Sudden Stops with Heterogenous Agents*

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Abstract

I introduce income heterogeneity into a small open economy model with an occasionally binding collateral constraint. Income heterogeneity generates poor households that borrow up to the constraint to smooth over their income shock. This differs from representative agent models that require a depressed aggregate state for the representative household to interact with the constraint. As a consequence, the model displays a higher average marginal propensity to consume and volatility of aggregate consumption. The improvements disappear when the collateral constraint is removed from the model. The model with income heterogeneity fails to generate sudden stops. This occurs as the income shock generates rich households that are able to consumption smooth throughout contractions.

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1 Introduction

Small open economies display more volatile behavior than developed economies. At the household level, this is expressed in a higher marginal propensity to consume (MPC) and portion of financially constrained households (Hong 2023b), (Cugat 2019). At the aggregate level, this is expressed in a higher standard deviation of aggregate consumption relative to output. Additionally, emerging markets display rare 'sudden stop' episodes, where asset inflows rapidly reverse despite a depression in output.

This paper studies the ability of household heterogeneity, generated by idiosyncratic income risk, to explain these phenomena. I show that heterogeneity increases the average marginal propensity to consume and the aggregate volatility of consumption. This occurs as income risk generates impoverished households for whom financial frictions generate nonlinear behavior, regardless of the aggregate state. This differs from representative agent (RA) models that require a depressed aggregate state for financial frictions to have a significant effect on behavior. I show that income heterogeneity decreases the intensity of sudden stops at the aggregate level and argue this occurs due to the asset structure of the model. The income shock generates a population of rich households that save their temporary increase in income into the single riskless asset. This places them far from the financial friction that affects impoverished households, allowing them to consumption smooth throughout both output contractions and sudden stops. The ability of rich households to consumption smooth during sudden stops produces a weaker decrease in aggregate consumption relative to the RA model and a muted trade balance response.

I start by building a RA small open economy model. Households receive an exogenous aggregate endowment and can borrow on the international market, subject to a collateral constraint that depends on the aggregate endowment. The collateral constraint represents a financial accelerator in which access to liquidity tightens during recessions, studied in Kiyotaki and Moore (1997) and Bernanke et al. (1999). From an emerging markets perspective, the financial accelerator is studied in Mendoza (2002), Mendoza (2010), Bianchi (2011).

In the RA model, the collateral constraint generates characteristic sudden stops events that feature a sharp decrease in consumption and a sudden reversal of the trade balance. The trade balance reverses because the collateral constraint binds, forcing households to save, despite the decrease in their income. Because the constraint only occasionally binds, however, the dramatic behavior of sudden stops does not have a significant effect on the average behavior of the model, as noted in Mendoza (2010). This leads the RA model to fail to explain excess consumption volatility and the high average MPC of households.

I then introduce income heterogeneity into the household problem. This leads to several differences between the models. At the household level, the HA model displays a higher average marginal propensity to consume and a higher proportion of constrained households. At the aggregate level, the HA model displays a higher volatility of consumption relative to output. This is the natural result of aggregate consumption incooperating the more volatile responses of constrained households. However, the HA model fails to generate a sudden stop as it displays a weaker drop in consumption and a muted response of the trade balance relative to the RA model.

The improvements of the HA model stem from the more frequent interaction of households with the collateral constraint. In the RA model, the constraint occasionally binds during a contractionary aggregate state. In the HA model, the income shock generates a population of households that are poor irrespective of the aggregate state. No differently than an RA household would borrow in response to a contractionary aggregate shock, these poor households take on debt to smooth over their temporary decrease in income. This leads to an average of 57% of households being constrained in the HA model, whereas households in the RA model are constrained in 12% of periods. This is closer to Cugat (2019)'s empirical estimate of 58% of households being hand to mouth. Constrained households display a high MPC as they wish to consume more but are prohibited by the collateral constraint. Nearly constrained households display a similarly high MPC as they wish to consume more but face significant risk in moving any closer to the constraint. These household contribute to an average annual MPC of 70% in the HA model, whereas the RA model has an average MPC of 25%. The HA estimate actually overshoots Hong (2023b)'s empirical estimate of 54.5% to 59.2%.

The improvements raise a serious question of why the HA model fails to generate a sudden stop. Recall that the canonical definition of a sudden stop in the RA model requires that all households are constrained. This implies that no household is able to consumption smooth, and all households are forced to increase their savings. Constrained households in the HA model display similar decreases in consumption and increases in savings. The income shock, however, generates a population of households that are rich regardless of the aggregate state. Because the income shock is transitory, rich households save a significant portion of their income increase into the single liquid asset. This places them far from the collateral constraint and allows them to behave like permanent income consumers throughout the sudden stop. Within the population of rich households, this is reflected in a lower percentage decrease in consumption relative to output and an increase in borrowing. The ability of rich households to consumption smooth has a significant affect on aggregate consumption as they make a disproportionate share of consumption relative to poor households. As a consequence, the HA model displays a weaker decrease in aggregate consumption relative to the RA model. Additionally, the increase in leverage taken on by rich households nullifies the forced savings of constrained households. The result is a dampened trade balance to output ratio that fails to reverse.

1.1 Related Literature

This paper contributes to the literature on sudden stops in emerging markets. In contrast to Cugat (2019), and like Villalvazo (2023), I find that household heterogeneity weakens the size of sudden stops.[1]This is because households in Cugat (2019)'s perturbated RA model is able to consumption smooth throughout the sudden stop. In contrast, no households in my RA model are able to consumption smooth during a sudden stop. My model is driven by endowment shocks, while Villalvazo (2023) is driven purely by interest rate shocks. Further work could incooperate both types of shocks. Computationally, my model is more similar to Mendoza (2010). Because I use a nonlinear global solution method, sudden stops are anticipated and not generated by large, unexpected shocks. My model differs from Mendoza (2010) in that prices are completely static, which implies that there is no Fisherian debt deflation to initiate or accelerate sudden stops. I interpret this as making my results conservative.

My paper suggests that single asset heterogenous agent models may understate the magnitude of financial crises relative to RA models. This occurs from two effects. In the first effect, rich households are able to consumption smooth during crises. This makes the simulated consumption response weaker than in RA models where all households are constrained during a crisis. Secondly, the response of rich or unconstrained households may nullify the responses of constrained households. In my model, this is expressed in a muted trade balance response. The first effect applies to single asset heterogenous agent models that attempt to explain dramatic aggregate behavior during contractions. This includes Auclert et al. (2021), where rich households are able to smooth over their decrease in real income following an exchange rate depreciation. Additionally, de Ferra et al. (2020) features net wealth rich households are not forced to deleverage after an exchange rate depreciation expands their gross debt holdings.[2] Both of these examples focus on decreases in real income or net asset holdings generated by an exchange rate depreciation. However, the core principle is that rich households have acquired liquidity before the crisis that allows them to consumption smooth throughout the crisis.

My paper contributes to the growing literature on the effects of heterogeneity in emerging markets. My implications are most similar to Hong (2023a). Namely, I show that financial frictions drive a higher MPC and excess volatility of consumption. This provides a contrast to the interest rate fluctuations of Neumeyer and Perri (2005), and the growth rate shocks of Aguiar and Gopinath (2007), neither of which are present in my model. Beyond Hong (2023a), I show that the removal of the collateral constraint erases the improvements of the HA economy. This supports that increases in the marginal propensity to consume are driven by households' interaction with financial frictions rather than a 'pure' effect of income risk.

I replicate Guntin et al. (2020)'s finding that single asset heterogenous agent models fail to capture the nearly one to one consumption and income elasticity of rich households during crises. I further show the single assest structure leads to two additional failures at the aggregate level: a weak response of consumption relative to the RA model, and a failure of the trade balance to reverse. Guntin et al. (2020) argue that this supports permanent income shocks as being the driver of steep drops in consumption during crises, rather than financial frictions. However, I argue this

[2]The interpretation for de Ferra et al. (2020) is more complicated as households' only choose their net wealth (the single asset) and the allocation across debt and capital is exogenously specified.

is a weakness of the single asset model and not financial frictions. The single asset model imposes that rich households can only save in a purely liquid asset, and saving mechanically places them far from the collateral constraint. From this perspective, the assumption of a single asset is quite restrictive.

My paper contributes to the literature of solution methods for heterogenous agent models. I leverage two assumptions to solve the model globally. I exploit the assumption, specific to small open economies, of a completely static interest rate. I additionally assume that households do not care about the distribution of earnings and asset holdings.[3] This allows me to solve the model globally by simply appending the aggregate state to the households' statespace. This differs from Reiter (2009) and Auclert et al. (2020) who linearize in aggregates. The method is adaptable to any model where the relevant aggregate states to the household problem are exogenous.

The rest of the paper is organized as follows. Section 2 details the quantitative models. Section 3 describes the calibration. Section 4 describes the results and Section 5 concludes.

2 Model

This section develops the quantitative RA and HA models. Both models receive an aggregate income endowment Y_t that follows a known stochastic process. The RA model features a continuum of identical households. The HA model features a continuum of households which are heterogenous due to an idiosyncratic, uninsurable income shock. The income shock generates a distribution of households with varying levels of wealth and income. In either economy, individual households can save or borrow in a bond at a constant interest rate r. Borrowing is limited by a collateral constraint that depends on the aggregate endowment of income.

Within the HA model I denote household terms using lower case letters x and aggregate terms using upper case letters X. Throughout the paper, I denote a generic household in the HA economy as household i. Within the RA economy, households follow the same policy functions as they are identical. This implies household terms coincide with aggregate terms. I therefore cast the RA

[3] Even if the distribution were appended to the household's statespace, their optimal policy functions would be independent of it as it does not provide any information about the (completely static) interest rate in the next period.

model directly in terms of aggregates. Section 2.1 describes the RA model and section 2.2 describes the HA model.

2.1 RA Model

The economy is populated by a continuum of identical households. The representative household has preferences over infinite consumption streams $\{C_t\}_{t=0}^{\infty}$ given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t). \tag{1}$$

Households satisfy the sequential budget constraint

$$C_t + (1+r)D_{t-1} = D_t + Y_t \tag{2}$$

where C_t is the choice of consumption, D_t is the quantity of debt held, and Y_t is the aggregate endowment. Borrowing is permitted, denoted by $D_t > 0$, but is subject to the collateral constraint

$$D_t \leq \kappa Y_t \tag{3}$$

where κ is a positive constant. The collateral constraint works as a financial accelerator, studied in Kiyotaki and Moore (1997) and Bernanke et al. (1999). The role of the collateral constraint in generating sudden stops in emerging markets is studied in Mendoza (2010). The dependence of the collateral constraint on an exogenous variable simplifies the dynamics that it generates. There is no debt deflation cycle, like in Mendoza (2010), that reduces the market value of collateral through prices. The lack of prices removes any possible price externalities, like that discussed in Bianchi (2011), that increase the frequency of financial crises. Finally, the lack of endogenous prices removes the possibility of the multiple equilibria, studied in Schmitt-Grohé and Uribe (2020).

The representative household's recursive problem can be expressed recursively as

$$V(Y_t, D_{t-1}) = \max_{C_t, D_t} u(C_t) + \beta E V(Y_{t+1}, D_t) \tag{4}$$

$$C_t + (1+r)D_{t-1} = D_t + Y_t \tag{5}$$

$$D_t \leq \kappa Y_t \tag{6}$$

We can write the first order conditions as

$$u'(C_t) = \mu_t + \beta(1+r)Eu'(C_{t+1}). \tag{7}$$

where μ_t denotes the inequality multiplier associated with (3).

2.2 HA Model

A continuum of households is indexed by $i \in [0, 1]$. Household i has preferences over infinite consumption streams $\{c_t^i\}_{t=0}^{\infty}$ given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i). \tag{8}$$

Household i satisfies the sequential budget constraint

$$c_t^i + (1+r)d_{t-1}^i = d_t^i + e_t^i Y_t \tag{9}$$

where d_t^i is household i's debt position and e_t^i is their idiosyncratic income shock. Rather than receiving the aggregate income Y_t, household i receives the personal income $e_t^i Y_t$. Household i can borrow subject to the collateral constraint[4]

$$d_t^i \leq \kappa Y_t. \tag{10}$$

[4] I also consider the case where borrowing is limited by personal income, $d_t^i \leq \kappa e_t^i Y_t$.

The recursive formulation of household i's problem can be expressed as

$$V(e_t^i, d_{t-1}^i; Y_t) = \max_{c_t^i, d_t^i} u(c_t^i) + \beta EV(e_{t+1}^i, d_t^i; Y_{t+1}) \tag{11}$$

$$c_t^i + (1+r)d_{t-1}^i = d_t + e_t^i Y_t \tag{12}$$

$$d_t^i \leq \kappa Y_t. \tag{13}$$

The Euler equation is

$$u'(c_t^i) = \mu_t^i + \beta(1+r)Eu'(c_{t+1}^i). \tag{14}$$

where μ_t^i is the inequality multiplier associated with the collateral constraint. I denote the solutions to the above problem by $c(e_t, d_{t-1}; Y_t)$ and $d'(e_t, d_{t-1}; Y_t)$.

Distribution and Aggregates The distribution of households is defined by the CDF function

$$\Psi_t(e, d) = Pr(e_t \leq e, d_{t-1} \leq d) \tag{15}$$

The distribution satisfies the law of motion

$$\Psi_t(e^*, d^*) = \int_{e,d} \pi(e' < e^*|e)1(d'(e, d; Y) < d^*)d\Psi_{t-1}(e, d). \tag{16}$$

Aggregate consumption and savings are defined as the sum of individual household policies:

$$C_t = \int c(e_t, d_{t-1}; Y_t)d\Psi_t(e_t, d_{t-1}) \tag{17}$$

$$D_t = \int d(e_t, d_{t-1}; Y_t)d\Psi_t(e_t, d_{t-1}) \tag{18}$$

2.3 Aggregates and Household Variables

The trade balance is defined as net exports of the consumption good

$$\mathrm{TB}_t = (1+r)D_{t-1} - D_t, \tag{19}$$

$$\tag{20}$$

and $\mathrm{TBY}_t = \mathrm{TB}_t / Y_t$ denotes the trade balance to output ratio.

The term BIND_t^{RA} denotes whether the RA household is constrained in period t.

$$\mathrm{BIND}_t^{RA} = \mathbb{1}(D_t = \kappa Y_t) \tag{21}$$

The term BIND_t^{HA} is the portion of constrained households in the economy:

$$\mathrm{BIND}_t^{HA} = \int \mathbb{1}(d(e_t, d_{t-1}; Y_t) = \kappa Y_t) d\Psi_t \tag{22}$$

MPCs in the HA and RA models are defined as

$$\mathrm{mpc}^{HA}(e^i, b^i; Y) = \frac{c(e^i, b^i + \epsilon; Y) - c(e^i, b^i; Y)}{\epsilon} \tag{23}$$

$$\mathrm{MPC}^{RA}(Y, B) = \frac{C(Y, B + \epsilon) - C(Y, B)}{\epsilon}. \tag{24}$$

2.4 Functional Forms

Households have CRRA utility given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{25}$$

Aggregate and idiosyncratic shocks both follow an AR(1) process in logs:

$$\log Y_t = \rho_Y \log Y_{t-1} + \epsilon_t^Y, \epsilon_t^Y \sim \mathcal{N}(0, \sigma_Y) \tag{26}$$

$$\log e_t^i = \rho_e \log e_{t-1}^i + \epsilon_t^e, \epsilon_t^e \sim \mathcal{N}(0, \sigma_e) \tag{27}$$

2.5 Equilibrium Definition

RA Decentralized Recursive Competitive Equilibrium. *Given an aggregate output process (26), a RA decentralized recursive competitive equilibrium is a value function $V(D, Y)$ and a set of policies $C(D, Y)$, $D'(D, Y)$ such that*

- $V(D, Y), C(D, Y), D'(D, Y)$ *satisfy equations (4) - (6).*

HA Decentralized Recursive Competitive Equilibrium. *Given an aggregate output process (26) and an idiosyncratic income process (27), an HA decentralized recursive competitive equilibrium is a value function $V(e, d; Y)$, a set of policies $c(e, d; Y), d'(e, d; Y)$ and a distribution $\Psi(e, d)$ such that*

- *V(e, d; Y), c(e, d;Y), d'(e, d;Y) solve the recursive problem given by equations (11) - (13).*
- *The distribution Ψ follows the law of motion given by equation (16).*

3 Calibration

The time period is one year. The parameters that calibrate the models can be separated into three groups: parameters that are identical across models, parameters that differ across models, and parameters that are exclusive to the HA model.

Table 1 lists the set of shared parameters. I set the risk aversion γ to one. I set the collateral constraint multiplier κ to $-1/3$. I arbitrarily set ρ_Y to 0.90 and set σ_Y to 0.022 to target an unconditional standard deviation of output of 0.05.

Table 2 lists the set of parameters that are specific to the HA model. This includes the persistence and standard deviation of the idiosyncratic income shock, ρ_e and σ_e, respectively. I set $\rho_e = .88$

and $\sigma_e = 0.26$ to match Guntin et al. (2020)'s annual calibration for Italy.

Parameter	Value	Source
σ	1	Standard
r	0.02	Standard
κ	$-.3$	Standard
ρ_Y	0.90	Arbitrary
σ_Y	0.022	Arbitrary

Table 1: Calibration, Shared Parameters

Parameter	Value	Source
ρ_e	0.88	Guntin et al. (2022)
σ_e	0.26	Guntin et al. (2022)

Table 2: Calibration, HA Parameters

Parameter	RA	HA	Target
β	0.979	0.912	NFA/GDP $= -11\%$

Table 3: Calibration, Differing Parameters

Table 3 lists the parameter that differs across models, β. I calibrate β to 0.979 in the RA model to target a household net foreign asset position to GDP ratio of -11%. To maintain the same ratio in the HA model, β needs to be recalibrated. To study the effects of no recalibration, I fix β and gradually increase σ_e from 0 to empirically realistic values. The RA model is featured as the limiting case when $\sigma_e = 0$. Without recalibration, the idiosyncratic income shock generates an extraordinary precautionary savings effect that leads households to save on the international market. This reduces the variance of aggregate consumption, the portion of constrained households, and the average MPC across households.

The size of precautionary savings is much larger than in production economies like Aiyagari (1994). In my economy, the rate of return of the asset is static. In Aiyagari (1994)'s production

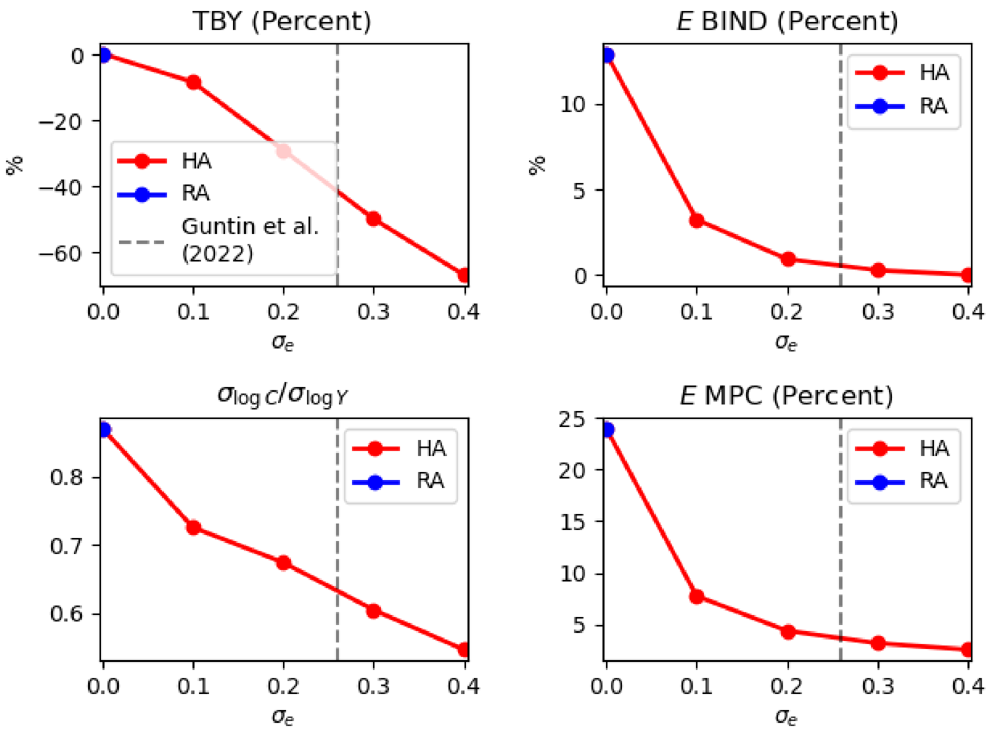

Figure 1: Effects of Increase in σ_e

economy, the rate of return decreases with savings. This makes the marginal value of saving less valuable as households save more. Further work could make the external interest rate decreasing in aggregate savings, like in García-Cicco et al. (2010). This would reduce the required decrease in β.

The model is solved over a discretized statespace, presented in Table 4. This requires discretizing the aggregate endowment Y_t, the idiosyncratic income shock e_t, and liquid asset position b_t. The RA model only features the aggregate and liquid asset states, whlie the HA model additionally features the idiosyncratic income shock state. I use the same grids and discretizations for the aggregate endowment and liquid asset in both models. Equations (26) and (27) are discretized using the Rouwenhurst method. I use 11 points for Y_t and 11 points for e_t^i. I use 300 points for the liquid asset grid. The liquid asset grid is generated using the method in Auclert et al. (2020)'s Sequence Space Jacobian package. This places more points near the borrowing constraint, where household policies are more nonlinear. I set the highest borrowing position to $\kappa\overline{Y}$, the maximum possible level of borrowing as allowed by the collateral constraint. I set the maximum savings position to -30.00, which is never reached in the simulation.

3.1 Simulation

I simulate both models for 300,000 periods and remove an initial 10,000 burn in periods. I additionally generate an income shock series $\{e_t^i\}_{t=0}^N$ to simulate the consumption and savings decisions of an individual household within the HA model.

The simulation methods are as follows:

RA Simulation Algorithm. *Simulate* $\{Y_t\}_{t=0}^{N+N_{burn}}$ *using (26). Pick some initial debt level* D_0*. For* $t = 1, \ldots, N + N_{burn}$*:*

1. *Set* $D_t = D'(D_{t-1}, Y_t)$*.*
2. *Compute* $C_t = C(D_{t-1}, Y_t)$*.*

Then drop the burn in periods to form the series $\{C_t\}_{t=0}^N, \{D_t\}_{t=0}^N$*.*

Object	Count/ Range	Description
n_Y	11	Number of grid points for Y_t
n_b	300	Number of grid points for b_t
n_e	11	Number of grid points for e_t
$[\underline{Y}, \overline{Y}]$	[.85, .17]	Range for output
$[\underline{b}, \overline{b}]$	[-.35, 30.0]	Range for savings
$[\underline{e}, \overline{e}]$	[.23, .3.29]	Range for income shock

Table 4: Discretization

HA Simulation Algorithm. *Simulate $\{Y_t\}_{t=0}^{N+N_{burn}}$ using (26). Pick some initial household distribution Ψ_0.*

For $t = 1, \ldots, N + N_{burn}$:

1. *Compute the updated distribution of household Ψ_{t+1} using equation (16).*

2. *Compute C_t, D_t and any statistics that depend on Ψ_t.*

Then drop the burn in periods to form the series $\{C_t\}_{t=0}^{N}, \{D_t\}_{t=0}^{N}$.

HA Household Simulation Algorithm. *Simulate $\{Y_t\}_{t=0}^{N+N_{burn}}$ and $\{e_t\}_{t=0}^{N+N_{burn}}$ using equations (26) and (27). Pick some initial position e_1, d_0.*

For $t = 1, \ldots, N + N_{burn}$:

1. *Set $d_t = d'(e_t, d_{t-1}; Y_t)$.*

2. *Compute $c_t = c(e_t, d_{t-1}; Y_t)$.*

Then drop the burn in periods to form the series $\{c_t\}_{t=0}^{N}, \{d_t\}_{t=0}^{N}$.

4 Results

This section compares the HA and RA models. I first compare the long run moments of each model and their response to a typical output shock. I then compare sudden stops in each model.

4.1 Moments

Figure 5 displays selected moments for each economy. I denote the relative volatility of consumption by σ_C/σ_Y where σ_C is the standard deviation of aggregate consumption and σ_Y is the standard deviation of aggregate output. The ratio in the RA model is 0.81. The HA model improves on this with a ratio of 0.93. Because output is an exogenous endowment, the ratios are driven entirely by the change in σ_C.

The trade balance to output ratio is procyclical in the RA model, with a correlation to output of 0.60. The HA model offers a weaker correlation of 0.45. The models differ significantly in their standard deviation of the trade balance to output ratio. The RA model exhibits a standard deviation of 2.8% while the HA economy displays a weaker standard deviation of 0.6%.

Why does the HA economy display a lower standard deviation of the trade balance to output ratio? Recall the aggregate resource constraint

$$C_t + TB_t = Y_t.$$

Because this is an endowment economy, the variance on the right hand side of the equation is fixed. The principle of consumption smoothing is to funnel variation from the right hand side of the equation into the trade balance TB_t, rather than consumption C_t. When unconstrained, the RA model is fairly successful at accomplishing this. There are cases when the RA model is constrained, and the majority of variation will channel into C_t. Because these cases are rare, however, they will not have a significant effect on long run moments, as commented in Mendoza (2010). This produces a less volatile consumption C_t and more volatile TB_t.

Mendoza (2010)'s comment on the small contribution of financial frictions does not apply in the HA model. This is because the idiosyncratic income shock generates poor households that are near the constraint at any point in time. No differently than they would respond to a transitory aggregate shock, these households take on leverage to smooth over their transitory income shock. This can be seen in Figure 2 which plots the long run distributions of income and slack for a generic household in the RA and HA economies. The left figure shows that the HA household experiences much more variance in income relative to the RA household. This highlights that fluctuations in aggregate income mask enormous variation in idiosyncratic income. The right figure displays the empirical cumulative density function (ECDF) of slack for the RA and HA households. I use an ECDF because the distribution of slack features a mass point at zero, and is continuous on nonzero values. The typical HA household is constrained in 56% of periods. In contrast, the RA household is constrained in 12% of periods. The HA household is also more likely to be close to but not on the constraint.

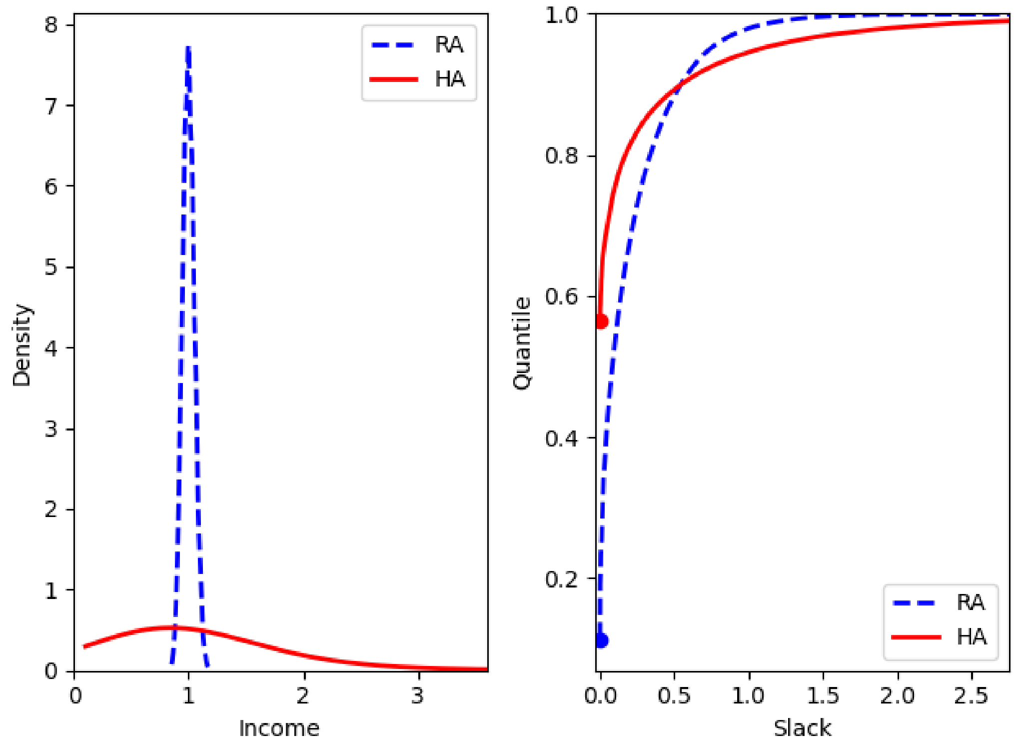

Figure 2: Income and Slack Distributions of HA and RA Households

The decrease in the volatility of the trade balance to output ratio may be viewed as a failure. However, I emphasize that the relationship between output (technology) shocks and the trade balance to output ratio is weak to begin with. García-Cicco et al. (2010)'s representative agent model estimates that stationary technology shocks (which drive output), account for only 1.3%

Model	σ_C/σ_Y	σ_C	σ_Y	$\rho(TBY, T)$	σ_{TBY}
HA	0.97	0.049	0.05	0.44	0.0028
RA	0.86	0.043	0.05	0.50	0.0201

Table 5: Moments

Model	EMPC	EBIND
HA	70.4%	56.9%
RA	24.6%	11.5%

Table 6: Household Moments

of the variation in the trade balance to output ratio. They find that the variation in the trade balance is almost entirely driven by preference shocks and interest rate shocks, neither of which are present in this model. Hong (2023a)'s heterogenous agent business cycle model estimates a larger contribution of 37%, however this reduces to 2% in an expanded model.

The Role of Financial Frictions To study what drives the higher volatility of consumption in the HA model, I resimulate the model without the collateral constraint. In this case, households face no formal constraint but always choose $d' < \frac{e^{\min}Y^{\min}}{r}$ to avoid any possibility of starvation.[5] Targeting NFA to GDP of -11% requires recalibrating β to 0.974. The larger calibration of β highlights the importance of financial frictions in generating risk, and hence precautionary savings, for households in the HA model. The model without financial frictions displays a standard deviation of consumption relative to output of 0.82, which is weaker than the result of the RA model that includes financial frictions. Additionally, the model without financial frictions displays an average MPC of 5.2%, significantly lower than either the RA or HA model.

4.2 Output Contraction

This section compares the responses of each economy to a typical output contraction. I define a contraction at time t as when output is one standard deviation below its mean. I then collect windows $[t, \ldots, t + 10]$ of aggregate output, aggregate consumption, the trade balance to output

[5] This is the analog of the natural debt limit for individual household income eY.

ratio, and the portion of constrained households.[6] Figure 3 displays the responses. Output is slightly more than 7% below its mean at impact, and gradually reverts to its mean. The HA economy displays a similar initial decrease in consumption, larger than the RA model's response of 4%. As dictated by the resource constraint, the RA economy's weaker decrease in consumption must be financed through a stronger deterioration of the trade balance.

What drives the stronger consumption response of the HA model? On impact, the RA model has a stronger initial proportion of 65% of households, whereas only 60% of HA households are constrained. However, the portion of constrained households is much more persistent in the HA model.

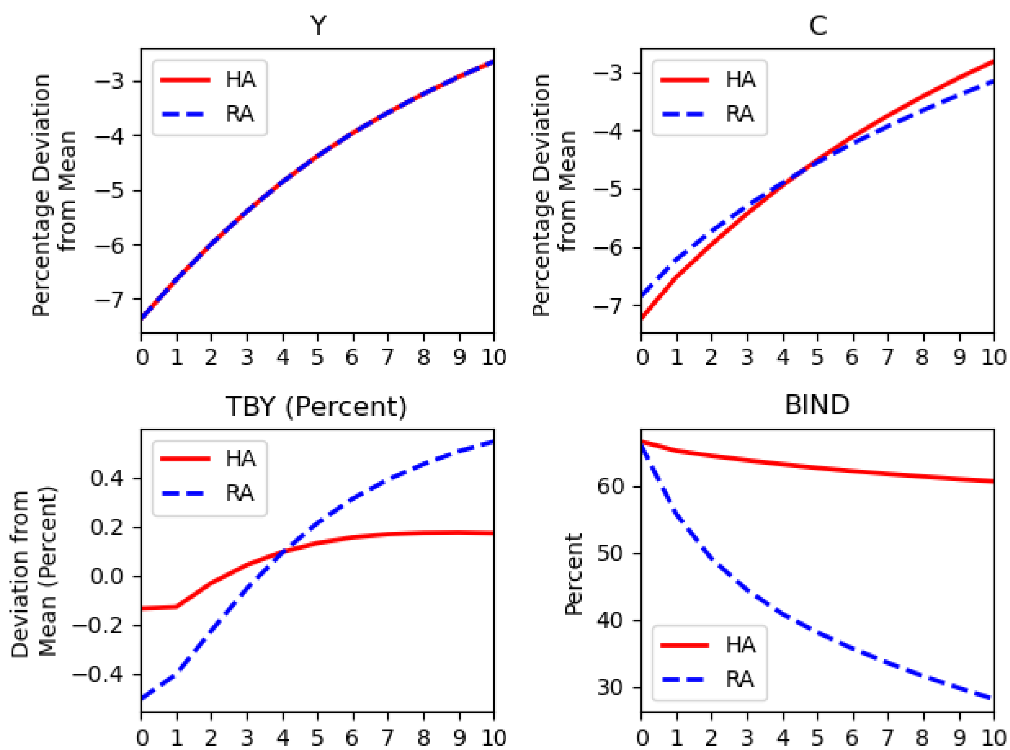

Figure 3: Output Contraction

Because the HA model features a nondegenerate distribution of income, it's possible to decom-

[6]Within the RA model, this is more strictly interpreted as the probability of all households being constrained at a given point in time.

pose aggregate responses by income groups. I separate households by income into the bottom 20%, middle 60%, and top 20%.[7] Throughout the remainder of the paper, I label these groups the 'poor', the 'middle class', and the 'rich'. Figure 4 displays the responses. The portion of constrained households is fairly static within each income group. 85% and 65% of poor and middle class households are constrained, respectively. Notably, none of the rich households ever borrow up the constraint throughout the output contraction. This occurs because rich households save a proportion of their increased income in the bond since they know their increase in income is transitory.

The upper right figure displays the percentage change in consumption of each income group, normalized by each income group's long run mean of consumption. The poor and middle households display the largest drops in consumption. This occurs because each income group contains a larger proportion of constrained households. In contrast, the rich display a weaker drop in consumption because none of them are constrained. This implies every rich household is able to consumption smooth.

The lower left figure displays the absolute contribution of each income group ot the change in aggregate consumption. We can see that, while the poor households display the most dramatic drop of consumption in percentage terms, their contribution to aggregate consumption is small. This occurs because poor households do not consume much to begin with. In contrast, rich households consume a large amount relative to their group size. This implies rich households will have a more significant effect on the response of aggregate consumption.

The bottom right figure displays the absolute change in each group's liquid asset holdings. The poor households display an increase in savings, induced by the contraction of the collateral constraint. Despite their size, middle class households display a muted response as it includes a mix of constrained and unconstrained households. Rich households significantly increase their borrowing. This reflects that rich households are unconstrained. As with consumption, the aggregate response of poor households is overwhelmed by that of rich households.

[7] A similar decomposition can be done based on asset holdings. This makes computations more complex because the liquid asset distribution changes every period, whereas the normalized distribution of income is completely static. Additionally, income and liquid asset holdings strongly predict one another.

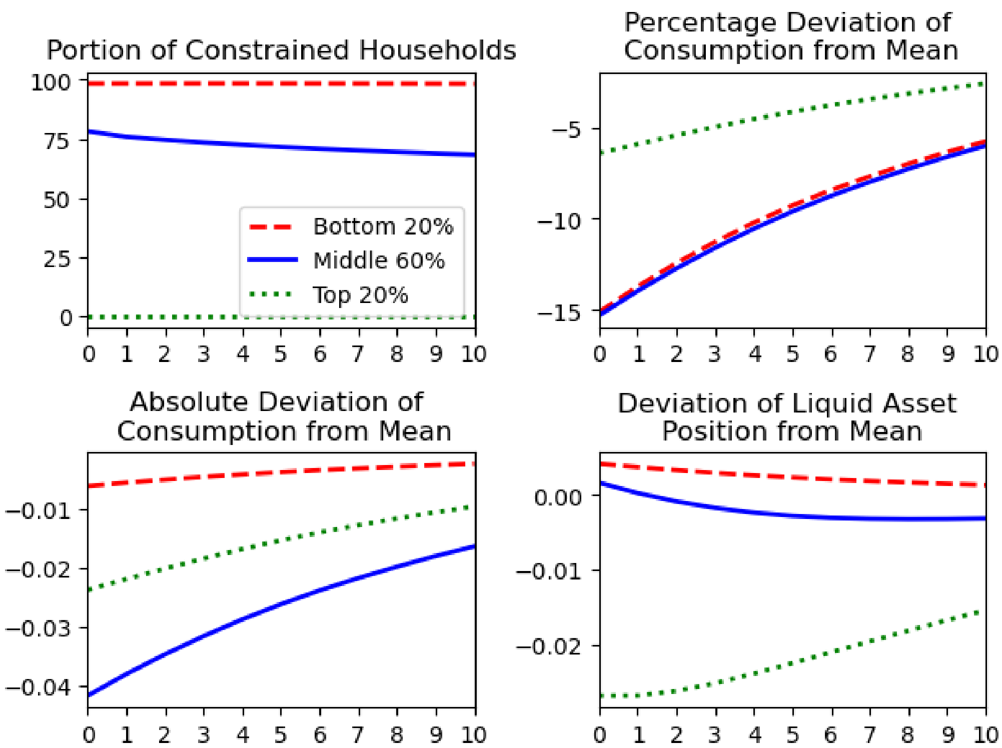

Figure 4: Decomposed HA Output Contraction

4.3 Sudden Stops

This section compares the response of each economy to a typical sudden stop event. Within the RA economy, I define a sudden stop as when the collateral constraint binds and the trade balance to output ratio is two percentage points above its long run mean. I collect windows 10 periods before and 20 periods after the sudden stop and average them. An immediate challenge in the HA economy is that the portion of constrained households is continuous and not binary. To take an agnostic approach, I feed the same output series into the HA economy and collect the same sudden stop windows as from the RA economy. This allows me to study the responses of the HA economy to output sequences that generate sudden stops in the RA economy. Using this method, the RA economy experiences a sudden stop in 2.1% of periods. This is lower than Mendoza (2010)'s frequency of sudden stops, but I emphasize that my model does not include debt deflation and is only driven by exogenous variations in output. Mendoza (2010) finds that when debt deflation is eliminated, the frequency of sudden stops strongly decreases. Additionally, Bianchi (2011)'s planner equilibrium, that does not feature overborrowing, only features crises in 0.4% of periods.[8]

Figure 5 plots the responses of output, aggregate consumption, the trade balance to output ratio, and the portion of constrained households during a sudden stop, for both the RA and HA economies. Because the same windows are collected, the economies feature identical output series. The RA economy exhibits the characteristic features of a sudden stop. Output decreases preceding the sudden stop, followed by a sudden decrease that triggers it. At time 0, the collateral constraint binds with certainty. This forces households to save, generating a reversal in the trade balance. The increase in savings can only be financed through a decrease in consumption. As a consequence, the decrease in consumption is larger than that of output.

The HA economy fails to generate a trade balance reversal that characterizes a sudden stop. Additionally, the response of consumption is weaker than that of the RA model and not disproportionate to output. It helps to compare the portion of constrained households in each economy. In the RA economy, the definition of the sudden stop requires that the representative household, and

[8]Bianchi defines a crisis as when the collateral constraint binds and net capital flows exceed one standard deviation of the ergodic distribution of capital outputflows.

hence every household, is constrained. This implies that no households are able to consumption smooth, and all households are forced to deleverage in response to the contraction of the collateral constraint.

In the HA economy only a continuous proportion of households are constrained. Theoretically, this proportion can approach one, but it never reaches more than 60% in equilibrium. This implies that a nontrivial proportion of households are unconstrained at the time of the sudden stop. These households are able to take on debt to smooth over the income contraction. , producing a weaker decrease in aggregate consumption. Additionally, the increase in leverage puts downward pressure on the trade balance to output ratio. This weakens the ability of the model to generate a trade balance reversal.

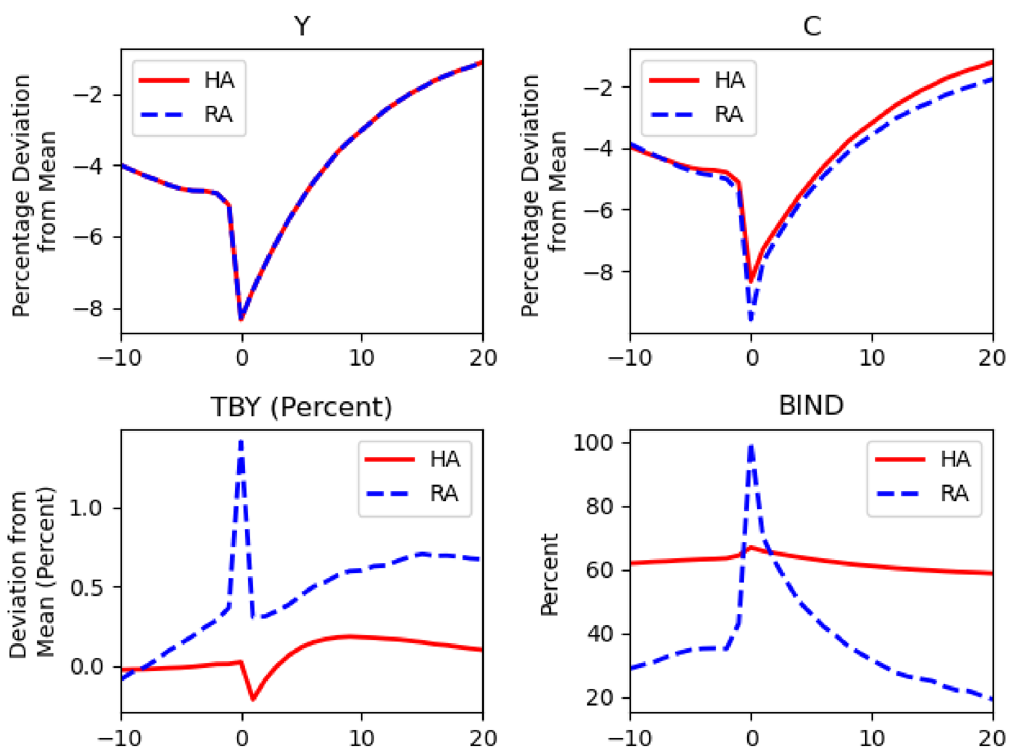

Figure 5: Sudden Stop in RA Economy

To study the effects of unconstrained households, I decompose the sudden stop by income group

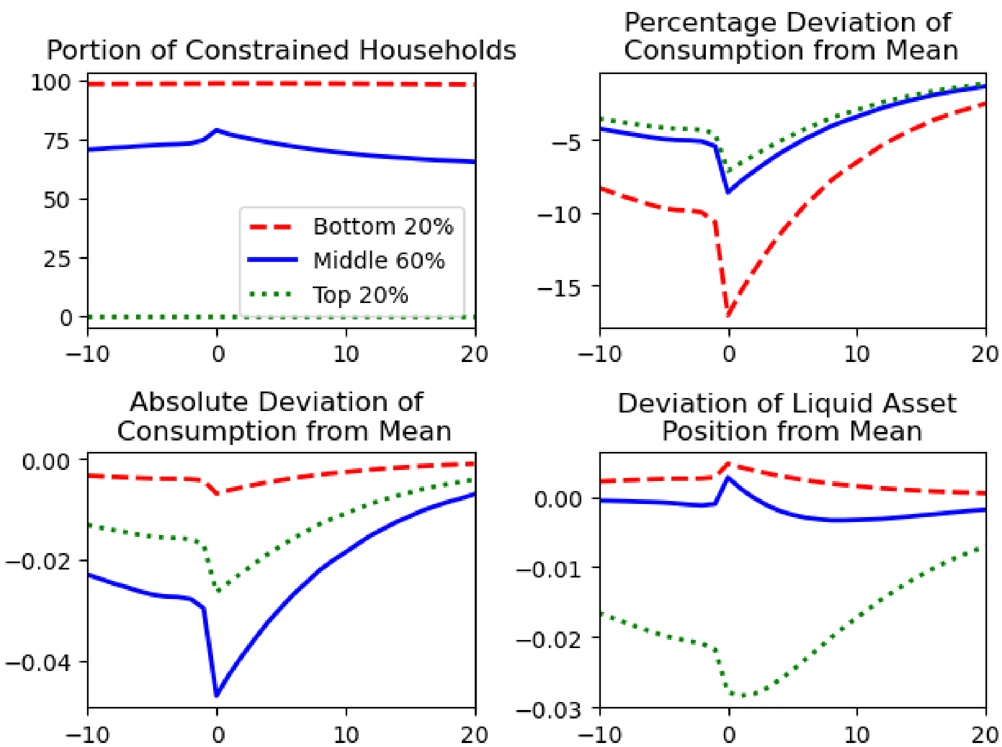

Figure 6: Decomposed Sudden Stop in HA Economy

into the bottom 20%, middle 60%, and top 20%. The upper left of figure 6 displays the portion of constrained households in each group throughout the sudden stop. The vast majority of poor households are constrained leading up to the sudden stop. This is due to the contractionary output shocks that precede it. The majority of middle class households are constrained preceding the sudden stop. On impact this proportion jumps by a small amount. Throughout the sudden stop event, none of the rich households are constrained. This implies that rich households are able to consumption smooth throughout the sudden stop.

The upper right panel displays the percentage change in consumption for each income group, normalized by each group's long run mean consumption. The poor households display a strong consumption response that is disproportionate to output. This occurs because the majority of poor households are unable to consumption smooth, like households in the RA model. Rich households display a weaker percentage change in consumption because they are able to consumption smooth. The response of the middle households is between the poor and rich.

The lower left panel displays the aggregate response of consumption, decomposed in absolute terms by income groups. While poor household experience a more dramatic decrease in consumption in percentage terms, they consume a small amount relative to their group size. This leads their aggregate response of consumption to be small. In contrast, rich households take up a disproportionate share of consumption relative to their group size. This implies that their consumption smoothing behavior is more strongly reflected in the aggregate consumption response.

The lower right panel displays the aggregate savings of each group throughout the sudden stop. Poor households display a small, positive savings that increases at impact. The savings is forced through the contraction of the collateral constraint. Middle households take a small debt position that decreases at the sudden stop. The decrease in debt follows from the forced deleveraging of constrained middle class households. In contrast, rich households significantly increase their debt position. This nullifies the increase in savings of both the middle class and poor households, driving the muted trade balance response.

4.4 Comparisons and Moving Foward

The failure of the HA model to generate a sudden stop differs from the results in Cugat (2019), who finds that heterogeneity amplifies the effects of a sudden stop. Our models differ in both the presence of financial frictions and the nature of household heterogeneity. Cugat (2019)'s RA benchmark is a perturbated model that does not feature occasionally binding financial constraints. Her HA model introduces hand to mouth households that do not have access to any assets. Lastly, the sudden stop is genereated through a sudden decrease in tradeable productivity and an increase in the external interest rate. During the sudden stop, households in Cugat (2019)'s RA benchmark have access to the bond throughout the contraction. This allows them to consumption smooth like the rich households in my HA model. Because her HA model includes a proportion of hand to mouth households, the aggregate response of consumption is larger. In contrast, the sudden stop definition in my RA benchmark imposes that all households in the economy are constrained. Like in Cugat (2019), only a proportion of households in my HA economy are constrained during the sudden stop. Because all households in my RA economy are constrained, the aggregate response of consumption is more dramatic.

A possible path foward is to introduce exogenous interest rate shocks that are correlated to output. During a sudden stop, the decrease in output would be associated with an increase in the interest rate. For constrained households, the change in the interest rate would be irrelevant to their borrowing position. For unconstrained and rich households, the change would put upward pressure on their savings. This would improve the muted response of the trade balance and assist in generating a reversal.

My model expands on Guntin et al. (2020)'s analysis of the failure of financial frictions to account for financial crises in heterogenous agent models. Guntin et al. (2020)'s model, and mine, predict that richer households have a smaller percentage response of consumption relative to income. This contradicts the data which supports a consumption-income elasticity of nearly one across all income deciles. My model replicates this result and shows that it occurs because rich households are far from the financial friction, allowing them to consumption smooth throughout output contractions and sudden stops. I go further by showing the model fails to account for the reversal of the trade balance because rich households increase their borrowing during simulated sudden stops. A possible avenue of research is to study how the liquid asset positions of rich and poor households change during financial crises using household level data.

Guntin et al. (2020) conclude that the Tequila Crisis and the experience of Italy during the Global Financial Criris were driven by permanent income shocks rather than financial frictions. I argue that the failure of HA stems from the asset structure of the model rather than financial frictions. Because the model features a single asset, households that save more are mechanically further from the financial friction. This produces a well known failure of single asset HA models: the lack of wealthy hand to mouth. While the HA model is superior to the RA model in matching average MPC, the MPC is too strongly declining in income.

The common method to generate the wealthy hand to mouth, implemented in Kaplan et al. (2018) and Hong (2023a), is to introduce a two asset environment with a liquid asset and an illiquid asset. In a typical setup households can only borrow in the liquid asset and keep the majority of their savings in the illiquid asset as it provides a higher return. Lastly, the illiquid asset features convex adjustment costs that make it unideal as a smoothing asset.

5 Conslusion

This paper studies the ability of household heterogeneity to explain empirical features of emerging markets. I show that heterogeneity increases the marginal propensity to consume and the portion of constrained households. This occurs because income risk generates poor households that take on leverage to smooth over their temporary income shock. These households interact more frequently with the financial constraint, producing nonlinear behavior. At the aggregate level, this is reflected in a higher volatility of consumption. Household heterogeneity weakens the ability of the model to generate sudden stops, reflected in a dampened response of aggregate consumption and the trade balance. I argue this occurs because the income shock generates rich households that effectively behave as permanent income consumers throughout crises.

I propose two paths foward to improve the model. The introduction of interest rates that increase during sudden stops would motivate rich households to save. Alternatively, introducing a two asset structure would allow for wealthy hand to mouth households that are unable to consumption smooth.

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