P452 Term Paper Report
Coupled water and heat flow in one dimensional unsaturated soil column

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Abstract

The flow of heat and water is highly coupled and occurs very commonly in nature. In this work the coupling due to dependency of soil thermal and hydraulic properties on soil water content has been studied and simulated. The coupled differential equations were solved numerically in python to get the water content and temperature relation with time when a fictitious boundary condition is applied. It was found that with the flow of water in unsaturated soil, the water content increased and subsequently the rate of heat flow also increased. The work can be further extended to account for the affect of water flow as a result of temperature getting lower than the freezing point resulting in freeze-thaw cycle. However, it could not be done due to time constraints.

1 Introduction

The soils in general can be seen as a porous matrix through which water and heat can flow following the fundamental energy and mass conservation laws. Since the flow of water can affect the thermal conductivity and in turn the temperature change of the soil can lead to freeze and thaw cycles. The correlation between the two flow and their common occurrence in nature makes it simulation an interesting as well as a challenging problem.

The commonly used model of soil considers it to be quasi-homogeneous consisting of a matrix of individual solid grains between which are interconnected pore spaces that can contain varying amount of water and air. The ratio of the volume of these pore spaces to the total volume of the soil is called porosity.

$$\phi = \frac{V_a + V_w}{Vs} \tag{1}$$

where V_a,V_w and V_s are the volume of air, water and mineral grains respectively. The amount of water present in the soils is given simply as the ratio between the volume of water to the volume of soil.

$$\theta = \frac{V_w}{Vs} \tag{2}$$

From the definition it can be seen that the water content 'θ' can take any value between 0 to ϕ. When the water content in a soil is equal to the porosity, it cannot hold any more water in it and is called saturated soil.

2 Theory

2.1 Heat flow

The flow of heat can be described by the Fourier's empirical heat law given as,

$$q_\mathrm{h} = C \cdot \Delta T_\mathrm{c} \tag{3}$$

where, C_c is the effective volumetric heat capacity given by,

$$C = C_\mathrm{w}\theta_\mathrm{w} + C_\mathrm{i}\theta_\mathrm{i} + C_\mathrm{s}\theta_\mathrm{s} + C_\mathrm{a}\theta_\mathrm{a} \tag{4}$$

where θ is volumetric fraction and subscripts w, i, s, and a represent the fractions of water, ice, soil solids and air fractions respectively.

The equation 3 can then be used to balance heat flow with the neighbouring cells. The one-dimensional conductive heat transport in variably saturated soils can be given by the heat balance equation,

$$q_\mathrm{h} = \sum_{\zeta=i-1}^{i+1} i,\zeta \cdot \frac{T_\zeta - T_i}{l_{i,\zeta}}, \zeta \neq i \tag{5}$$

where, subscripts i and ζ refer to the cell and its active neighbours, q_h is the net heat flux for the i_{th} cell, T is cell temperature $(^\circ\mathrm{C})$, $C_{i,\zeta}$ is average effective thermal conductivity of the region between the i_{th} and the ζ_{th} cells , and $l_{i,\zeta}$ is the distance between the centres of the i_{th} and the ζ_{th} cells (m).

2.2 Water flow

Buckingham–Darcy's equation is used to describe the flow of water under hydraulic head gradients wherein it is recognized that the soil matrix potential (Ψ) and hydraulic conductivity (K_h) are functions of liquid water content (θ). The dependency of Ψ and K_h on θ can be expressed as a constitutive relationship dependent on the soil being considered and is given as,

$$\psi(\theta) = \psi_{ac} \cdot \left(\frac{\phi}{\theta}\right)^b \tag{6}$$

$$K_h(\theta) = K_s \cdot \left(\frac{\theta}{\phi}\right)^c \tag{7}$$

where, ψ_{ac} is the potential for saturated soil and has negative value, K_s is the hydraulic conductivity of saturated soil, b is the pore size distribution index, c is the pore disconnectedness index, $c = 2 \times b$. The figure 1 represents the general trend of matrix potential and hydraulic conductivity with water content.

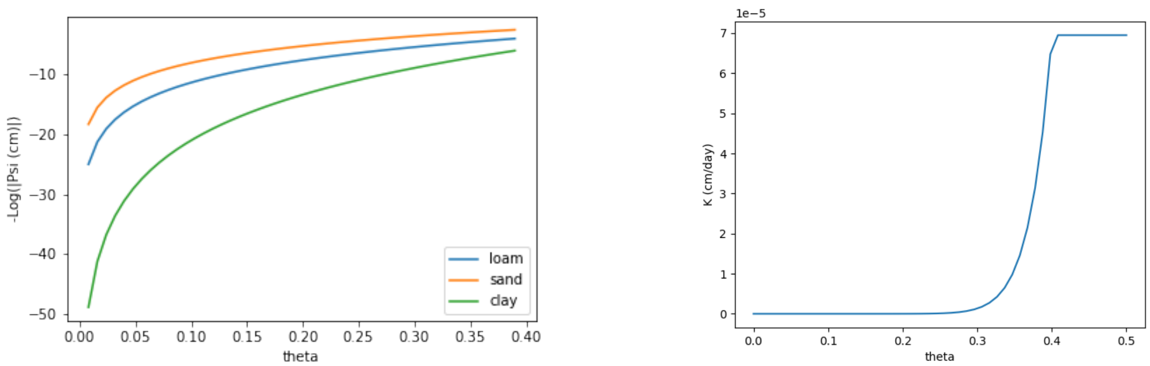

Figure 1: Matrix potential (left) and hydraulic conductivity (right) as a function of water content

In the figure 1, it can be seen that the hydraulic conductivity increases with increase in water content till the saturation point after which it gets constant. This is because the unsaturated soil retains most of the water that flows through it, however when the saturation is reached it does not holds anymore water and passes it completely. The Buckingham–Darcy's equation is then given as,

$$q_z = -K_h(\theta) \cdot \frac{d[z + \psi(\theta)]}{dz} \tag{8}$$

Adapting the equation for the discrete model we get the following form of the equation as,

$$q_w = \sum_{\zeta=i-1}^{i+1} k_{i,\zeta} \cdot \frac{(\psi + z)_\zeta - (\psi + z)_i}{l_{i,\zeta}}, \zeta \neq i \tag{9}$$

where, z is the cell elevation and k here represents the average hydraulic conductivity of the region between the $\mathrm{i}_t h$ and the ζ_{th} cells.

3 Algorithm

The one-dimensional column of cell was taken to be the sum of many small individual cell such that for each cell the water content and temperature can be assumed to be constant. Similarly, the total duration of simulation was also taken to be the sum of many small time intervals such that any physical parameter remains constant in that interval.

1. The soil column was initialized with a fixed water content less than the saturation value. The initial temperature for each such cell was also assumed to be constant for the entire column.
2. The boundary condition for water flow was provided by assuming a saturated soil cell at the top. For heat flow, the temperature of the top cell was kept fixed at a constant temperature.
3. The heat flux and water flux was calculated using the equations 5 and equation 9 respectively. In essence, the thermal conduction and hydraulic conduction codes run simultaneously and are not affected by each other in the same time step.
4. The calculated value of flux were then used to update the water content and temperature profile of the column. The updated water content was then used to update the effective heat capacity of the cell for next iteration.

4 Results

The water content was initialized at 0.2 and the top cell was fixed at the saturation value. It was found that water content increased to the saturation value of 0.4 gradually. The increase in water content was faster for the cell on top and so the flow was less and the water seeped through the layers gradually as seen in figure 2.

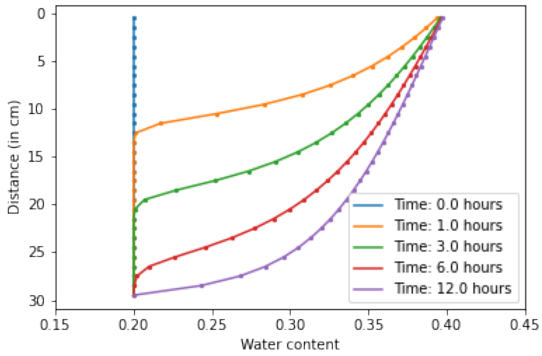

Figure 2: Water content with time when saturate boundary condition is applied

The flow of water across the column changed the effective heat capacity of the column and in turn affected the heat flow. Initially, the cells were assumed to be at constant temperature of 20° and the top cell was fixed to be at 40°. The change in temperature profile with time was as seen in figure 3.

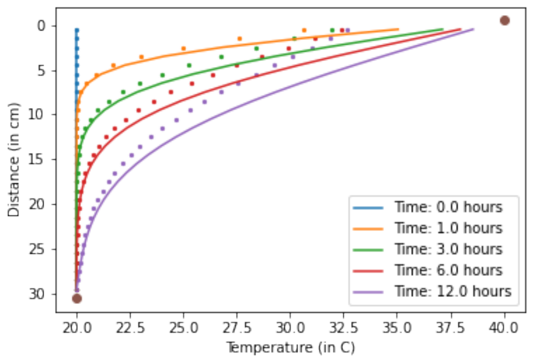

Figure 3: Temperature with time with and without considering water flow

The solid lines in figure 3 represents the heat flow in presence of water flow while in contrast the dotted lines are the ones assuming that there was no water flow and water content was the same as was initialized. It can be clearly seen that the increase in water content increased the speed of heat flow and the temperature increased rapidly than compared with no water flow. Hence the coupling between the two was confirmed.

5 Conclusion

The coupled equations for water flow based on Buckingham Darcy's law and heat flow based on Fourier's heat low was solved numerically by using a modified finite difference approach. The power law equation of Campbell (1974) was used to find the approximate analytical relation of matrix potential and hydraulic conductivity with water content. The heat flow was observed to be significantly coupled with the water flow. The obtained results were found in agreement with the previously known works and observations.

Due to the time constraint, the freeze and thaw cycle could not be simulated. It would have involved the other way coupling where the heat flow would have affected the water flow as the temperature would have fallen beyond freezing point. Furthermore, the temperature of water being introduced in the system was not considered which can play a major role in the temperature profile of the column.

References

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