Heat transfer coupled with water in soil

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1 Abstract

The transfer of heat and transfer of water in soil is a coupled process.Water moving through soil carries with it heat, and soil water content influences soil thermal properties. The coupled transfer is modelled in this paper using finite volume method. Two experiments is done to test the results where the numerical solution is compared with trend in experiment result and in the second model is compared with analytical solution. The results were successfully reproduced .

2 Introduction

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The evolution of heat and moisture content in soil help in various aspects like understanding the living condition of vegetation and microorganisms in soil.In this study a 1 dimensional soil column is taken for the study where the coupled heat transfer with water will be numerically calculated using fundamental laws, the heat balance equation or Fourier law and mass balance equation Buckingham–Darcy equation.The Buckingham-Darcy equation defines the motion of water in unsaturated soil. The soil temperature of the soil is mainly affected by three process conduction, convection and radiation. Conduction is the transfer of heat energy from one particle to another through motion. In convection the heat is transferred through fluids Radiation is the transfer of heat as electromagnetic radiation. For the subsurface condition heat is balanced by conduction and convection.The movement of water is governed by the forces on the soil water which is calculated using the soil water potentials. The potential describes the tendency of the water to flow or move freely in the soil. The total soil water potential is the sum of several other potentials. For the considered 1 dimension soil column we are considering the matric potential and the gravitational potential.

3 Formulae used

3.1 Heat transfer by conduction:

Discretization is the process through which we can transform continuous variables, models or functions into a discrete form by creating a set of contiguous intervals (or bins) that go across the range of our desired variable/model/function. Here finite volume discretization is used. The soil column is discretised into cells where each component is computed for the cell. The time is also discretised. The model find the dependent variable for each cell at discretised time intervals.

The temperature change in 1 dimension can be given by using a energy balance, for this accumulation energy and the total flux i.e the influx and out-flux need to be found.

The heat balance equation is given as,

$$qh = \sum_{j=i-1}^{i+1} \lambda_{ij} \frac{T_j - T_i}{l_{ij}} \tag{1}$$

i is the active cell and j is the neighbouring cell. qh is the net flux in the ith cell where λ_{ij} is the average effective thermal conductivity. T is the cell temperature. $l - ij$ is the length between the centers of the active and neighbour cell. The relationship between the temperature change in the cell and the heat flux is,

$$\frac{qh.\Delta t}{l_i} = C_i.\Delta T$$

where, C$_i = C_w\theta_w + C_a\theta_a + C_s\theta_s(2)$

C_i is the effective volumetric heat capacity and subscripts w,a,s denotes water, air and soil.

3.2 Water flow in soil:

The flow of water in unsaturated soil is given by the Buckingham–Darcy's equation. The Darcy's law can only explain the flow of water in saturated soil. Richard's equation combine the Darcy's equation and continuity equation. This describes the flow under hydraulic head gradient, the driving force that causes groundwater to move in the direction of maximum decreasing total head. By using this equation the matric potential, which measures the attraction of water to the soil and hydraulic conductivity as a function of the liquid water content.

Darcy's equation is:

$$q_s = -K_s.S_f \tag{3}$$

here the flux is given as the product of a scalar saturated hydraulic conductivity and the driving gradient. The potential here is the sum total of gravitational potential, solute potential and pressure potential.In the Buckingham–Darcy's equation the driving gradient include the matric potential. The pressure potential is taken zero as it is unsaturated condition and the hydraulic conductivity is a function of water content. The textural properties of the soil that dictates characteristics like water conductivity is given by the Mualem–van Genuchten.

Matric potential as a function of water content is,

$$\psi(\theta_w) = \frac{[S_e^{-1/m} - 1]^{1/n}}{\alpha} \tag{4}$$

$$S_e = \frac{\theta_w - \theta_{res}}{\theta_{sat} - \theta_{res}} \tag{5}$$

$$k(\theta_w) = K_s.(S_e)^{0.5}.[1 - (1 - (S_e)^{1/m})^m]^2 \tag{6}$$

$$q_w = \sum_{j=i-1}^{i+1} k_{ij} \frac{(\psi + z)_j - (\psi + z)_i}{l_{ij}} \tag{7}$$

3.3 transfer of heat coupled with water

: The moisture transfer carries some heat and the rising moisture content can increase the heat conductivity. In the code the heat flux and water flux is found at a time interval, separately and the temperature and water content is updated from their respective outputs. This updated water content is fed to the heat capacity equation. This loop is run for the entire time range.

4 Simulation details

For performing the simulations numpy, scipy,panda and matplotlib packages has been used.

4.1 Algorithm

4.1.1 Heat flux

The partial differential equation is solved as discretized to form algebraic equation.Then the forward Euler time advancement scheme is applied. The discretised equation is

$$T[i] = \alpha(\frac{T_i - T_{i-1}}{dx^2} + \frac{T_{i+1} - T_i}{dx^2}) \tag{8}$$

4.2 Water flux

The Buckingham–Darcy's equation is discretized as,

$$q = -K_{mid}.(\frac{(\psi_{i+1} - \psi i)}{dz} + 1.0) \tag{9}$$

$$\psi = -\frac{(q[i+1] - q[i])}{dz}/C \tag{10}$$

This gives a set of coupled ordinary differential equations that is solved using the odeint of scipy. This is using the LSODA algorithm where it uses dense or banded Jacobian when the problem is stiff, but it automatically selects between nonstiff (Adams) and stiff (BDF) methods. It uses the nonstiff method initially, and dynamically monitors data in order to decide which method to use.

4.2.1 Coupling

: Inputs parameters for the heat flux function and water flux function is provided. These include the time interval and spatial interval in time and spatial discretization, soil properties etc.Numpy and panda is used for data structure, matplotlib is used for plotting

The heat and water flux functions are run separately. The outputs are saved.

The input parameter of effective heat capacity is updated using the output of the above functions.This is run through out the desired time.

4.3 First experiment

: The heat transfer was coupled with the moisture content and the result is compared to that published in Hua.Jin et.al(2018).Initial temperature and moisture both are uniform through the one-dimensional soil column. Te temperature of the soil at zero distance is assumed to be the heat source temperature, due to the neglect of the heat transfer time. Since the soil column is assumed to have a long length, both the temperature and moisture at the outer of it will be constant and uninfuenced.

Parameters	Experiment 1
Soil thermal conductivity (J/(min·m·°C))	48
Soil particle density (kg/m^3)	2650
Heat capacity of soil(J/(kg·°C))	1739
Length of soil column (m)	1
Number of distance element	100
Time step (min)	1
Total time (h)	24
Initial soil temperature (°C)	20.6
Temperature of heat source (°C)	33.5
Porosity	0.4403
Water thermal conductivity (J/(min·m·°C))	35.94
Water density (kg/m^3)	1000
Heat capacity of moisture (J/(kg·°C))	4180
Initial moisture content (m^3/m^3)	0.08

Figure 1: Parameters used

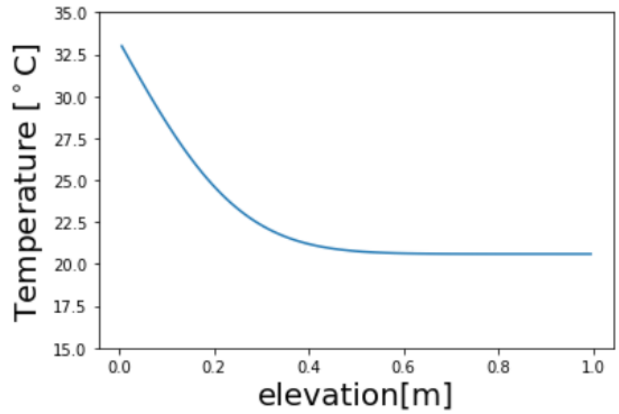

Figure 2: numerically solved temperature profile for 1m soil column after 24 hours. Boundary condition 33.5°C

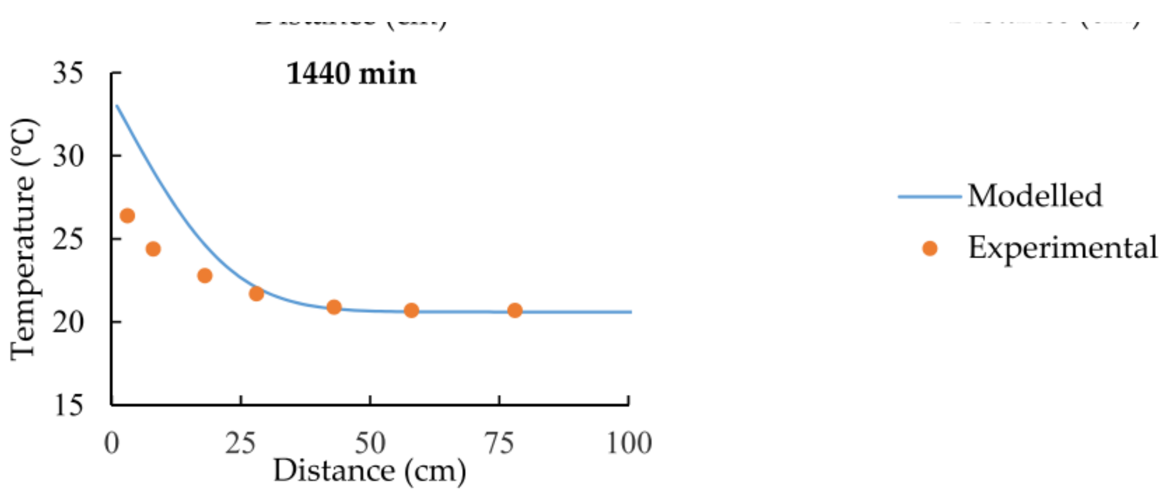

Figure 3: Hua Jin et.al(2018) temperature profile for 1m soil column after 24 hours. Boundary condition 33.5°C

4.4 second experiment

: Stallman's analytical solution (1965) to the subsurface temperature profile in a semi-infinite porous medium in response to a sinusoidal surface temperature provides a test of the model The temperature variation is given by

$$T(z_0, t) = T_s ur + A.sin\frac{2.\pi.t}{\tau} \tag{11}$$

Symbol	Parameter	Value
η	porosity	0.40
λ	bulk thermal conductivity	$2.0\,\mathrm{J\,s^{-1}\,m^{-1}\,{}^{\circ}C^{-1}}$
C_{w}	volumetric heat capacity of water	$4\,174\,000\,\mathrm{J\,m^{-3}\,{}^{\circ}C^{-1}}$
C_{s}	volumetric heat capacity of soil solids	$2\,104\,000\,\mathrm{J\,m^{-3}\,{}^{\circ}C^{-1}}$
ρ_{w}	density of water	$1000\,\mathrm{kg\,m^{-3}}$
ρ_{s}	density of soil solids	$2630\,\mathrm{kg\,m^{-3}}$
l	length of cell	0.01 m
t	length of time step in CA solution	1 s
q_{f}	specific flux	$4\times10^{-7}\,\mathrm{m\,s^{-1}}$ downward
τ	period of oscillation of temperature at the ground surface	24 h
A	amplitude of the temperature variation at the ground surface	5 °C
T_{surf}	average ambient temperature at the ground surface	20 °C
T_{∞}	ambient temperature at depth	20 °C

Figure 4: R.M.Nagare et.al(2015) temperature profile for 1.5m soil column after 24 hours.

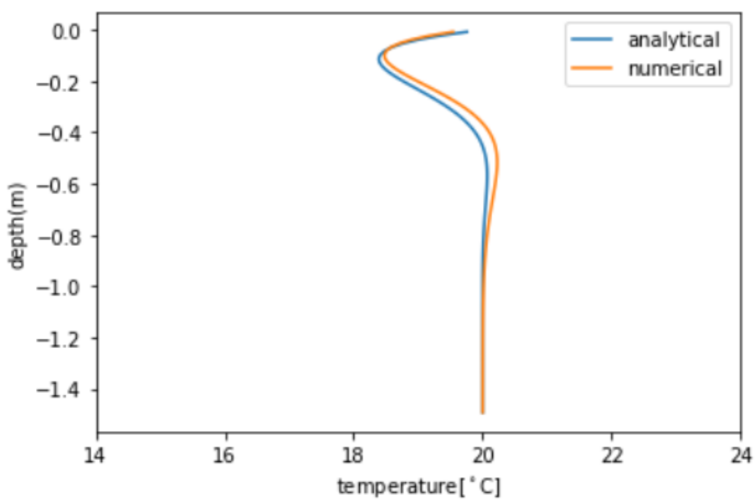

Figure 5: The numerical plot and analytical plot

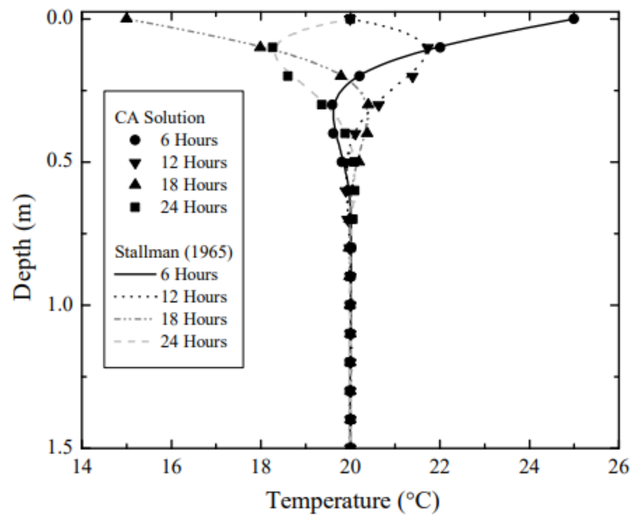

Figure 6: R.M.Nagare et.al(2015) result

5 Conclusion

In this the coupled heat transfer equation was solved numerically using the concept of cellular automata, finite volume method and euler method. The coupling of the water and heat transfer equation was done by updating the effective heat capacity which is the sum of the heat capacity of soil water. The model that was created was compared to two results one with variable boundary condition and other fixed boundary condition. Both models were able to reproduced published results from other literature. The thermal conductivity is taken here constant even though the moisture content affects this. This is done as proper value for the desired condition was not able to find. It is seen here that this approximation didnt deviate the result to large oder.

6 Reference

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