

CSC 3110 – Homework 4

1. Find the order of growth for solutions of the following recurrences:
 - a. $T(n) = 5T(n/3) + n$

1. $T(n) = 5T\left(\frac{n}{3}\right) + n$
 ~~$a=5$~~ $a=5$ $f(n) \in \Theta(n^d)$ $d \geq 0$
 $b=3$
 $d=1$

$5 > 3^1 \checkmark$ $T(n) \in \Theta(n^{\log_3 5}) \approx n^{1.46}$
 $5 = 3^1 \times$
 $5 < 3^1 \times$

b. $T(n) = 9T(n/3) + n^2$

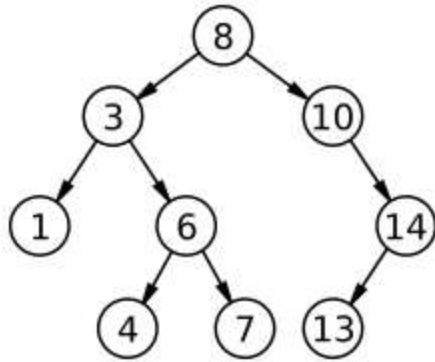
b) $T(n) = 9T\left(\frac{n}{3}\right) + n^2$
 $a = 9$ $f(n) \in \Theta(n^d)$ $d \geq 0$
 $b = 3$
 $d = 2$
 $S < 3^2$ ✓ $T(n) \in \Theta(n^2)$
 $S = 3^2$ ✗
 $S > 3^2$ ✗

c. $T(n) = 10T(n/3) + n^3$

c) $T(n) = 10T(n/3) + n^3$
 $a = 10$ $f(n) \in \Theta(n^d)$ $d \geq 0$
 $b = 3$
 $D = 3$

$10 < 3^3 \checkmark T(n) \in \Theta(n^3)$
 $10 = 3^3 \quad \times$
 $10 > 3^3 \quad \times$

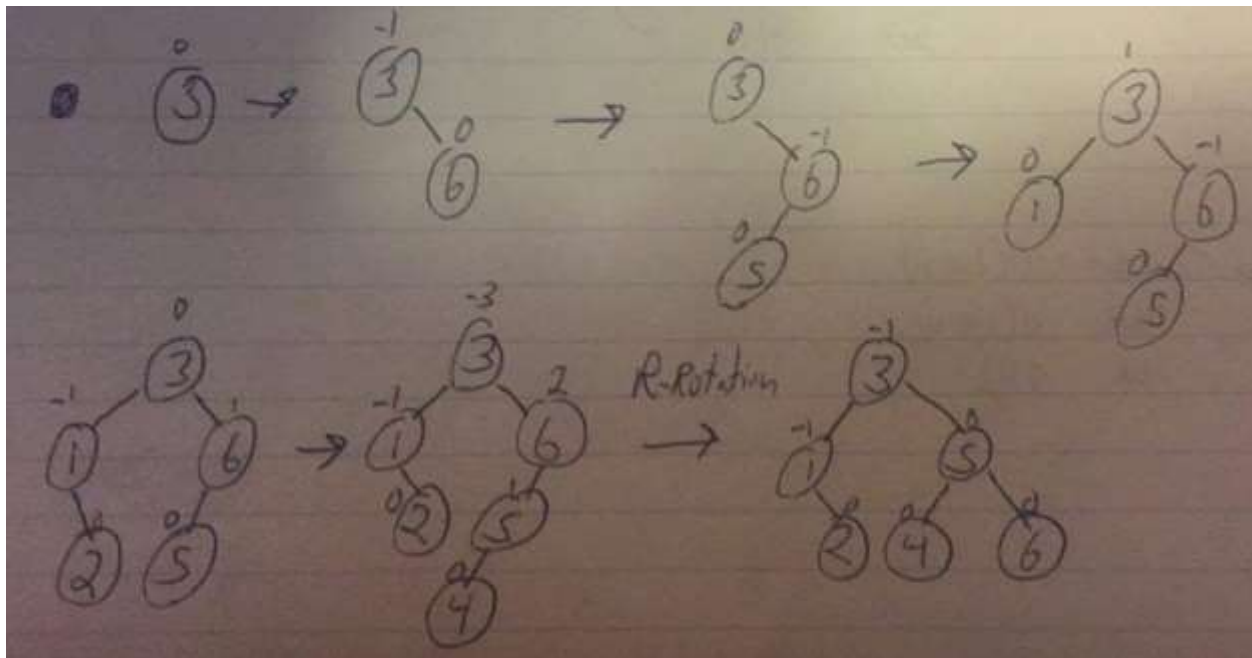
2. Traverse the following binary tree in



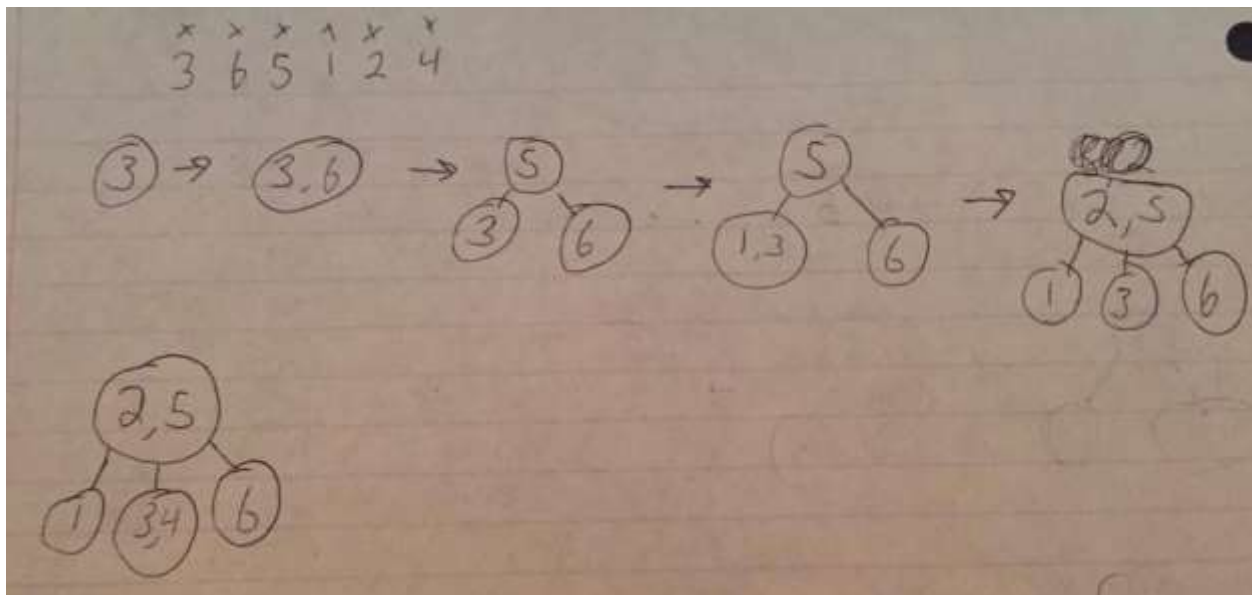
- a. Preorder = 8,3,1,6,4,7,10,14,13
 - b. Inorder = 1,3,4,6,7,8,10,13,14
 - c. Postorder = 1,4,7,6,3,13,14,10,8
3. Indicate the time efficiency classes of the three main operations of the priority queue implemented as
- a. Unsorted array
 - i. Insert = $O(1)$
 - ii. Delete = $O(n)$
 - iii. Peek = $O(n)$
 - b. Sorted array
 - i. Insert = $O(\log n)$
 - ii. Delete = $O(1)$
 - iii. Peek = $O(1)$
 - c. Binary Search Tree
 - i. Insert = $O(\log n)$
 - ii. Delete = $O(\log n)$
 - iii. Peek = $O(\log n)$
 - d. AVL tree
 - i. Insert = $O(\log n)$
 - ii. Delete = $O(\log n)$
 - iii. Peek = $O(\log n)$
 - e. Heap
 - i. Insert = $O(\log n)$
 - ii. Delete = $O(\log n)$
 - iii. Peek = $O(1)$

4. Construct both an AVL tree and a 2-3 tree for the values 3 6 5 1 2 4

a. AVL tree



b. 2-3 Tree



5. Apply Quicksort using both Lomuto's and Hoare's partitioning algorithms on 3 6 5 1 2 4
a. Lomuto's

Partition the array
if $S = k-1$ r
else if $S > L+k$
else partition other

0	1	2	3	4	5
3	6	5	1	2	4

S i

[3 6 5 1 2 4]
 S i

[3 6 5 1 2 4]
 S i

[3 1 5 6 2 4]
 S i

[3 1 5 6 2 4]
 S i

[3 1 2 6 5 4]
 S i

[3 1 2 6 5 4]
 S i

[2 1 3 6 5 4] // sort 2 sub arrays using Lom

$$P=2 \quad \begin{bmatrix} 2 & 1 & | & 3 & 6 & 5 & 4 \\ s & i & & & & & \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & | & 3 & 6 & 5 & 4 \\ s & i & & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & | & 3 & 6 & 5 & 4 \\ s & i & & & & & \end{bmatrix}$$

// sort other side

$$P=6 \quad \begin{bmatrix} 1 & 2 & 3 & | & 6 & 5 & 4 \\ s & i & & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 6 & 5 & 4 \\ s & i & & & & & \end{bmatrix}$$

~~1 2 3 4 5 6~~

~~1 2 3 4 5 6~~

~~1 2 3 4 5 6~~

$$\begin{bmatrix} 1 & 2 & 3 & | & 6 & 5 & 4 \\ s & i & & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & | & 6 & 5 & 4 \\ s & i & & & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

b. Hoare's

$P=3$

$$\begin{bmatrix} 3 & 6 & 5 & 1 & 2 & 4 \end{bmatrix}$$

$\quad \quad i \quad \quad \quad j$

$$\begin{bmatrix} 3 & 2 & 5 & 1 & 6 & 4 \end{bmatrix}$$

$\quad \quad i \quad \quad j \quad \quad \quad$

$$\begin{bmatrix} 3 & 2 & 1 & 5 & 6 & 4 \end{bmatrix}$$

$\quad \quad j \quad \quad i$

$P=1$

$$\begin{bmatrix} 1 & 2 & | & 3 & 5 & 6 & 4 \end{bmatrix} \quad i=j$$

$\quad \quad i \quad \quad \quad \quad \quad$

$P=5$

$$\begin{bmatrix} 1 & 2 & | & 3 & 5 & 6 & 4 \end{bmatrix}$$

$\quad \quad \quad \quad \quad i \quad \quad j$

$$\begin{bmatrix} 1 & 2 & | & 3 & 5 & 4 & 6 \end{bmatrix}$$

$\quad \quad \quad \quad \quad j \quad \quad i$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$