ATMIYA INSTITUTE OF TECHNOLOGY & SCIENCE

Sub: Statistical Methods(630003,2630003)

GTU Examples(Jan-2011 to May-2014)

Date:

1. Define the Exponential probability distribution. Give the properties of exponential Probability distribution.

GTU-Jan-2011 [02]

Answer:

Probability density function $f(x) = \frac{1}{\mu}e^{\frac{-x}{\mu}}$

Properties

- It is continuous distribution.
- It is a family of curve.
- It is skewed to the right.
- Its apex is always x=0.
- the curve steadily decreases as x sets larger.

2. A random variable X denotes the number of accidents per week at the traffic signal situated 0 In a crowded locality of a city and probability distribution is given below:

x	0	1	2	3
P(x)	0.4	0.3	0.2	0.1

Find the probability that there is at most one and at least two accidents per week and draw The sketch graph of probability distribution. (Without graph paper)

GTU-Jan-2011 [07]

Solution:

$P(x \leq 1)$=P(0)+P(1)=0.4+0.3=0.7

$P(x \geq 2)$=P(2)+P(3)=0.2+0.1=0.3

3. For events A and B,$P(A^{'} \cap B) = 0.1$,$P(A \cap B^{'})$=0.4,$P(A^{'} \cup B^{'})$=0.6,then find following:

(a) P(A)

(b) P(B)

(c) $P(A \cup B)$

(d) $P(A^{'} \cup B)$

GTU-Jan-2011 [07]

Solution:

Now, $P(A^{'} \cup B^{'}) = 0.6$

$\therefore P((A \cap B)^{'}) = 0.6$

$\therefore$ 1-$P(A \cap B) = 0.6$

$\therefore P(A \cap B) = 0.4$

Also,$P(A \cap B^{'}) = P(A - B) = 0.4$

Now,$P(A - B) = P(A) - P(A \cap B)$

$\therefore$ 0.4=P(A)-0.4

$\therefore$ P(A)=0.8

Also$P(A^{'} \cap B) = P(B - A) = 0.1$

Now,$P(B - A) = P(B) - P(A \cap B)$

$\therefore$ 0.1=P(B)-0.4

$\therefore$ P(B)=0.5

Now,$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cup B) = 0.8 + 0.5 - 0.4$

$\therefore P(A \cup B) = 0.9$

Now,$P(A^{'} \cup B) = P((A \cap B^{'})^{'})$

$\therefore P(A^{'} \cup B) = 1 - P(A \cap B^{'})$

$\therefore P(A^{'} \cup B) = 1 - 0.4$

$\therefore P(A^{'} \cup B)$=0.6

4. In a city of some western country 70 % of the married persons take divorce. What is The probability that at least three among four persons will take divorce?

GTU-Jan-2011 [03]

Solution:

We have,

p=0.7

n=4

$P(x) =_n C^x p^x (1-p)^{n-x}$

$P(x \geq 3)$ =?

Now,$P(x \geq 3) = P(3) + P(4)$

$\therefore P(x \geq 3)$=$_4C^3(0.7)^3(1-0.7)^{4-3}$+$_4C^4(0.7)^4(1-0.7)^{4-4}$

$\therefore P(x \geq 3) = 0.4116 + 0.2401 = 0.6517$

5. The probability function of a binomial distribution is f(x) = $_6C^x p^x(1-p)^{6-x}$, x = 0,1,2,3,. . . ..6.

If 3f(2) = 2f(3) find the value of p.

GTU-Jan-2011 [04]

Solution:

We have,

3f(2)=2f(3)

$\therefore 3\binom{6}{2}p^2(1-p)^{6-2} = 2\binom{6}{3}p^3(1-p)^{6-3}$

$\therefore 3(15)p^2(1-p)^4 = 2(20)p^3(1-p)^3$

$\therefore 45(1-p) = 40p$

$\therefore 45 - 45p = 40p$

$\therefore 85p = 45$

$\therefore p = \frac{45}{85} = \frac{9}{17}$

6. Define the normal distribution. Marks of large number of students are distribution normally with mean 50 and standard Deviation 12. If a student is selected at random what is the probability that his marks will be Between

   (a) 38 and 62

   (b) 26 and 74

   (Given area between z = 0 and z = 1 is 0.34135 & z = 0 and z = 2 is 0.47725)

   GTU-Jan-2011 [07]

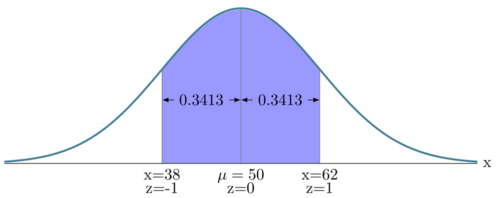
   Solution:

   We have,

   $\mu = 50$

   $\sigma = 12$

   (a):$P(38 \le x \le 62) = ?$



   $z = \frac{x-\mu}{\sigma}$

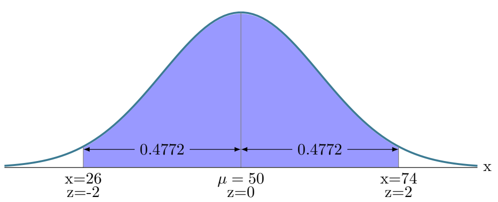
   $\therefore z = \frac{38-50}{12} = -1, z = \frac{62-50}{12} = 1$

   Now, $P(38 \le x \le 62) = P(-1 \le z \le 1)$

   $\therefore P(38 \le x \le 62) = P(-1 \le z \le 0) + P(0 \le z \le 1)$

   $\therefore P(38 \le x \le 62) = 0.3413 + 0.3413 = 0.6826.$

   (b):$P(26 \le x \le 74)$



   $z = \frac{x-\mu}{\sigma}$

   $\therefore z = \frac{26-50}{12} = -2, z = \frac{74-50}{12} = 2$

   Now, $P(26 \le x \le 74) = P(-2 \le z \le 2)$

   $\therefore P(26 \le x \le 74) = P(-2 \le z \le 0) + P(0 \le z \le 2)$

   $\therefore P(26 \le x \le 74) = 0.4772 + 0.4772 = 0.9544.$

7. Answer the following(One Mark Each)

(a) Standard deviation for binomial distribution if n=10 and p=0.3.

(b) If P(A)=a, P(B)=b, and $P(A \cap B)$=c then find the value of $P(A^{'} \cap B^{'})$.

(c) If ${}_nP^r$ = 336 & ${}_nC^r$= 56 then find the value of n and r.

(d) If A and B are mutually exclusive events then what is the value of $P(A \cap B)$

(e) Following are the wages of 8 workers in rupees: 50, 62, 40, 70, 45, 56, 32 and 45 then If one of the workers is selected at random, what is the probability that his wage would be lower than the average wages?

GTU-Jun-2011 [05]

Solution:

(a): Standard deviation $\sigma = \sqrt{n.p.(1-p)} = \sqrt{(10)(0.3)(0.7)} = 1.4491$

(b):$P(A^{'} \cap B^{'}) = P((A \cup B)^{'}) = 1 - P((A \cup B)^{'}) = 1 - [P(A) + P(B) - P(A \cap B) = 1 - [a + b - c] = 1 - a - b + c$

(c): Now ${}_nP^r = 336$,and ${}_nC^r = 56$

$\therefore \frac{n!}{(n-r)!} = 336$ and $\frac{n!}{r!(n-r)!} = 56$

$\therefore \frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{336}{56}$

$\therefore r! = 6$

$\therefore$ r=3

Now ${}_nP^r = 336$

$\therefore {}_nP^3 = 336$

$\therefore \frac{n!}{(n-3)!} = 336$

$\therefore \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 336$

$\therefore n(n-1)(n-2) = 8.7.6$

$\therefore$ n=8.

(d):if A and B are mutually exclusive then $A \cap B = \phi$

$\therefore$ if A and B are mutually exclusive then $P(A \cap B) = 0$

(e): Average of 50,62,40,70,45,56,32 and 45 is 50.

$\therefore$ no of data below 50 are 4 (40,45,32,45).

$\therefore$ probability that his wage rate is lower than average=$\frac{4}{8} = 0.5$

8. Write the following terms for binomial distribution

Properties,Mean and Variance.

GTU-Jun-2011 [04]

Answer:

Properties

- There must be n identical trials.
- There must be only two outcomes.One outcomes refer to success and other refer to failure.
- Probability of success and failure do not change trial by trial.
- Result of any trial does not dependent on any other trial.

Mean

$\mu = np$

Variance

σ^2=n.p.(1-p)

9. Write the Characteristics of the Normal Distribution.

GTU-Jun-2011 & May-2014 [04] &[03]

Answer:

Properties

- It is continuous distribution.
- It is symmetrical distribution about its mean.
- It is asymptotic to the horizontal axis.
- It is a family of curve.

10. Define mutually exclusive event, independent event, marginal probability.

GTU-Jun-2011 [03]

Answer:

Mutually exclusive events

Two events A and B are said to be mutually exclusive if $A \cap B = \phi$

Independent events

Two events A and B are said to be independent events if either P(A/B)=P(A) or P(B/A)=P(B)

Marginal probability

The probability of one variable taking a specific value irrespective of the values of the others is said to be marginal probability.

11. Data on the 30 largest bond funds provides 1-year and 5-year percentage returns for the period ending March 31, 2000. Suppose we consider a 1-year return in excess of 2% to be high and a 5-year return in excess of 44 % to be high. 15 of the funds had a 1-year return in excess of 2%, 12 of the funds had a 5-year return in excess of 44% and six of the funds had both a 1-year return in excess of 2% and a 5-year return in excess of 44%.

    (a) What is the probability that a fund has a high 1-year return or a high, 5 year return or both?

    (b) What is the probability that a fund has neither a high 1-year return nor a high 5-year return?

GTU-Jun-2011 [02]

Solution:

Events are

A=An event that selected bond is 1-year in excess of 2% to be high.

B=An event that selected bond is 5-year in excess of 44% to be high.

$\therefore P(A) = \frac{15}{30} = 0.5$

$\therefore P(B) - \frac{12}{30} = 0.4$

$\therefore P(A \cap B) = \frac{6}{30} = 0.2$

(a):$P(A \cup B)$ =?

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cup B) = 0.5 + 0.4 - 0.2 = 0.7$

(b):$P(A^{'} \cap B^{'})$ =? Now, $P(A^{'} \cap B^{'}) = P((A \cup B^{'})$

$\therefore P(A^{'} \cap B^{'}) = 1 - P((A \cup B^{'})$

$\therefore P(A^{'} \cap B^{'}) = 1 - 0.7$=0.3

12. A shipment of 10 items has two defective and eight non defective items. In the inspection of the shipment, a sample of items will be selected and tested. If a defective item is found, the shipment of 10 items will be rejected.

    (a) If a sample of three items is selected, what is the probability that the shipment will be rejected?

    (b) If a sample of four items is selected, what is the probability that the shipment will be rejected?

    (c) If a sample of five items is selected, what is the probability that the shipment will be rejected?

GTU-Jun-2011 [03]

Solution:

p=$\frac{2}{10} = 0.2$

(a):n=3,$P(x \geq 1)$ =?

Now,$P(x \geq 1) = P(1) + P(2)$

$\therefore P(x \geq 1) =_3 C^1 (0.2)^1 (0.8)^{3-1} +_3 C^2 (0.2)^1 (0.8)^{3-2}$

$\therefore P(x \geq 1) = 0.3840 + 0.0960$

$\therefore P(x \geq 1) = 0.4800$

(b):n=4,$P(x \geq 1)$

Now,$P(x \geq 1) = P(1) + P(2)$

$\therefore P(x \geq 1) =_4 C^1 (0.2)^0 (0.8)^{4-1} +_4 C^2 (0.2)^0 (0.8)^{4-2}$

$\therefore P(x \geq 1) = 0.4096 + 0.1536$

$\therefore P(x \geq 1) = 0.5632$

(c):n=5,$P(x \geq 1)$

$P(x \geq 1) = P(1) + P(2)$

$\therefore P(x \geq 1) =_5 C^1 (0.2)^0 (0.8)^{5-1} +_5 C^2 (0.2)^0 (0.8)^{5-2}$

$\therefore P(x \geq 1) = 04096 + 0.2048$

$\therefore P(x \geq 1) = 0.6144$

13. Phone calls arrive at the rate of 48 per hour at the reservation desk for Regional airways.

    (a) Compute the probability of receiving three calls in a five minute interval of time.

    (b) Compute the probability of receiving exactly ten calls in fifteen minutes.

    (c) If no calls are being processed, what is the probability that the agent can take three minutes for personal time without being interrupted by a call?

GTU-Jun-2011 [03]

Solution:

We have, μ=48 calls/hour

(a):P(x=3 calls/5-mins)=?

Now μ=48 calls/hour

$\therefore \mu$=4 calls/5-min

$P(x) = \frac{\mu^x . e^{-\mu}}{x!}$

$\therefore$ P(4)=$\frac{4^3 e^{-4}}{3!}$ =0.1954

(b):P(x=10 calls/15-min)=?

Now μ=48 calls/hour

$\therefore$ μ=(12 calls/15-min)

$P(x) = \frac{\mu^x . e^{-\mu}}{x!}$

$\therefore$ P(12)=$\frac{12^{10} e^{-12}}{10!}$ =0.1048

(c):P(x=0 calls/3-mins)=?

Now μ=48 calls/hour

$\therefore$ μ=2.4 calls/3-mins

$P(x) = \frac{\mu^x . e^{-\mu}}{x!}$

$\therefore$ P(0)=$\frac{2.4^0 e^{-2.4}}{0!}$ = 0.0907

14. Fifty percent of Americans think we are in a recession, even though technically we have not had two straight quarters of negative growth. For a sample of 20 Americans, make the following calculations.
    (a) Compute the probability that exactly 12 people think we are in a recession.
    (b) Compute the probability that no more than 5 people think we are in a recession.
    (c) How many people would you expect to say we are in a recession?
    (d) Compute the variance and standard deviation of the number of people who think we are in a recession

GTU-Jun-2011 [04]

Solution:

We have,

n=20

p=0.50

(a):P(x=12)=?

Now,$P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore$ P(12)= ${}_{20}C^{12}(0.5)^{12}(1-0.5)^{20-12}$=0.1201

(b):$P(x \le 5)$

Now,$P(x \le 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$\therefore P(x \le 5)={}_{20}C^5(0.5)^5(1-0.5)^{20-5}+{}_{20}C^4(0.5)^4(1-0.5)^{20-4}+{}_{20}C^3(0.5)^3(1-0.5)^{20-3}+{}_{20}C^3(0.5)^3(1-0.5)^{20-3}$
$+ {}_{20}C^2(0.5)^2(1-0.5)^{20-2} + {}_{20}C^1(0.5)^1(1-0.5)^{20-1} + {}_{20}C^0(0.5)^0(1-0.5)^{20-0}$

$\therefore P(x \le 5) = 0.0000 + 0.0000 + 0.0001 + 0.0010 + 0.0046 + 0.0147$

$\therefore P(x \le 5) = 0.02048$

(c):$\mu = np$

$\therefore \mu = (20)(0.5) = 10$

(d):$\sigma^2 = np(1-p)$

$\therefore \sigma^2 = (20)(0.5)(1-0.5) = 5$

Now,$\sigma = \sqrt{np(1-p)}$

$\therefore \sigma = \sqrt{5} = 2.2361$

15. A GMAC MBA new-matriculates survey provided the following data for 2018 students.

Age group	Applied to more than one school	
	Yes	No
23 & under	207	201
24-26	299	379
27-30	185	268
31-35	66	193
36 & Over	51	169

(a) Given that a person applied to more than one school, what is the probability that the person is 24-26 years old ?

(b) Given that a person is in the 36-and-over age group, what is the probability that the person applied to more than one school?

(c) What is the probability that a person is 24-26 years old or applied to more than one school?

GTU-Jun-2011 [04]

Solution:

Events are

A=An Event that a selected student whose age group is 24-26.

B=An Event that a selected student whose age group is 36 & over.

C=An Event that a selected applied for more than one school.

(a):$P(A/C)$ =?

Now $P(A/C) = \frac{P(A \cap C)}{P(C)}$

$\therefore P(A/C) = \frac{207/2018}{808/2018}$

$\therefore P(A/C) = \frac{299}{808}$

(b):$P(C/B)$ =?

Now $P(C/B) = \frac{P(B \cap C)}{P(B)}$

$\therefore P(C/B) = \frac{51/2018}{220/2018} = \frac{51}{220}$

(c):$P(A \cup C)$ =?

Now $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$\therefore P(A \cup C) = \frac{678}{2018} + \frac{808}{2018} - \frac{299}{2018}$

$\therefore P(A \cup C) = \frac{1187}{2018}$

16. Define Mutually Exclusive Events

GTU-Dec-2011 [01]

Answer:

Two events are said to be mutually exclusive events if $A \cap B = \phi$

17. True of false

"The set of all possible outcomes of an experiment is called the sample space for the experiment"

GTU-Dec-2011 [01]

Answer:

True

18. A company is considering upgrading its computer system, and a major portion of the upgrade is a new operating system. The company has asked an engineer for an evaluation of the operating system. Suppose the probability of a favorable evaluation is 0.65. If the probability the company will upgrade its system given a favorable evaluation is 0.85, what is the probability that the company will upgrade and receive a favorable evaluation?

GTU-Dec-2011 [07]

Solution:

Events are

F=An Event for favorable evaluation.

U=An event that company will upgrade its system.

$\therefore$ P(F)=0.65,P(U/F)=0.85 and $P(U \cap F)$ =?

Now,$P(U/F) = \frac{P(U \cap F)}{P(F)}$

$\therefore P(U \cap F) = P(U/F).P(F)$

$\therefore P(U \cap F) = (0.85)(0.65) = 0.5525$

19. A physical therapist at Gujarat Technological University knows that the football team will play 40% of its games on artificial turf this season. He also knows that a football player's chances of incurring a knee injury are 50% higher if he is playing on artificial turf instead of grass. If a player's probability of knee injury on artificial turf is 0.42, what is the probability that:

   (a) A randomly selected football player incurs a knee injury?

   (b) A randomly selected football player with a knee injury incurred the injury playing on grass?

GTU-Dec-2011 [07]

Solution:

Events are

A=An event in which football team will play his game on artificial turf.

I=An event in which player incur knee injury.

G=An event in which football team will play his game on grass.

We have, P(A)=0.40

$\therefore$P(G)=0.60 P(I/A)=0.42

P(I/A)=P(I/G)+50 % of P(I/G)=$\frac{150}{100}$P(I/G)

$\therefore$ P(I/G)=$\frac{100P(I/A)}{150}$

$\therefore$ P(I/G)=$\frac{(100)(0.42)}{150} = 0.28$

(a):P(I)=?

Now,$P(I) = P(I \cap A) + P(I \cap G)$

$\therefore$ P(I)=P(I/G).P(G)+P(I/A).P(A)

$\therefore$ P(I)=(0.28)(0.60)+(0.42)(0.40)=0.336

(b):P(G/I)=?

Now,P(G/I)=$\frac{P(I \cap G)}{P(I)}$

$\therefore$ P(G/I)=$\frac{P(I/G).P(G)}{P(I)}$

$\therefore$ P(G/I)=$\frac{(0.28)(0.60)}{0.40}=0.5$

20. Assuming that half of the Indian population is vegetarian. Estimate how many investigators out of 100 will report that 3 or less are vegetarian in the sample of 10 individuals.

GTU-Dec-2011& May-2014 [04] & [07]

Solution:

We have

n=10

p=0.5

$P(x) =_n C^x p^x (1-p)^{n-x} P(x \le 3) =?$

Now $P(x \le 3) = P(0) + P(1) + P(2) + P(3)$

$\therefore$ $P(x \le 3)$=$_{10}C^3(0.5)^3(1-0.5)^{10-3}$ + $_{10}C^2(0.5)^2(1-0.5)^{10-2}$ + $_{10}C^1(0.5)^1(1-0.5)^{10-1}$ + $_{10}C^0(0.5)^0(1-0.5)^{10-0}$

$\therefore$ $P(x \le 3) = 0.009 + 0.0097 + 0.0439 + 0.1171 = 0.1417$

$\therefore$ No of report that 3 or less are vegetarian are (100)(0.1417)$\approx$ 15

21. If the prices of new cars increase an average of four times every 3 years, find the probability of five or more price hikes in a randomly selected period of 3 years.

GTU-Dec-2011 [05]

Solution:

We have μ=4 times/3 years,

$P(x \ge 5/3$ years)=?

$P(x) = \frac{\mu^x e^{-\mu}}{x!}$

Now $P(x \ge 5) = P(5) + P(6) + P(7) + P(8) + ...$

$\therefore$ $P(x \ge 5) = 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

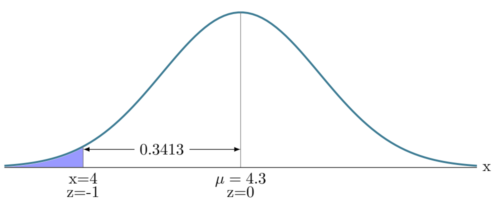
$\therefore$ $P(x \ge 5)$ =1- $\frac{4^0e^{-4}}{0!}$- $\frac{4^1e^{-4}}{1!}$-$\frac{4^2e^{-4}}{2!}$-$\frac{4^3e^{-4}}{3!}$-$\frac{4^4e^{-4}}{4!}$

$P(x \ge 5)$=1-0.0183-0.0733-0.1465-0.1954-0.1954=0.3711

22. In a production process the diameter of items is distributed normally with mean 4.3 cm and variance 0.09. 200 items are having less than 4 cm diameter. Estimate the total number of items in the production.

GTU-Dec-2011 [05]

Solution:



We have

μ=4.3cm

σ^2=0.09

$\therefore \sigma = 0.3$

$P(x \geq 4) = ?$

$z = \frac{x-\mu}{\sigma}$

$z = \frac{4-4.3}{0.3} = -1.0$ Now $P(x \leq 4) = P(z \leq -1)$

$\therefore P(x \leq 4) = 0.5 - P(-1 \leq z \leq 0)$

$\therefore P(x \leq 4) = 0.5 - 0.3413 = 0.1587$

i.e. 15.87 % of the item having diameter less than 4cm diameter which are 200(given)

$\therefore$ If total items are N then 15.87 % of N=200

$\therefore$ 200= $\frac{15.87 \times N}{100}$

[1mm] $\therefore$N=$\frac{200 \times 100}{15.87} \approx 1260$

23. The probability that a bomb dropped from an aero plane will hit the target is 0.4. Five bombs are dropped from the aero plane to destroy a bridge. 2 bombs are sufficient to destroy the bridge. What is the probability that the bridge will be destroyed?

GTU-Dec-2011 [04]

Solution:

We have, p=0.4

n=5

$P(x \geq 2) = ?$

$P(x \geq 2) = P(2) + P(3) + P(4) + P(5)$

$P(x \geq 2)$=${}_5C^2(0.4)^2(1-0.4)^{5-2}$+${}_5C^3(0.4)^3(1-0.4)^{5-3}$+${}_5C^4(0.4)^4(1-0.4)^{5-4}$+${}_5C^5(0.4)^5(1-0.4)^{5-5}$

$P(x \geq 2) = 0.3456 + 0.2304 + 0.00768 + 0.0102 = 0.6630$

24. Past police records indicate a mean of 5 accidents per month at a dangerous intersection. Find out the probability of 3 accidents in a given period of one month.

GTU-Dec-2011 [05]

Solution:

We have,

μ=5 accidents/ month

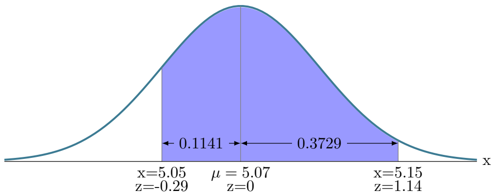
P(x=3 accidents/month)

Now,$P(x) = \frac{\mu^x e^{-\mu}}{x!}$

$\therefore$ P(3)=$\frac{5^3 e^{-5}}{3!}$=0.1404

25. A machinery company has received a big order to produce electric motors for a manufacturing company. In order to fit in its bearing, the drive shaft of the motor must have a diameter of 5.1 ± 0.05 (inches). The company's purchasing agent realizes that there is a large stock of steel rods in inventory with a mean diameter of 5.07", and a standard deviation of 0.07". What is the probability of a steel rod from inventory fitting the bearing? **GTU-Dec-2011 [05]**

Solution:



We have,

$\mu = 5.07$

$\sigma = 0.07$

$P(5.05 \le x \le 5.15) = ?$

$z = \frac{x-\mu}{\sigma}$

[2.5mm] $z = \frac{5.05-5.07}{0.07} = -0.29, z = \frac{5.15-5.07}{-.07} = 1.14$

[2.5mm] Now, $P(5.05 \le x \le 5.15) = P(-.29 \le z \le 1.14)$

$\therefore P(5.05 \le x \le 5.15) = P(-0.29 \le z \le 0) + P(0 \le z \le 1.14)$

$\therefore P(5.05 \le x \le 5.15) = 0.1141 + 0.3729 = 0.4870$

26. True or False(One Mark Each)

    (a) If two events A and B are independent, conditional probability $P(A/B) = P(A \cap B)$.

    Answer: False

    If A nd B are independent events then P(A/B)=P(A).

    (b) Random variable 'occurrences of an event over a period of time or space follows exponential distribution.

    Answer: False

    Random variable 'occurrences of an event over a period of time or space follows Poisson distribution.

GTU-May-2012 [02]

27. Survey shows that 40 % of the students are using 3G mobile. In a random sample of 10 students, what is the probability that two students have 3G mobile?

GTU-May-2012 [03]

Solution:

We have,

p=0.40,

n=10,

P(x=2)=?

Now $P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore$ P(2)= ${}_{10}C^2 (0.4)^2 (1-0.4)^{10-2}$=0.1209

28. A psychologist determined that the number of sessions required to obtain the trust of patient is either 1, 2 or 3. Let X be a random variable indicating the number of sessions required to gain patient's trust. For given probability function $f(x) = \frac{x}{6}$.where x=1, 2 or 3; compute expected value and variance of X.

GTU-May-2012 [04]

Solution:

As $f(x) = \frac{x}{6}$

$\therefore$ f(1)=$\frac{1}{6}$,f(2)=$\frac{2}{6}$=$\frac{1}{3}$,f(3)=$\frac{3}{6}$=$\frac{1}{2}$

x	f(x)	x.f(x)	$(x-\mu)^2$	(x-μ)^2).f(x)
1	1/6	1/6	16/9	8/27
2	1/3	2/3	1/9	1/27
3	1/2	3/2	4/9	2/9
		$\sum x.f(x)$=7/3		$\sum(x-\mu)^2.f(x)$=5/9

Now,$\mu = \sum x.f(x) \therefore \mu = \frac{7}{3}$

Also $\sigma^2 = \sum(x-\mu)^2.f(x)$

$\therefore \sigma^2 = \frac{5}{9}$

29. For two mutually exclusive events A and B having $P(A)$=0.3 and $P(B)$=0.4, compute $P(A \cup B)$ and $P(A/B)$.

GTU-May-2012 [03]

Solution:

As A and B are mutually exclusive hence $A \cap B = \phi$

$\therefore P(A \cap B) = 0$

Now,$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cup B) = 0.3 + 0.4 = 0.7$

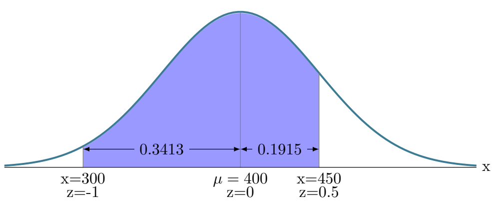
Also,$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\therefore P(A/B) = \frac{0}{0.4} = 0$

30. Assume that admission test scores are normally distributed with mean 400 and σ = 100 marks. Find the probability of scores between 300 and 450.

GTU-May-2012 [04]

Solution:



We have,

$\mu = 400$

$\sigma = 100$

$P(300 \le x \le 450)$ =?

$z = \frac{x-\mu}{\sigma}$

$z = \frac{300-400}{100} = 1, z = \frac{450-400}{100} = 0.5$

Now,$P(300 \le x \le 450) = P(-1 \le z \le 0.5)$

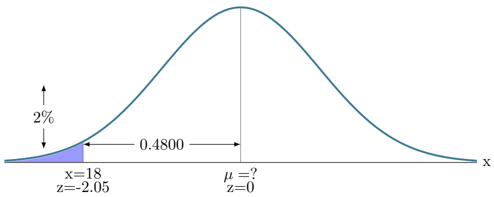
$\therefore P(300 \le x \le 450) = P(-1 \le z \le 0) + P(0 \le z \le 0.5)$

$\therefore P(300 \le x \le 450) = 0.3413 + 0.1915 = 0.5328$

31. A machine fills containers with a particular product. Filled weights have a normal distribution with $\sigma = 0.6$ ounce. If only 2% of the containers hold less than 18 ounces, what is the population mean of weight.

GTU-May-2012 [04]

Solution:



We have,

σ=0.6

$P(x \leq 18)$=0.02

μ=? $\therefore$ $P(18 \leq x \leq \mu)$=05-0.02=04800

$z = \frac{x-\mu}{\sigma}$

$\therefore$ -2.05=$\frac{18-\mu}{0.6}$

$\therefore$ (-2.05)(0.6)=18-μ

$\therefore$ μ=19.23

32. Customer arrivals at a bank are random and independent. The probability of an arrival in any one-minute period is same as that in any other one-minute period. Assuming mean arrival rate of five customers per minute, find the probability of

   (a) Exactly three arrivals in one-minute period
   (b) No arrivals in half-minute period
   (c) Three minutes for next customer to arrive

GTU-May-2012 [07]

Solution:

We have,

μ=5 customers/min

(a):P(x=3 customers/min)=?

Now $P(x) = \frac{\mu^x e^{-\mu}}{x!}$

$\therefore$ P(3)=$\frac{5^3 e^{-5}}{3!}$=0.1404

(b):P(x=0 customers/half min)=?

μ=5 customers/min

$\therefore$ μ=2.5 customers/half min

Now $P(x) = \frac{\mu^x e^{-\mu}}{x!}$

$\therefore$ P(0)=$\frac{2.5^0 e^{-2.5}}{0!}$=0.0821

[3mm] **(c):**$P(x \geq 3/\text{customer})$=?

μ=5 customers/min

$\therefore$ μ=0.2 mins/customer

Now,$P(x \geq x_o) = e^{\frac{-x_o}{\mu}}$

$\therefore$ $P(x \geq 3) = e^{\frac{-3}{0.2}} = 0.0000$

33. If the probability that a fluorescent light has a useful life of at least 500 hours is 0.85, find the probability that among 20 such lights

    (a) At least 18 will have a useful life of at least 500 hours

    (b) Exactly 15 will have a useful life of at least 500 hours

    (c) None will have a useful life of at least 500 hours

GTU-May-2012 [07]

Solution:

We have,

p=0.85,

n=20

(a):$P(x \geq 18)$=?

$P(x) =_n C^x p^x (1-p)^{n-x}$

Now,$P(x \geq 18) = P(18) + P(19) + P(20)$

$\therefore$ $P(x \geq 18)$=${}_{20}C^{18}(0.85)^{18}(1-0.85)^{20-18}$+${}_{20}C^{19}(0.85)^{19}(1-0.85)^{20-19}$+${}_{20}C^{20}(0.85)^{20}(1-0.85)^{20-20}$

$\therefore$ $P(x \geq 18) = 0.2293 + 0.1367 + 0.0387 = 0.4047$

(b):P(x=15)=?

$P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore$ P(15)=${}_{20}C^{15}(0.85)^{15}(1-0.85)^{20-15}$=0.1028

(c):P(x=0)=?

$P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore$ P(0)=${}_{20}C^{0}(0.85)^{0}(1-0.85)^{20-0}$=0.0000

34. Consider a binomial experiment with and n=20 and p=0.70 Compute $P(12)$,$P(x \geq 16)$,$E(x)$and $Var(x)$

GTU-Dec-2012 [04]

Solution:

Now,$P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore$ P(12)= ${}_{20}C^{12}(0.70)^{12}(1-0.70)^{20-12}$ =0.1143

Also $P(x \geq 16) = P(16) + P(17) + P(18) + P(19) + P(20)$

$\therefore$ $P(x \geq 16)$= ${}_{20}C^{16}(0.7)^{16}(1-0.7)^{20-16}$ + ${}_{20}C^{17}(0.7)^{17}(1-0.7)^{20-17}$ + ${}_{20}C^{18}(0.7)^{18}(1-0.7)^{20-18}$ + ${}_{20}C^{19}(0.7)^{19}(1-0.7)^{20-19}$ + ${}_{20}C^{20}(0.7)^{20}(1-0.7)^{20-20}$

$\therefore$ $P(x \geq 16)$=0.1304+0.0716+0.0278+0.0068+0.0007=0.2374

35. For two events A and B, with $P(A) = 0.50$,$P(B) = 0.60$ and $P(A \cap B) = 0.40$ Find

    (a) $P(A/B)$

    (b) $P(B/A)$

(c) Are and independent? Why or Why not?

GTU-Dec-2012 [07]

Solution:

We have,

P(A)=0.50

P(B)=0.60

$P(A \cap B) = 0.40$

(a):P(A/B)=?

Now,$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\therefore P(A/B) = \frac{0.40}{0.60} = 0.6667$

[4mm] **(b):**P(B/A)=?

Now,$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\therefore P(B/A) = \frac{0.40}{0.50} = 0.80$

(c):A and B are not independent because $P(A/B) \neq P(A)$ and $P(B/A) \neq P(B)$

36. Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.

    (a) Compute the probability of no arrivals in a 1-minute period.
    (b) Compute the probability that three or fewer passengers arrive in a 1-minute period.
    (c) Compute the probability of no arrivals in a 15-second period.
    (d) Compute the probability of at least one arrival in a 15-second period.

GTU-Dec-2012 & GTU-May-2014 [07] & [04]

Solution:

We have, μ=10 passengers/min

(a):P(x=0/min)

=? $P(x) = \frac{\mu^x e^{-\mu}}{x!}$

$\therefore$ P(0)=$\frac{10^0 e^{-10}}{0!}$=0.0000

(b):$P(x \le 3)$/min=?

$P(x) = \frac{\mu^x e^{-\mu}}{x!}$

Now, $P(x \le 3)$=P(3)+P(2)+P(1)+P(0)

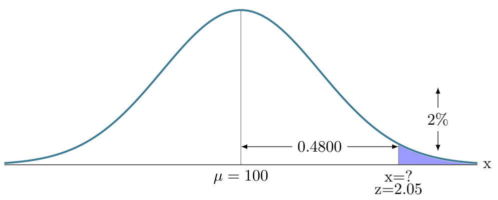
$\therefore P(x \le 3)$=$\frac{10^3 e^{-10}}{3!}+\frac{10^2 e^{-10}}{2!}+\frac{10^1 e^{-10}}{1!}+\frac{10^0 e^{-10}}{0!}$

$\therefore P(x \le 3)$=0.0076+0.0023+.0005+.0000=0.0104

37. A person must score in the upper 2% of the population on an IQ test to qualify for membership in GTU-IQ-Club. If IQ scores are normally distributed with a mean of 100 and a standard deviation of 15, what score must a person have to qualify for GTU-IQ-Club?

GTU-Dec-2012 [07]

Solution:



We have,

μ=100

σ=15

$P(x \geq x_o)$=0.02

x_o=?

Now,$P(x \geq x_o)$=0.02

$\therefore P(\mu \leq x \leq x_o)$=0.4800

Now according to z-distribution table z-value corresponding to x_0=2.55

Now $z = \frac{x_0 - \mu}{\sigma}$

$\therefore 2.55 = \frac{x_0 - 100}{15}$

$\therefore (2.55)(15) = x_0 - 100$

$\therefore x_0 = (2.55)(15) + 100 = 138.25$

$\therefore$A person must score atleast 138.25 to qualify for GTU-IQ Club.

38. The lifetime (hours) of an electronic device is a random variable with the following exponential probability density function. $f(x) = \frac{1}{30} e^{\frac{-x}{30}}$ for $x > 0$

    (a) What is the mean lifetime of the device?

    (b) What is the probability that the device will fail in the first 25 hours of operation?

    (c) What is the probability that the device will operate 100 or more hours before failure?

GTU-Dec-2012 [07]

Solution:

As $f(x) = \frac{1}{30} e^{\frac{-x}{30}} \therefore \mu$=30 hours

(a):μ=30

(b):$P(x \leq 25)$=?

Now,$P(x \leq x_0) = 1 - e^{\frac{-x_0}{\mu}}$ $P(x \leq x_o) = 1 - e^{\frac{-x_o}{\mu}}$

$\therefore P(x \leq 25) = 1 - e^{\frac{-25}{30}}$=0.5654

(c): $P(x \geq x_o) = e^{\frac{-x_o}{\mu}}$

$P(x \geq 100) = e^{\frac{-100}{30}}$ =0.0356

39. Fill in the blank(One Mark Each)

    (a) $P(A/B)$= ......if events A and B are independent.(0, P(A))

    (b) Mean and variance of....... variate is same. (Binomial, Poisson)

GTU-May-2013 [02]

Answer:

(a): P(A)

(b): Poisson

40. Write necessary conditions to use normal approximation for binomial distribution.

GTU-May-2013 [03]

Answer:

Conditions are

- n.p≥5 and n.(1-p)≥5.
- $\mu \pm 3\sigma$ must lie between 0 to n.

41. The average number of calls received by a switchboard in a 30 minute period is 15.

    (a) What is the probability that the switchboard will receive exactly 10 calls between 10:00 and 10:30?

    (b) What is the probability that the switchboard will receive fewer than 3 calls between 10:00 and 10:15?

GTU-May-2013 [04]

Solution:

We have,

μ=15 calls/30 mins

(a): P(x=10 calls/30-mins)=?

Now, $P(x) = \frac{\mu^x e^{-\mu}}{x!}$

P(10)=$\frac{15^{10}e^{-15}}{10!}$=0.0486

(b): $P(x \le 3/15\text{-mins})$=?

Now, μ=15 calls/30 min

$\therefore$ μ=7.5 calls/15 mins

Now, $P(x \le 3)$=P(3)+P(2)+P(1)+P(0)

$\therefore P(x \le 3)=\frac{7.5^3e^{-7.5}}{3!}+\frac{7.5^2e^{-7.5}}{2!}+\frac{7.5^1e^{-7.5}}{1!}+\frac{7.5^0e^{-7.5}}{0!}$

$\therefore P(x \le 3)$=0.0389+0.0156+0.0041+0.0006=0.0592

42. A local bottling company has determined the number of machine breakdowns per month and their respective probabilities as shown below. Compute expected number and variance of machine breakdowns per month.

Number of breakdowns	0	1	2	3	4
Probability	0.12	0.38	0.25	0.18	0.07

GTU-May-2013 [03]

Solution:

x	P(x)	x.P(x)	$(x-\mu)^2$	$(x-\mu)^2$.P(x)
0	0.12	0.00	2.89	0.3468
1	0.38	0.38	0.49	0.1862
2	0.25	0.50	0.09	0.0225
3	0.18	0.54	1.69	0.3042
4	0.07	0.28	5.29	0.3703
		$\sum x.P(x)$=1.70		$\sum(x-\mu)^2.P(x)$=1.23

Now,$\mu = \sum x.P(x)$

$\therefore \mu$=1.7

Also, $\sigma^2 = \sum (x-\mu)^2.P(x)$

$\therefore \sigma^2$=1.23

43. The daily dinner bills in a local restaurant are normally distributed with a mean Rs.30 and a standard deviation Rs.5.

    (a) What is the probability that a randomly selected bill will be at least Rs. 35?

    (b) What is the probability that a randomly selected bill will be between Rs. 28 and Rs. 35?

GTU-May-2013 [04]

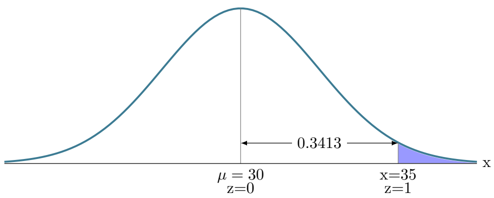
Solution:

We have,

μ=30

σ=5

(a):$P(x \geq 35)$=?



$z = \frac{x-\mu}{\sigma}$

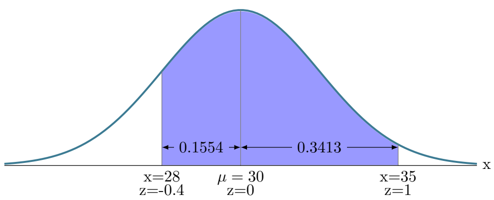
$\therefore z = \frac{35-30}{5}$=1

Now,$P(x \geq 35 = P(z \geq 1)$

$\therefore P(x \geq 35 = 0.5 - P(0 \leq z \leq 1)$

$\therefore P(x \geq 35$=0.5-0.3413=01587

(b):$P(28 \leq x \leq 35)$=?



$z = \frac{x-\mu}{\sigma}$

$\therefore z = \frac{28-30}{5}$=-0.4,$z = \frac{35-30}{5}$=1

Now,$P(28 \leq x \leq 35) = P(-.4 \leq z \leq 1)$

$\therefore P(28 \leq x \leq 35) = P(-0.4 \leq z \leq 0) + P(0 \leq z \leq 1)$

$\therefore P(28 \leq x \leq 35) = 0.1554 + 0.3413 = 0.4967$

44. List properties of normal distribution.

GTU-May-2013 [03]

Answer:

Properties

- It is continuous distribution.
- It is symmetrical distribution about its mean.
- It is asymptotic to the horizontal axis.
- It is a family of curve.

45. Ten percent of the items produced by a machine are defective. Out of 8 items chosen at random,

    (a) Find the probability of less than 2 defective items.

    (b) Find the probability of 4 defective items.

GTU-May-2013 [04]

Solution:

We have,

p=0.10

n=8

(a):$P(x<2)$=?

$P(x) = _n C^x p^x (1-p)^{n-x}$

Now,$P(x<2)$=P(1)+P(0)

$\therefore P(x<2)$=$_8C^1(0.10)^1(1-0.10)^{8-1}$+$_8C^0(0.10)^0(1-0.10)^{8-0}$

$\therefore P(x<2)$=0.3826+0.4304=0.8130

(b):P(x=4)=?

Now, $P(x) = _n C^x p^x (1-p)^{n-x}$

$\therefore$ P(4)=$_8C^4(0.10)^4(1-0.10)^{8-4}$=0.0046

46. The monthly earnings of computer systems analysts are normally distributed with a mean of Rs.24,300. If only 5 percent of the systems analysts have a monthly income of more than Rs. 26,140, what is the value of the standard deviation of the monthly earnings of the computer systems analysts?

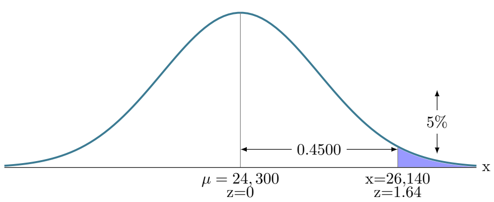
GTU-May-2013 [03]

Solution:

We have, μ=24,300

$P(x \geq 26,140)$=0.05

σ=?



Now,$P(x \geq 26,140)$=0.05

$\therefore P(24,300 \leq x \leq 26,140)$=0.5-0.05=0.45

$\therefore$ z-value corresponding to 26,140 is z=1.64

Now, $z = \frac{x-\mu}{\sigma}$

$\therefore 1.64 = \frac{26140-24300}{\sigma}$

$\therefore 1.64\sigma=1840$

$\therefore \sigma \approx 1122$

47. As a company manager for ABC Corporation, there is a 0.40 probability that you will be promoted this year. There is a 0.72 probability that you will get either promotion or raise or both. The probability of getting both promotion and raise is 0.25.

    (a) What is the probability that you will get a raise?

    (b) If you get a promotion, what is the probability that you will also get a raise?

GTU-May-2013 [04]

Solution:

Events are

A=An event in which selected manager will be promoted.

B=An event in which selected manager will be raised

$\therefore$ P(A)=0.40,$P(A\cup B)$=0.72,P(B)=0.25

(a):P(B)=?

Now,$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore$ 0.72=0.40+P(B)-0.25

$\therefore$ P(B)=0.57

(b):P(B/A)=?

P(B/A)=$\frac{P(A\cap B)}{P(A)}$

$\therefore$ P(B/A)=$\frac{0.25}{0.40}$=0.625

48. The time it takes a mechanic to change the oil in a car is exponentially distributed with a mean of 5 minutes. What is the probability that it will take a mechanic less than 6 minutes to change oil?

GTU-May-2013 [03]

Solution:

We have, μ=5 minutes

$P(x\leq 6)$=?

Now,$P(x\leq x_o)=1-e^{\frac{-x_o}{\mu}}$

$\therefore P(x\leq 6)=1-e^{\frac{-6}{5}}$ =0.5654

49. Define Mutually exclusive events

GTU-Dec-2013 [01]

Answer: If A and B be two events then A ans B are said to be mutually exclusive if $A\cap B=\phi$

50. Given the probabilities of three events, A, B, and C, occurring are P(A) = 0.35, P(B) = 0.45, and P(C) = 0.2. Assuming that A, B, or C has occurred, the probabilities of another event, X, occurring are P(X/A) = 0.8, P(X/B) = 0.65, and P(X/C) = 0.3. Find P(A/X), P(B/X), and P(C/X).

GTU-Dec-2013 [07]

Solution:

We have, P(A)=0.35

P(B)=0.45

P(C)=0.2

P(X/A)=0.8

P(X/B)=0.65

P(X/C)=0.3

P(X)=?

Now,$P(X) = P(X \cap A) + P(X \cap B) + P(X \cap C)$

$\therefore P(X) = P(X/A).P(A) + P(X/B).P(B) + P(X/C).P(C)$

$\therefore P(X) = (0.8)(0.35) + (0.65)(0.45) + (0.3)(0.2) = 0.2975$

51. If the prices of new cars increase an average of four times every 3 years, find the probability of:

    (a) No price hikes in a randomly selected period of 3 years
    (b) Two price hikes
    (c) Four price hikes

GTU-Dec-2013 [07]

Solution:

We have,

μ=4 times /3 years

Now $P(x) = \frac{\mu^x e^{-\mu}}{x!}$

(a):P(x=0 times/ 3 years)=?

$\therefore$ P(0)=$\frac{4^0 e^{-4}}{0!}$=0.0183

[3mm] **(b):**P(2 times/ 3 years)=?

P(2)=$\frac{4^2 e^{-4}}{2!}$=0.1465

[3mm] **(c)**P(4 times/3 years)=?

P(4)=$\frac{4^4 e^{-4}}{4!}$=0.1954

52. The weights of items produced by a company are normally distributed with a mean of 45 gm and a standard deviation of 3 gm. What is the probability that a randomly selected item from the production will weigh at least 42 gm?

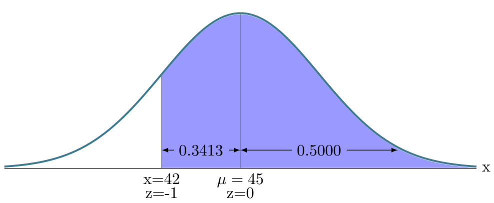
GTU-Dec-2013 [07]

Solution:

We have,

μ=45 σ=3

$P(x \geq 42)$



$z = \frac{x-\mu}{\sigma}$

$\therefore z = \frac{42-45}{3}$= -1

$P(x \geq 42) = P(z \geq -1)$

$\therefore P(x \geq 42) = P(-1 \leq z \leq 0) + 0.5$

$\therefore P(x \geq 42) = 0.3413 + 0.5 = 0.8413$

53. In a city of some Western country 70% of the married persons have children. What is the probability that at least three among four persons will have children?

GTU-Dec-2013 [07]

Solution:

We have, p=0.70

n=4

$P(x \geq 3)$=?

$P(x) =_n C^x p^x (1-p)^{n-x}$

Now $P(x \geq 3)$=P(3)+P(4)

$\therefore P(x \geq 3)$= $_4C^3(0.70)^3(1-0.70)^{4-3}$+$_4C^4(0.70)^4(1-0.70)^{4-4}$

$\therefore P(x \geq 3)$=0.4116+0.2401=0.6517

54. Passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute. Compute the probability of no arrivals in a 15 second period.

GTU-Dec-2013 [07]

Solution:

We have, μ=10 passenger/min

P(x=0 passenger/15 seconds)=?

μ=10 passenger/min

$\therefore \mu$=2.5 passenger/15 seconds

Now $P(x) = \frac{\mu^x e^{-\mu}}{x!}$

$\therefore$ P(0)=$\frac{2.5^0 e^{-2.5}}{0!}$=0.0821

55. A farmer owns 60 acres of wheat fields. Based on past experience, he knows that the yield from each individual acre is normally distributed with mean 120 quintals and standard deviation 12 quintals. Help the farmer plan for his next year's crop by finding:

   (a) The expected mean of the yields from his 60 acres of wheat.
   (b) The standard deviation of the sample mean of the yields from his 60 acres.
   (c) The probability that the mean yield per acre will exceed 123.8 quintals.
   (d) The probability that the mean yield per acre will fall between 117 and 122 quintals.

GTU-Dec-2013 [07]

Solution:

We have

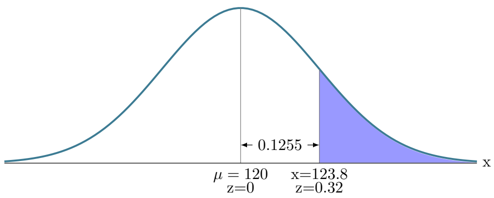
μ=120

σ=12

(a): Expected Value =μ=120

(b): Standard deviation = σ=12

(c):$P(x \geq 123.8)$=?



$z = \frac{x-\mu}{\sigma}$

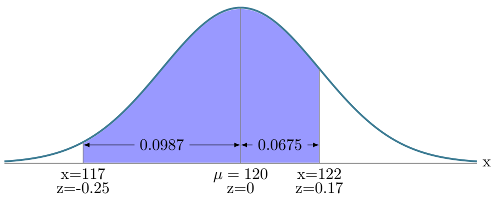
$\therefore z = \frac{123.8-120}{12}$=0.32

Now,$P(x \geq 123.8) = P(z \geq 0.32)$

$\therefore P(x \geq 123.8) = 0.5 - P(0 \leq z \leq o.32)$

$\therefore P(x \geq 123.8)$=0.5-0.1255=0.3745

(d):$P(117 \leq x \leq 122)$=?



Now,$P(117 \leq x \leq 122) = P(-0.25 \leq z \leq 0.17)$

$\therefore P(117 \leq x \leq 122) = P(-0.25 \leq z \leq 0) + P(0 \leq z \leq 0.17)$

$\therefore P(117 \leq x \leq 122)$=0.0987+0.06750=0.1662

56. According to the Sleep Foundation, the average night's sleep is 6.8 hours (Fortune, March 20, 2006). Assume the standard deviation is 0.6 hours and that the probability distribution is normal.

    (a) What is the probability that a randomly selected person sleeps more than 8 hours?

    (b) What is the probability that a randomly selected person sleeps 6 hours or less?

    (c) Doctors suggest getting between 7 and 9 hours of sleep each night. What percentage of the population gets this much sleep?

GTU-Dec-2013[2630003] [07]

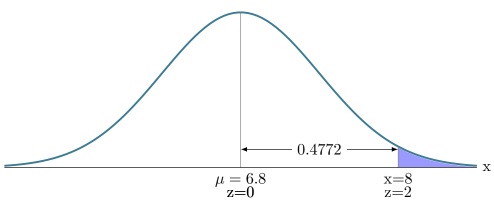
Solution:

We have,

μ=6.8

σ=0.6

(a):$P(>8)$=?

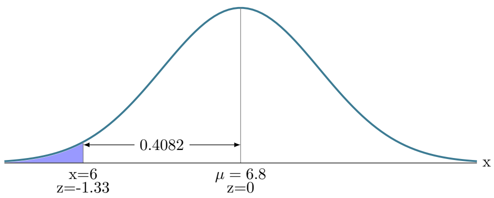


Now,z=$\frac{x-\mu}{\sigma}$

$\therefore$ z= $\frac{8-6.8}{0.6} = 2$

$\therefore P(x > 8) = P(z > 2) = 0.5 - P(0 < z < 2) = 0.5 - 0.4772 = 0.0228$

b:$P(x \leq 6)$ =?

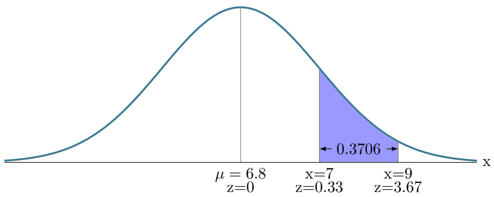


Now,z=$\frac{x-\mu}{\sigma}$

$\therefore$ z= $\frac{6-6.8}{0.6} = -1.33$

$\therefore P(x > 8) = P(z > 2) = 0.5 - P(0 < z < 2) = 0.5 - 0.4082 = 0.0918$

c:$P(7 < x < 9)$=?



Now,z=$\frac{x-\mu}{\sigma}$

$\therefore$ z= $\frac{7-6.8}{0.6} = 0.33$,z= $\frac{9-6.8}{0.6} = 3.67$

$\therefore P(7 < x < 9) = P(0.33 < z > 3.67) = 0.4999 - 0.1293 = 0.3706$

57. Define following(One Mark Each)

    (a) Mutually Exclusive Events
    (b) Exhaustive Events
    (c) Equally Likely Events

GTU-Dec-2013[2630003] [03]

Answer:

Mutually exclusive events

Two events A and B are said to be mutually exclusive events if $A \cap B = \phi$.

Mutually exhaustive events

Two events A and B are said to be mutually exhaustive events if $A \cup B = U$.

Equally likely events

Two or more events are said to be equally likely events if the probabilities of their occurrence are equal.

58. The probability that a person has a mobile phone is 0.60, the probability that a person has a credit-card is 0.50 and the probability that a person has mobile phone and a credit card is 0.20. Find the probability that

    (a) The person has mobile phone but not credit card.
    (b) The person has at least one of them.

GTU-Dec-2013[2630003] [04]

Solution:

Events are,

A=An event in which selected person has mobile phone.

B=An event in which selected person has credit card.

$\therefore$ P(A)=0.60,P(B)=0.50,$P(A \cap B) = 0.20$

(a):P(A-B)=?

Now P(A-B)=P(A)-$P(A) \cap B$

$\therefore$P(A-B)=0.60-0.20=0.40

(b):$P(A \cup B)$=?

Now,$P(A \cup B)$=P(A)+P(B)-$P(A \cap B)$

$\therefore$ $P(A \cup B)$=0.60+0.50-0.20=0.90

59. The probabilities of X, Y and Z becoming managers are 4/9, 2/9 and 1/3 respectively. The probabilities that the Bonus scheme will be introduced if X, Y and Z becoming managers are 3/10, 1/2 and 4/5 respectively.

    (a) What is the probability that the bonus scheme will be introduced?

    (b) If the Bonus scheme has been introduced, what is the probability that the manager appointed was X?

GTU-Dec-2013[2630003] [07]

Solution:

Events are,

X=An event in which X become manager.

Y=An event in which Y become manager.

Z=An event in which Z become manager.

B=An event in which bonus scheme will be introduce.

$\therefore$ P(X)=$\frac{4}{9}$,P(Y)=$\frac{2}{9}$,P(Z)=$\frac{1}{3}$,P(B/X)=$\frac{3}{10}$,P(B/Y)=$\frac{1}{2}$,P(B/Z)=$\frac{4}{5}$,

(a):P(B)=?

Now,P(B)=$P(X \cap B) + P(Y \cap B) + P(Z \cap B)$

$\therefore$ P(B)=P(B/X).P(X)+P(B/Y).P(Y)+P(B/Z).P(Z)

$\therefore$ P(B)=$\left(\frac{3}{10}\right)\left(\frac{4}{9}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{9}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{3}\right) = \frac{23}{45}$

[3mm] **(b):**P(X/B)=?

P(X/B)=$\frac{P(X \cap B)}{P(B)}$

$\therefore$ P(X/B)=$\frac{P(B/X).P(X)))}{P(B)}$

$\therefore$ P(X/B)=$\frac{\frac{3}{10} \cdot \frac{4}{9}}{\frac{23}{45}}$

$\therefore$ P(X/B)=$\frac{6}{23}$

60. A volunteer ambulance service handles 0 to 5 service calls on any given day. The probability distribution for the number of service calls is as follows:

Number of Service Calls	0	1	2	3	4	5
Probability	0.10	0.15	0.30	0.20	0.15	0.10

    (a) What is the expected number of service calls?

(b) What is the variance in the number of service calls? What is the standard deviation?

GTU-Dec-2013[2630003] [04]

Solution:

x	P(x)	x.P(x)	$(x-\mu)^2$	$(x-\mu)^2$.P(x)
0	0.10	0	6.0025	0.6202
1	0.15	0.15	2.1025	0.3254
2	0.30	0.60	0.2025	0.0607
3	0.20	0.60	0.3025	0.0605
4	0.15	0.60	2.4025	0.3604
5	0.10	0.50	6.5025	0.6502
		$\Sigma(x.P(x))$=2.45	•	$\Sigma((x-\mu)^2.P(x)) = 2.0774$

a:Expected Value$\mu = \Sigma(x.P(x)) = 2.45$

b:Variance $\sigma^2 = \Sigma((x-\mu)^2.P(x)) = 2.0774$

Standard deviation $\sigma = \sqrt{\sigma^2} = \sqrt{2.0774} = 1.441$

61. A particular train reaches the destination in time is 75 percent of the times. A person travels 5 times in that train. Find the probability that he will reach the destination in time, for all the 5 times.

GTU-Dec-2013[2630003] [03]

Solution:

We have,

p=0.75

n=5

P(x=5)=?

Now $P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore$ P(5)=$_5C^5(0.75)^5(1-0.75)^{5-5}$=0.2373

62. The time required to pass through security screening at the airport can be annoying to travelers. The mean wait time during peak periods at Cincinnati/Northern Kentucky International Airport is 12.1 minutes. Assume the time to pass through security screening follows an exponential distribution.

(a) What is the probability it will take less than 10 minutes to pass through security screening during a peak period?

(b) What is the probability it will take more than 20 minutes to pass through security screening during a peak period?

(c) What is the probability it will take between 10 and 20 minutes to pass through security screening during a peak period?

(d) It is 8:00 A.M. (a peak period) and you just entered the security line. To catch your plane you must be at the gate within 30 minutes. If it takes 12 minutes from the time you lear security until you reach your gate, what is the probability you will miss your flight?

GTU-Dec-2013[2630003] & May-2014[2630003] [07] &[07]

Solution:

We have,

$\mu = 12.1 mins$

a:$P(x<10)=?$

Now,$P(x<x_0)=1-e^{\frac{-x_0}{\mu}}$

$\therefore P(x<10)=1-e^{\frac{-10}{12.1}}=0.5623$

b:$P(x>20)=?$

Now,$P(x>x_0)=e^{\frac{-x_0}{\mu}}$

$\therefore P(x>20)=e^{\frac{-20}{12.1}}=0.1915$

c:$P(10<x<20)=?$

Now,$P(10<x<20)=P(x>10)-P(x>20)$

$\therefore P(10<x<20)=e^{\frac{-10}{12.1}}-e^{\frac{-20}{12.1}}=0.4376-0.1915=0.2461$

d:To catch the plane you must be at he gate within 20 minutes.Now if it take 12 minutes from the time you lear security until you reach the gate

$\therefore$To catch the plane the time required to pass through security screening must be at most 18 minutes

$\therefore$If you miss the flight then the time taken in security screening must be at least 18 minutes

$\therefore$Required Probability is $P(x>18)$

Now,$P(x>x_0)=e^{\frac{-x_0}{\mu}}$

$\therefore P(x>18)=e^{\frac{-18}{12.1}}=0.2259$

63. True or false(One Mark Each)

    (a) For some events A and B, it is possible that P(A)=0.22 and $P(A\cup B)$ =0.12

    (b) If P(A)=0.25, P(B)=0.20, $P(A\cap B)=0.05$, then A and B must be Independent events.

GTU-May-2014 [02]

Solution:

a:False

Because $P(A\cup B)$ is always greater than or equal P(A)

b:True

A and B are independent if P(A/B)=P(A) or P(B/A)=P(B)

Now,P(A/B)=$\frac{P(A\cap B)}{P(B)}=\frac{0.05}{0.20}=0.25=P(A)$

64. For two events A and B, given that P(A)= 0.50, P(B)=0.60, $P(A\cap B)$=0.40.Find

    (a) P(A/B)

    (b) P(B/A)

    (c) $P(A\cup B)$

GTU-May-2014 [03]

Solution:

We have, P(A)=0.5

P(B)=0.6

$P(A\cap B)$=0.4

(a):P(A/B)=?

P(A/B)=$\frac{P(A\cap B)}{P(B)}$

$\therefore$ P(A/B)=$\frac{0.4}{0.5}$=0.8

(b):P(B/A)=? P(B/A)=$\frac{P(A\cap B)}{P(A)}$

$\therefore$ P(B/A)=$\frac{0.4}{0.6}$=0.67

(c)$P(A\cup B)$ =?

$P(A\cap B)$=P(A)+P(B)-$P(A\cap B)$

$\therefore$ $P(A\cap B)$=0.5+0.6-0.4=0.7

65. A survey found that 33% of the students have their own credit card.

    (a) In a sample of six students, what is the probability that two will have their own credit card?

    (b) In a sample of 10 students, what is the probability that none will have their own credit card?

GTU-May-2014 [04]

Solution:

We have, p=0.35

(a):n=6,P(x=2)=?

Now, $P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore {}_6C^2(0.35)^2(1-0.35)^{6-2}$ =0.3280

(b):n=10,P(x=0)=?

Now $P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore$ P(2)=${}_{10}C^0(0.35)^0(1-0.35)^{10-0}$ =0.01346

66. For the following probability distribution of a random variable x, Compute $E(x)$ and $Var(x)$.

x	1	2	4	7	8
f(x)	0.1	0.2	0.3	0.3	0.1

GTU-May-2014 [03]

Solution:

x	f(x)	x.f(x)	$(x-\mu)^2$	$(x-\mu)^2.f(x)$
1	0.1	0.1	12.96	1.296
2	0.2	0.4	6.76	1.352
4	0.3	1.2	0.36	0.108
7	0.3	2.1	5.76	1.728
8	0.1	0.8	11.56	1.156
		$\Sigma(x.f(x)) = 4.6$		$\Sigma((x-\mu)^2.f(x)) = 5.64$

Now,$\mu = \Sigma(x.f(x)) = 4.6$

Var(x)=$\sigma^2 = \Sigma((x-\mu)^2.f(x)) = 5.64$

67. Forty percent of business travelers carry either a cell phone or a laptop with them. For a sample of 15 business travelers,

    (a) Compute the probability that exactly three of the travelers carry a cell phone or a laptop.

    (b) Compute the probability that at least two of the travelers carry a cell Phone or a laptop.

GTU-May-2014 [04]

Solution:

We have,

p=0.4

n=15

(a):P(x=3)=?

Now, $P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore {}_{15}C^3 (0.4)^3 (1-0.4)^{15-3}$=0.06338

(b):$P(x \geq 2)$ $P(x) =_n C^x p^x (1-p)^{n-x}$

Now,$P(x \geq 2)$=P(2)+P(3)+P(4)+.....+P(15)

$\therefore P(x \geq 2)$=1-[P(0)+P(1)]

$\therefore P(x \geq 2)$ =1 -[${}_{15}C^0 (0.4)^0 (1-0.4)^{15-0}$+${}_{15}C^1 (0.4)^1 (1-0.4)^{15-1}$]

$\therefore P(x \geq 2)$=1-[0.0004+0.00470]=0.9949

68. The lifetime (in hours) of an electric device is a random variable with the Exponential probability density function,$f(x) = \frac{1}{50} e^{\frac{-x}{50}}, x > 0$

    (a) What is the mean lifetime of the device?

    (b) What is the probability that the device will fail in the first 25 hours of operation?

GTU-May-2014 [03]

Solution:

According to probability density function $\mu = 50$hours

a:$\mu = 50$hours

b:$P(x < 25)$

Now $P(x < x_0) = 1 - e^{\frac{-x_0}{\mu}}$

$\therefore P(x < 25) = 1 - e^{\frac{-25}{50}} = 0.3934$

69. Assume that the test scores from a college admissions test are normally distributed, with a mean of 450 and a standard deviation of 100.

    (a) What percentage of the people taking the test score between 400 and 500?

    (b) If a particular university will not admit anyone scoring below 480,what percentage of the persons taking the test would be acceptable to the university?

GTU-May-2014 [04]

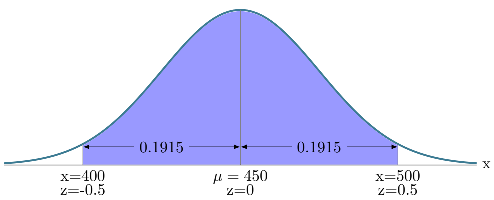
Solution:

We have,

μ=450,

σ=100

(a):$P(400 \leq x \leq 500)$=?



$z = \frac{x-\mu}{\sigma}$

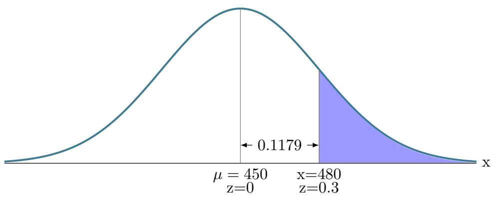
$\therefore$ z=$\frac{400-450}{100}$= -0.5,z=$\frac{500-450}{100}$=0.5

Now,$P(400 \le x \le 500)$=$P(-0.5 \le z \le 0.5)$

$\therefore P(400 \le x \le 500)$=$P(-0.5 \le z \le 0) + P(0 \le 0.5)$

$\therefore P(400 \le x \le 500)$=0.1915+0.1915=0.3830

(b):$P(x \ge 480)$



$z = \frac{x-\mu}{\sigma}$

$\therefore$ z=$\frac{480-450}{100}$=0.3

Now,$P(x \ge 480)$=$P(z \ge 0.3)$ $\therefore P(x \ge 480)$=0.5-$P(0 \le z \le 0.3)$

$\therefore P(x \ge 480)$=0.5-0.1179=0.3821

70. A new automated production process averages 1.5 breakdowns per day. Assume that breakdowns occur randomly, that the probability of a breakdown is the same for any two time intervals of equal length, and that breakdowns in one period are independent of breakdowns in other periods.

    (a) What is the probability of having exactly two breakdowns during a day?

    (b) What is the probability of having three or more breakdowns during a day?

GTU-May-2014 [04]

Solution:

We have,

μ=1.5 breakdowns/day

(a):P(x=2 breakdowns/day)

Now,$P(x) = \frac{\mu^x e^{-\mu}}{x!}$

$\therefore$ P(2)= $\frac{1.5^2 e^{-1.5}}{2!}$ =0.2510

(b):$P(x \ge 3)$=?

$P(x) = \frac{\mu^x e^{-\mu}}{x!}$

Now,$P(x \ge 3)$ =P(3)+P(4)+P(5).......

$\therefore P(x \ge 3)$ = 1-[P(0)+P(1)+P(2)]

$\therefore P(x \ge 3)$ =1-$\frac{1.5^0 e^{-1.5}}{0!}$-$\frac{1.5^1 e^{-1.5}}{1!}$-$\frac{1.5^2 e^{-1.5}}{2!}$

$\therefore P(x \ge 3)$=1-0.2231-0.3347-0.2510=0.1912

71. The time between arrivals of vehicles at a particular intersection follows an exponential probability distribution with a mean of a 12 seconds.

    (a) What is the probability that the arrival time between vehicles is 12 seconds or less?

    (b) What is the probability of 30 or more seconds between vehicle arrivals?

GTU-May-2014 [03]

Solution:

We have,

$\mu = 12$seconds

a:$P(x \leq 12) =?$

Now,$P(x \leq x_0) = 1 - e^{\frac{-x_0}{\mu}}$

$\therefore P(x \leq 12) = 1 - e^{\frac{-12}{12}} = 0.6321$

b:$P(x \geq 30) =?$

Now,$P(x \geq x_0) = e^{\frac{-x_0}{\mu}}$

$\therefore P(x \geq 30) = e^{\frac{-30}{12}} = 0.9179$

72. A machine fills containers with a particular product. The standard deviation of filling weights is known from past data to be 0.6 ounce. If only 2% of the containers hold less than 18 ounces, what is the mean filling weight for the machine? Assume that the filling weights have a normal distribution.

GTU-May-2014 [04]

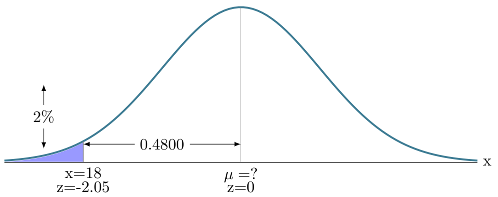
Solution:

We have,

σ=0.6

$P(x \leq 18)$=0.02

μ=?



Now,$P(x \leq 18)$=0.02

$\therefore P(18 \leq x \leq \mu)$=0.5-0.02=0.4800

Hence,z-value corresponding to x=18 is z=-2.05

Now, $z = \frac{x-\mu}{\sigma}$

$\therefore$ -2.05=$\frac{18-\mu}{0.6}$

$\therefore$ (-2.05)(0.6)=18-μ

$\therefore \mu$=16.77

73. Write Properties of Poisson distribution.

GTU-May-2014 [03]

74. The time needed to complete a final examination in a particular college course is normally distributed with a mean of 80 minutes and a standard deviation of 10 minutes. Answer the following questions.

    (a) What is the probability of completing the exam in one hour or less?

    (b) What is the probability that a student will complete the exam in more than 60 minutes but less than 75 minutes?

    (c) Assume that the class has 60 students and that the examination period is 90 minutes in length. How many students do you expect will be unable to complete the exam in the allotted time?

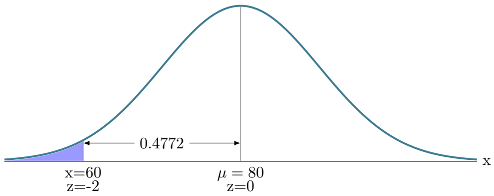
Solution:

We have,

μ=80

σ=10

(a):$P(x \leq 60)$=?



$z = \frac{x-\mu}{\sigma}$

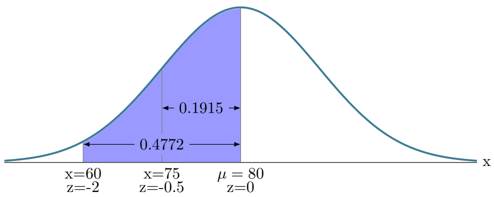
$\therefore$ z=$\frac{60-80}{10}$=-2

Now,$P(x \leq 60) = P(z \leq -2)$

$\therefore$ $P(x \leq 60)$ =0.5-$P(-2 \leq z \leq 0)$

$\therefore$ $P(x \leq 60)$=0.4-0.4772=0.0228

(b):$P(60 \leq x \leq 75)$=?



$z = \frac{x-\mu}{\sigma}$

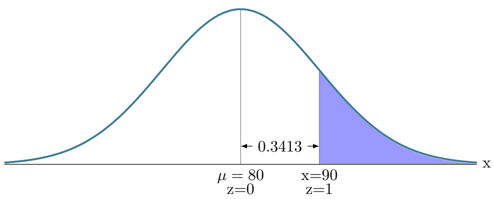
$\therefore$ z=$\frac{60-80}{10}$=-2,z=$\frac{75-80}{10}$=-0.5

Now,$P(60 \leq x \leq 75) = P(-2 \leq z \leq -0.5)$

$\therefore$ $P(60 \leq x \leq 75) = P(-2 \leq z \leq 0) - P(-0.5 \leq z \leq 0)$

$\therefore$ $P(60 \leq x \leq 75)$ =0.4772-0.1915=0.2857

(c)n=60,$P(x \geq 90)$=?



$z = \frac{x-\mu}{\sigma}$

$\therefore$ z=$\frac{90-80}{10}$=1

Now,$P(x \geq 90) = P(z \leq 1)$

$\therefore$ $P(x \geq 90) = 0.5 - P(0 \leq z \leq 1)$

$\therefore$ $P(x \geq 90)$ =0.5-0.3413=1587

Hence expected no of students will be unable to complete exam in allotted time are =(60)(0.1587)≈10

75. Nine percent of undergraduate students carry credit card balances greater than \$7000. Suppose 10

undergraduate students are selected randomly to be interviewed about credit card usage.

(a) What is the probability that two of the students will have a credit card balance greater than \$7000?

(b) What is the probability that none will have a credit card balance greater than \$7000?

(c) What is the probability that at least three will have a credit card balance greater than \$7000?

GTU-May-2014[2630003] [07]

Solution:

We have,

p=0.09

n=10

(a):$P(x=2)$=?

Now,$P(x) =_n C^x p^x(1-p)^{n-x}$

$\therefore$ P(2)=$_{10}C^2(0.09)^2(1-0.09)^{10-2}$=0.1714

(b):$P(x=0)$=?

Now, $P(x) =_n C^x p^x(1-p)^{n-x}$

$\therefore$ P(0)= $_{10}C^0(0.09)^0(1-0.09)^{10-0}$=0.3894

(c):$P(x\geq 3)$=?

$P(x) =_n C^x p^x(1-p)^{n-x}$

Now,$P(x\geq 3)$=P(3)+P(4)+P(5)+P(6)+.......+P(20)

$\therefore$,$P(x\geq 3)$ =1-[P(0)+P(1)+P(2)]

$\therefore$,$P(x\geq 3)$ =1-[$_{10}C^0(0.09)^0(1-0.09)^{10-0}$+$_{10}C^1(0.09)^1(1-0.09)^{10-1}$+$_{10}C^2(0.09)^2(1-0.09)^{10-2}$]

$\therefore$ $P(x\geq 3)$=1-[0.3894+0.3851+0.1714]=0.0540

76. If A, B and C are mutually exclusive and exhaustive events and if P[A]=3P[B]=4P[C], then Find P [$B\cup C$]
If P [A] =$\frac{1}{2}$, P[B] = $\frac{2}{5}$, P[A/B] = $\frac{1}{5}$, find

(a) P[B/A]

(b) P[$A'\cap B'$]

GTU-Dec-2014 [03]

Solution:

let P(A)=x

$\therefore P(B)=\frac{x}{3}$ and $P(c)=\frac{x}{4}$

Now,A,B and C are mutually exclusive and exhaustive events

$\therefore$P(A)+P(B)+P(C)=1

$\therefore x+\frac{x}{3}+\frac{x}{4}=1$

$\therefore \frac{19x}{12}=1$

$\therefore x=\frac{12}{19}$

$\therefore P(A)=\frac{12}{19}, P(B)=\frac{12}{57}, P(C)=\frac{12}{76}=\frac{3}{19}$

As A,B and C are mutually exclusive we have $P(B\cup C)=P(B)+P(C)=\frac{12}{57}+\frac{3}{19}=\frac{7}{19}$

let $P(A)=\frac{1}{2}, P(B)=\frac{2}{5}, P(A/B)=\frac{1}{5}$

a:P(B/A)=?

Now,$P(A/B) = \frac{P(A\cap B)}{P(B)}$

$\frac{1}{5} = \frac{P(A\cap B))}{\frac{2}{5}}$

$\therefore P(A\cap B) = \frac{1}{5}.\frac{2}{5} = \frac{2}{25}$

P(B/A)=$\frac{P(A\cap B)}{P(A)} = \frac{\frac{2}{25}}{\frac{1}{2}} = \frac{4}{25}$

b:$P(A'\cap B')$=?

Now, $P(A'\cap B') = P(A\cup B)' = 1 - P(A\cup B) = 1 - [P(A) + P(B) - P(A\cap B)] = 1 - \left[\frac{1}{2} + \frac{2}{5} - \frac{2}{25}\right] = \frac{9}{50}$

77. A discrete random variable (x) has mean = 6 and standard deviation = $\sqrt{2}$. If it is assumed the random variable is Binomial, what is the probability that $5 \le x \le 7$?

GTU-Dec-2014 [04]

Solution:

$P(5 \le 7)$ =?

We have,

$\mu = 6$ and $\sigma = \sqrt{2}$

Now,According to binomial distribution $\mu = np$ and $\sigma = \sqrt{np(1-p)}$

$\therefore$ np=6 and $\sqrt{np(1-p)}$=2

$\therefore \sqrt{6(1-p)} = \sqrt{2}$

$\therefore$ 6(1-p)=2

$\therefore$ 1-p=$\frac{2}{6}$

$\therefore$ p=$\frac{4}{6} = \frac{2}{3}$

Now,np=6

$\therefore$ n=9

$P(5 \le 7) = P(5) + P(6) + P(7)$

$P(x) =_n C^x p^x (1-p)^{n-x}$

$\therefore P(5 \le 7)$=${}_9C^5(\frac{2}{3})^5(1-\frac{2}{3})^{9-5}$+${}_9C^6(\frac{2}{3})^6(1-\frac{2}{3})^{9-6}$+${}_9C^7(\frac{2}{3})^7(1-\frac{2}{3})^{9-7}$

$\therefore P(5 \le 7) = 0.2048 + 0.2731 + 0.2341 = 0.7120$

78. A person has some taxi cars and the average demand of cars per day is 3. Find the probability that on any one day not more than 2 cars are used.(Given $e^{-3} = 0.0498$)

GTU-Dec-2014 [04]

Solution:

We have,$\mu = 3$cars/day and $P(x \le 2)$ =?

Now,$P(x \le 2) = P(2) + P(1) + P(0)$

$P(x) = \frac{\mu^x e^{-\mu}}{x!}$

$\therefore P(x \le 2)$=$\frac{2^3 e^{-2}}{3!}$+$\frac{1^3 e^{-1}}{3!}$+$\frac{0^3 e^{-0}}{3!}$

$\therefore P(x \le 2)$=0.2240+0.1494+0.0498=0.4232

79. Compute E(x) and standard deviation of the following probability distribution:

x	2	4	7	8
P(x)	0.20	0.30	0.40	0.10

GTU-Dec-2014 [03]

Solution:

x	P(x)	x.P(x)	$(x-\mu)^2$	$(x-\mu)^2.P(x)$
2	0.2	0.4	10.24	2.048
4	0.3	1.2	1.44	0.432
7	0.4	2.8	3.24	1.296
8	0.1	0.8	7.84	0.784
		$\Sigma x.P(x) = 5.2$		$\Sigma(x-\mu)^2.P(x) = 4.56$

Now,$E(x) = \mu = \Sigma(x.P(x)) = 5.2$ and standard deviation $\sigma = \sqrt{(x-\mu)^2.p(x)} = 4.56$

80. Phone calls arrive at the rate of 48 per hour at the reservation desk for Regional Airways and follow the Poisson Distribution.

    (a) Compute the probability of receiving three calls in a 5-minute interval of time.

    (b) Compute the probability of receiving exactly 10 calls in 15 minutes.

    (c) Suppose no calls are currently on hold. If the agent takes 5 minutes to complete the current call, how many callers do you expect to be waiting by that time? What is the probability that none will be waiting?

    (d) If no calls are currently being processed, what is the probability that the agent can take 3 minutes for personal time without being interrupted by a call?

GTU-Dec-2014 [04]

81. The average stock price of 50 companies belonging to an industry is Rs. 30, and the standard deviation is Rs. 8.20. Assume the stock prices are normally distributed.

    (a) What is the probability a company will have a stock price of at least Rs. 40?

    (b) What is the probability a company will have a stock price no higher than Rs. 20?

    (c) How high does a stock price have to be to put a company in the top 10%?

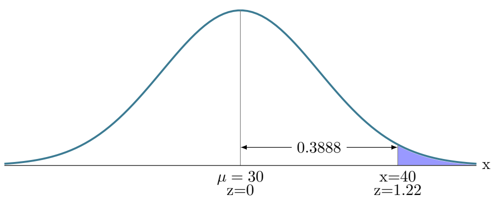
GTU-Dec-2014 [07]

Solution:

We have,

$\mu = 30, \sigma = 8.2$

a:$P(x \geq 40)$



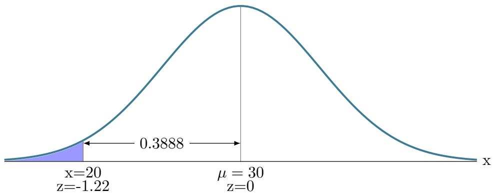
Now,$z = \frac{x-\mu}{\sigma}$

$\therefore$ z=$\frac{40-30}{8.2}$=1.22

$\therefore P(x \geq 40) = P(z \geq 1.22) = 0.5 - 0.3888 = 0.1112$

b:$P(x < 20)$

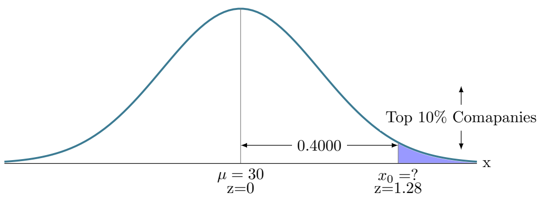
Now,$z = \frac{x-\mu}{\sigma}$



$\therefore$ z=$\frac{20-30}{8.2}$=-1.22

$\therefore P(x < 20) = P(z < 1.22) = 0.5 - 0.3888 = 0.1112$

c: Find x_0 such that $P(x > x_0)$=0.10



If $P(x > x_0) = 0.10$ then $P(\mu < x < x_0)$=0.4

$\therefore$ z-value corresponding to x_0 is approximately 1.28

Now, $z = \frac{x-\mu}{\sigma}$

$\therefore$ 1.28=$\frac{x_0-\mu}{\sigma}$

$\therefore 1.28 = \frac{x_0-30}{8.2}$

$\therefore x_0 = 70.49$

$\therefore$ Stoke price of atleast Rs 70.49 will put company in top 10% company.

82. Suppose that IQ scores of students have a bell-shaped distribution with a mean of 100 and a standard deviation of 15.

    (a) What percentage of people should have an IQ score between 85 and 115?

    (b) What percentage of people should have an IQ score between 70 and 130?

GTU-Dec-2014[2630003] [07]

Solution:

We have,

$\mu = 100, \sigma = 15$

a:

Now,$z = \frac{x-\mu}{\sigma}$

$\therefore z = \frac{85-100}{15} = -1, z = \frac{115-100}{15} = 1$

Now,Approximately 68% of the data values will be within one standard deviation of the mean.

$\therefore$ Approximately 68% percentage of people should have an IQ score between 85 and 115,

b:

$z = \frac{x-\mu}{\sigma}$

$\therefore z = \frac{70-100}{15} = -2, z = \frac{130-100}{15} = 2$

Now,Approximately 95% of the data values will be within two standard deviation of the mean.

$\therefore$ Approximately 95% percentage of people should have an IQ score between 70 and 130,

83. What is meant by equally likely events?

GTU-Dec-2014[2630003] [01]

Solution:

Equally likely events:

84. The average stock price for companies making up the top 100 companies is Rs. 30 and the standard deviation is Rs. 8.2. Assume the stock prices are normally distributed.

    (a) What is the probability a company will have a stock price of at least Rs. 40?

    (b) What is the probability a company will have stock price less than Rs. 20?

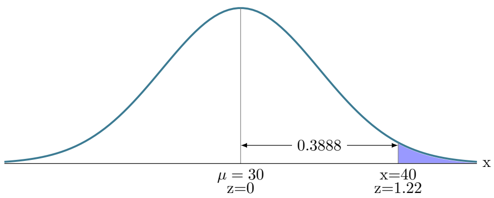
GTU-Dec-2014[2630003] [07]

Solution:

We have,

$\mu = 30, \sigma = 8.2$

a:$P(x \geq 40)$



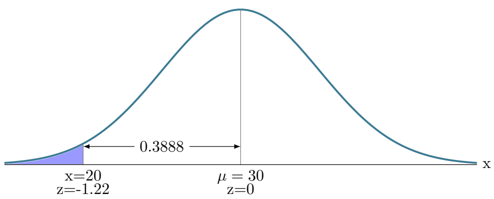
Now, $z = \frac{x-\mu}{\sigma}$

$\therefore$ z=$\frac{40-30}{8.2}$=1.22

$\therefore P(x \geq 40) = P(z \geq 1.22) = 0.5 - 0.3888 = 0.1112$

b:$P(x < 20)$

Now, $z = \frac{x-\mu}{\sigma}$



$\therefore$ z=$\frac{20-30}{8.2}$=-1.22

$\therefore P(x < 20) = P(z < 1.22) = 0.5 - 0.3888 = 0.1112$

85. The time between arrivals of vehicles at a particular intersection follows an exponential probability distribution with a mean of 12 seconds.

    (a) What is the probability that the arrival time between vehicles is less than or equal to 12 seconds?

    (b) What is the probability that the arrival time between vehicles is less than or equal to 6 seconds.

GTU-Dec-2014[2630003] [07]

Solution:

We have,

$\mu = 12 seconds$

a:$P(x \leq 12)$=?

Now,$P(x \leq x_0) = 1 - e^{\frac{-x_0}{\mu}}$

$\therefore P(x \leq 12) = 1 - e^{\frac{-12}{12}} = 0.6322$

b:$P(x \leq 6)$=?

Now,$P(x \leq x_0) = 1 - e^{\frac{-x_0}{\mu}}$

$\therefore P(x \leq 6) = 1 - e^{\frac{-12}{12}} = 0.3934$

86. Write True/False with justification

    The normal distribution with X=0 and σ =1 is known as standard normal distribution.

GTU-May-2015 [01]

Solution: