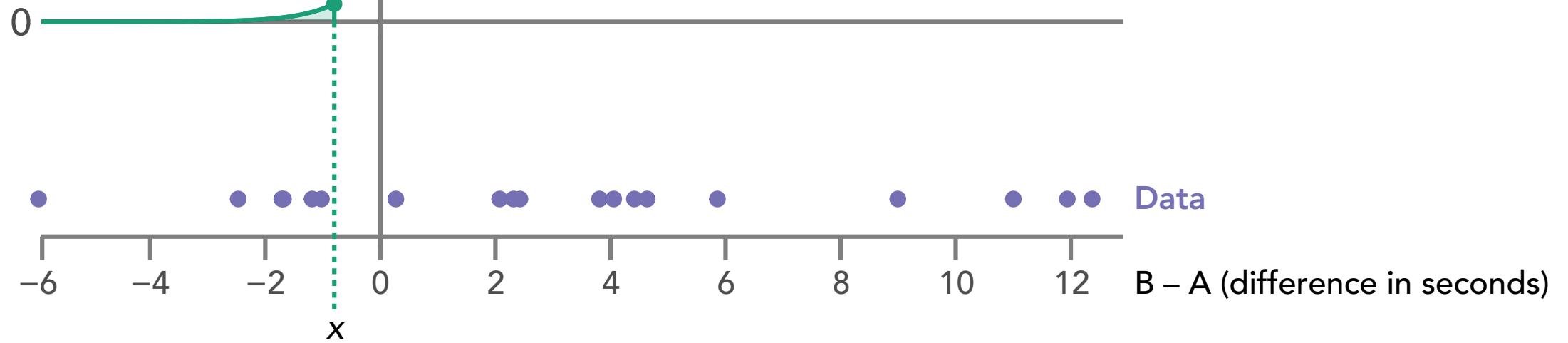
$\leftarrow$  B is faster

A is faster  $\rightarrow$

I want:

$P(\text{mean difference} \mid \text{data})$

$P(\text{data} \mid \text{mean difference} = x)$



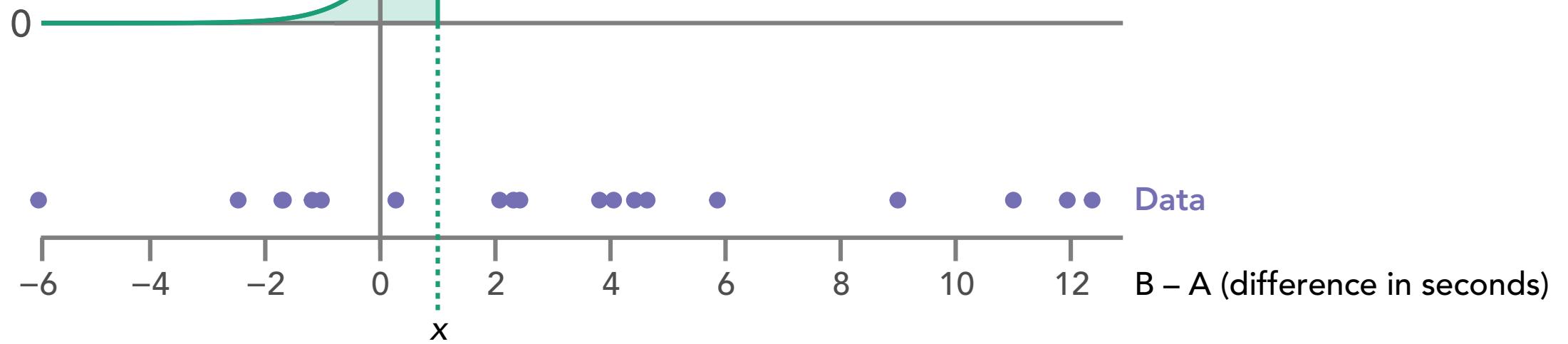
← B is faster

A is faster →

I want:

$P(\text{mean difference} \mid \text{data})$

$P(\text{data} \mid \text{mean difference} = x)$



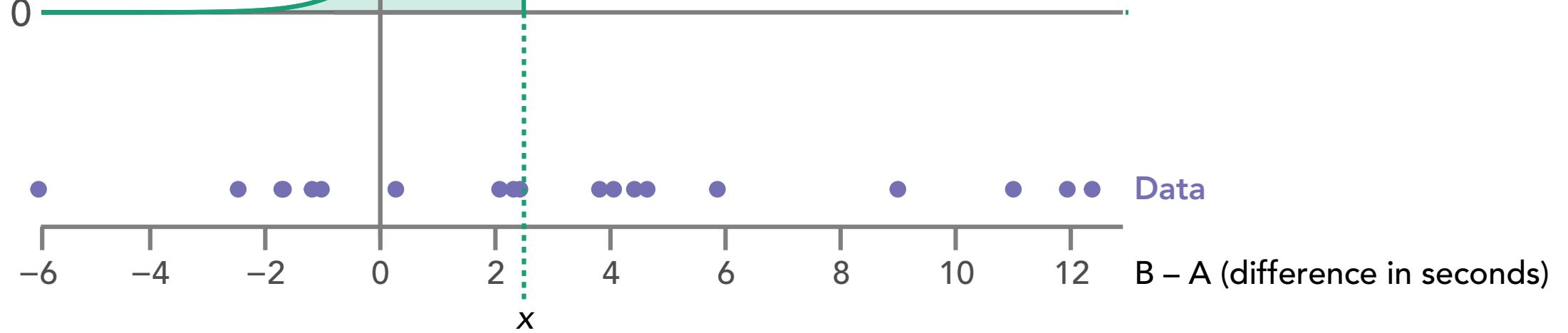
← B is faster

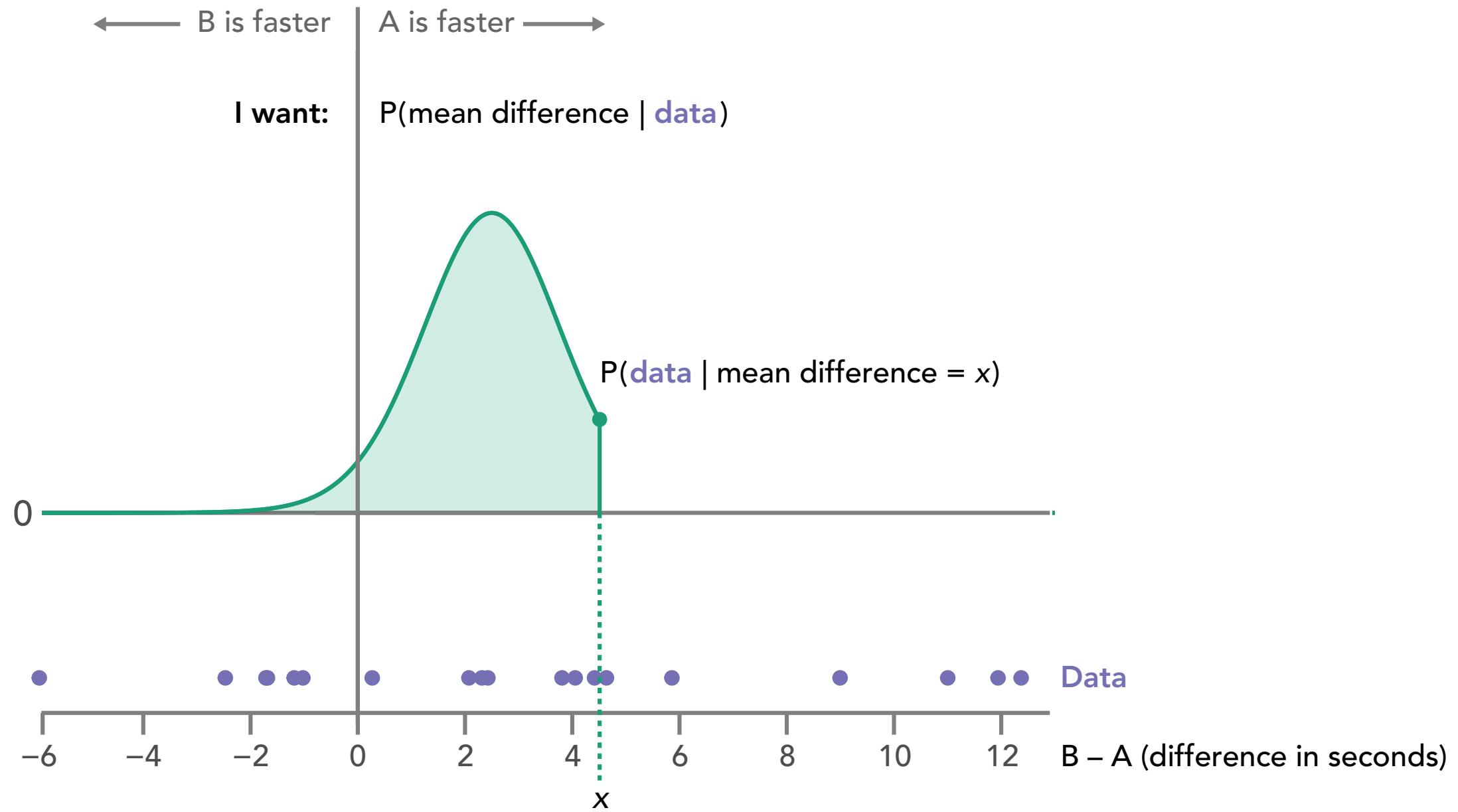
A is faster →

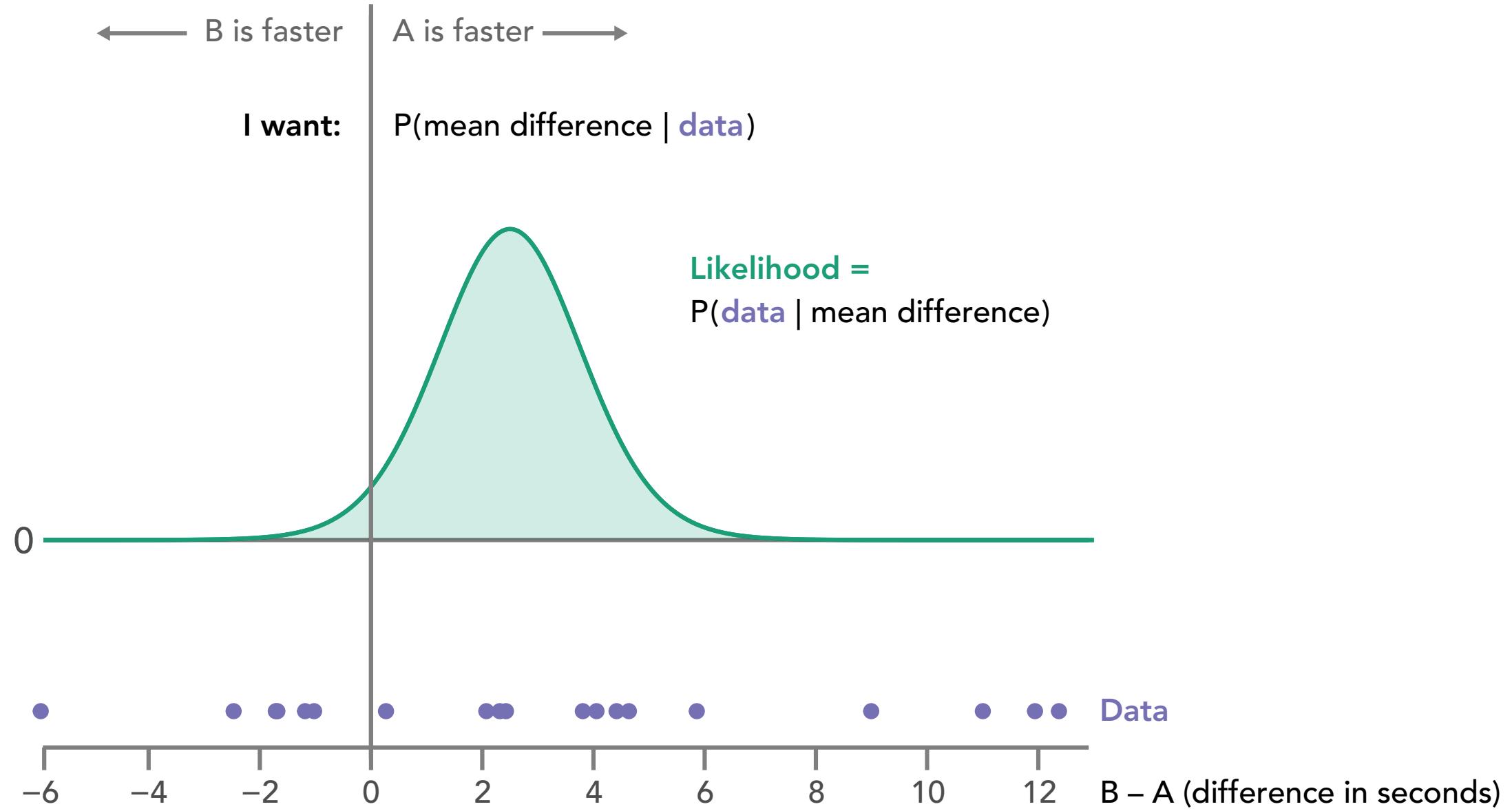
I want:

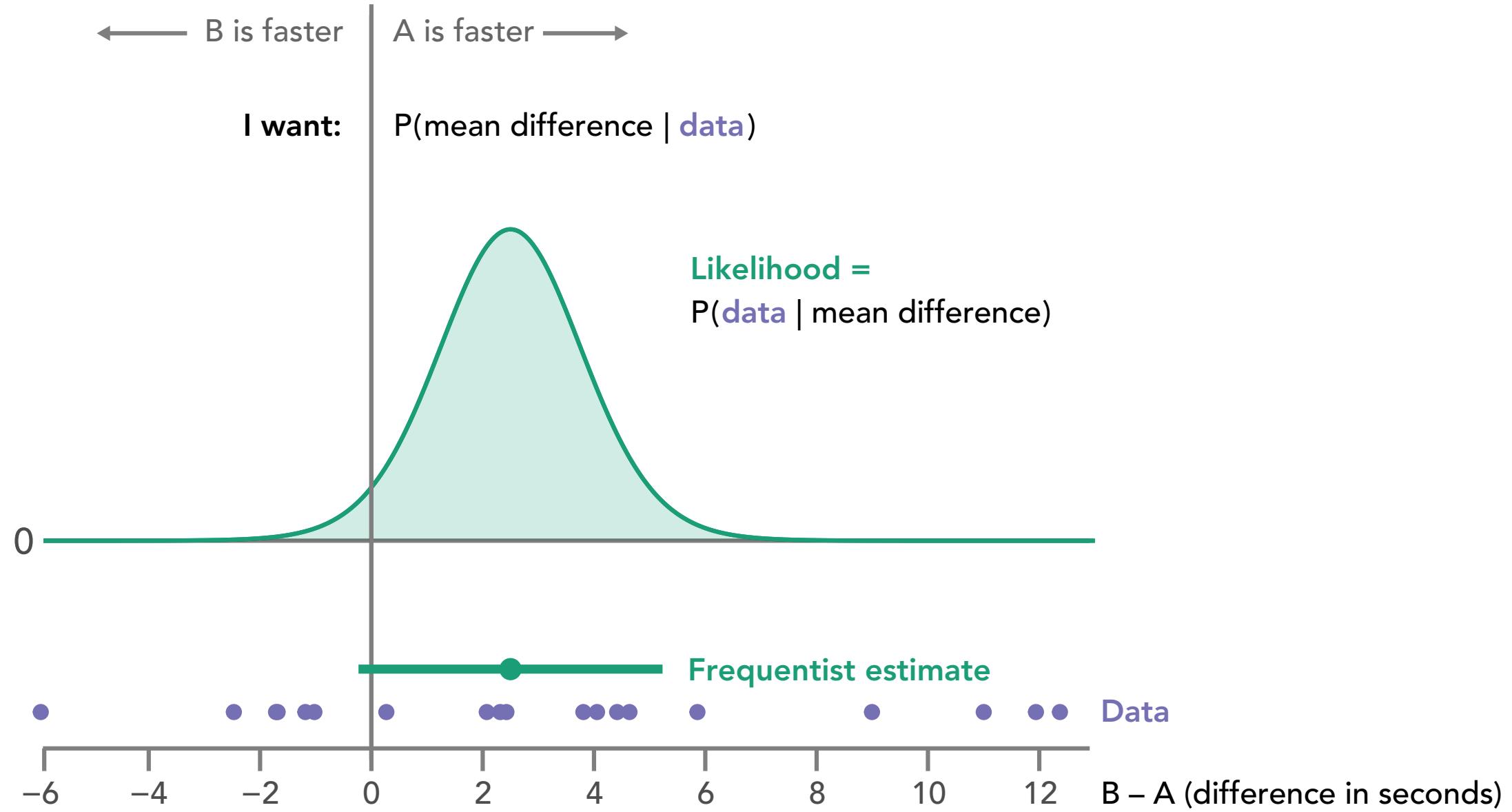
$P(\text{mean difference} \mid \text{data})$

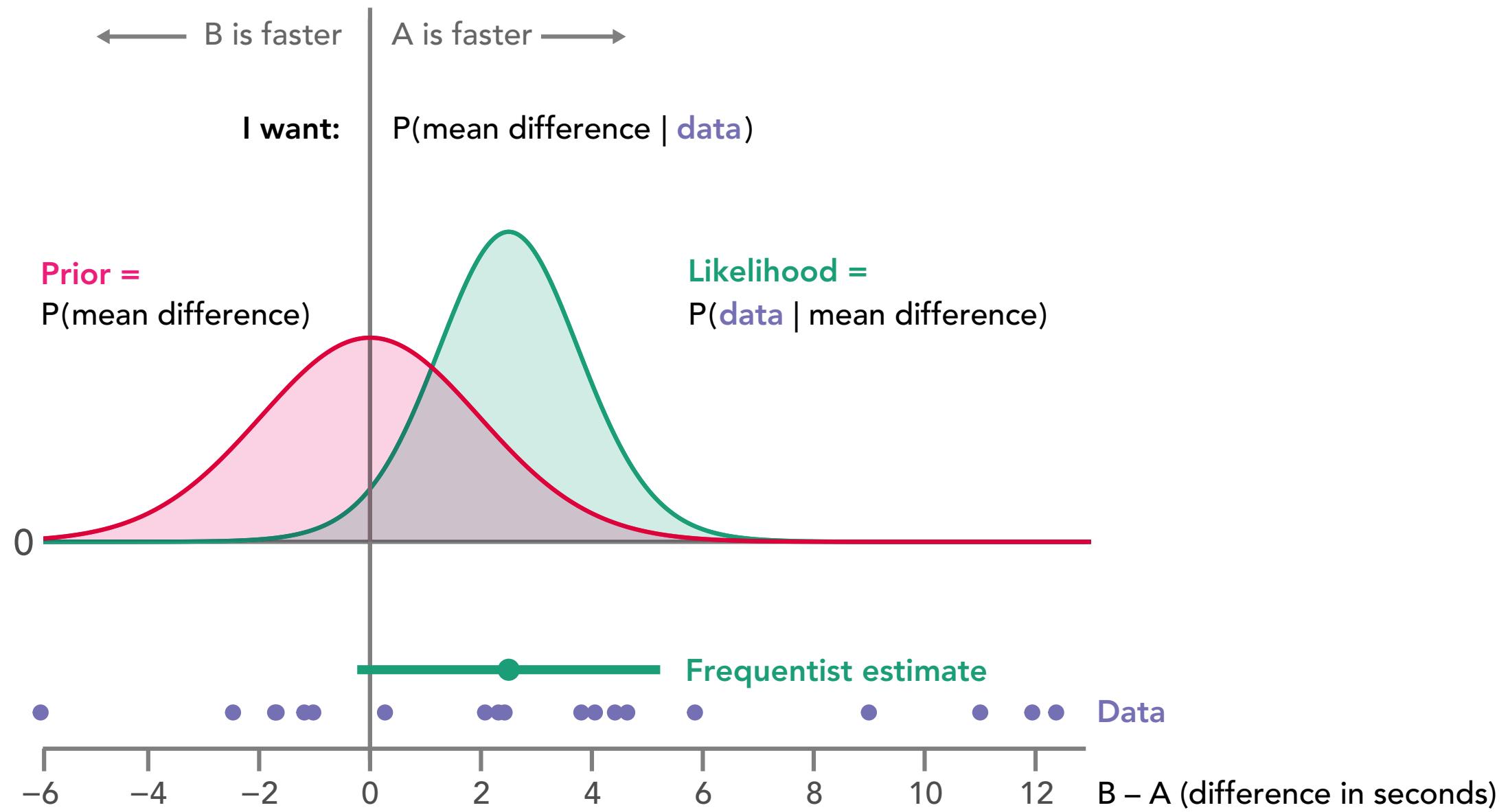
$P(\text{data} \mid \text{mean difference} = x)$

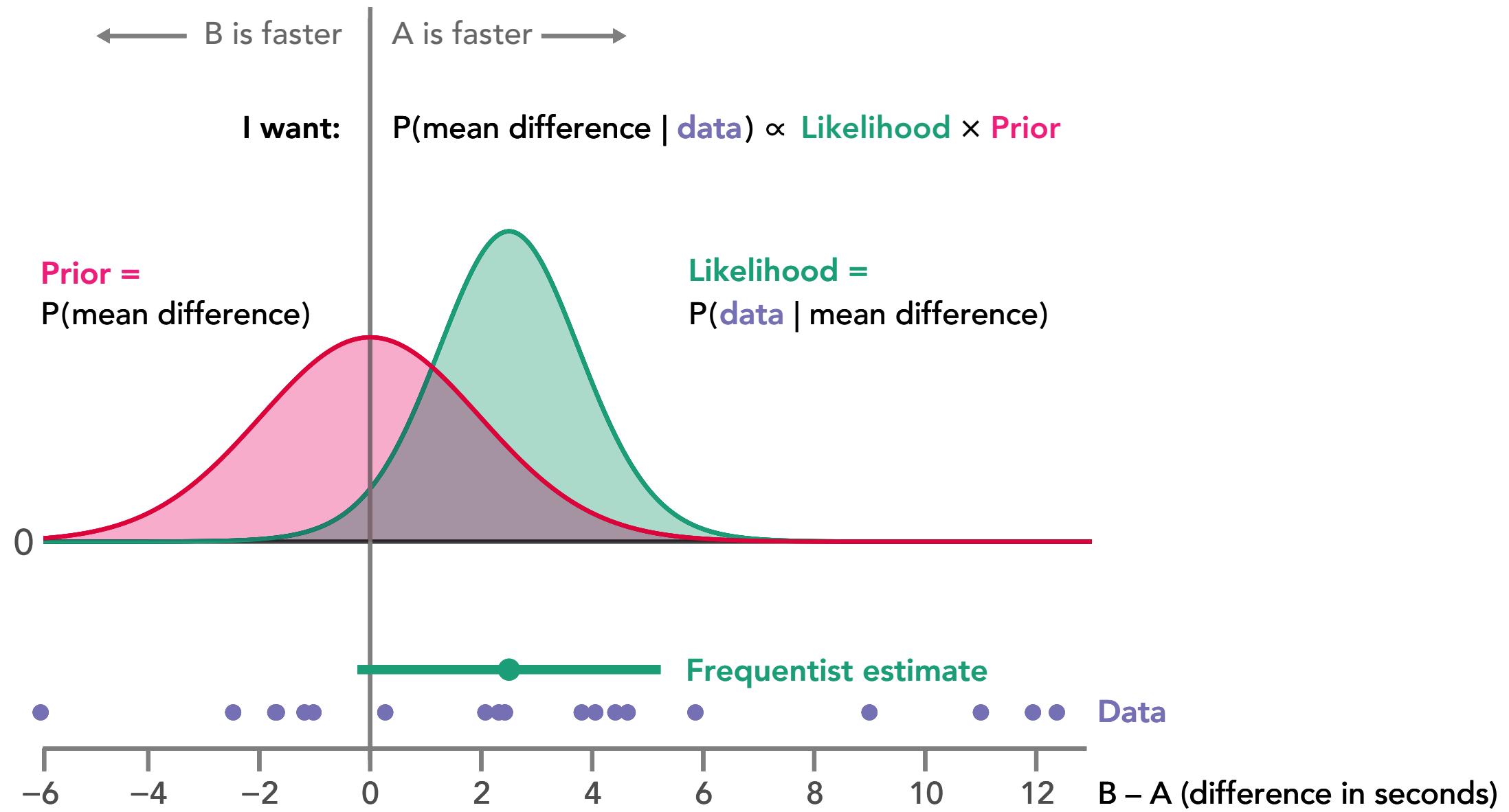


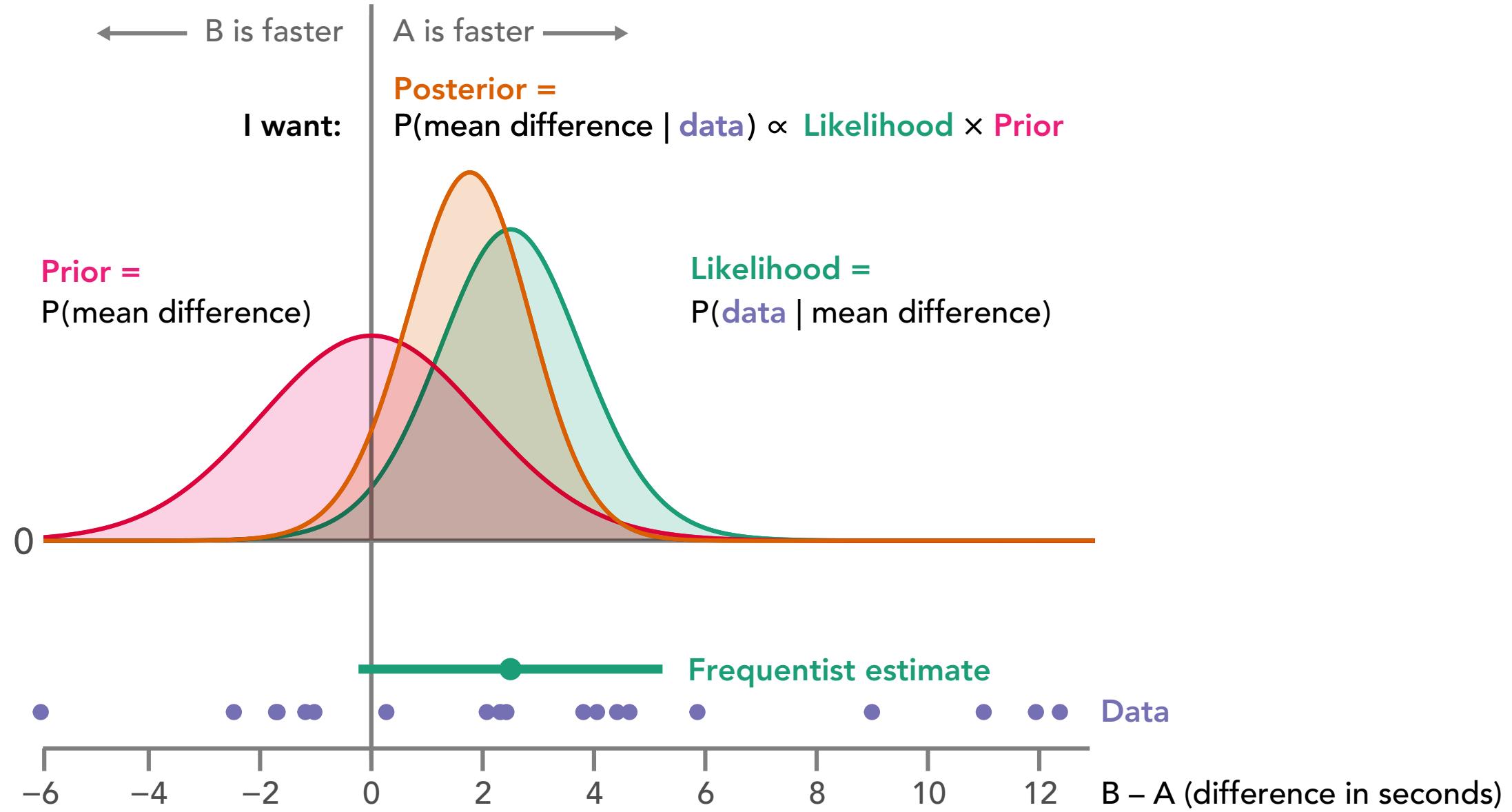


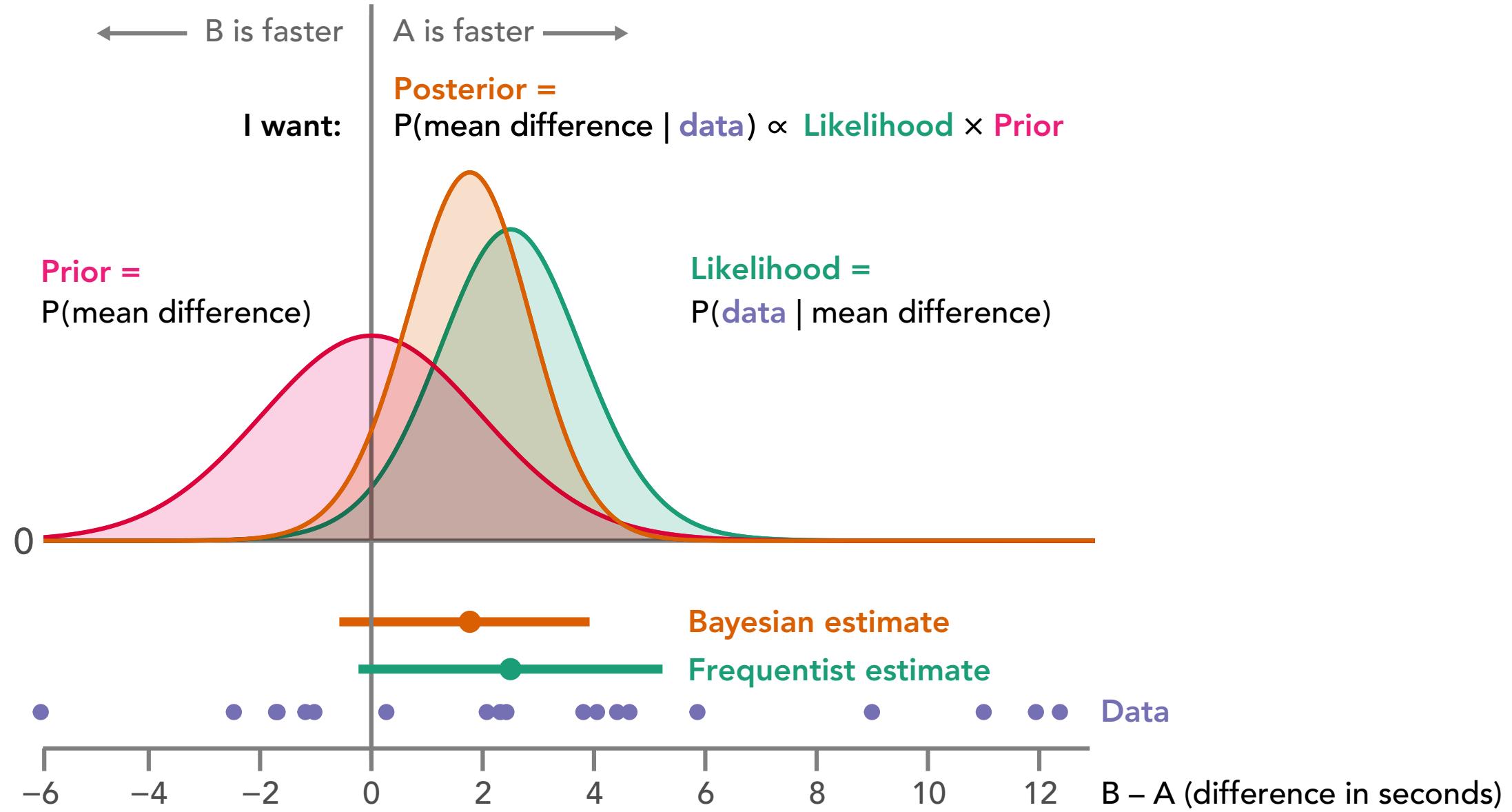


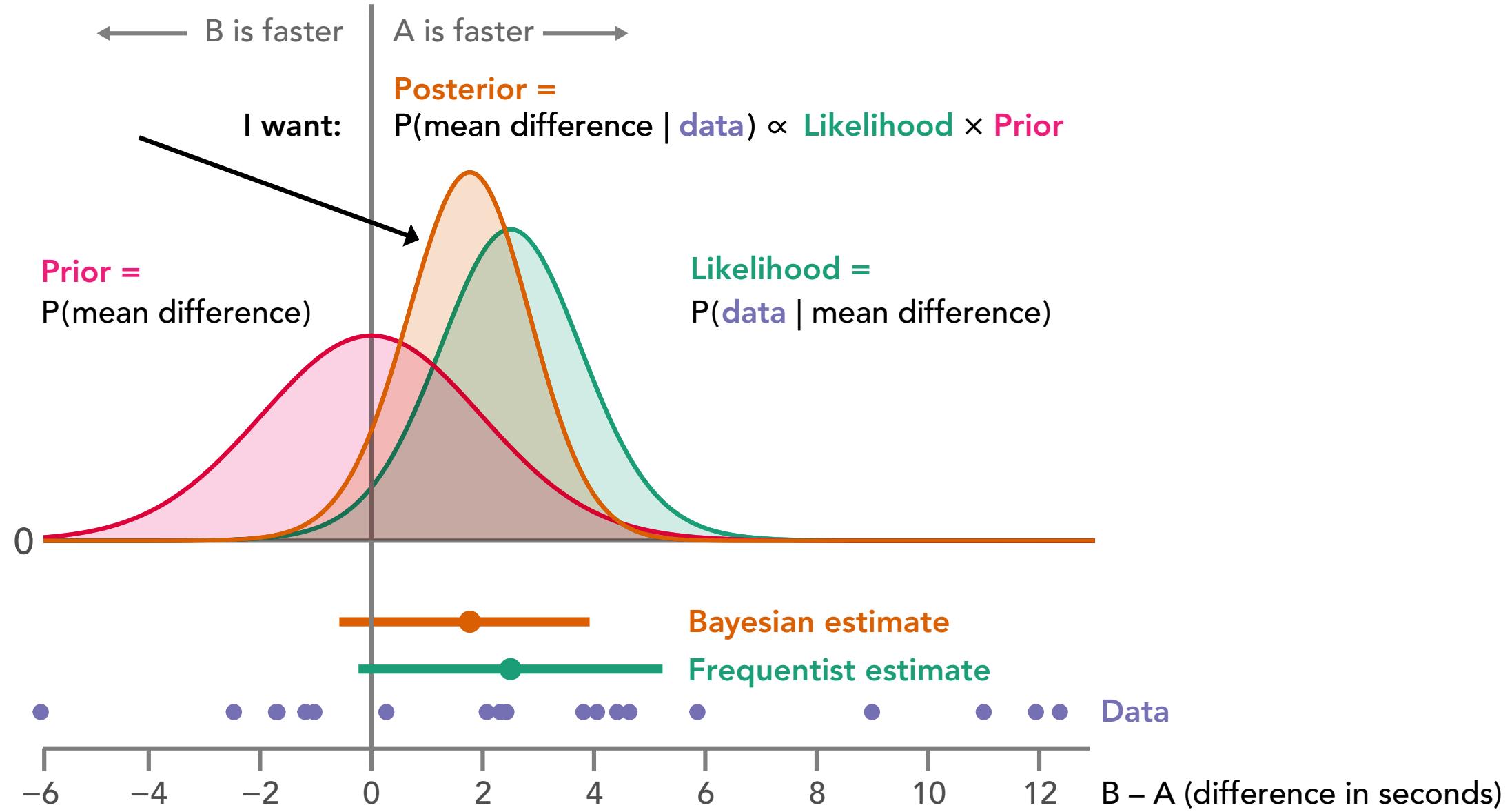




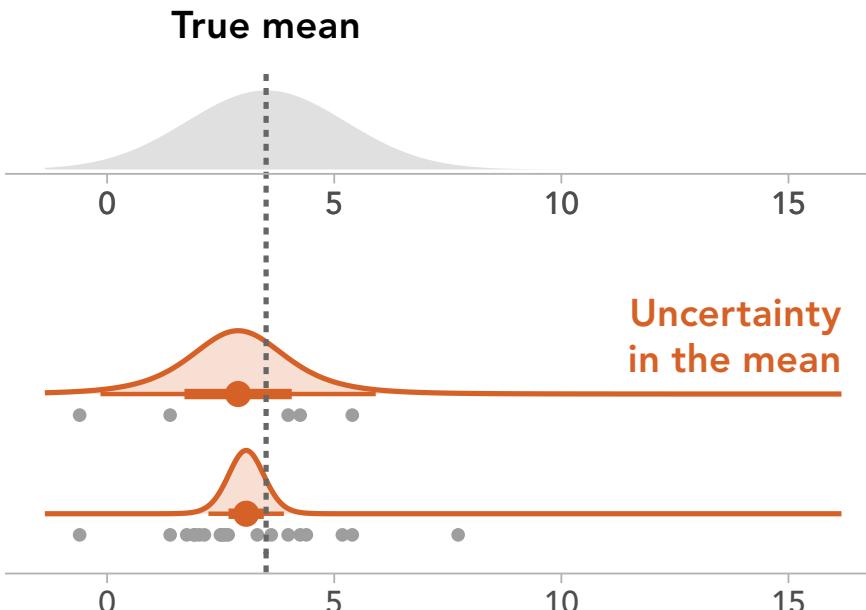




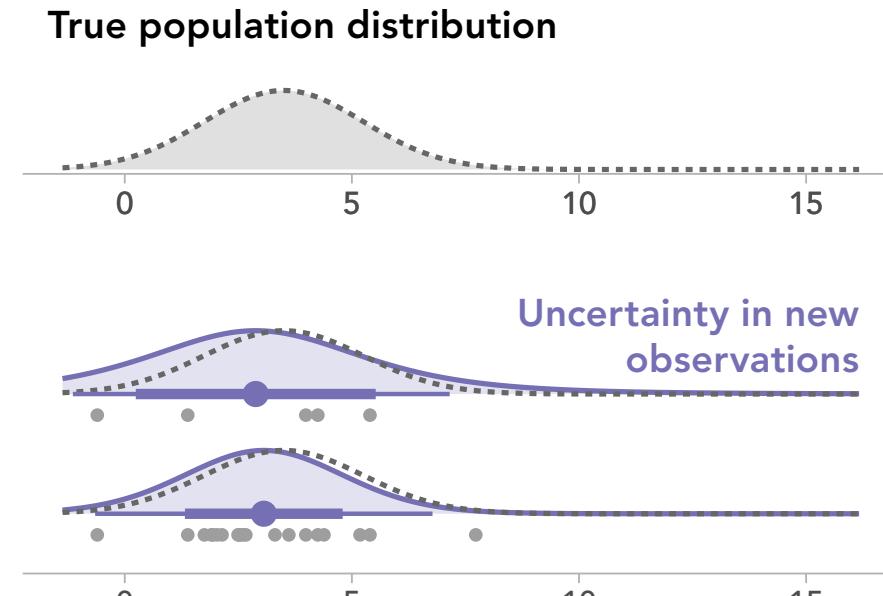




# Parameter uncertainty



# Predictive uncertainty



## True (unknown) population

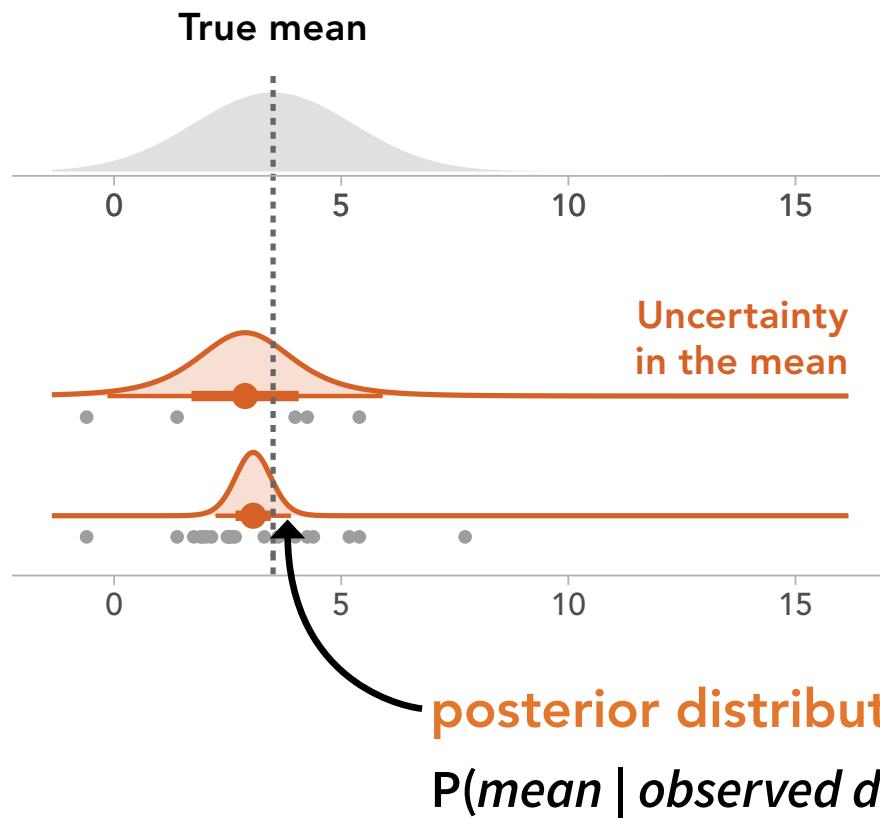
## B – A (difference in seconds)

## 5 samples

20 samples

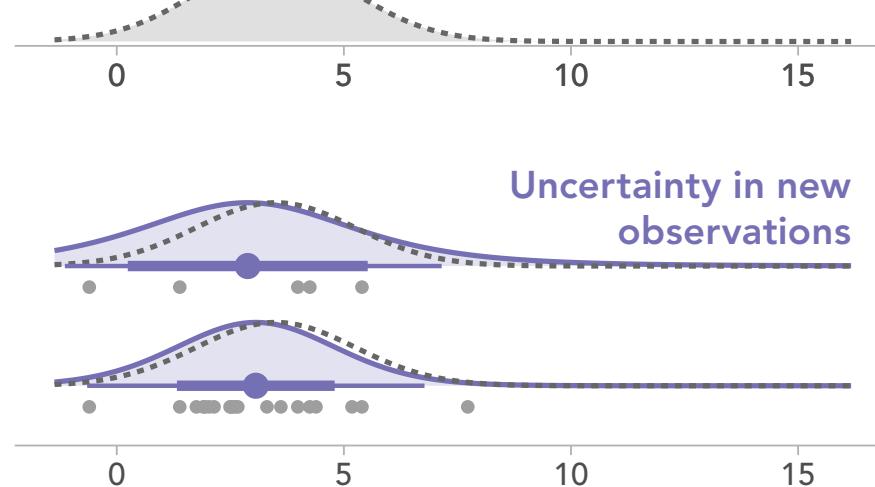
### B – A (difference in seconds)

## Parameter uncertainty



## Predictive uncertainty

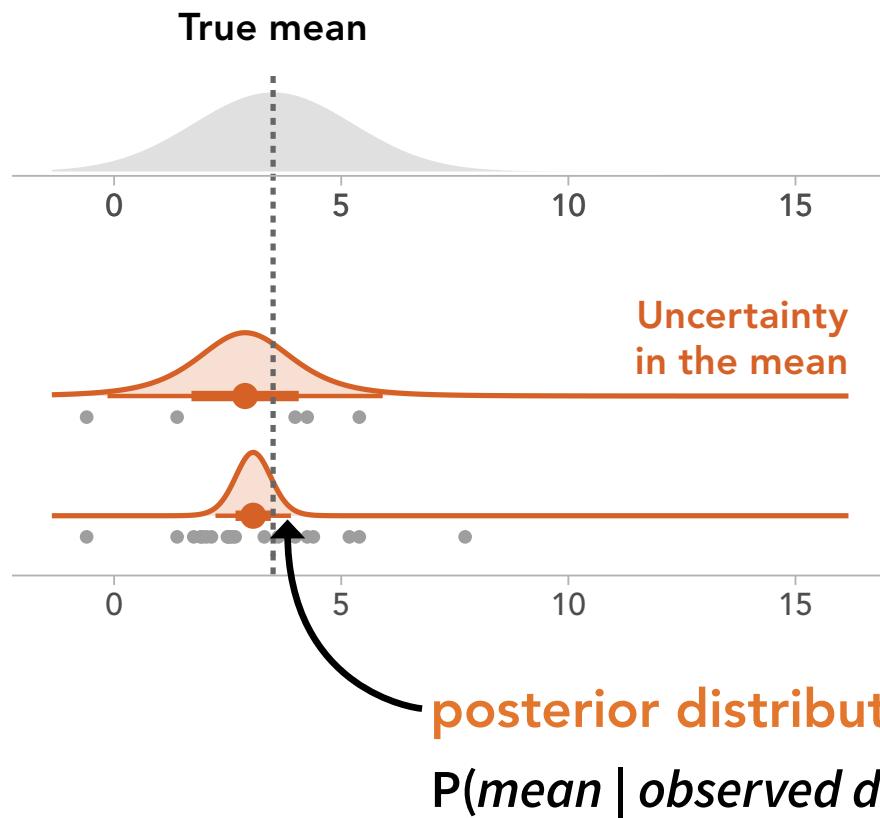
### True population distribution



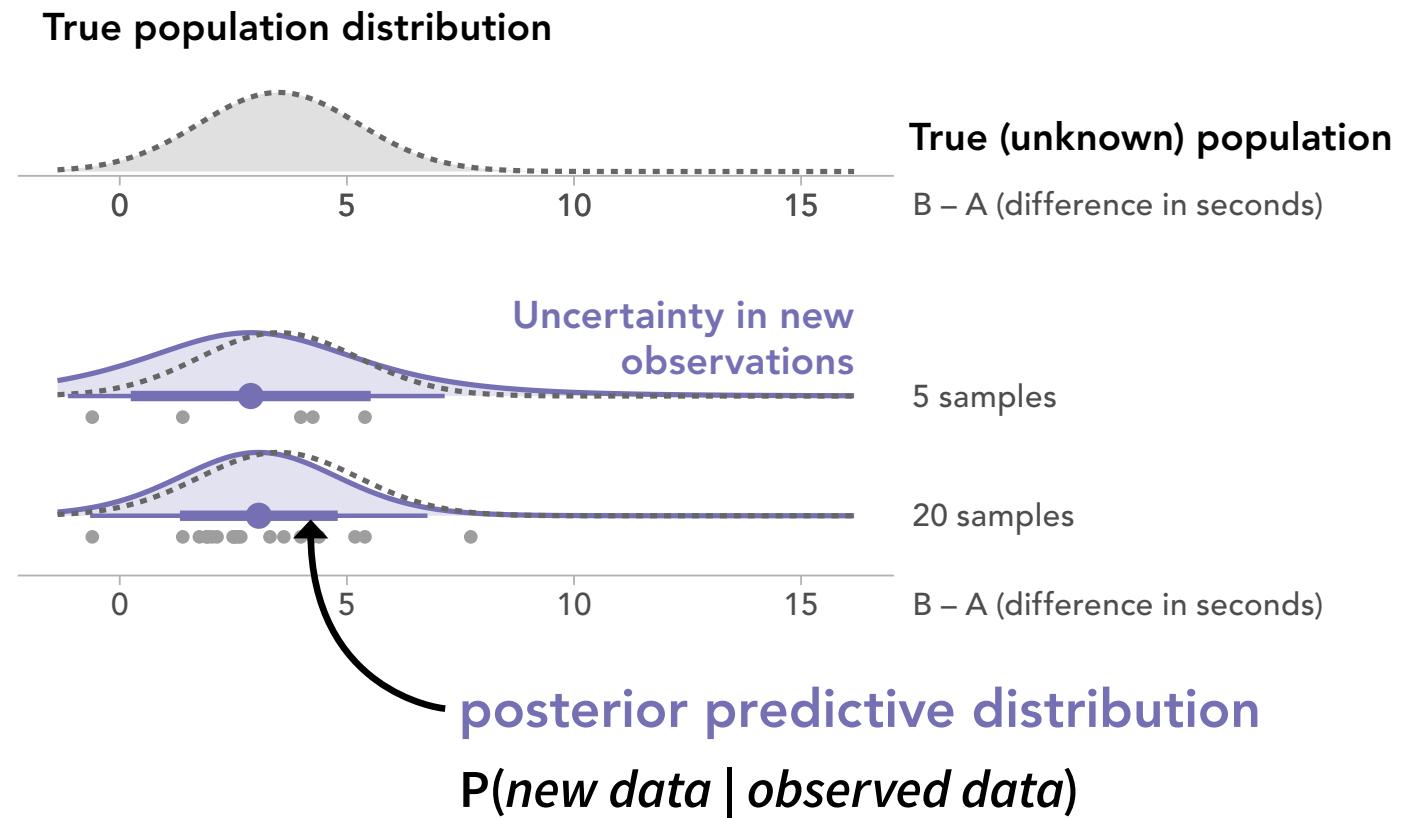
True (unknown) population  
B – A (difference in seconds)

5 samples  
20 samples  
B – A (difference in seconds)

## Parameter uncertainty



## Predictive uncertainty



# Summing up

# **Summing up**

**Everything** we say about the **small world** is contingent upon **large world** assumptions

# Summing up

**Everything** we say about the **small world** is contingent upon **large world** assumptions

This includes **confidence** and **probability** statements

# In the frequentist small world...

We talk about **confidence distributions**  
(or **sampling distributions**)

And make uncertainty statements using  
**confidence intervals, standard errors, ...**

# In the Bayesian small world...

We talk about **posterior distributions**

And make uncertainty statements using  
**credible intervals, probabilities, ...**

# In the rest of this class

We'll be mostly agnostic to Bayesian / frequentist

We'll assume you have quantified your uncertainty with  
a **confidence distribution** or a **probability distribution**...

# In the rest of this class

We'll be mostly agnostic to Bayesian / frequentist

We'll assume you have quantified your uncertainty with  
a **confidence distribution** or a **probability distribution**...

...then talk about **visual representations** for it

# Pragmatic small world uncertainty

SIADS 542: Presenting uncertainty – Week 1, Lecture 2

Matthew Kay

Assistant Professor

School of Information

University of Michigan