ATMIYA INSTITUTE OF TECHNOLOGY & SCIENCE

Sub: Statistical Methods(630003,2630003)

GTU Examples(Jan-2011 to May-2014)

Date:

1. Define the chi-square test. A die is thrown 150 times and the following results are obtained.

Number turned up	1	2	3	4	5	6
Frequency	19	23	28	17	32	31

Test the hypothesis that the die is unbiased at 5 % level of significance (At 5 % level of significance for 5 d.f.$\chi^2 = 11.07$)

GTU -Jan-2011 [07]

Solution:

No turned up	O	E	$\left(\frac{(O-E)^2}{E}\right)$
1	19	25	1.44
2	23	25	0.16
3	28	25	0.36
4	17	25	2.56
5	32	25	1.96
6	31	25	1.44
			$\sum\left(\frac{(O-E)^2}{E}\right)$=7.92

H_o:Die is unbiased

H_a:Die is biased

Now,$\chi^2 = \sum\left(\frac{(O-E)^2}{E}\right)$

df=n-1=6-1=5

$\chi^2_{5,0.05}$=16.750

Since calculated χ^2 value is less than table χ^2 value.

$\therefore$ H_o is accepted.

2. A man buys 50 electric bulbs of 'Philips' and 50 electric bulbs of 'HMT' Brand.He fined that 'Philips' bulbs give an average life of 1500 hrs. With standard deviation of 60 hrs. and 'HMT' bulbs gave an average life of 1512 hours with standard deviation of 80 hrs. is there a significant difference in the mean life of two brands of bulbs ?

GTU -Jan-2011 [07]

Solution:

We have,

$\bar{x}_1 = 1500$ $\bar{x}_2 = 1512$

$\sigma_1 = 60$ $\sigma_2 = 80$

$n_1 = 50$ $n_2 = 50$

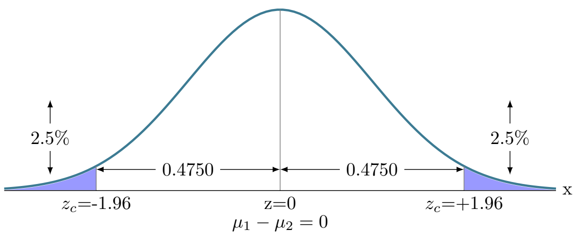
$\alpha = 0.05$

$H_o : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 \neq 0$

Now,$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

$\therefore z = \frac{(1500 - 1512) - 0}{\sqrt{\frac{60^2}{50} + \frac{80^2}{50}}}$ =-0.85



Since calculated z-value is less than table value,

$\therefore H_o$ is accepted.

3. Consider the hypothesis, $H_o : \mu = 22$, $H_a : \mu \neq 22$ a sample of 75 is used and the population standard deviation is 10. Use α= 0.01,compute p-value and state your conclusion for $\bar{x} = 23$

GTU -Jun-2011 [01]

Solution:

We have,

$\bar{x} = 23$

$\sigma = 10$

n=75

$\alpha = 0.01$

$H_o : \mu = 22$

$H_a : \mu \neq 22$

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$\therefore z = \frac{23 - 22}{\frac{10}{\sqrt{75}}}$=0.87

Now,p-value=2(0.5-0.3078)=0.3844

Since calculated p-value is greater than α

$\therefore H_0$ is accepted.

4. Write the formula for finding test statistic for small sample for hypothesis test about $\mu_1 - \mu_2$,σ_1 and σ_2 unknown.

GTU -Jun-2011 [01]

Answer:

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

5. Explain Type I and Type II error with example.

GTU -Jun-2011 [04]

6. Define the following(One Mark Each)
   (a) p-value,
   (b) Degree of freedom
   (c) Level of Significance.

GTU -Jun-2011 [03]

7. The National Association of Home Builders provided data on the cost of the most popular home remodeling projects. Sample data on cost in thousands of dollars for two types of remodeling projects are as follows:

Kitchen Master	25.2	17.4	22.8	21.9	19.7	23.0	19.7	16.9	21.8	23.6
Bedroom	18.0	22.9	26.4	24.8	26.9	17.8	24.6	21.0		

   (a) Develop a point estimate of the difference between the population mean remodeling costs for the two types of projects.
   (b) Develop a 90% confidence interval for the difference between the two population means.

GTU -Jun-2011 [07]

x_1	x_2	x_1^2	x_2^2
25.2	18.0	635.04	324
17.4	22.9	302.76	524.41
22.8	26.4	519.84	696.96
21.9	24.8	479.61	615.04
19.7	26.9	388.09	723.61
23.0	17.8	529.00	316.84
19.7	24.6	288.09	605.16
16.9	21.0	285.61	441.00
21.8			
23.6			

$\sum x_1=212$ $\quad\sum x_2=182.4$ $\quad\sum x_1^2=4560.24$ $\quad\sum x_2^2=4247.02$

$\bar{x}_1=\frac{\sum x_1}{n_1}$, $\quad\bar{x}_2=\frac{\sum x_2}{n_2}$

$\therefore\bar{x}_1=\frac{212}{10}$=21.2, $\quad\bar{x}_2=\frac{178.8}{8}$=22.8

(a):$\bar{x}_1-\bar{x}_2$=?

$\bar{x}_1 - \bar{x}_2$=21.2-22.8=-1.6

(b):

$s_1 = \sqrt{\frac{\Sigma x_1^2 - \frac{(\Sigma x_1)^2}{n_1}}{n_1 - 1}}, \quad s_2 = \sqrt{\frac{\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n_2}}{n_2 - 1}}$

$\therefore s_1 = \sqrt{\frac{4560.24 - \frac{(212)^2}{10}}{10-1}} = 2.70, \quad s_2 = \sqrt{\frac{4247.02 - \frac{(184.4)^2}{8}}{8-1}}$=3.55

s=$\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$\therefore$ s=$\sqrt{\frac{2.70^2(10-1)+3.55^2(8-1)}{10+8-2}}\sqrt{\left(\frac{1}{10}+\frac{1}{8}\right)}$=0..83

Now,

df=$n_1 + n_2 - 2$=10+8-2=16

$1 - \alpha$=0.90

$\therefore \alpha = 0.10$

$\therefore \alpha/2 = 0.05$

$\therefore t_{\alpha/2} = t_{0.05}$16=1.746

M.E=$t_{\alpha/2}\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$\therefore$ M.E=(1.746)(0.83)=1.45

Confidence interval is given by

$(\bar{x}_1 - \bar{x}_2) \pm$ M.E

$\therefore$ Confidence interval is

$(21.2 - 22.8) \pm 1.45$=-3.05 & -0.15

$\therefore$ P$(-0.15 \le (\mu_1 - \mu_2) \le -3.05)$=0.90

8. Consider the following hypothesis test:

   $H_o : \mu = 15, H_a : \mu \neq 15$

   A sample of 50 provided a sample mean of 14.15. The population standard deviation is 3.

   (a) Compute the value of the test statistic
   (b) What is the p- value?
   (c) At $\alpha = 0.05$, what is your conclusion?
   (d) What is the rejection rule using the critical value? What is your conclusion?

GTU -Jun-2011 [04]

Solution:

We have,

$\bar{x} = 14.15$

$\sigma = 3$

n=50

$\alpha = 0.05$

$H_o : \mu = 15$

$H_a : \mu \neq 15$

(a):

$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

$\therefore z = \frac{14.15-15}{\frac{3}{\sqrt{50}}}$=-2

(b):

p-value =2(0.5-0.4772)=0.0456

(c): Since p-value is less than α

$\therefore H_0$ is rejected.

(d):

If p-value< $\alpha \Rightarrow$ Reject H_o

If p-value≥ $\alpha \Rightarrow$ Accepted H_o

9. Consider the following hypothesis test:

   $H_o : \mu_1 - \mu_2 = 0, H_o : \mu_1 - \mu_2 \neq 0$

   The following results are from independent samples taken from two populations.

Sample 1	$n_1 = 35$	$\bar{x_1} = 13.6$	$s_1 = 5.2$
Sample 2	$n_2 = 40$	$\bar{x_1} = 10.1$	$s_2 = 8.5$

   (a) What is the value of the test statistic?

   (b) What is the degrees of freedom for the t- distribution?

   (c) What is the p- value?

   (d) At $\alpha = 0.05$, what is your conclusion?

GTU -Jun-2011 [03]

Solution:

We have,

$\bar{x}_1 = 13.6$ $\bar{x}_2 = 10.1$

$s_1 = 5.2$ $s_2 = 8.5$

$n_1 = 35$ $n_2 = 40$

$\alpha = 0.05$

(a):$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

$\therefore t = \frac{(13.6-10.1)-0}{\sqrt{\frac{5.2^2(35-1)+8.5^2(40-1)}{35+40-2}\left(\frac{1}{35}+\frac{1}{40}\right)}}$ =1.9633

(b):df=$n_1 + n_2 - 2$=35+40-2=63

(c):$t_{0.025,63}$=1.998 & $t_0.05, 63$=1.669

Now, $1.669 < 1.9633 < 1.998$

$\therefore 0.025 < p/2 < 0.05$

$\therefore$ 0.05 <p-value< 0.1

(d): p-value> α

$\therefore H_o$ accepted.

10. One of the questions on the Business Week Subscriber Study was, "In the past 12 months, when traveling for business, what type of airline ticket didyou purchase most often?" The data obtained are shown in the following contingency table.

	Type of Flight	
Type of Ticket	Domestic Flights	International Flights
First Class	29	22
Business/Executive class	95	121
Full fare economy/coach class	518	135

Use $\alpha = 0.05$ and test for the independence of type of flight and type of ticket, What is your conclusion?

GTU -Jun-2011 [04]

Solution:

	Type of Flight		
Type of Ticket	Domestic Flights	International Flights	Total
First Class	29	22	51
Business/Executive class	95	121	216
Full fare economy/coach class	518	135	653
Total	642	278	920

$e_{11} = \frac{51\times642}{920}$=35.59

$\therefore e_{12}$=51-35.59=25.41

$e_{21} = \frac{216\times642}{920}$=150.73

$\therefore e_{22}$=216-150.73=65.27

e_{31}=642-35.59-150.73=455.68

$\therefore e_{32}$=653-455.68=197.32

Expected Frequencies are:

	Type of Flight		
Type of Ticket	Domestic Flights	International Flights	Total
First Class	35.59	25.41	51
Business/Executive class	150.73	65.27	216
Full fare economy/coach class	455.68	197.32	653
Total	642	278	920

$\chi^2 = \Sigma\Sigma\left(\frac{(O-E)^2}{E}\right)$

$\therefore \chi^2$=$\left(\frac{(29-35.59)^2}{35.59}\right)\left(\frac{(22-25.41)^2}{25.41}\right)+\left(\frac{(95-150.73)^2}{150.73}\right)+\left(\frac{(121-65.27)^2}{65.27}\right)+\left(\frac{(518-455.68)^2}{455.68}\right)+\left(\frac{(135-197.32)^2}{197.32}\right)$

$\therefore \chi^2$=12.20+0.45+20.60+47.58+8.52+19.68=109.03

df=(r-1)(c-1)=(3-1)(2-1)=2

Now,$\chi^2_{0.05,2} = 10.5966$

Since calculated χ^2 value is greater than table χ^2 value.

$\therefore H_o$ is rejected.

11. The following data are from matched samples taken from two populations.

Element	1	2	3	4	5	6	7
Population-1	11	7	9	12	13	15	15
Population-2	8	8	6	7	10	15	14

(a) What is the point estimate of the difference between two population means?

(b) Provide a 95% confidence interval for the difference between two population means.

GTU -Jun-2011 [03]

Solution:

Element	x_1	x_2	x_1^2	x_2^2
1	11	121	8	64
2	7	49	8	64
3	9	81	6	36
4	12	144	7	49
5	13	169	10	100
6	15	225	15	225
7	14	196	14	196
	$\sum x_1 = 81$	$\sum x_2 = 985$	$\sum x_1^2 = 68$	$\sum x_2^2 = 734$

$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{81}{7} = 11.57, \quad \bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{68}{7} = 9.71$

$s_1 = \sqrt{\frac{\Sigma x_1^2 - \frac{(\Sigma x_1)^2}{n_1}}{n_1 - 1}}, \quad s_2 = \sqrt{\frac{\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n_2}}{n_2 - 1}}$

$\therefore s_1 = \sqrt{\frac{985 - \frac{(81)^2}{7}}{7-1}} = 2.82, \quad s_2 = \sqrt{\frac{734 - \frac{(68)^2}{7}}{7-1}} = 3.49$

(a) $(\bar{x}_1 - \bar{x}_2)$=?

$(\bar{x}_1 - \bar{x}_2)$=1.86

(b): $s=\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$\therefore s=\sqrt{\frac{2.82^2(7-1)+3.49^2(7-1)}{7+7-2}}\sqrt{\left(\frac{1}{7}+\frac{1}{7}\right)}=0.94$

df=$n_1 + n_2 - 2$=7+7-2=12

1-α=0.95

$\therefore \alpha = 0.05$

$\therefore \alpha/2 = 0.025$

$\therefore t_{\alpha/2} = t_{0.025,12}$=2.179

M.E=$t_{\alpha/2}\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

$\therefore$ M.E=(2.179)(0.94)=2.048

Confidence interval is given by

$(\bar{x}_1 - \bar{x}_{12}) \pm$ M.E

Confidence interval is

(1.86) $\pm$ 2.048=-0.188 & 3.908

$\therefore$P$(-0.188 \le (\mu_1 - \mu_{12}) \le 3.908)$=0.95

12. Young Adult magazine states the following hypotheses about the mean of its subscribers.

    $H_o : \mu = 28, H_a : \mu \neq 28$

    (a) What would it mean to make a Type II error in this solution?

    (b) The population standard deviation is assumed known as $\sigma = 6$ years and the sample size is 100. With $\alpha = 0.05$ what is the probability of accepting H_o for μ equal to 26, 27, 29 and 30?

    (c) What is the power at $\mu = 26$? What does this result tell you?

GTU -Jun-2011 [03]

13. A study by Consumer Report showed that 64% of supermarket shoppers believe super market brands to be as good as national name brands. To investigate whether this result applies to its own product, the manufacturer of national name brand ketchup asked sample shoppers whether they believed that supermarket ketchup was as good as the national brand ketchup.

    (a) If a sample of 100 shoppers showed 52 stating that the supermarket brand was as good as the national brand, what is the p-value?

    (b) If $\alpha = 0.05$ what is your conclusion?

    (c) Should the national brand ketchup manufacturer be pleased with this conclusion? Explain.

GTU -Jun-2011 [04]

Solution:

We have,

$\bar{p} = \frac{52}{100} = 0.52$

n=100

$\alpha = 0.05$

(a):

$z = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

$\therefore z = \frac{0.52-0.64}{\sqrt{\frac{0.64(1-0.64)}{100}}}$=-2.5

P-value=2(0.5-0.4938)=0.0124

(b):since p-value is less than α

$\therefore H_o$ is rejected.

14. In a sample of 500 people from a village, 280 are found to be rice eaters and the rest are wheat eaters. Can we assume that both the food articles are equally popular?

GTU -Dec-2011 [04]

Solution:

We have,

$\bar{p} = \frac{280}{500} = 0.56$

n=500

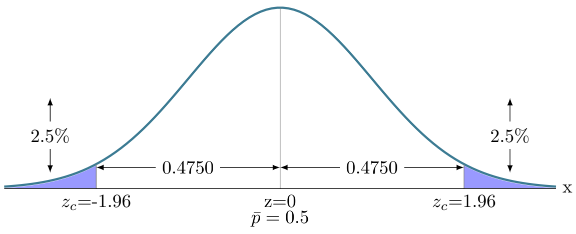
$\alpha = 0.05$

$H_o : p = 0.5$

$H_a : p \neq 0.5$

Now,$z = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

$\therefore z = \frac{0.56-0.5}{\sqrt{\frac{0.5(1-0.5)}{500}}}$=2.68



Since calculated z-value is beyond the table value,

$\therefore H_o$ is rejected.

15. A manufacturer supplies the rear axles for trucks. These axles must be able to withstand 80000 pounds per square inch in stress tests, but an excessively strong axle raises production costs significantly. Experience indicates that the standard deviation of the strength of its axles is 4000 pounds per square inch. The manufacturer selects a sample of 100 axles from production, tests them, and finds that the mean stress capacity of the sample is 79600 pounds per square inch. Test the hypothesis at 5% level that the sample has come from the same population.

GTU -Dec-2011 [05]

Solution:

We have,

$\bar{x} = 79600$

$\sigma = 4000$

n=100

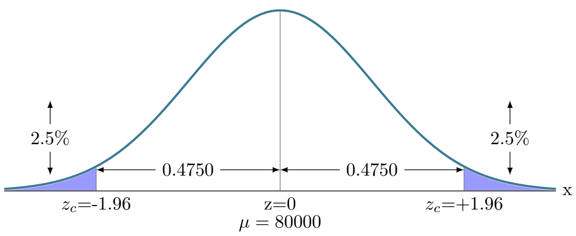
$\alpha = 0.05$

$H_o : \mu = 80000$

$H_a : \mu \neq 80000$

Now,$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

$\therefore z = \frac{79600-80000}{\frac{4000}{\sqrt{100}}}$=-1



Since calculated z-value is less than table value,

$\therefore H_o$ is accepted.

16. In a hospital 480 female and 520 male babies were born in a week. Do these figures confirm the hypothesis that males and females are born in equal numbers?

GTU -Dec-2011 [04]

Solution:

We have,

$\bar{p} = \frac{480}{1000} = 0.48$

n=1000

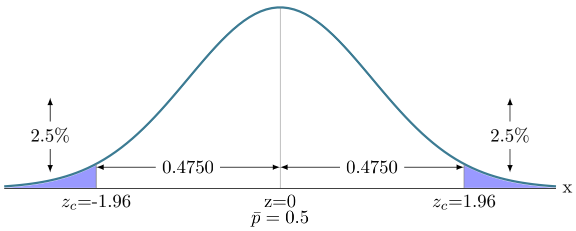
$\alpha = 0.05$

$H_o : p = 0.5$

$H_a : p \neq 0.5$

Now, $z = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

$z = \frac{0.48-0.5}{\sqrt{\frac{0.5(1-0.5)}{1000}}}$=-1.26



Since calculated z-value is less than table value,

$\therefore H_o$ is accepted.

17. Nine computer-components dealers in major metropolitan areas were asked for their prices in \$ on two similar colour inkjet printers. The results of this survey are given below. At $\alpha = 0.05$, is it reasonable to assert that, on average, the Apson printer is less expensive than the Okaydata printer?

Dealer	1	2	3	4	5	6	7	8	9
Apson	250	319	285	260	305	295	289	309	275
Okaydata	270	325	269	275	289	285	295	325	300

GTU -Dec-2011 [05]

Solution:

x_1	x_1^2	x_2	x_2^2
250	62500	270	72900
319	101761	325	105625
285	81225	269	72361
260	67600	275	75625
305	93025	289	83521
295	87025	285	81225
289	83521	295	87025
309	95481	325	105625
275	75625	500	250000
$\sum x_1$=2587	$\sum x_1^2$ =747763	$\sum x_2$=2633	$\sum x_2^2$=773907=21.23

$\bar{x}_1 = \frac{\sum x_1}{n_1}, \quad \bar{x}_2 = \frac{\sum x_2}{n_2}$

$\therefore \bar{x}_1 = \frac{2587}{9}$=287.44, $\quad \bar{x}_2 = \frac{2633}{9}$=292.55

$s_1 = \sqrt{\frac{\Sigma x_1^2 - \frac{(\Sigma x_1)^2}{n_1}}{n_1 - 1}}, \quad s_2 = \sqrt{\frac{\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n_2}}{n_2 - 1}}$

$\therefore s_1 = \sqrt{\frac{747763 - \frac{(2587)^2}{9}}{9-1}}$=22.76, $\quad s_2 = \sqrt{\frac{773907 - \frac{(2633)^2}{9}}{9-1}}$=21.23

We have,

$\bar{x}_1 = 287.44 \qquad \bar{x}_2 = 292.55$

$s_1 = 22.76 \qquad s_2 = 21.23$

$n_1 = 9 \qquad n_2 = 9$

$\alpha = 0.05$

$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}} \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$H_o : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 \neq 0$

$\therefore t = \frac{(287.44 - 292.55) - 0}{\sqrt{\frac{22.76^2(9-1) + 21.23^2(9-1)}{9+9-2}} \left(\frac{1}{9} + \frac{1}{9}\right)}$=-0.49

Now,df=$n_1 + n_2 - 2$=9+9-2=16 $t_{0.025,16} = 2.1199$

Since calculated t-value is less than table value,

$\therefore H_o$ is accepted.

18. Students Safe Driving has targeted seat-belt usage as a positive step to reduce accidents and injuries. Before a major campaign at one college, 44 percent of 150 drivers entering the college parking lot were using their seat belts. After the seat-belt awareness program, the proportion using seat-belts had risen to 52 percent in a sample of 200 vehicles. At a 0.04 significance level, can the students conclude that their campaign was effective?

GTU -Dec-2011 [04]

Solution:

We have,

$\bar{p}_1 = 0.44 \qquad \bar{p}_2 = 0.52$

$n_1 = 150 \qquad n_2 = 200$

$\alpha = 0.04$

$H_o : \bar{p}_1 - \bar{p}_2 \geq 0$

$H_a : \bar{p}_1 - \bar{p}_2 < 0$

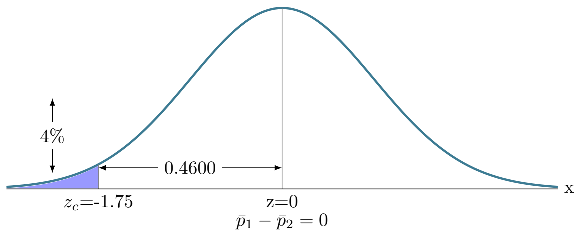
$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1+n_2}$

$\bar{p} = \frac{(0.44)(150)++(0.52)(200)}{150+200}$ =0.48

$\therefore \bar{q}$=1-$\bar{p}$=1-0.48=0.52

Now, $z = \frac{(\bar{p}_1-\bar{p}_2)-(p_1-p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

$z = \frac{(0.44-0.52)-0}{\sqrt{(0.48)(0.52)\left(\frac{1}{150}+\frac{1}{200}\right)}}$=-1.48



Since calculated z-value is less than table value,

$\therefore H_o$ is accepted.

19. A brand manager is concerned that her brand's share may be unevenly distributed throughout the country. In a survey in which the country was divided into four geographical regions, a random sampling of 100 consumers in each region was surveyed, with the following results:

	Region				Total
	NE	NW	SE	SW	
Purchase the brand	40	55	45	50	190
Do not purchase	60	45	55	50	210
Total	100	100	100	100	400

Develop a table of observed and expected frequencies and calculate the sample χ^2 value. State the null and alternative hypotheses and test whether brand share is the same across the four regions at α = 0.05

GTU -Dec-2011 [05]

Solution:

$e_{11} = \frac{190\times100}{400}$=47.5

$e_{12} = \frac{190\times100}{400}$=47.5

$e_{13} = \frac{190\times100}{400}$=47.5

$\therefore e_{14} = 190 - e_{11} - e_{12} - e_{13}$=47.5

$e_{21} = \frac{210\times100}{400}$=52.5

$e_{11} = \frac{210\times100}{400}$=52.5

$e_{11} = \frac{210\times100}{400}$=52.5

$\therefore e_{24} = 210 - e_{21} - e_{22} - e_{23}$=52.5

Expected frequencies are:

	Region				
	NE	NW	SE	SW	Total
Purchase the brand	47.5	47.5	47.5	47.5	190
Do not purchase	52.5	52.5	52.5	52.5	210
Total	100	100	100	100	400

$\chi^2 = \sum\sum\left(\frac{(O-E)^2}{E}\right)$

$\therefore \chi^2$=$\left(\frac{(40-47.5)^2}{47.5}\right)+\left(\frac{(55-47.5)^2}{47.5}\right)+\left(\frac{(45-47.5)^2}{47.5}\right)+\left(\frac{(50-47.5)^2}{47.5}\right)+\left(\frac{(60-52.5)^2}{52.5}\right)+\left(\frac{(45-52.5)^2}{52.5}\right)+\left(\frac{(55-52.5)^2}{52.5}\right)+\left(\frac{(50-52.5)^2}{52.5}\right)$

$\therefore \chi^2$=1.18+1.18+0.13+0.13+1.071+1.071+0.1190+0.1190=5

df=(r-1)(c-1)=(3-1)(4-1)=6

Now,$\chi^2_{0.05,6}$=18.5475

Since calculated χ^2 value is less than table χ^2 value.

$\therefore H_o$ is accepted.

20. The data below are a random sample of 9 firms chosen from the "Digest of Earnings Reports" in The Wall Street Journal on September 2011.

    (a) Find the mean change in earnings per share between 2010 and 2011.
    (b) Find the standard deviation of the change and the standard error of the mean.
    (c) Were average earnings per share different in 2010 and 2011? Test at $\alpha = 0.02$.

Firm	1	2	3	4	5	6	7	8	9
2010 earnings	1.38	1.26	3.64	3.50	2.47	3.21	1.05	1.98	2.72
2011 earnings	2.48	1.50	4.59	3.06	2.11	2.80	1.59	0.92	0.47

GTU -Dec-2011 [05]

Solution:

x_1	x_1^2	x_2	x_2^2
1.38	1.9044	2.48	6.1504
1.26	1.5876	1.50	2.25
3.64	13.2496	4.59	21.0681
3.50	12.25	3.06	9.3636
2.47	6.1009	2.11	4.4521
3.21	10.3041	2.80	7.84
1.05	1.1025	1.59	2.5281
1.98	3.9204	0.92	0.8464
2.72	7.3984	0.47	0.2209
$\Sigma x_1 = 21.21$	Σx_1^2=57.8179	Σx_2=19.52	$\Sigma x_2^2 = 54.7196$

Now,$\bar{x}_1 = \frac{\sum x_1}{n_1}, \bar{x}_2 = \frac{\sum x_2}{n_2}$

$\bar{x}_1 = \frac{21.21}{9}$=2.36 ,$\bar{x}_2 = \frac{19.52}{9}$=2.17

Also,$s_1 = \sqrt{\frac{\Sigma x_1^2 - \frac{(\Sigma x_1)^2}{n_1}}{n_1 - 1}}, s_2 = \sqrt{\frac{\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n_2}}{n_2 - 1}}$

$\therefore s_1 = \sqrt{\frac{57.8179 - \frac{(21.21)^2}{9}}{9-1}}, s_2 = \sqrt{\frac{54.7196 - \frac{(19.52)^2}{9}}{9-1}}$

$\therefore s_1$=0.98,$s_2 = 1.24$

We have,

$\bar{x}_1 = 2.36$	$\bar{x}_2 = 2.17$
$s_1 = 0.98$	$s_2 = 1.24$
$n_1 = 9$	$n_2 = 9$

$\alpha = 0.02$

$H_o : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 \neq 0$

Now,

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}} \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = \frac{(2.36-2.17)-0}{\sqrt{\frac{0.98^2(9-1)+1.24^2(9-1)}{9+9-2}\left(\frac{1}{9}+\frac{1}{9}\right)}} = 0.33$$

Now,df=$n_1 + n_2 - 2$=9+9-2=16 and $t_{0.01,16}$= 2.5834

Since calculated t-value is less than table value,

$\therefore H_o$ is accepted.

21. A manufacturer of pet foods was wondering whether cat owners and dog owners reacted differently to premium pet foods. They commissioned a consumer survey that yielded the following data:

Pet	Owners Surveyed	No. Using Premium Food
Cat	280	152
Dog	190	81

Is it reasonable to conclude, at $\alpha = 0.02$, that cat owners are more likely than dog owners to feed their pets premium food?

GTU -Dec-2011 [04]

Solution:

Pet	Owners Surveyed	No. Using Premium Food	Total
Cat	280	152	432
Dog	190	81	271
Total	470	233	703

$e_{11} = \frac{432\times470}{703}$=288.81

$\therefore e_{12}$=432-288.81=143.19

$e_{21} = \frac{271\times470}{703}$=181.18

$\therefore e_{22}$=271-181.18=89.82

$\chi^2 = \sum\sum\left(\frac{(O-E)^2}{E}\right)$

$\therefore \chi^2 = \left(\frac{(280-288.81)^2}{288.81}\right)+\left(\frac{(152-143.19)^2}{143.19}\right)+\left(\frac{(190-181.18)^2}{181.18}\right)+\left(\frac{(81-89.82)^2}{89.82}\right)$

$\therefore \chi^2$=0.2787+0.5420+0.4293+0.8660=2.1160

22. An advertising firm is trying to determine the demographics for a new product. They have randomly selected 75 people in each 5 different age groups and introduced the product to them. The results of the survey are given below:

	Age group				
Future Activity	18-29	30-39	40-49	50-59	60-69
Purchase frequently	12	18	17	22	32
Seldom purchase	18	25	29	24	30
Never purchase	45	32	29	29	13

Develop a table of observed and expected frequencies, and calculate χ^2 value. For the level of significance is 0.01, should the null hypothesis be rejected?

GTU -Dec-2011 [05]

Solution:

	Age group					
Future Activity	18-29	30-39	40-49	50-59	60-69	Total
Purchase frequently	12	18	17	22	32	101
Seldom purchase	18	25	29	24	30	126
Never purchase	45	32	29	29	13	148
Total	75	75	75	75	75	375

$e_{11} = \frac{101\times75}{375} = 20.2$

$e_{12} = \frac{101\times75}{375} = 20.2$

$e_{13} = \frac{101 \times 75}{375} = 20.2$

$e_{14} = \frac{101 \times 75}{375} = 20.2$

$\therefore e_{15} = 101 - 20.2 - 20.2 - 20.2 - 20.2 = 20.2$

$e_{21} = \frac{126 \times 75}{375} = 25.2$

$e_{22} = \frac{126 \times 75}{375} = 25.2$

$e_{23} = \frac{126 \times 75}{375} = 25.2$

$e_{24} = \frac{126 \times 75}{375} = 25.2$

$\therefore e_{25} = 126 - 25.2 - 25.2 - 25.2 - 25.2 = 25.2$

$e_{31} = \frac{148 \times 75}{375} = 29.6$

$e_{32} = \frac{148 \times 75}{375} = 29.6$

$e_{33} = \frac{148 \times 75}{375} = 29.6$

$e_{34} = \frac{148 \times 75}{375} = 29.6$

$\therefore e_{35} = 148 - 29.6 - 29.6 - 29.6 - 29.6 = 29.6$

Expected frequencies are:

	Age group					
Future Activity	18-29	30-39	40-49	50-59	60-69	Total
Purchase frequently	20.2	20.2	20.2	20.2	20.2	101
Seldom purchase	25.2	25.2	25.2	25.2	25.2	126
Never purchase	29.6	29.6	29.6	29.6	29.6	148
Total	75	75	75	75	75	375

$\chi^2 = \sum\sum\left(\frac{(O-E)^2}{E}\right)$

$\therefore \chi^2 = \left(\frac{(12-20.2)^2}{20.2}\right) + \left(\frac{(18-20.2)^2}{20.2}\right) + \left(\frac{(17-20.2)^2}{20.2}\right) + \left(\frac{(22-20.2)^2}{20.2}\right) + \left(\frac{(32-20.2)^2}{20.2}\right) + \left(\frac{(18-25.2)^2}{25.2}\right) + \left(\frac{(25-25.2)^2}{25.2}\right) + \left(\frac{(29-25.2)^2}{25.2}\right) + \left(\frac{(24-25.2)^2}{25.2}\right)$
$+ \left(\frac{(20-25.2)^2}{25.2}\right) + \left(\frac{(45-29.6)^2}{29.6}\right) + \left(\frac{(32-29.6)^2}{29.6}\right) + \left(\frac{(29-29.6)^2}{29.6}\right) + \left(\frac{(29-29.6)^2}{29.6}\right) + \left(\frac{(13-29.6)^2}{29.6}\right)$

$\therefore \chi^2 = 3.3287 + 0.2396 + 0.5069 + 0.1603 + 6.8930 + 2.0571 + 0.0016$

+0.5730+0.0571+1.073+8.012+0.1945+0.0122+9.3094=32.41

df=(r-1)(c-1)=(3-1)(5-1)=8

Now, $\chi^2_{0.025,8} = 17.535$

Since calculated χ^2 value is greater than table χ^2 value.

$\therefore H_o$ is rejected.

23. Write the necessary conditions for using t-distribution.

GTU -May-2012 [03]

24. Company manufactures car tyres. Mean life of tyre is 42000 km with a standard deviation of 3000 km. Company changes the production process to improve the quality. After this change, a test sample of 20 new tyres has a mean life of 43500 km with same s.d. as before. Do you think that the new car tyres are significantly superior to the earlier one?

GTU -May-2012 [04]

Solution:

We have,

$\bar{x} = 43500$

$\sigma = 3000$

n=20

$\alpha = 0.05$

$H_o : \mu \leq 42000$

$H_a : \mu > 42000$

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

$\therefore t = \frac{43500-42000}{\frac{3000}{\sqrt{20}}}$ =2.23

Now df=n-1=20-1=19

25. Describe type-I and type-II error.

GTU -May-2012 [03]

26. It is claimed that 20% of Indian consumers used internet to buy gifts during Diwali festival. In a sample of 900 customers this year, it is found that 15% used internet to buy gifts during Diwali. Test the claim at $\alpha = 0.05$.

GTU -May-2012 [04]

Solution:

We have,

$\bar{p} = 0.15$

n=900

$\alpha = 0.05$

$H_o : p = 0.20$

$H_a : p \neq 0.20$

$z = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$

$\therefore z = \frac{0.15-0.20}{\sqrt{\frac{0.20(1-0.20)}{900}}}$=-3.75

Since calculated z-value is beyond the table value,

$\therefore H_o$ is rejected.

27. The following data shows sales made by salespeople from two different cities.

    City A: 59,68,44,71,63,46,69,54,48

    City B : 50,36,62,52,70,41

    Assuming the populations sampled to be approximately normal having the same variance, test whether there is any significant difference between the means of these samples.

GTU -May-2012 [07]

x_1	x_2	x_1^2	x_2^2
59	50	3481	2500
68	36	4624	1296
44	62	1936	3844
71	52	5041	2704
63	70	3969	4900
46	41	2116	1681
69	$\sum x_2 = 311$	4761	$\sum x_2^2 = 16925$
54		2916	
48		2304	
$\sum x_1 = 522$		$\sum x_1^2 = 31148$	

$\bar{x}_1 = \frac{\sum x_1}{n_1}, \quad \bar{x}_2 = \frac{\sum x_2}{n_2}$

$\therefore \bar{x}_1 = \frac{522}{9}$=58, $\quad \bar{x}_2 = \frac{311}{6}$=51.83

$s_1 = \sqrt{\frac{\sum x_1^2 - \frac{(\sum x_1)^2}{n_1}}{n_1 - 1}}, \quad s_2 = \sqrt{\frac{\sum x_2^2 - \frac{(\sum x_2)^2}{n_2}}{n_2 - 1}}$

$\therefore s_1 = \sqrt{\frac{31148 - \frac{(522)^2}{9}}{9-1}}$=10.44 , $\quad s_2 = \sqrt{\frac{16925 - \frac{(311)^2}{6}}{6-1}}$=12.69

We have,

$\bar{x}_1 = 58 \qquad \bar{x}_2 = 51.83$

$s_1 = 10.44 \qquad s_2 = 12.69$

$n_1 = 9 \qquad n_2 = 6$

$\alpha = 0.05$

$H_o : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 \neq 0$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$\therefore t = \frac{(58-51.83)-0}{\sqrt{\frac{10.44^2(9-1)+12.69^2(6-1)}{9+6-2}\left(\frac{1}{9}+\frac{1}{6}\right)}} = 0.1723$$

df=$n_1 + n_2 - 2$=9+6-2=13

$t_{0.025,13} = 2.160$

Since calculated t-value is less than table value,

$\therefore H_o$ is accepted.

28. A company has recently created a new hair dryer A with fewer parts than the current hair dryer B. 300 units of each type of hair dryer were tested. 50 units of type A and 75 units of type B failed in a performance test. Can you conclude that new hair dryer is more reliable?

GTU -May-2012 [07]

Solution:

We have,

$\bar{p}_1 = \frac{50}{300} = 0.17$ $\bar{p}_2 = \frac{75}{300} = 0.25$

$n_1 = 300$ $n_2 = 300$

$\alpha = 0.05$

$H_o : \bar{p}_1 - \bar{p}_2 = 0$

$H_o : \bar{p}_1 - \bar{p}_2 < 0$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1+n_2}$$

$$\therefore \bar{p} = \frac{(0.17)(300)++(0.25)(300)}{300+300} = \frac{x_1+x_2}{n_1+n_2} = \frac{50+75}{300+300} = 0.21$$

$\therefore \bar{q} = 1 - \bar{p} = 1 - 0.21 = 0.79$

$$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$\therefore z = \frac{(0.17-0.25)-0}{\sqrt{(0.21)(0.79)\left(\frac{1}{300}+\frac{1}{300}\right)}} = -2.40$$

Since calculated z-value is beyond the table value,

$\therefore H_o$ is rejected.

29. Using the following data, test the hypothesis that the drug is no better than sugar pills for curing cold.

	HELPED	HARMED	NO EFFECT
DRUG	50	12	18
SUGAR PILLS	40	14	26

Solution:

	HELPED	HARMED	NO EFFECT	TOTAL
DRUG	50	12	18	80
SUGAR PILLS	40	14	26	80
TOTAL	90	26	44	160

$e_{11} = \frac{80\times90}{160} = 45$

$e_{12} = \frac{80\times26}{160} = 13$

$\therefore e_{13} = 80-45-13 = 22$

$e_{21} = \frac{80\times90}{160} = 45$

$e_{22} = \frac{80\times26}{160} = 13$

$\therefore e_{23} = 80-45-13 = 22$

Expected frequencies are:

$\chi^2 = \sum\sum\left(\frac{(O-E)^2}{E}\right)$

$\therefore \chi^2 = \left(\frac{(50-45)^2}{45}\right)+\left(\frac{(12-13)^2}{13}\right)+\left(\frac{(18-22)^2}{22}\right)+\left(\frac{(40-45)^2}{45}\right)+\left(\frac{(14-13)^2}{13}\right)+\left(\frac{(26-22)^2}{22}\right)$

$\therefore \chi^2$=0.5555+0.07692+0.7273+0.5555+0.07692+0.7273=2.7194

df=(r-1)(c-1)=(2-1)(3-1)=2

$\chi^2_{0.05,2}$=5.991

Since calculated χ^2 value is less than table χ^2 value.

$\therefore H_o$ is accepted.

30. A typist in a company commits the following number of mistakes per page in typing 432 pages. Does this information verify that the mistakes are distributed according to Poisson law?

No. of mistakes per page	0	1	2	3	4	5
No. of pages	223	142	48	15	4	0

GTU -May-2012 [07]

31. Define and Differentiate

- Type-I and Type-II errors
- Tow-tailed and One-tailed tests

32. A factory is producing 50000 pairs of shoes daily. From a sample of 500 pairs, 2% were found to be of sub-standard quality. Estimate the number of pairs that can be reasonably expected to be spoiled in the daily production and assign limits at 95% level of confidence.

GTU -Dec-2012 [07]

33. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured a third class, 90 were placed in second class and 20 got a first class. Are these figures commensurate with the general examination result which is the ratio of 4 : 3 : 2 : 1 for the various categories respectively?

GTU -Dec-2012 [07]

Solution:

Category	O	E
Failed	220	200
Third Class	170	150
Second Class	90	100
First Class	20	50
Total	500	500

$\chi^2 = \sum\sum\left(\frac{(O-E)^2}{E}\right)$

$\therefore \chi^2 = \left(\frac{(220-200)^2}{200}\right) + \left(\frac{(170-150)^2}{150}\right) + \left(\frac{(90-100)^2}{100}\right) + \left(\frac{(20-50)^2}{50}\right)$

$\therefore \chi^2$=2.0000+2.6667+1.0000+18.0000=23.6667

df=n-1=4-1=3

$\chi^2_{0.05,3}$=7.815

Since calculated χ^2 value is greater than table χ^2 value.

$\therefore H_o$ is rejected.

34. In a random sample of 500 persons belonging to urban area 200 are found to be commuters of public transport. In another sample of 400 persons belonging to rural area 200 are found to be commuters of public transport. Discuss whether the data reveal a significant difference between urban and rural area so far as the proportion of commuters of public transport is concerned at 1% level of significance.

GTU -Dec-2012 [07]

Solution:

We have,

$\bar{p}_1 = \frac{200}{500} = 0.44$ $\bar{p}_2 = \frac{200}{400} = 0.5$

$n_1 = 500$ $n_2 = 400$

$\alpha = 0.01$

$H_o : p_1 - p_2 = 0$

$H_a : p_1 - p_2 \neq 0$

$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1+n_2}$

$\therefore \bar{p} = \frac{(0.44)(0.5)++(500)(400)}{0.5+400}$ =0.44

$\therefore \bar{q} = 1 - \bar{p} = 1 - 0.44 = 0.56$

$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$\therefore z = \frac{(0.4-0.5)-0}{\sqrt{(0.44)(0.56)\left(\frac{1}{500} + \frac{1}{400}\right)}}$

35. A set of 5 coins is tossed 3200 times and the number of heads appearing each time is noted. The results are given below:

No. of heads	0	1	2	3	4	5
Frequency	80	570	1100	900	50	50

Test the hypothesis that the coins are unbiased.

36. A sample of 225 account balances of a credit company showed an average balance of Rs.15,000 with a standard deviation of Rs.625. Formulate the hypotheses and compute the test statistic that can be used to determine whether the mean of all account balances is significantly different from \$14,500.

GTU -May-2013 [04]

Solution:

We have,

$\bar{x} = 15000$

$\sigma = 625$

n=225

$\alpha = 0.05$

$H_o : \mu = 14500$

$H_a : \mu \neq 14500$

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$\therefore z = \frac{15000-14500}{\frac{625}{\sqrt{225}}}$ =12

Since calculated z-value is beyond the table value,

$\therefore H_o$ is rejected.

37. The following information was obtained from samples regarding the productivity score (out of 10) of 5 and 7 individuals using two different methods of production.

Method1	8	10	14	10	13
Method2	12	15	11	14	16

Is there a significant difference between the productivity of the two methods? take $\alpha = 0.05$.

GTU -May-2013 [07]

Solution:

x_1	x_2	x_1^2	x_2^2
8	64	12	144
10	100	15	225
14	196	11	121
10	100	14	196
13	169	16	256
$\sum x_1$=55	$\sum x_1^2$=629	$\sum x_2$=68	$\sum x_2^2$=942

$\bar{x}_1 = \frac{\sum x_1}{n_1}, \quad \bar{x}_2 = \frac{\sum x_2}{n_2}$

$\therefore \bar{x}_{55} = \frac{5}{=} 11, \quad \bar{x}_{68} = \frac{5}{=} 13.6$

$s_1 = \sqrt{\frac{\sum x_1^2 - \frac{(\sum x_1)^2}{n_1}}{n_1 - 1}}, \quad s_2 = \sqrt{\frac{\sum x_2^2 - \frac{(\sum x_2)^2}{n_2}}{n_2 - 1}}$

$s_1 = \sqrt{\frac{55 - \frac{(629)^2}{5}}{5-1}} = 2.45, \quad s_2 = \sqrt{\frac{68 - \frac{(942)^2}{5}}{5-1}} = 2.07$

We have,

$\bar{x}_1 = 11 \qquad \bar{x}_2 = 13.6$

$s_1 = 2.45 \qquad s_2 = 2.07$

$n_1 = 5 \qquad n_2 = 5$

$\alpha = 0.05$

$H_o : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 \neq 0$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}} \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = \frac{(11 - 13.6) - 0}{\sqrt{\frac{2.45^2(5-1) + 2.07^2(5-1)}{5+5-2}\left(\frac{1}{5} + \frac{1}{5}\right)}} = \text{-1.81}$$

df=$n_1 + n_2 - 2$=5+5-2=8

Since calculated t-value is less than table value,

$\therefore H_o$ is accepted.

38. The table below gives beverage preferences for random samples of teens and adults.

	Teens	Adults	Total
Coffee	50	200	250
Tea	100	150	250
Soft Drink	200	200	400
Other	50	50	100

Test for independence between age (i.e., adult and teen) and drink preferences at $\alpha = 0.05$.

GTU -May-2013 [07]

Solution:

$e_{11} = \frac{250 \times 400}{1000} = 100$

$\therefore e_{12} = 250 - 100 = 150$

$e_{21} = \frac{250 \times 400}{1000} = 100$

$\therefore e_{22} = 250 - 100 = 150$

$e_{31} = \frac{400 \times 400}{1000} = 160$

$\therefore e_{32} = 400 - 160 = 240$

$e_{41} = \frac{100 \times 400}{1000} = 40$

$\therefore e_{42} = 100 - 40 = 60$

Expected frequencies:

	Teens	Adults	Total
Coffee	100	150	250
Tea	100	150	250
Soft Drink	160	240	400
Other	40	60	100
Total	400	600	1000

$\chi^2 = \sum\sum\left(\frac{(O-E)^2}{E}\right)$

$\therefore \chi^2 = \left(\frac{(50-100)^2}{100}\right) + \left(\frac{(200-150)^2}{150}\right) + \left(\frac{(100-100)^2}{100}\right) + \left(\frac{(150-150)^2}{150}\right) + \left(\frac{(200-160)^2}{160}\right) + \left(\frac{(200-240)^2}{240}\right) + \left(\frac{(50-40)^2}{40}\right) + \left(\frac{(50-60)^2}{60}\right)$

$\therefore \chi^2 = 25 + 16.67 + 0.0000 + 0.0000 + 10 + 6.67 + 2.5 + 1.67 = 62.51$

df=(r-1)(c-1)=(2-1)(4-1)=3

$\chi^2_{0.05,3} = 7.815$

Since calculated χ^2 value is greater than table χ^2 value.

$\therefore H_o$ is rejected.

39. The sales (in thousand Rs) data of an item in six shops before and after a special promotional campaign are as under:

Shops	A	B	C	D	E	F
Before campaign	55	25	35	50	50	40
After campaign	60	22	30	55	58	45

Did the campaign make any significant difference in sale?

GTU -May-2013 [07]

Solution:

Before	After	d	d^2
55	60	2	4
25	22	-3	9
35	30	-5	25
50	55	5	25
50	58	8	64
40	45	5	25
		$\sum d = 12$	$\sum d^2 = 152$

40. The number of defects per unit in a sample of manufactured product was found as follows:

No. of defects	0	1	2	3	4
No. of units	200	90	20	8	2

Fit Poisson distribution to the data and test the goodness of the fit

GTU -May-2013 [07]

41. A TV-documentary on overeating claimed that Indians are about 10 pounds overweight on average. To test this claim, eighteen randomly selected individuals were examined; their average excess weight was found to be 12.4 pounds, and the standard deviation was 2.7 pounds. At a significance level of 0.01, is there any reason to doubt the validity of the claimed 10- pound value?

GTU -Dec-2013 [07]

Solution:

We have,

$\bar{x} = 12.4$

s=18

n=2.7

$\alpha = 0.01$

$H_o : \mu = 10$

$H_a : \mu \neq 10$

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

$\therefore t = \frac{12.4-10}{\frac{2.7}{\sqrt{18}}}$=1.53

df=n-1=18-1=17

42. A credit insurance organization has developed a new high-tech method of training new sales personnel. The company sampled 16 employees who were trained the original way and found average daily sales to be Rs.688 and the sample standard deviation was Rs.32.63. They also sampled 11 employees who were trained using the new method and found average daily sales to be Rs.706 and the sample standard deviation was Rs.24.84. At $\alpha = 0.05$, can the company conclude that average daily sales have increased under the new plan?

GTU -Dec-2013 [07]

Solution:

We have,

$\bar{x}_1 = 688$ $\bar{x}_2 = 706$

$s_1 = 32.63$ $s_2 = 24.84$

$n_1 = 16$ $n_2 = 11$

$\alpha = 0.05$

$H_o : \mu_1 - \mu_2 = 0$

$H_o : \mu_1 - \mu_2 < 0$

$$t = \frac{(\bar{x}_1 - \bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$t = \frac{(688-706)-0}{\sqrt{\frac{32.63^2(24.84-1)+16^2(11-1)}{24.84+11-2}\left(\frac{1}{24.84}+\frac{1}{11}\right)}}$$

43. An advertising firm is trying to determine the demographics for a new product. They have randomly selected 75 people in each 5 different age groups and introduced the product to them. The results of the survey are given below:

	Age group				
Future Activity	18-29	30-39	40-49	50-59	60-69
Purchase frequently	12	18	17	22	32
Seldom purchase	18	25	29	24	30
Never purchase	45	32	29	29	13

Develop a table of observed and expected frequencies, and calculate χ^2 value. For the level of significance is 0.01, should the null hypothesis be rejected?

GTU -Dec-2013 [07]

44. Two different areas of a large eastern city are being considered as sites for day-care centers. Of 200 households surveyed in one section, the proportion in which the mother worked full-time was 0.52. In another section, 40 percent of the 100 households surveyed had mothers working at full-time jobs. At the 0.04 level of significance, is there a significant difference in the proportions of working mothers in the two areas of the city?

GTU -Dec-2013 [07]

45. A brand manager is concerned that her brand's share may be unevenly distributed throughout the country. In a survey in which the country was divided into four geographical regions, a random sampling of 100 consumers in each region was surveyed, with the following results:

	Region				
	NE	NW	SE	SW	Total
Purchase the brand	40	55	45	50	190
Do not purchase	60	45	55	50	210
Total	100	100	100	100	400

Develop a table of observed and expected frequencies and calculate the sample χ^2 value. State the null and alternative hypotheses and test whether brand share is the same across the four regions at α = 0.05.

GTU -Dec-2013 [07]

46. For a random sample of 10 persons, fed on diet A, the increased weight in pounds in a certain period were:

10, 6, 16, 17, 13, 12, 8, 14, 15, 9

For another random sample of 12 persons, fed on diet B, the increase in the same period were:

7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17

Test whether the diets A and B differ significantly as regards their effect on increase in weight at 5% level of significance.

GTU -Dec-2013[2630003] [07]

47. A machine produced 20 defective articles in a batch of 400. After overhauling it produced 10 defectives in a batch of 300. Test at 5% level of significance, Has the machine improved?

GTU -Dec-2013[2630003] [04]

48. The number of defects per unit in a sample of 330 units of a manufactured product was found as follows:

Number of defects	0	1	2	3	4
Number of Units	214	92	20	3	1

Fit a Poisson Distribution to the data and test for goodness of fit at 5% level of significance.

GTU -Dec-2013[2630003] [07]

49. Give Difference between

    (a) One tailed and two tailed test.

    (b) Type-I and Type-II Error.

GTU -Dec-2013[2630003] [03]

50. Eleven sales executive trainees are assigned selling jobs right after their recruitment. After a fortnight they are withdrawn from their field duties and given a month's training for executive sales. Sales executed by them in thousands of rupees before and after the training, in the same period are listed below:

Sales Before Training	23	20	19	21	18	20	18	17	23	16	19
Sales After Training	24	19	21	18	20	22	20	20	23	20	27

Test at 5% level of significance, Do these data indicate that the training has contributed to their performance?

GTU -Dec-2013[2630003] [07]