Single Patch

```
In[1]:= dS[s_, e_, i_] := r s (1 - (s + e + i) / k) - β s i
        dE[s_, e_, i_] := β s i - (σ + μ + r (s + e + i) / k) e
        dI[s_, e_, i_] := σ e - (ν + μ + r (s + e + i) / k) i
```

In[4]:=
```
Solve[0 == r s (1 - (s + e + i) / k) - β s i &&
0 == β s i - (σ + μ + r (s + e + i) / k) e &&
0 == σ e - (ν + μ + r (s + e + i) / k) i, {s, e, i}]
```

Out[4]=

$$
\begin{aligned}
&\Bigl\{\{s \to 0,\ e \to 0,\ i \to 0\},\ \{s \to k,\ e \to 0,\ i \to 0\},\\
&\Bigl\{s \to 0,\ e \to 0,\ i \to \frac{-k\,\mu - k\,\nu}{r}\Bigr\},\ \Bigl\{s \to 0,\ e \to -\frac{k\,(\nu-\sigma)\,(\mu+\sigma)}{r\,\nu},\ i \to -\frac{k\,\sigma\,(\mu+\sigma)}{r\,\nu}\Bigr\},\\
&\Bigl\{s \to \frac{1}{r^2 + r\,\mu - k\,\beta\,\sigma}\Bigl(k\,r^2 + k\,r\,\mu - k^2\,\beta\,\sigma - \frac{k\,r^4}{r^2 + r\,\mu - k\,\beta\,\sigma} - \frac{2\,k\,r^3\,\mu}{r^2 + r\,\mu - k\,\beta\,\sigma} - \frac{k\,r^2\,\mu^2}{r^2 + r\,\mu - k\,\beta\,\sigma} +\\
&\quad \frac{r^4\,\nu}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r^2\,\mu\,\nu}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{2\,r^3\,\mu\,\nu}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r\,\mu^2\,\nu}{r^2 + r\,\mu - k\,\beta\,\sigma} +\\
&\quad \frac{r^2\,\mu^2\,\nu}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r^2\,\nu^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^3\,\nu^2}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r\,\mu\,\nu^2}{r^2 + r\,\mu - k\,\beta\,\sigma} +\\
&\quad \frac{r^2\,\mu\,\nu^2}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{2\,k^2\,r^2\,\beta\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r^2\,\mu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{2\,k^2\,r\,\beta\,\mu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r\,\mu^2\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} +\\
&\quad \frac{k^2\,\beta\,\mu^2\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^3\,\nu\,\sigma}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{2\,k\,r\,\mu\,\nu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^2\,\mu\,\nu\,\sigma}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} +\\
&\quad \frac{k^2\,\beta\,\mu\,\nu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r\,\nu^2\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^2\,\nu^2\,\sigma}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r^2\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} - \frac{k^3\,\beta^2\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} +\\
&\quad \frac{k\,r\,\mu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k^2\,\beta\,\mu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r\,\nu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k^2\,\beta\,\nu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma}\Bigr),\\
&\ e \to \frac{1}{r^2 + r\,\mu - k\,\beta\,\sigma}\Bigl(-\frac{r^3\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} -\\
&\quad \frac{k\,r\,\mu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{r^2 + r\,\mu - k\,\beta\,\sigma} -\\
&\quad \frac{r^2\,\mu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} -\\
&\quad \frac{k\,r\,\nu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{r^2 + r\,\mu - k\,\beta\,\sigma} -\\
&\quad \frac{r^2\,\nu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)}\Bigr),\\
&\ i \to \frac{r\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)}\Bigr\}\Bigr\}
\end{aligned}
$$

In[5]:= `% /. {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73} // FullSimplify`

Out[5]=

$$\left\{\{s \to 0, e \to 0, i \to 0\}, \{s \to k, e \to 0, i \to 0\}, \{s \to 0, e \to 0, i \to -147.\,k\}, \{s \to 0, e \to -22.1918\,k, i \to -4.80822\,k\}, \left\{s \to \frac{0.000218507 + (0.0407286 + 0.954087\,k)\,k}{(-0.000480769 + k)^2}, e \to \frac{-0.000221501 + (-0.0349783 + 0.0353365\,k)\,k}{(-0.000480769 + k)^2}, i \to \frac{-0.00622596 + 0.00625\,k}{-0.000480769 + k}\right\}\right\}$$

In[6]:=
```
J = {{D[dS[s, e, i], s], D[dS[s, e, i], e], D[dS[s, e, i], i]},
{D[dE[s, e, i], s], D[dE[s, e, i], e], D[dE[s, e, i], i]},
{D[dI[s, e, i], s], D[dI[s, e, i], e], D[dI[s, e, i], i]}} // FullSimplify
```

Out[6]=

$$\left\{\left\{-\frac{r\,(e + i - k + 2\,s)}{k} - i\,\beta, -\frac{r\,s}{k}, -\frac{s\,(r + k\,\beta)}{k}\right\}, \left\{-\frac{e\,r}{k} + i\,\beta, -\frac{r\,(2\,e + i + s) + k\,(\mu + \sigma)}{k}, -\frac{e\,r}{k} + s\,\beta\right\}, \left\{-\frac{i\,r}{k}, -\frac{i\,r}{k} + \sigma, -\frac{r\,(e + 2\,i + s) + k\,(\mu + \nu)}{k}\right\}\right\}$$

In[7]:= `Eigenvalues[J] /.`

$$
\begin{aligned}
\Bigg\{ s \to{} & \frac{1}{r^2 + r\,\mu - k\,\beta\,\sigma}\Bigg(k\,r^2 + k\,r\,\mu - k^2\,\beta\,\sigma - \frac{k\,r^4}{r^2 + r\,\mu - k\,\beta\,\sigma} - \frac{2\,k\,r^3\,\mu}{r^2 + r\,\mu - k\,\beta\,\sigma} - \frac{k\,r^2\,\mu^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \\
& \frac{r^4\,\nu}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r^2\,\mu\,\nu}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{2\,r^3\,\mu\,\nu}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r\,\mu^2\,\nu}{r^2 + r\,\mu - k\,\beta\,\sigma} + \\
& \frac{r^2\,\mu^2\,\nu}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r^2\,\nu^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^3\,\nu^2}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r\,\mu\,\nu^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \\
& \frac{r^2\,\mu\,\nu^2}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{2\,k^2\,r^2\,\beta\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r^2\,\mu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{2\,k^2\,r\,\beta\,\mu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r\,\mu^2\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \\
& \frac{k^2\,\beta\,\mu^2\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^3\,\nu\,\sigma}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{2\,k\,r\,\mu\,\nu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^2\,\mu\,\nu\,\sigma}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \\
& \frac{k^2\,\beta\,\mu\,\nu\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r\,\nu^2\,\sigma}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{r^2\,\nu^2\,\sigma}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} + \frac{k\,r^2\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} - \\
& \frac{k^3\,\beta^2\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r\,\mu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k^2\,\beta\,\mu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k\,r\,\nu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma} + \frac{k^2\,\beta\,\nu\,\sigma^2}{r^2 + r\,\mu - k\,\beta\,\sigma}\Bigg), \\
e \to{} & \frac{1}{r^2 + r\,\mu - k\,\beta\,\sigma}\Bigg(-\frac{r^3\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} - \\
& \frac{k\,r\,\mu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{r^2 + r\,\mu - k\,\beta\,\sigma} - \\
& \frac{r^2\,\mu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)} - \\
& \frac{k\,r\,\nu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{r^2 + r\,\mu - k\,\beta\,\sigma} - \\
& \frac{r^2\,\nu\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)}\Bigg), \\
i \to{} & \frac{r\,(r^2 + 2\,r\,\mu + \mu^2 + r\,\nu + \mu\,\nu + r\,\sigma - k\,\beta\,\sigma + \mu\,\sigma + \nu\,\sigma)}{\beta\,(r^2 + r\,\mu - k\,\beta\,\sigma)}\Bigg\} \;\text{// FullSimplify}
\end{aligned}
$$

Out[7]=

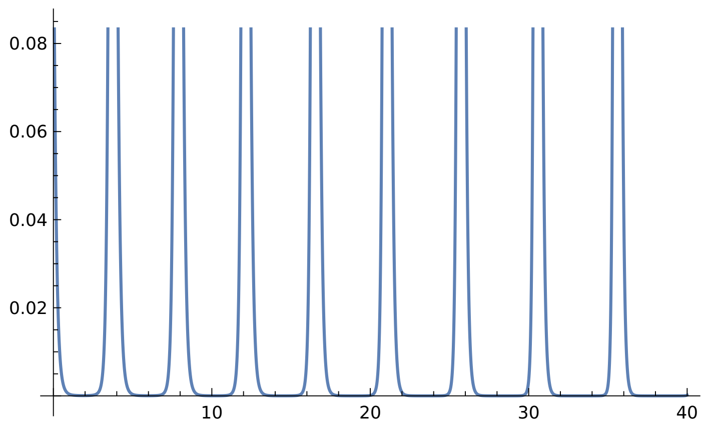
$$
\left\{\frac{\mathrm{Root}\left[\frac{k^{11}\,r^7\,\nu}{\beta\,(\cdots 1 \cdots)^2} + \frac{k^{12}\,r^5\,\mu\,\nu}{\cdots 1 \cdots^{2}} + \cdots 94 \cdots + (\cdots 1 \cdots)\;\cdots 1 \cdots + (\cdots 1 \cdots)\,\#1^3 \,\&, 1\right]}{k^4},\right.
$$

$$
\left.\frac{\mathrm{Root}[\cdots 1 \cdots \&, 2]}{k^4},\; \frac{\mathrm{Root}\left[\frac{\cdots 1 \cdots}{\beta\;\cdots 1 \cdots} + \cdots 96 \cdots + \cdots 1 \cdots \&, 3\right]}{k^4}\right\}
$$

large output | show less | show more | show all | set size limit...

In[8]:=
```
% /. {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73, k → 15} // FullSimplify
```
Out[8]=
```
{-87.1276, 0.0140128 - 2.30897 ⅈ, 0.0140128 + 2.30897 ⅈ}
```

In[9]:=
```
sol = NDSolve[
{s '[t] == r s [t] (1 - (s[t] + e[t] + i[t]) / k) - β s[t] × i[t],
e '[t] == β s[t] × i[t] - (σ + μ + r (s[t] + e[t] + i[t]) / k) e[t],
i '[t] == σ e[t] - (ν + μ + r (s[t] + e[t] + i[t]) / k) i[t],
s[0] == 0.5, e[0] == 0, i[0] == 0.1} /.
    {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73, k → 15}, {s, e, i}, {t, 0, 40},
StartingStepSize → 1 / 100 000, Method → "ExplicitRungeKutta"];
Plot[Evaluate[(i[t] + e[t]) / (s[t] + e[t] + i[t]) /. sol], {t, 0, 40}]
```

Out[10]=


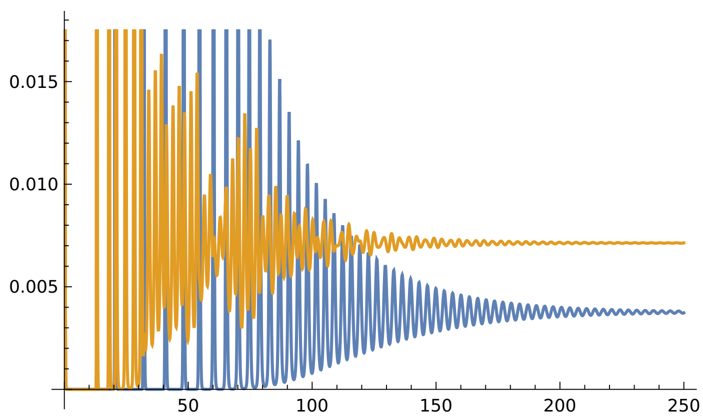
Metapopulation

In[11]:=
```
dS1[S1_, E1_, I1_] := r S1 (1 - (S1 + E1 + I1) / K1) - β S1 I1 - δ S1
dE1[S1_, E1_, I1_] := β S1 I1 - (σ + μ + r (S1 + E1 + I1) / K1) E1 - δ E1
dI1[S1_, E1_, I1_] := σ E1 - (ν + μ + r (S1 + E1 + I1) / K1) I1 - δ I1
dS2[S2_, E2_, I2_, S1_] := r S2 (1 - (S2 + E2 + I2) / K2) - β S2 I2 + δ S1
dE2[S2_, E2_, I2_, E1_] := β S2 I2 - (σ + μ + r (S2 + E2 + I2) / K2) E2 + δ E1
dI2[S2_, E2_, I2_, I1_] := σ E2 - (ν + μ + r (S2 + E2 + I2) / K2) I2 + δ I1
```

In[17]:=

```
sol = NDSolve[
{s1'[t] == r s1[t] (1 - (s1[t] + e1[t] + i1[t]) / k1) - β s1[t] × i1[t] - δ s1[t],
e1'[t] == β s1[t] × i1[t] - (σ + μ + r (s1[t] + e1[t] + i1[t]) / k1) e1[t] - δ e1[t],
i1'[t] == σ e1[t] - (ν + μ + r (s1[t] + e1[t] + i1[t]) / k1) i1[t] - δ i1[t],
s2'[t] == r s2[t] (1 - (s2[t] + e2[t] + i2[t]) / k2) - β s2[t] × i2[t] + δ s1[t],
e2'[t] == β s2[t] × i2[t] - (σ + μ + r (s2[t] + e2[t] + i2[t]) / k2) e2[t] + δ e1[t],
i2'[t] == σ e2[t] - (ν + μ + r (s2[t] + e2[t] + i2[t]) / k2) i2[t] + δ i1[t],
s1[0] == 5, e1[0] == 0, i1[0] == 1, s2[0] == 5, e2[0] == 0, i2[0] == 1} /.
    {r → 0.5, β → 79.69, σ → 13, μ → 0.5, ν → 73, k1 → 5, k2 → 15, δ → 0.1},
{s1, e1, i1, s2, e2, i2}, {t, 0, 250},
StartingStepSize → 1/100 000, Method → "ExplicitRungeKutta"];
Plot[{Evaluate[i1[t] /. sol], Evaluate[i2[t] /. sol]}, {t, 0, 250}]
```

Out[18]=



5->15

In[19]:=

```
Solve[0 == r S1 (1 - (S1 + E1 + I1) / K1) - β S1 I1 - δ S1 ×
0 == β S1 I1 - (σ + μ + r (S1 + E1 + I1) / K1) E1 - δ E1 ×
0 == σ E1 - (ν + μ + r (S1 + E1 + I1) / K1) I1 - δ I1 ×
0 == r S2 (1 - (S2 + E2 + I2) / K2) - β S2 I2 + δ S1 ×
0 == β S2 I2 - (σ + μ + r (S2 + E2 + I2) / K2) E2 + δ E1 ×
0 == σ E2 - (ν + μ + r (S2 + E2 + I2) / K2) I2 + δ I1, {S1, E1, I1, S2, E2, I2}]
```

Out[19]=

$$\left\{\{S1 \to K1,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to 0,\ I2 \to 0\},\ \cdots 23 \cdots,\ \left\{S1 \to \frac{K1\, r^2 + \cdots 35 \cdots + \frac{K1^2\, \beta\, \nu\, \sigma^2}{r^2 + r\, \mu - K1\, \beta\, \sigma}}{r^2 + r\, \mu - K1\, \beta\, \sigma},\ E1 \to \frac{\cdots 1 \cdots}{r^2 + \cdots 1 \cdots \cdots 1 \cdots \cdots 1 \cdots},\ \cdots 2 \cdots,\ E2 \to \cdots 1 \cdots,\ I2 \to \frac{K2\, r^3 + \cdots 11 \cdots + \frac{K2 \cdots 3 \cdots (\cdots 1 \cdots)}{2 (\cdots 1 \cdots)}}{K2\, r^2\, \beta - r^2\, \nu - K2\, r\, \beta\, \nu - K2\, r\, \beta\, \sigma - K2^2\, \beta^2\, \sigma}\right\}\right\}$$

large output | **show less** | **show more** | **show all** | **set size limit...**

In[20]:= **% /. {r → 0.5, β → 80, σ → 13, μ → 0.5, v → 73, K1 → 5, K2 → 15} // FullSimplify**

Out[20]=
$$\{\{S1 \to 5, E1 \to 0, I1 \to 0, S2 \to 0, E2 \to 0, I2 \to 0\},$$
$$\{S1 \to 5, E1 \to 0, I1 \to 0, S2 \to 0, E2 \to 0, I2 \to -2205.\},$$
$$\{S1 \to 5, E1 \to 0, I1 \to 0, S2 \to 0, E2 \to -332.877, I2 \to -72.1233\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to 0, S2 \to 0, E2 \to 0, I2 \to 0\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to 0, S2 \to 0, E2 \to 0, I2 \to -2205.\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to 0, S2 \to 0, E2 \to -332.877, I2 \to -72.1233\},$$
$$\{S1 \to 5, E1 \to 0, I1 \to 0, S2 \to 15, E2 \to 0, I2 \to 0\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to 0, S2 \to 15, E2 \to 0, I2 \to 0\},$$
$$\{S1 \to 5, E1 \to 0, I1 \to 0, S2 \to 0.956864, E2 \to 0.0330058, I2 \to 0.00583512\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to 0, S2 \to 0.956864, E2 \to 0.0330058, I2 \to 0.00583512\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to -735., S2 \to 0, E2 \to 0, I2 \to -1102.5 - 2.\ \sqrt{303877. - 5512.5\,\delta}\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to -735., S2 \to 0, E2 \to 0, I2 \to 2.\left(-551.25 + \sqrt{303877. - 5512.5\,\delta}\right)\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to -735., S2 \to 0, E2 \to -332.877 + 10.0685\,\delta, I2 \to -72.1233 - 10.0685\,\delta\},$$
$$\{S1 \to 0, E1 \to 0, I1 \to -735.,$$
$$S2 \to 7.97843 - 5.13675\times10^{-11}\sqrt{1.86849\times10^{22} - 9.04973\times10^{24}\,\delta} - 56.5638\,\delta,$$
$$E2 \to 0.0165029 + 1.2073\times10^{-13}\sqrt{1.86849\times10^{22} - 9.04973\times10^{24}\,\delta} + 56.5638\,\delta,$$
$$I2 \to 0.00291756 + 2.13439\times10^{-14}\sqrt{1.86849\times10^{22} - 9.04973\times10^{24}\,\delta}\}, \{S1 \to 0, E1 \to 0,$$
$$I1 \to -735., S2 \to 7.97843 + 5.13675\times10^{-11}\sqrt{1.86849\times10^{22} - 9.04973\times10^{24}\,\delta} - 56.5638\,\delta,$$
$$E2 \to 0.0165029 - 1.2073\times10^{-13}\sqrt{1.86849\times10^{22} - 9.04973\times10^{24}\,\delta} + 56.5638\,\delta,$$
$$I2 \to 0.00291756 - 2.13439\times10^{-14}\sqrt{1.86849\times10^{22} - 9.04973\times10^{24}\,\delta}\},$$
$$\{S1 \to 0, E1 \to -110.959, I1 \to -24.0411, S2 \to 0, E2 \to -332.877 + 0.32933\,\delta,$$
$$I2 \to -72.1233 - 0.32933\,\delta\}, \{S1 \to 0, E1 \to -110.959, I1 \to -24.0411, S2 \to 0, E2 \to 0,$$
$$I2 \to -1102.5 - 0.0273973\sqrt{1.61936\times10^{9} - 960863.\,\delta}\}, \{S1 \to 0, E1 \to -110.959,$$
$$I1 \to -24.0411, S2 \to 0, E2 \to 0, I2 \to 0.0273973\left(-40241.3 + \sqrt{1.61936\times10^{9} - 960863.\,\delta}\right)\},$$
$$\{S1 \to 0, E1 \to -110.959, I1 \to -24.0411,$$
$$S2 \to 7.97843 - 1.85014\,\delta - 5.13675\times10^{-11}\sqrt{1.86849\times10^{22} + \delta\left(-2.96007\times10^{23} + 262144.\,\delta\right)},$$
$$E2 \to 0.0165029 + 1.85014\,\delta + 1.2073\times10^{-13}\sqrt{1.86849\times10^{22} + \delta\left(-2.96007\times10^{23} + 262144.\,\delta\right)},$$
$$I2 \to 0.00291756 + 1.93805\times10^{-19}\,\delta +$$
$$2.13439\times10^{-14}\sqrt{1.86849\times10^{22} + \delta\left(-2.96007\times10^{23} + 262144.\,\delta\right)}\},$$
$$\{S1 \to 0, E1 \to -110.959, I1 \to -24.0411,$$
$$S2 \to 7.97843 - 1.85014\,\delta + 5.13675\times10^{-11}\sqrt{1.86849\times10^{22} + \delta\left(-2.96007\times10^{23} + 262144.\,\delta\right)},$$
$$E2 \to 0.0165029 + 1.85014\,\delta - 1.2073\times10^{-13}\sqrt{1.86849\times10^{22} + \delta\left(-2.96007\times10^{23} + 262144.\,\delta\right)},$$
$$I2 \to 0.00291756 + 1.93805\times10^{-19}\,\delta -$$

$$2.13439\times10^{-14}\ \sqrt{1.86849\times10^{22}+\delta\left(-2.96007\times10^{23}+262\,144.\ \delta\right)}\Big\},$$

$$\Big\{\mathrm{S1}\to0.962426,\ \mathrm{E1}\to0.0283375,\ \mathrm{I1}\to0.00500529,\ \mathrm{S2}\to0,$$

$$\mathrm{E2}\to-332.877+\left(-0.0000685656-3.19066\times10^{-27}\ \delta\right)\delta+\frac{3.8147\times10^{-6}}{-6.73987\times10^{6}+1.\ \delta},$$

$$\mathrm{I2}\to-72.1233+0.0000685656\ \delta\Big\},$$

$$\Big\{\mathrm{S1}\to0.962426,\ \mathrm{E1}\to0.0283375,\ \mathrm{I1}\to0.00500529,\ \mathrm{S2}\to0,$$

$$\mathrm{E2}\to\frac{\delta\left(2.27374\times10^{-12}+3.91654\times10^{-20}\ \delta-4.61991\times10^{-21}\ \sqrt{2.1031\times10^{17}+2.59809\times10^{10}\ \delta}\right)}{-6.73987\times10^{6}+1.\ \delta},$$

$$\mathrm{I2}\to-1102.5+2.40408\times10^{-6}\ \sqrt{2.1031\times10^{17}+2.59809\times10^{10}\ \delta}\Big\},$$

$$\Big\{\mathrm{S1}\to0.962426,\ \mathrm{E1}\to0.0283375,\ \mathrm{I1}\to0.00500529,\ \mathrm{S2}\to0,$$

$$\mathrm{E2}\to\frac{\delta\left(2.27374\times10^{-12}+3.91654\times10^{-20}\ \delta+4.61991\times10^{-21}\ \sqrt{2.1031\times10^{17}+2.59809\times10^{10}\ \delta}\right)}{-6.73987\times10^{6}+1.\ \delta},$$

$$\mathrm{I2}\to-2.40408\times10^{-6}\left(4.58596\times10^{8}+\sqrt{2.1031\times10^{17}+2.59809\times10^{10}\ \delta}\right)\Big\},$$

$$\Big\{\mathrm{S1}\to0.962426,\ \mathrm{E1}\to0.0283375,\ \mathrm{I1}\to0.00500529,\ \mathrm{S2}\to7.97843+0.000385195\ \delta+$$
$$4.93966\times10^{-15}\ \sqrt{2.02057\times10^{30}+\left(6.66439\times10^{27}-2.09715\times10^{6}\ \delta\right)\delta},\ \mathrm{E2}\to0.0165029-$$
$$0.000385195\ \delta-1.16098\times10^{-17}\ \sqrt{2.02057\times10^{30}+\left(6.66439\times10^{27}-2.09715\times10^{6}\ \delta\right)\delta},$$
$$\mathrm{I2}\to0.00291756-1.18289\times10^{-22}\ \delta-2.0525\times10^{-18}$$
$$\sqrt{2.02057\times10^{30}+\left(6.66439\times10^{27}-2.09715\times10^{6}\ \delta\right)\delta}\Big\},\ \Big\{\mathrm{S1}\to0.962426,$$
$$\mathrm{E1}\to0.0283375,\ \mathrm{I1}\to0.00500529,\ \mathrm{S2}\to7.97843+0.000385195\ \delta-4.93966\times10^{-15}$$
$$\sqrt{2.02057\times10^{30}+\left(6.66439\times10^{27}-2.09715\times10^{6}\ \delta\right)\delta},\ \mathrm{E2}\to0.0165029-0.000385195\ \delta+$$
$$1.16098\times10^{-17}\ \sqrt{2.02057\times10^{30}+\left(6.66439\times10^{27}-2.09715\times10^{6}\ \delta\right)\delta},\ \mathrm{I2}\to0.00291756-$$
$$1.18289\times10^{-22}\ \delta+2.0525\times10^{-18}\ \sqrt{2.02057\times10^{30}+\left(6.66439\times10^{27}-2.09715\times10^{6}\ \delta\right)\delta}\Big\}\Big\}$$

In[21]:=
```
J = {
{D[dS1[s1, e1, i1], s1], D[dS1[s1, e1, i1], e1], D[dS1[s1, e1, i1], i1],
    D[dS1[s1, e1, i1], s2], D[dS1[s1, e1, i1], e2], D[dS1[s1, e1, i1], i2]},
{D[dE1[s1, e1, i1], s1], D[dE1[s1, e1, i1], e1], D[dE1[s1, e1, i1], i1],
    D[dE1[s1, e1, i1], s2], D[dE1[s1, e1, i1], e2], D[dE1[s1, e1, i1], i2]},
{D[dI1[s1, e1, i1], s1], D[dI1[s1, e1, i1], e1], D[dI1[s1, e1, i1], i1],
    D[dI1[s1, e1, i1], s2], D[dI1[s1, e1, i1], e2], D[dI1[s1, e1, i1], i2]},
{D[dS2[s2, e2, i2, s1], s1], D[dS2[s2, e2, i2, s1], e1], D[dS2[s2, e2, i2, s1], i1],
    D[dS2[s2, e2, i2, s1], s2], D[dS2[s2, e2, i2, s1], e2], D[dS2[s2, e2, i2, s1], i2]},
{D[dE2[s2, e2, i2, e1], s1], D[dE2[s2, e2, i2, e1], e1], D[dE2[s2, e2, i2, e1], i1],
    D[dE2[s2, e2, i2, e1], s2], D[dE2[s2, e2, i2, e1], e2], D[dE2[s2, e2, i2, e1], i2]},
{D[dI2[s2, e2, i2, i1], s1], D[dI2[s2, e2, i2, i1], e1], D[dI2[s2, e2, i2, i1], i1],
    D[dI2[s2, e2, i2, i1], s2], D[dI2[s2, e2, i2, i1], e2], D[dI2[s2, e2, i2, i1], i2]}
} // FullSimplify
```

Out[21]=
$$\left\{\left\{-\frac{e1\,r+i1\,r-K1\,r+2\,r\,s1+i1\,K1\,\beta+K1\,\delta}{K1},-\frac{r\,s1}{K1},-\frac{s1\,(r+K1\,\beta)}{K1},0,0,0\right\},\right.$$
$$\left\{-\frac{e1\,r}{K1}+i1\,\beta,-\frac{r\,(2\,e1+i1+s1)+K1\,(\delta+\mu+\sigma)}{K1},-\frac{e1\,r}{K1}+s1\,\beta,0,0,0\right\},$$
$$\left\{-\frac{i1\,r}{K1},-\frac{i1\,r}{K1}+\sigma,-\frac{r\,(e1+2\,i1+s1)+K1\,(\delta+\mu+v)}{K1},0,0,0\right\},$$
$$\left\{\delta,0,0,-\frac{r\,(e2+i2-K2+2\,s2)}{K2}-i2\,\beta,-\frac{r\,s2}{K2},-\frac{s2\,(r+K2\,\beta)}{K2}\right\},$$
$$\left\{0,\delta,0,-\frac{e2\,r}{K2}+i2\,\beta,-\frac{r\,(2\,e2+i2+s2)+K2\,(\mu+\sigma)}{K2},-\frac{e2\,r}{K2}+s2\,\beta\right\},$$
$$\left.\left\{0,0,\delta,-\frac{i2\,r}{K2},-\frac{i2\,r}{K2}+\sigma,-\frac{r\,(e2+2\,i2+s2)+K2\,(\mu+v)}{K2}\right\}\right\}$$

In[22]:=
```
{{S1 → 5, E1 → 0, I1 → 0, S2 → 0.9568640827497324`, E2 → 0.033005784874229914`,
   I2 → 0.005835122920606429 5`}, {S1 → 0, E1 → 0, I1 → 0, S2 → 0.9568640827497324`,
   E2 → 0.033005784874229914`, I2 → 0.0058351229206064295`},
  {S1 → 0.9624260618496701`, E1 → 0.0283374731056132`, I1 → 0.005005288970093278`,
   S2 → 7.978432041374867` + 0.00038519500039084476` δ - 4.939660235979592`*^-15
      √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ) ,
   E2 → 0.016502892437114957` - 0.0003851950003908446` δ + 1.1609754691632573`*^-17
      √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ) ,
   I2 → 0.0029175614603032143` - 1.1828929751645508`*^-22 δ + 2.0524991592203083`*^-18
      √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ) }} /. {δ →
   0.1}
```

Out[22]=
```
{{S1 → 5, E1 → 0, I1 → 0, S2 → 0.956864, E2 → 0.0330058, I2 → 0.00583512},
 {S1 → 0, E1 → 0, I1 → 0, S2 → 0.956864, E2 → 0.0330058, I2 → 0.00583512}, {S1 → 0.962426,
  E1 → 0.0283375, I1 → 0.00500529, S2 → 0.955745, E2 → 0.03297, I2 → 0.0058356}}
```

In[23]:=
```
J = {
{D[dS1[s1, e1, i1], s1], D[dS1[s1, e1, i1], e1], D[dS1[s1, e1, i1], i1],
    D[dS1[s1, e1, i1], s2], D[dS1[s1, e1, i1], e2], D[dS1[s1, e1, i1], i2]},
{D[dE1[s1, e1, i1], s1], D[dE1[s1, e1, i1], e1], D[dE1[s1, e1, i1], i1],
    D[dE1[s1, e1, i1], s2], D[dE1[s1, e1, i1], e2], D[dE1[s1, e1, i1], i2]},
{D[dI1[s1, e1, i1], s1], D[dI1[s1, e1, i1], e1], D[dI1[s1, e1, i1], i1],
    D[dI1[s1, e1, i1], s2], D[dI1[s1, e1, i1], e2], D[dI1[s1, e1, i1], i2]},
{D[dS2[s2, e2, i2, s1], s1], D[dS2[s2, e2, i2, s1], e1], D[dS2[s2, e2, i2, s1], i1],
    D[dS2[s2, e2, i2, s1], s2], D[dS2[s2, e2, i2, s1], e2], D[dS2[s2, e2, i2, s1], i2]},
{D[dE2[s2, e2, i2, e1], s1], D[dE2[s2, e2, i2, e1], e1], D[dE2[s2, e2, i2, e1], i1],
    D[dE2[s2, e2, i2, e1], s2], D[dE2[s2, e2, i2, e1], e2], D[dE2[s2, e2, i2, e1], i2]},
{D[dI2[s2, e2, i2, i1], s1], D[dI2[s2, e2, i2, i1], e1], D[dI2[s2, e2, i2, i1], i1],
    D[dI2[s2, e2, i2, i1], s2], D[dI2[s2, e2, i2, i1], e2], D[dI2[s2, e2, i2, i1], i2]}
} /. {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73, K1 → 5, K2 → 15} /.
   {s1 → 0.9624260618496701`, e1 → 0.0283374731056132`, i1 → 0.005005288970093278`,
    s2 → 7.978432041374867` + 0.00038519500039084476` δ - 4.939660235979592`*^-15
      √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ),
    e2 → 0.016502892437114957` - 0.0003851950003908446` δ + 1.1609754691632573`*^-17
      √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ), i2 →
     0.0029175614603032143` - 1.1828929751645508`*^-22 δ + 2.0524991592203083`*^-18
      √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ)} //
  FullSimplify // MatrixForm
```

Out[23]//MatrixForm=

$$\begin{pmatrix}
-0.0962426-\delta & -0.0962426 & -77.0903 & 0 \\
0.397589 & -13.6024-\delta & 76.9913 & 0 \\
-0.000500529 & 12.9995 & -73.6001-\delta & 0 \\
\delta & 0 & 0 & -0.265948-0.0000128398\,\delta+1.64655\times 10^{-16}\sqrt{2.\text{(}} \\
0 & \delta & 0 & 0.232855+0.0000128398\,\delta+1.63813\times 10^{-16}\sqrt{2.\text{(}} \\
0 & 0 & \delta & -0.000097252+3.94298\times 10^{-24}\,\delta-6.84166\times 10^{-20}\sqrt{\text{(}}
\end{pmatrix}$$

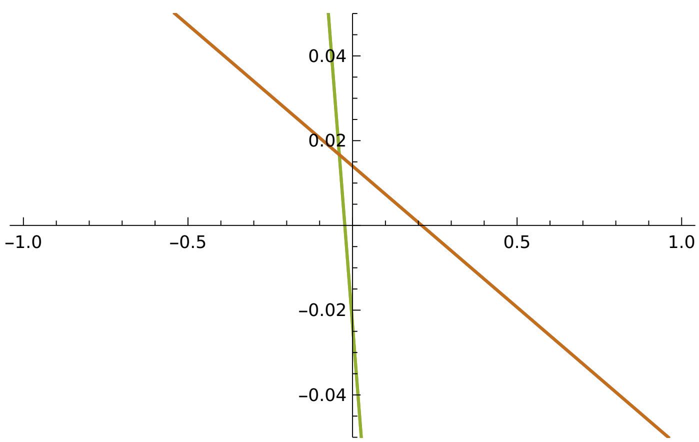
In[24]:=
```
mat[δ_] := {
{D[dS1[s1, e1, i1], s1], D[dS1[s1, e1, i1], e1], D[dS1[s1, e1, i1], i1],
    D[dS1[s1, e1, i1], s2], D[dS1[s1, e1, i1], e2], D[dS1[s1, e1, i1], i2]},
{D[dE1[s1, e1, i1], s1], D[dE1[s1, e1, i1], e1], D[dE1[s1, e1, i1], i1],
    D[dE1[s1, e1, i1], s2], D[dE1[s1, e1, i1], e2], D[dE1[s1, e1, i1], i2]},
{D[dI1[s1, e1, i1], s1], D[dI1[s1, e1, i1], e1], D[dI1[s1, e1, i1], i1],
    D[dI1[s1, e1, i1], s2], D[dI1[s1, e1, i1], e2], D[dI1[s1, e1, i1], i2]},
{D[dS2[s2, e2, i2, s1], s1], D[dS2[s2, e2, i2, s1], e1], D[dS2[s2, e2, i2, s1], i1],
    D[dS2[s2, e2, i2, s1], s2], D[dS2[s2, e2, i2, s1], e2], D[dS2[s2, e2, i2, s1], i2]},
{D[dE2[s2, e2, i2, e1], s1], D[dE2[s2, e2, i2, e1], e1], D[dE2[s2, e2, i2, e1], i1],
    D[dE2[s2, e2, i2, e1], s2], D[dE2[s2, e2, i2, e1], e2], D[dE2[s2, e2, i2, e1], i2]},
{D[dI2[s2, e2, i2, i1], s1], D[dI2[s2, e2, i2, i1], e1], D[dI2[s2, e2, i2, i1], i1],
    D[dI2[s2, e2, i2, i1], s2], D[dI2[s2, e2, i2, i1], e2], D[dI2[s2, e2, i2, i1], i2]}
} /. {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73, K1 → 5, K2 → 15} /.
  {s1 → 0.9624260618496701`, e1 → 0.0283374731056132`, i1 → 0.005005288970093278`,
   s2 → 7.978432041374867` + 0.00038519500039084476` δ - 4.939660235979592`*^-15
       √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ),
   e2 → 0.016502892437114957` - 0.0003851950003908446` δ + 1.1609754691632573`*^-17
       √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ),
   i2 → 0.0029175614603032143` - 1.1828929751645508`*^-22 δ + 2.0524991592203083`*^-18
       √(2.0205707008194935`*^30 + (6.664392307946753`*^27 - 2.097152`*^6 δ) δ)}
```

In[25]:=
```
Evaluate[Eigenvalues[mat[δ]]] /. δ → 0.1
```

Out[25]=
```
{-87.3519, -0.123438 - 2.14302 ⅈ, -0.123438 + 2.14302 ⅈ,
 -87.1141, 0.00734601 - 2.30802 ⅈ, 0.00734601 + 2.30802 ⅈ}
```

In[26]:=
```
Plot[Evaluate[Re @ Eigenvalues[mat[δ]]], {δ, -1, 1}, PlotRange → {-0.05, 0.05}]
```

Out[26]=



In[27]:=

15->5

In[28]:=
```
Solve[0 == r S1 (1 - (S1 + E1 + I1) / K1) - β S1 I1 - δ S1 &&
0 == β S1 I1 - (σ + μ + r (S1 + E1 + I1) / K1) E1 - δ E1 &&
0 == σ E1 - (ν + μ + r (S1 + E1 + I1) / K1) I1 - δ I1 &&
0 == r S2 (1 - (S2 + E2 + I2) / K2) - β S2 I2 + δ S1 &&
0 == β S2 I2 - (σ + μ + r (S2 + E2 + I2) / K2) E2 + δ E1 &&
0 == σ E2 - (ν + μ + r (S2 + E2 + I2) / K2) I2 + δ I1, {S1, E1, I1, S2, E2, I2}]
```

Out[28]=

$$\Big\{\{S1 \to K1,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to 0,\ I2 \to 0\},$$

$$\cdots 23 \cdots,\ \Big\{S1 \to \frac{K1\, r^2 + \cdots 35 \cdots + \frac{K1^2\, \beta\, \nu\, \sigma^2}{r^2 + r\,\mu - K1\,\beta\,\sigma}}{r^2 + r\,\mu - K1\,\beta\,\sigma},\ E1 \to \frac{\cdots 1 \cdots}{r^2 + \cdots 1 \cdots \cdots 1 \cdots \cdots 1 \cdots},$$

$$\cdots 2 \cdots,\ E2 \to \cdots 1 \cdots,\ I2 \to \frac{K2\, r^3 + \cdots 11 \cdots + \frac{K2 \cdots 3 \cdots (\cdots 1 \cdots)}{2(\cdots 1 \cdots)}}{K2\, r^2\, \beta - r^2\, \nu - K2\, r\, \beta\, \nu - K2\, r\, \beta\, \sigma - K2^2\, \beta^2\, \sigma}\Big\}\Big\}$$

large output | **show less** | **show more** | **show all** | **set size limit...**

In[29]:=
```
% /. {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73, K1 → 15, K2 → 5} // FullSimplify
```

Out[29]=

$$\{\{S1 \to 15,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to 0,\ I2 \to 0\},$$
$$\{S1 \to 15,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to 0,\ I2 \to -735.\},$$
$$\{S1 \to 15,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to -110.959,\ I2 \to -24.0411\},$$
$$\{S1 \to 0,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to 0,\ I2 \to 0\},$$
$$\{S1 \to 0,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to 0,\ I2 \to -735.\},$$
$$\{S1 \to 0,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0,\ E2 \to -110.959,\ I2 \to -24.0411\},$$
$$\{S1 \to 15,\ E1 \to 0,\ I1 \to 0,\ S2 \to 5,\ E2 \to 0,\ I2 \to 0\},$$
$$\{S1 \to 0,\ E1 \to 0,\ I1 \to 0,\ S2 \to 5,\ E2 \to 0,\ I2 \to 0\},$$
$$\{S1 \to 15,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0.962426,\ E2 \to 0.0283375,\ I2 \to 0.00500529\},$$
$$\{S1 \to 0,\ E1 \to 0,\ I1 \to 0,\ S2 \to 0.962426,\ E2 \to 0.0283375,\ I2 \to 0.00500529\},$$
$$\left\{S1 \to 0,\ E1 \to 0,\ I1 \to -2205.,\ S2 \to 0,\ E2 \to 0,\ I2 \to -367.5 - 2.\ \sqrt{33764.1 - 5512.5\,\delta}\right\},$$
$$\left\{S1 \to 0,\ E1 \to 0,\ I1 \to -2205.,\ S2 \to 0,\ E2 \to 0,\ I2 \to 2.\ \left(-183.75 + \sqrt{33764.1 - 5512.5\,\delta}\right)\right\},$$
$$\{S1 \to 0,\ E1 \to 0,\ I1 \to -2205.,\ S2 \to 0,\ E2 \to -110.959 + 30.2055\,\delta,\ I2 \to -24.0411 - 30.2055\,\delta\},$$
$$\Big\{S1 \to 0,\ E1 \to 0,\ I1 \to -2205.,$$
$$S2 \to 2.98121 - 4.62367 \times 10^{-10}\ \sqrt{1.90637 \times 10^{19} - 1.12944 \times 10^{23}\,\delta} - 169.844\,\delta,$$
$$E2 \to 0.0141687 + 3.2451 \times 10^{-12}\ \sqrt{1.90637 \times 10^{19} - 1.12944 \times 10^{23}\,\delta} + 169.844\,\delta,$$
$$I2 \to 0.00250264 + 5.73186 \times 10^{-13}\ \sqrt{1.90637 \times 10^{19} - 1.12944 \times 10^{23}\,\delta} + 2.7756 \times 10^{-17}\,\delta\Big\},$$
$$\Big\{S1 \to 0,\ E1 \to 0,\ I1 \to -2205.,$$
$$S2 \to 2.98121 + 4.62367 \times 10^{-10}\ \sqrt{1.90637 \times 10^{19} - 1.12944 \times 10^{23}\,\delta} - 169.844\,\delta,$$

$E2 \to 0.0141687 - 3.2451 \times 10^{-12} \sqrt{1.90637 \times 10^{19} - 1.12944 \times 10^{23}\,\delta} + 169.844\,\delta,$
$I2 \to 0.00250264 - 5.73186 \times 10^{-13} \sqrt{1.90637 \times 10^{19} - 1.12944 \times 10^{23}\,\delta} + 2.7756 \times 10^{-17}\,\delta\Big\},$
$\{S1 \to 0,\ E1 \to -332.877,\ I1 \to -72.1233,\ S2 \to 0,\ E2 \to -110.959 + 0.98799\,\delta,$
$I2 \to -24.0411 - 0.98799\,\delta\},\ \Big\{S1 \to 0,\ E1 \to -332.877,\ I1 \to -72.1233,\ S2 \to 0,\ E2 \to 0,$
$I2 \to -367.5 - 0.0273973 \sqrt{1.79929 \times 10^{8} - 960\,863.\,\delta}\Big\},\ \Big\{S1 \to 0,\ E1 \to -332.877,$
$I1 \to -72.1233,\ S2 \to 0,\ E2 \to 0,\ I2 \to 0.0273973\left(-13\,413.8 + \sqrt{1.79929 \times 10^{8} - 960\,863.\,\delta}\right)\Big\},$
$\Big\{S1 \to 0,\ E1 \to -332.877,\ I1 \to -72.1233,$
$S2 \to 2.98121 - 5.55541\,\delta - 4.62367 \times 10^{-10} \sqrt{1.90637 \times 10^{19} + \delta\left(-3.69429 \times 10^{21} + 16\,384.\,\delta\right)},$
$E2 \to 0.0141687 + 5.55541\,\delta + 3.2451 \times 10^{-12} \sqrt{1.90637 \times 10^{19} + \delta\left(-3.69429 \times 10^{21} + 16\,384.\,\delta\right)},$
$I2 \to 0.00250264 + 1.73475 \times 10^{-18}\,\delta +$
$5.73186 \times 10^{-13} \sqrt{1.90637 \times 10^{19} + \delta\left(-3.69429 \times 10^{21} + 16\,384.\,\delta\right)}\Big\},$
$\Big\{S1 \to 0,\ E1 \to -332.877,\ I1 \to -72.1233,$
$S2 \to 2.98121 - 5.55541\,\delta + 4.62367 \times 10^{-10} \sqrt{1.90637 \times 10^{19} + \delta\left(-3.69429 \times 10^{21} + 16\,384.\,\delta\right)},$
$E2 \to 0.0141687 + 5.55541\,\delta - 3.2451 \times 10^{-12} \sqrt{1.90637 \times 10^{19} + \delta\left(-3.69429 \times 10^{21} + 16\,384.\,\delta\right)},$
$I2 \to 0.00250264 + 1.73475 \times 10^{-18}\,\delta -$
$5.73186 \times 10^{-13} \sqrt{1.90637 \times 10^{19} + \delta\left(-3.69429 \times 10^{21} + 16\,384.\,\delta\right)}\Big\},$
$\Big\{S1 \to 0.956864,\ E1 \to 0.0330058,\ I1 \to 0.00583512,\ S2 \to 0,$
$E2 \to -110.959 + \left(-0.0000799332 - 1.67395 \times 10^{-26}\,\delta\right)\delta - \frac{4.47035 \times 10^{-7}}{-1.92712 \times 10^{6} + 1.\,\delta},$
$I2 \to -24.0411 + 0.0000799332\,\delta\Big\},$
$\Big\{S1 \to 0.956864,\ E1 \to 0.0330058,\ I1 \to 0.00583512,\ S2 \to 0,\ E2 \to \frac{1}{-1.92712 \times 10^{6} + 1.\,\delta}$
$\Big(9.53674 \times 10^{-7} - 1.45579 \times 10^{-15} \sqrt{2.10337 \times 10^{17} + 9.08764 \times 10^{10}\,\delta} +$
$\delta\left(-3.41061 \times 10^{-13} + 6.94177 \times 10^{-22} \sqrt{2.10337 \times 10^{17} + 9.08764 \times 10^{10}\,\delta}\right)\Big),$
$I2 \to -367.5 + 8.01308 \times 10^{-7} \sqrt{2.10337 \times 10^{17} + 9.08764 \times 10^{10}\,\delta}\Big\},$
$\Big\{S1 \to 0.956864,\ E1 \to 0.0330058,\ I1 \to 0.00583512,\ S2 \to 0,\ E2 \to \frac{1}{-1.92712 \times 10^{6} + 1.\,\delta}$
$\Big(9.53674 \times 10^{-7} + 1.45579 \times 10^{-15} \sqrt{2.10337 \times 10^{17} + 9.08764 \times 10^{10}\,\delta} +$
$\delta\left(-3.41061 \times 10^{-13} - 6.94177 \times 10^{-22} \sqrt{2.10337 \times 10^{17} + 9.08764 \times 10^{10}\,\delta}\right)\Big),$
$I2 \to -8.01308 \times 10^{-7}\left(4.58625 \times 10^{8} + \sqrt{2.10337 \times 10^{17} + 9.08764 \times 10^{10}\,\delta}\right)\Big\},$
$\Big\{S1 \to 0.956864,\ E1 \to 0.0330058,\ I1 \to 0.00583512,\ S2 \to 2.98121 + 0.00044946\,\delta +$

$1.48199\times10^{-14}\ \sqrt{1.85562\times10^{28}+\left(2.90929\times10^{26}-262\,144.\ \delta\right)\delta}$, E2 →
$0.0141687-0.00044946\,\delta-1.04013\times10^{-16}\ \sqrt{1.85562\times10^{28}+\left(2.90929\times10^{26}-262\,144.\ \delta\right)\delta}$,
I2 → $0.00250264+1.05881\times10^{-22}\,\delta-$
$1.83719\times10^{-17}\ \sqrt{1.85562\times10^{28}+\left(2.90929\times10^{26}-262\,144.\ \delta\right)\delta}\Big\}$,
$\Big\{$S1 → 0.956864, E1 → 0.0330058, I1 → 0.00583512, S2 → $2.98121+0.00044946\,\delta-$
$1.48199\times10^{-14}\ \sqrt{1.85562\times10^{28}+\left(2.90929\times10^{26}-262\,144.\ \delta\right)\delta}$, E2 →
$0.0141687-0.00044946\,\delta+1.04013\times10^{-16}\ \sqrt{1.85562\times10^{28}+\left(2.90929\times10^{26}-262\,144.\ \delta\right)\delta}$,
I2 → $0.00250264+1.05881\times10^{-22}\,\delta+$
$1.83719\times10^{-17}\ \sqrt{1.85562\times10^{28}+\left(2.90929\times10^{26}-262\,144.\ \delta\right)\delta}\Big\}\Big\}$

In[30]:=
```
{{S1 → 0.9568640827497336`, E1 → 0.033005784874229914`, I1 → 0.005835122920606429 5`,
  S2 → 2.981213030924836` + 0.00044945989608671496` δ - 1.4819930733948197`*^-14
    √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ) ,
  E2 → 0.014168736552806597` - 0.00044945989608671506` δ + 1.0401280447948825`*^-16
    √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ) ,
  I2 → 0.0025026444850466388` + 1.058808223737652`*^-22 δ + 1.8371932496215616`*^-17
    √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ) }} /. {δ → 0.1}
```

Out[30]= {{S1 → 0.956864, E1 → 0.0330058,
I1 → 0.00583512, S2 → 0.960889, E2 → 0.0283036, I2 → 0.00500725}}

In[31]:=

```
J = {
  {D[dS1[s1, e1, i1], s1], D[dS1[s1, e1, i1], e1], D[dS1[s1, e1, i1], i1],
        D[dS1[s1, e1, i1], s2], D[dS1[s1, e1, i1], e2], D[dS1[s1, e1, i1], i2]},
  {D[dE1[s1, e1, i1], s1], D[dE1[s1, e1, i1], e1], D[dE1[s1, e1, i1], i1],
        D[dE1[s1, e1, i1], s2], D[dE1[s1, e1, i1], e2], D[dE1[s1, e1, i1], i2]},
  {D[dI1[s1, e1, i1], s1], D[dI1[s1, e1, i1], e1], D[dI1[s1, e1, i1], i1],
        D[dI1[s1, e1, i1], s2], D[dI1[s1, e1, i1], e2], D[dI1[s1, e1, i1], i2]},
  {D[dS2[s2, e2, i2, s1], s1], D[dS2[s2, e2, i2, s1], e1], D[dS2[s2, e2, i2, s1], i1],
        D[dS2[s2, e2, i2, s1], s2], D[dS2[s2, e2, i2, s1], e2], D[dS2[s2, e2, i2, s1], i2]},
  {D[dE2[s2, e2, i2, e1], s1], D[dE2[s2, e2, i2, e1], e1], D[dE2[s2, e2, i2, e1], i1],
        D[dE2[s2, e2, i2, e1], s2], D[dE2[s2, e2, i2, e1], e2], D[dE2[s2, e2, i2, e1], i2]},
  {D[dI2[s2, e2, i2, i1], s1], D[dI2[s2, e2, i2, i1], e1], D[dI2[s2, e2, i2, i1], i1],
        D[dI2[s2, e2, i2, i1], s2], D[dI2[s2, e2, i2, i1], e2], D[dI2[s2, e2, i2, i1], i2]}
  } /. {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73, K1 → 15, K2 → 5} /. {s1 → 0.9568640827497336`,
     e1 → 0.033005784874229914`, i1 → 0.005835122920606429 5`,
     s2 → 2.981213030924836` + 0.00044945989608671496` δ - 1.4819930733948197`*^-14
        √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ) , e2 →
      0.014168736552806597` - 0.00044945989608671506` δ + 1.0401280447948825`*^-16
        √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ) , i2 →
      0.0025026444850466388` + 1.058808223737652`*^-22 δ + 1.8371932496215616`*^-17
        √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ)} //
   FullSimplify // MatrixForm
```

Out[31]//MatrixForm=

$$\begin{pmatrix} -0.0318955-\delta & -0.0318955 & -76.581 & 0 \\ 0.46571 & -13.5343-\delta & 76.548 & 0 \\ -0.000194504 & 12.9998 & -73.5334-\delta & 0 \\ \delta & 0 & 0 & -0.298121-0.000044946\,\delta+1.48199\times10^{-15}\sqrt{1.8} \\ 0 & \delta & 0 & 0.198795+0.000044946\,\delta+1.45935\times10^{-15}\sqrt{1.8} \\ 0 & 0 & \delta & -0.000250264-1.05881\times10^{-23}\,\delta-1.83719\times10^{-18}\sqrt{} \end{pmatrix}$$

```
In[32]:= mat[δ_] := {
{D[dS1[s1, e1, i1], s1], D[dS1[s1, e1, i1], e1], D[dS1[s1, e1, i1], i1],
    D[dS1[s1, e1, i1], s2], D[dS1[s1, e1, i1], e2], D[dS1[s1, e1, i1], i2]},
{D[dE1[s1, e1, i1], s1], D[dE1[s1, e1, i1], e1], D[dE1[s1, e1, i1], i1],
    D[dE1[s1, e1, i1], s2], D[dE1[s1, e1, i1], e2], D[dE1[s1, e1, i1], i2]},
{D[dI1[s1, e1, i1], s1], D[dI1[s1, e1, i1], e1], D[dI1[s1, e1, i1], i1],
    D[dI1[s1, e1, i1], s2], D[dI1[s1, e1, i1], e2], D[dI1[s1, e1, i1], i2]},
{D[dS2[s2, e2, i2, s1], s1], D[dS2[s2, e2, i2, s1], e1], D[dS2[s2, e2, i2, s1], i1],
    D[dS2[s2, e2, i2, s1], s2], D[dS2[s2, e2, i2, s1], e2], D[dS2[s2, e2, i2, s1], i2]},
{D[dE2[s2, e2, i2, e1], s1], D[dE2[s2, e2, i2, e1], e1], D[dE2[s2, e2, i2, e1], i1],
    D[dE2[s2, e2, i2, e1], s2], D[dE2[s2, e2, i2, e1], e2], D[dE2[s2, e2, i2, e1], i2]},
{D[dI2[s2, e2, i2, i1], s1], D[dI2[s2, e2, i2, i1], e1], D[dI2[s2, e2, i2, i1], i1],
    D[dI2[s2, e2, i2, i1], s2], D[dI2[s2, e2, i2, i1], e2], D[dI2[s2, e2, i2, i1], i2]}
} /. {r → 0.5, β → 80, σ → 13, μ → 0.5, ν → 73, K1 → 15, K2 → 5} /.
  {s1 → 0.9568640827497336`, e1 → 0.033005784874229914`, i1 → 0.005835122920606429 5`,
   s2 → 2.981213030924836` + 0.00044945989608671496` δ - 1.4819930733948197`*^-14
      √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ),
   e2 → 0.014168736552806597` - 0.00044945989608671506` δ + 1.0401280447948825`*^-16
      √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ),
   i2 → 0.0025026444850466388` + 1.058808223737652`*^-22 δ + 1.8371932496215616`*^-17
      √(1.855618261745987`*^28 + (2.9092885491909936`*^26 - 262144.` δ) δ)}

In[33]:= Plot[Evaluate[Re @ Eigenvalues[mat[δ]]], {δ, -0.1, 0.1}, PlotRange → {-0.1, 0.1}]
```

Out[33]=